**Elementary School Mathematics Teachers’ Attitudes and Knowledge Regarding students’ Errors, and the Use of Errors in Mathematics teaching in an Elementary School**

**Abstract**

This study deals with the attitudes and knowledge of elementary school mathematics teachers regarding pupils’ errors, and the use of errors in mathematics teaching in a primary school. Most existing research on the subject focuses primarily on teachers who teach high school, and less on teachers who teach primary school, hence the importance of this study.

Over the years, changes have occurred in how teachers regard pupils’ errors in mathematics class, as well as in the ways teachers address errors. This is in accordance with the approach that considers errors as an impetus toward meaningful learning, and holds that using errors motivates inquiry, enables teachers to follow the pupils’ thought processes, and can be a tool for diagnosing difficulties and finding suitable ways to address them. Beyond that, error-based instruction encourages pupils toward mathematical discourse, develops justification and explanation skills, and contributes toward the development of meta-cognitive processes. In addition, dealing with errors helps pupils develop a number sense, as well as emotional and social resilience, merely by exposure to both their own errors and those of others.

Research shows that teachers have a significant impact on their pupils’ achievements, as well as on their emotional and social adaptation. Teachers’ attitudes also influence both the didactic decisions they make and their methods of instruction. Teachers were found to hold positive attitudes toward use of errors to motivate learning, but often qualify that as suitable primarily for the stronger students or for the more advanced levels of mathematical learning. Further, there is often a gap between the teachers’ attitudes and the implementation of these ideas. Teachers explain this gap within the context of several factors, among them organizational constraints (such as lack of time and the need to reach certain goals), as well as the lack of appropriate training.

Given the above, the purpose of the current study was to characterize primary school mathematics perspectives teachers regarding using errors in mathematics instruction, as well as their statement concerning the ways in which they integrate errors in their teaching. In addition, this study examined the teachers’ knowledge of pedagogical content regarding use of errors in math classes, specifically in the context of tasks relating to fractions. The way in which teachers integrated errors in their instruction method was examined according to the ways in which they suggested approaching the mathematical tasks (simple fractions) with which they were presented.

The research questions were: (i) What are the perspectives of teachers in grades 4-6 regarding use of pupils’ errors in their mathematics instruction? (ii) What methods do teachers in grades 4-6 report using pupils’ errors in mathematics lessons? (iii) What is the pedagogical content knowledge of teachers in grades 4-6 regarding errors in fractions?

The study population included 30 math teachers who teach grades 4-6 and are in different stages of professional development, as reflected by their years of experience at work. The teachers answered two close-ended questionnaires: one related to their views regarding the use of errors in math instruction, and the second focused on the ways in which they used errors in their math instruction. In addition, the teachers answered an open questionnaire dealing with knowledge of pedagogical content on the topic of simple fractions, knowledge about typical errors, possible causes for errors, and ways to address them. To broaden and deepen the information gathered from the questionnaires, six teachers in different stages of professional development were interviewed, as measured by their years of experience teaching math. The interviews were partially structured and focused on the research questions.

The study results show that the teachers express positive attitudes toward using pupils’ errors in math instruction, and view pupils’ errors as a tool to advance meaningful learning processes, enable learning through experience, and establish problem-solving and comprehension skills. Alongside the positive views toward the use of pupils’ errors, dissenting opinions were raised as well. These opinions questioned the use of errors in math instruction in certain age groups, and viewed it as dependent upon levels of learning (among strong or weak students), the stage at which the topic is being taught, and the pupils’ emotional preparedness. The interviews with the teachers showed that use of errors can promote norms of positive discourse in class, and, as a result, contribute to the creation of a positive social climate in class. In addition, the teachers thought that dealing with errors develops the pupils’ emotional resilience, by virtue of their exposure to the fact that everyone makes mistakes. Some of the teachers expressed ambivalence about the use of errors, noting factors such as the social climate in class or the pupils’ emotional state.

The teachers stated that they use errors in their math instruction to drive learning more often than their use of errors in traditional approaches, which address each error and each erring pupil individually. The interviews show that the teachers address errors both incidentally and by taking the initiative to do so.

Regarding the emotional and social aspects of using errors, and in accordance with the presented opinions, the teachers also considered their use of errors in math instruction while taking the emotional and social aspects into account. This finding refines the question of which errors can and should be exposed in class, and how to do so in a way which does not embarrass pupils.

Regarding the knowledge of pedagogical content, the teachers demonstrated knowledge of typical errors in the simple fractions tasks they were shown. In addition, the teachers were able to note possible reasons for these errors. The teachers noted the generalization of natural numbers, conceptual misunderstandings, and erroneous use of procedures as reasons for simple fractions errors. The majority of teachers noted more than one reason, and most cited, as a reason for errors, partial mastery of procedures, where non-mastery of a procedure was curiously perceived as the essence of the error and not the causative reason.

Regarding suitable methods of addressing errors, the teachers noted the use of illustrative techniques to reveal the contradiction that arises from an erroneous solution. The teachers emphasized that they would base their instruction on understanding, rather than on rules. They saw the need for reviewing previous knowledge, establishing the meaning of fractions, and concurrently reviewing concepts and procedures to address errors in fractions. In general, given the methods for addressing fractions errors, it seems that the teachers felt the need to address errors in a way which promotes the pupils’ conceptual understanding.

The interviews yielded a gap between ideals and reality regarding the use of errors. The teachers noted that despite their awareness of the importance of using errors in teaching, and despite their knowledge of pedagogical content regarding typical errors and the ways to address them, they do not use errors in their math instruction as often as they would like. This, according to the teachers, is due to organizational constraints, the lack of proper material in the instruction manuals, and the lack of suitable training. This demonstrates the importance of exposing the teachers to a variety of possibilities for using errors, and to expand their pedagogical knowledge in this area. In light of this, this study may contribute toward future improvement of teaching and learning methods, within the context of the potential embedded in the use of errors in math lessons.

It is important to also note the limitations of this study, such as a small sample size (30 teachers), out of which only six were interviewed. Despite this, the results reinforce the need for, and importance of, creating training programs for teachers (in continuing education programs and in the classroom), and for students studying math instruction, which encourages the use of errors in math instruction. As noted above, the teachers expressed the need for support, counselling, and direction on the use of errors in math instruction, on exposure to different ways of using errors, and, in particular, they expressed the difficulty of combining the organizational constraints with their ability to deal with errors in the manner and frequency they desired.