**On Peirce’s Philosophy and Realist Epistemology**

**ABSTRACT:**

Charles S. Peirce attempted to develop his semiotic theory of the interpretation of cognitive signs, which originate in our basic perceptual operations, that quasi-prove the truth of perceptual judgments representing reality. The essential problem was to explain how, by a cognitive interpretation of the sequence of perceptual signs, we can represent external physical reality and reflectively represent our cognitive mind’s operations of signs. With his introspective method of phaneroscopy, Peirce shows how, without going outside our cognitions, we can represent external reality. In this way, Peirce avoids the phenomenologies of Berkeley, Hume, and Kant, as well as of modern analytic philosophy and hermeneutics. Peirce showed that with the trio of semiotic interpretation–the abductive logic of discovery of hypotheses, the deductive logic of necessary inference, and the inductive logic of evaluation–we can reach a complete proof of the true representation of reality. This semiotic logic of reasoning is the epistemic logic representing our human confrontation with reality, by which we can achieve knowledge and conduct our behavior appropriately. However, Peirce did not complete his realistic revolution to eliminate the previously accepted nominalistic and idealistic epistemologies of formal logic and pure mathematics. Here, I inquire why he did not complete his historical realist epistemological revolution and, following that inquiry, I attempt to reconstruct it.

**Key Words:** Peirce**,** philosophy, realism, epistemology, semiotics, pragmaticism, Kant, nominalism

**INTRODUCTION**

1. **Peirce on Kant’s Nominalism and on How He Gradually Moved to his Realist Pragmaticism**

The present writer was a pure Kantist until he was forced by successive steps into Pragmaticism. The Kantist has only to abjure from the bottom of his heart the proposition that a thing-in-itself can, however indirectly, be conceived; and then correct the details of Kant’s doctrine, and he will find himself to have become a Critical Common-Sensist. (Peirce, *EP*II: #25, 353-4, 1905)

*What is philosophy? It is our general picture by which we analyze and explain life in nature, where epistemology is the structure of this explanation*. (Peirce)

*Science is the method and the rational operations by which we obtain knowledge of reality and of how it may be adjusted to improve our lives within it. This is the Peircean conception of the normative sciences*. (Peirce)

My acquaintance with the writings of Peirce began in the two years (1970-1972) I spent at Brandeis University, working on my PhD. At the time, I attended several lectures and met a number of philosophers who suggested that I read and work on Peirce, since until then I had worked especially on Spinoza and his philosophy. When I returned to Israel, I began teaching philosophy at the University of Haifa and concentrated on Peircean philosophy and on some of his contemporaries. In 1976, I delivered my paper “Peirce on Realism, Reality and Existence” at the *C.S. Peirce Bicentennial International Congress*, which later appeared in K.L. Ketner et al., eds., *The Proceedings of the C.S. Peirce Bicentennial International Congress*, (Amsterdam 1976), Texas Tech University Press, 1981: 247-251.

### From then on, I worked intensively on Peirce. In his writings, I found a new philosophical perspective compared to the main streams of twentieth-century philosophy, such as Logical Positivism, Analytic Philosophy, Ordinary Language philosophy, Russell and Wittgenstein’s philosophies, etc. In 1979, I was accepted to Harvard University as a research scholar, when Hilary Putnam was the chair of the department of philosophy. I participated in departmental activities, engaged with members of the department, and worked on Peircean philosophy in the five volumes of his Collected Papers, and especially on his yet unpublished manuscripts in the Houghton Library. The gist of my research concerned Peirce’s philosophical evolution from the Kantian approach to that of his later writings, which counter the Kantian Copernican Revolution, and aim to close the *gap* between Hume’s empirical phenomenology and Descartes and Leibniz’s rational axiomatic approaches, which Kant tried to connect mechanically in his transcendental philosophy. In his later writings, between 1893 to 1913, Peirce endeavored to overcome this impossibility by shifting from a Kantian nominalism to his pragmaticist realist epistemology (Peirce, *EP*II: #25, 353-4, 1905; #27, 1906; Nesher, 2002-2021). Indeed, I found that most of the philosophers who considered themselves Pragmatists were acquainted only with Peirce’s early writings and remained neo-Kantians, including William James, John Dewey, and other twentieth-century philosophers.

We can understand how Peirce, in his criticism of Kantian transcendental nominalism, developed his own *semiotics* from phenomenological introspection and attempted to explain our confrontation with reality in our perceptual judgments from our basic experience. In order to avoid the gap in Kant’s transcendental epistemology, between the *formal empty concepts* and the *material blind objects*, Peirce endeavored to begin from basic perceptual experience as he cognizes and explains the sequence of interpretations of signs which are synthesized into the thought of perceptual judgment and prove the true representation of Kantian *noumenal*-external reality. Thus, Peirce showed how abstract cognitions can develop from the sensual intuitions of experience, without any need to assume the transcendental, *nominalistic*, *empty formal wordings* (e.g., Peirce, *EP*I: #5, 1871).

In his philosophical inquiries, Peirce endeavored to discover and develop his own theory of cognitive signs that are discovered in our basic perceptual operations of sequential interpretations of those signs, which terminate in the conceptual sign and in the quasi-proof of the truth of the perceptual judgment representing reality. The essential problem was to discover and explain how, by a cognitive interpretation of the sequence of perceptual signs, we can indirectly represent external physical reality and reflectively represent our cognitive operations of signs. In his endeavor to develop his pragmaticist epistemology, Peirce began from basic perceptual experiences and, through phenomenological introspection, or phaneroscopy, cognized and explained the sequence of sign interpretations: The iconic feeling is interpreted by the indexical reaction to this feeling and synthesized into the symbolic thought of perceptual judgment. In this relationship to interpretation, the incoherency and coherency of the iconic feeling sign, the image of an eventual object, presents the *ego* expectation, and the indexical emotional reaction to the first sign, with which it can contrast or fit, is the *non-ego* according to Peirce. Hence, the latter either disappoints the expectation, and thus may be understood as representing reality negatively, or, rather, fulfills it and thus represents external reality positively (Nesher, 2002b: III). With his *phaneroscopy*, Peirce shows how, without going outside our cognitions, we can represent external reality. With this explanation, Peirce can avoid the different phenomenologies of Berkeley, Hume, and Kant, as well as of modern analytic philosophy and hermeneutics (Marty, 1982; Nesher, 2002b: VI, 2004a/b).

**2. Peirce on the Transition from the Phaneroscopical Inquiry into Perceptual Operations to the Complete Proof the Truth of Perceptual Judgments Representing Reality, as the Basis of the Pragmaticist Theory of Truth**

Peirce developed his semiotics into the epistemic logic of our perceptual confrontation with reality, manifested in the duality of the *ego* and *non-ego*, by interpreting our genuine signs as complete proofs of the true representation of external reality, conditioning the *validity* of the interpretation and the *soundness* of the proofs.

**[1] The Confrontation in Physical Reality by Coherent Interpretation of Meanings of the Three Inferences in the Quasi-Proof of the Truth of Perceptual Judgment Representing Reality**:

**Validity** of MeaningInterpretation and **Soundness** in Proving theTruthof **Perceptual Judgment**

*Percept sign**Deductive coherent inference* *Perceptual judgment***,** CIn is AAb

**Ab(C, A➞C)➾A)** + **Dd**((A➞C), A)➞C) + **In**((AAb, CIn) =❥Pr. m/n (AAb➞Cin)) = **Falsity or Truth**

[Initial Sign] **Icon Index Icon, Index Symbol:** **Perceptual Judgment** ⇘ ⇙ ▲

**Truth Conditions** = Duality= Comparison ❙ *Ego non-Ego* ❙

⇙ ⇘ ❙

**Logical Reality: *Confrontation in Reality*** ❙

*Incoherency Coherency* ❙

**Internal Proof Conditions:** ▼ ▼ ❙

Hesitation Assurance ➠ Assertion

❙ **Representing** Physical Reality

❙ **Object**P by **Description** **(AAb)**

▼ “This [CIn] is a stone [AAb]”

**Physical Reality**

We find that through our cognitive clash with reality, we first become conscious of external reality: this is our negative knowledge of reality, whereby we cognize the existence of something that contradicts our expectation, yet we still do not have a positive true representation of it.

And what do we mean by the real? It is a conception which we must first have had when we discovered that there was an unreal, an illusion; that is, when we first correct ourselves (Peirce, *CP*: 5.311, 1868).

The proof of the negative knowledge of external reality is the perceptual cognitive operation in which we discover our error, which cannot come only from ourselves. This explanation can be considered a philosophical proof of the existence of something external, which is independent of the way we present it. Further, when we interpret the *coherency* of themeanings of iconic and indexical signs, we can prove our positive knowledge of this external reality. Hence, *semiotics* can be understood as the *epistemic logic representing our confrontation with reality*;it is the *methodeutic* of all our *true representations of external reality*,as developed above (Nesher, 2018, 2016-2021, cf. Peirce, *EPI*: #8, 136–137, 1878).

**3. Pragmaticist Realism: Can Mathematical Reasoning Be Sound Without Being a True Representation of Reality?**

**3.1. The *Gap* between the Nominalist/Platonist Epistemology of Mathematics and Realist Empirical Sciences**

Peirce revolutionized philosophy by developing a realistic epistemology of the true representation of reality, in contrast to Cartesian metaphysical realism and Kantian transcendental phenomenalism. Peirce developed his semiotics as the eventual epistemic logic representing our human cognitive confrontation with external *reality*, thus providing proof of the truth of our true cognitive representation of *reality* (Nesher, 1981, 2002b: II, X, 2005). Hence, unlike nominalism, we can realistically quasi-prove the truth of our perceptual judgments and, upon them, prove that true scientific theories represent reality, with their *general natural kinds* and *general laws of nature*. This realist epistemology is the basis of all our knowledge of reality. However, since pure mathematics and formal logic do not confront reality experientially, according to Peirce, he cannot explain how such subjective reasonings can *determine* the *meaning* and the *truth* of their formalisms (Nesher, 2016).

Every reasoning takes place in some mind. It would not be that mind’s reasoning unless it satisfied that mind’s feeling of logicality…. But as long as it does that, nothing can be gained by criticizing the reasoning any further, since there is no other possible sign by which we could know that it was good than that feeling of logicality in the reasoner’s mind.... Consequently, since every reasoning satisfies the reasoner’s feeling of logically, every reasoning is as good as any reasoning can be. That is, there is no distinction of good and bad reasoning. (Peirce, *EP*II: #17, 243–244, 1903)

In his mature realism, Peirce understands that our reasoning cannot be sound without proving its true representation of external reality, but this is incompatible with his conceptions of pure mathematics and formal logic as, we might say, pure formally-closed games (Nesher, 2011, 2012). Accepting Peirce’s understanding that validity cannot be determined only by the reasoner’s feeling, one is surprised that his conception of pure mathematics is itself based on subjective feeling without any objective criteria by which to prove its valid meaning and sound truth (Peirce, *CP*: 4.227–245, 1902; Murphey, 1961: XII). Later in life, Peirce considered theoretic, aesthetics, and ethics as normative sciences, as distinct from logic and mathematics as pure cognitions separated from experienced reality.

Yet the maxim of Pragmatism does not bestow a single smile upon beauty, upon moral virtue, or upon abstract truth; – the three things that alone raise Humanity above Animality. (Peirce, *EP*II: #31, 465, 1913)

Historically there have been prominent examples of an alliance between nominalism and Platonism. … The reason of this odd conjunction of doctrines may perhaps be guessed at. The nominalist, by isolating his reality so entirely from mental influence as he has done, has made it something which the mind cannot conceived; he has created the so often talked of “improportion between the mind and the thing-in-itself.” And it is to overcome the various difficulties to which this gives rise that he supposes this *noumenon*, which,being totally unknown, the imagination can play about as it pleases, to be the emanation of archetypical ideas. The reality thus receives an intelligible nature again, and the peculiar inconveniences of nominalism are to some degree avoided. (Peirce, *EP*I: #5, 100, 1878; cf. *EP*II: #18, 260, 1903).

This is Kant’s difficulty with his nominalism, yet it seems that Peirce accepted “this odd conjunction of doctrines,” of *ideal realism* and *phenomenal nominalism*, with respect to pure mathematics and formal logic.

**[**2**] the Interpretation of Signs to Determine Their Meanings to Be Clear by Their Coherency and to Prove the Truth of Their Interpretation to Be Distinct in Sound Reasoning Representing Reality:**

⎡—Reflective Self-Control Feeling of **Cognitive** Operations of *Interpretation* and *Representation*—

The Sequence of the Cognitive *Interpretation* of the **Signs** of **the Mind** (Direct Relations)

--------------------------------------------------------------------------------------------------➤

Iconic Indexical Symbolic

**Sign** Mind=➤Feeling Quality=➤Emotional Reaction=➤Thought Reasoning: Perceptual Judgment

Self-controlling the coherence of *meanings* as **clear** and proving the *true* meaning interpretation as D**istinct *Valid*** ❙ ***Sound*** *Reasoning* by being *True*

❙ = *Representation* of Reality

(Indirectly)

▼ ***True***

**Physical Object**

We have, hitherto, not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear, but they may be ever so clear without being true. (Peirce, *EP*I: #8, 141, 1878)

Meaning is *clear* by its *coherent* interpretation, anditis*distinct*by being proved to be a *true interpretation* by a *true representation of reality*, such that the true proposition enables our self-controlled conduct in reality.

In a nutshell, it is the Jamesian position according to which behavior is based on the semiotic interpretation of meanings; if our interpretations lead us to accomplish our intentions, they can be considered *behaviorally* true. Although James’ formulation seems to echo Peirce’s initial explanation of the Pragmatic Maxim of 1878, in his later work (circa1898-1907) Peirce elaborated his realist epistemology and named it *Pragmaticism*, to distinguish it from James’s *Pragmatism*. Thus, according to Peirce, proving the true interpretation of cognitive meanings is connected to the proof of the true representation of reality and this is the condition for successful conduct in such known reality (Nesher, 1983, 2018). However, epistemologically, James’s pragmatism is rather similar to Peirce’s early pragmatism, in being nominalist and “pure Kantist.” Yet it is in conflict with Peirce’s mature realist pragmaticism, as Peirce admitted:

The present writer was a pure Kantist until he was forced by successive steps into Pragmaticism. The Kantist has only to abjure from the bottom of his heart the proposition that a thing-in-itself can, however indirectly, be conceived; and then correct the details of Kant’s doctrine, and he will find himself to have become a Critical Common-Sensist. (Peirce, *EP*II: #25, 353-4, 1905)

Moreover, philosophers of our time, who call themselves pragmatists, such as Popper, Davidson, Putnam, Hintikka, Reacher, and many others, are based on Peirce’s early writings and like early-Peirce have, unfortunately, remained pure Kantists and nominalists.

Pragmaticism makes the ultimate intellectual purport of what you please to consist in conceived conditional resolutions, or their substance; and therefore, the conditional propositions, with their hypothetical antecedents, in which such resolution consist, being of the ultimate nature of meaning, must be capable of being true, that is, of expressing whatever there be which is such as the proposition expresses, independently of being thought to be so in any judgment, or being represented to be so in any other symbol of any man or men. (Peirce, *CP*: 5.453, 1905)

Kant’s *transcendental logic* provides the justification of the confrontation of the *transcendental empty concepts* with the *blind objects* of *sensual intuition* that gives meaning to these concepts. But, in the end, Kant was unsuccessful in his enterprise due to the unbridgeable gap in his epistemology between the transcendental *formal* components and the sensual *material* components of cognition, as he admitted in a letter to a friend (in 1798I). Thus, we see that the form of the *empty concepts* cannot have any meaning without closing the gap between the transcendental empty language and the blind objects of sensual intuition. In this way, we can understand Peirce’s conception of Kantian nominalism, namely, of names of concepts without content or meaning, which led Peirce to his contra-Copernican Revolution, with his realist epistemology.

To say, as the article of January of 1878 seems to intend, that it is just as an arbitrary “usage of speech” choses to arrange its thoughts, is as much as to decide against the reality of the property, since the real is that which is such as it is regardless of how it is, in any time, thought to be. (Peirce, *CP*: 5.457, 1905)

This emphasizes the transition of Peirce’s epistemology from pure Kantianism to the realist epistemology of his pragmaticism between 1878 and 1905.

**3.2. Kant On Formal Logic and Pure Mathematics**

However, in contrast to Kant’s basic conception of logical judgment and its upper transcendental portion in schema [3], Kant presents his conception of pure mathematics as based on the pure intuition of the understanding.

Now, the intuition which pure mathematics lays at the foundation of all its cognitions and judgments which appear at once apodictic and necessary are space and time. For mathematics must first present all its concepts in intuition, and pure mathematics in pure intuition; that is, it must construct them. If it proceeded in any other way, it would be impossible to take a single step; for mathematics proceeds, not analytically by dissection of concepts, but synthetically, and if pure intuition be wanting there is nothing in which the matter for synthetical judgments *a priori* can be given. Geometry is based upon the pure intuition of space. Arithmetic achieves it concept of number by the successive addition of units in time... (Kant, Prolegomena (1783): 282-283; Hintikka, 1973; schema [4])

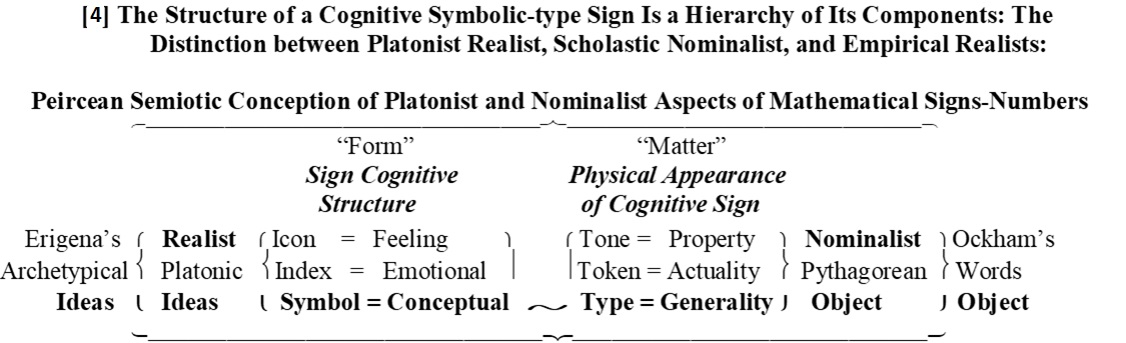
We can elaborate the realist conception of mathematics based on the Peircean semiotics into epistemic logic, which can show how mathematics can be an empirical science (Nesher, 2010, 2011, 2016, 2017, 2018). The following schema [4] binds the experiential components of Kantian transcendental epistemology, upon which, according of my interpretation, Peirce constructs his realist epistemology to revolutionize Kant’s Copernican Revolution. The Kantian conception of knowledge is based on pure concepts and empirical sensations: the evolvement of empirical concepts from blind sensual intuitions and the empty pure concepts through their synthesis in perceptual judgment, and the pure mathematics in pure intuition:

**[7] Transcendental Logic and Pure Mathematics in Pure Intuition**

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This schema can explain the synthesis of the *blind object* with the *empty pure concept* that makes the concept meaningful and the object determinate, and thus determines the *empirical object* by subsuming it under the *empirical concept*. However, the evolvement of the empirical concepts in perception from sensual intuitions and pure concepts along with the imagination to their synthesis by the schematism into perceptual judgment reveals Kant’s difficulty with the epistemology of empirical concepts and logical judgment (Kant, *CPuR*: #24-B150-151). However, Kant’s transcendental epistemology is based on the *mystical* conception of *schematism*, to bridge between the *form* of the *empty concepts a*nd the *matter* of the *blind objects*. Without it, his philosophical system cannot hold. The component of pure *a priori* knowledge includes the conception of *pure mathematics*, and yet formalism cannot work without empirical matter as the meaning of the pure forms. However, since Kant assumed that mathematics is pure science based on transcendental pure intuition, he had difficulties explaining this intuition, and in his *Critique of Pure Reason* B-1787 he empirically explains basic mathematical intuitions by empirically counting fingers or dots.

In thinking merely that union of seven and five, I have by no means already thought the concept of twelve; and no matter how long I dissect my concept of such possible sum, still I shall never find in it that twelve. We must go beyond these concepts and avail our ourselves of the intuition corresponding to one of the two: e.g., our five fingers or (as *Segner* does in his *Arithmetic*) five dots. In this way we must gradually add, the units of the five given in intuition. …. For then it is very evident that, no matter how much we twist and turn our concepts, we can never find the [number of the] sum by merely dissecting our concepts, i.e., without availing ourselves of intuition. (Kant, *CPuR*: B14-15)

The first epistemological difficulty is with numbers, whether they are ideas or objects, and this can be seen from the semantic structure of the signs-symbols: The*realist* platonic *ideas* are on the left and the *nominalist* phenomenal*object* is on the right side of the schema below. The epistemological difficulties of mathematics are how to determine *what numbers are*, objects of signs or signs of objects, and what *mathematics* and mathematical *proof* amount to (Russell, 1901). 

**Mathematical Reality Upon it the Pragmaticist Structure of Cognitive Symbolic-Signs Operating**

The number signs cannot be objects or concepts of empirical experience, but are the discovered signs, components of the human empirical operations of counting, grouping, and measuring physical objects (Nesher, 2011). The discovery of the concepts of these operations of enumeration and grouping, which contain natural numbers, and the further *discovery* oftheir expansion through abstraction and generalization, constitute our mathematical hypotheses, which will be evaluated and proved upon extended mathematical reality (Krantz, 2011). Hence, by proving the truth of perceptual facts representing our mathematical-arithmetical operations with signs-numbers upon physical objects, we represent mathematical reality. The interesting epistemological question is whether Kant himself, with his brilliant philosophical intuitions, can come close to explaining mathematical experience in reality, to explain our knowledge of mathematical reality, without turning to the Platonist enigmatic suggestion? It seems that in the following paragraphs from his Prolegomena (1783), Kant comes close to an empirical explanation of our knowledge, which Peirce reconstructed to develop his realist epistemology. The Kantian conception of formal logic and pure mathematics works only in deductive inferences, as axiomatic closed systems, which can infer without any proof of truth, and therefore differ from the sciences that can prove the truth of their hypotheses in respect to reality.

The alternative epistemology of mathematical proofs is at a crossroad from the pure formal game to empirical theory. The number signs cannot be of objects of empirical experience but are the discovered sign-components of the human empirical operations of counting, grouping, and measuring physical objects. (Nesher, 2011). The *discovery* of the concepts of these operations of enumeration contains natural numbers, and the further *discovery* oftheir expansion through abstraction and generalization constitutes our mathematical hypotheses, which will be evaluated and proved upon extended mathematical reality (Krantz, 2011). Hence, by proving the truth of perceptual facts representing mathematical operations we represent mathematical reality.

**[5]** **The Double Layer of Mathematical Operations: (1) Counting Physical Objects, (2) Perceptually Quasi-proving the Truth of Discovering the Numerical Signs and of Operating with them**

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By understanding that mathematical reality consists of perceptually self-controlled numerical operations on physical objects, we can see how Peirce, as well as Gödel, confuse the meaning-content of mathematical signs with abstract Platonist mathematical *forms* as objects. The arithmetical numbers are neither *physical objects* nor *abstract concepts* but the *conceptual components of our quantitative operations on physical objects* as the mathematical reality upon which we prove the truth or the falsity of our abstract mathematical hypotheses (Nesher, 2011).

**3.3. Gödel in Late, from Pure Platonism to a Realist Epistemology of the Mathematical Science**

Mathematics without operationally measuring the predicted and the eventually observed true facts of reality cannot be true and cannot be “on a much firmer ground” than physics without “a testable prediction.” Both have to prove their own truths upon “their different ways of approaching the world.”

However mathematical intuition in addition creates the conviction that, if these formulas express observable facts and were obtained by applying mathematics to verified physical laws (or if they express ascertainable mathematical facts), then these facts will be brought out by observation (or computation). (Gödel, 1953/9-III: #16; cf. #13-15 & n. 34)

How may one understand this hinted explication of the relationship between intuitive mathematical truth representing its own reality and its application to physical theories to enable their observable predictions(Gödel, 1953II: #15)? At the end, mathematics is neither the *queen* of science nor its *servant* but its *quantitative* *backbone*—that is, the quantified formulations of scientific theoretical hypotheses and their operations in scientific observations—without which physical and other theories cannot be evaluated experimentally. The explanation of the puzzlement as to why mathematics is considered *exact* or *pure* science while being empirical like other experimental sciences, is the relative simplicity of its represented reality with respect to physical and psychological realities. This empirical explanation can be seen in Gödel’s late philosophical writings on the foundations of mathematics:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.... This whole consideration incidentally shows that the philosophical implications of the mathematical facts explained do not lie entirely on the side of rationalistic or idealistic philosophy, but that in one respect they favor the empiricist viewpoint. It is true that only the second alternative points in this direction. (Gödel, 1951: 313)

Hence, we can know experientially the mathematical facts of the mathematical empirical reality.

**4. Epistemic Logic, Representing our Confrontation with Reality, is the Methodology of all our Knowledge**

## **4.1. Epistemic Logic is the Methodology of Perceptual and Scientific Operations in Proving True Representation of Reality to Guide Human Conduct**

Pragmaticistically, every cognitive operation consists of descriptive and normative components that compose both the rules of habit of our cognitive operations and the rational norms embedded in every rational judgment, including scientific theories, that promote our rational conduct in self-controlling ourselves in reality (Peirce, *CP* 1.281, c. 1902, *EP*II: #14, 198–199, 1903; Nesher 1982: 80–82, 1983b, 1990: 24–26).

That which any true proposition asserts is real, in the sense of being as it is regardless of what you and I may think about it. Let this proposition be a general conditional proposition as to the future, and it is a real general such as is calculated really to influence human conduct; and such the pragmaticist holds to be the rational purport of every concept. (Peirce, *CP* 5.432, 1905)

From this pragmaticist conception of semiotics, it is essential to understand the epistemological deficiency of syntactic and semantic axiomatic formal systems. Formal systems cannot explain human cognitive operations of proving our true representation of reality to guide human conduct (Nesher 2004b, 2011).

In order to gain a clear understanding of the origin of the various signs used in logical algebra and the reasons of the fundamental formulae, we ought to begin by considering how logic itself arises. (Peirce, *EP*I: #13, 200, 1880)

The epistemic difference between formal logic and epistemic logic lies in their different proof conditions, the formal system being hermetically closed upon its fixed formal proof conditions, which are detached from external reality. Epistemic logic is only relatively closed upon its proof conditions, being the method of complete proof and thus also quasi-proving the truth of our perceptual judgments as our basic facts. Thus, formal systems are complete and sterile, and human perception and science based on epistemic logic are incomplete but true in representing reality relative to accepted proof conditions (e.g., Peirce, *CP* 4.582, 1906).

In the first place, all our knowledge rests upon perceptual judgments … Now consider any other judgment I may make. That is a conclusion of inferences ultimately based on perceptual judgments, and since these are indisputable, all the truth which my judgment can have must consist in the logical correctness of those inferences … To say that a proposition is certainly true means simply that it never can be found out to be false, or in other words that it is derived by logically correct arguments from veracious perceptual judgments. Consequently, the only difference between material truth and the logical correctness of argumentation is that the *latter* refers to a single line of argument and the *former* to all the arguments which could have a given proposition or its denial as their conclusion. … These three kinds of reasonings are Abduction, Induction, and Deduction. (Peirce, *EPII*: #14, 204-205, 1903)

This is the distinction between formal logical inferences being isolated from reality and unable to be true about it and the epistemic logic of complete proof, be it true or false, which consists of the trio of abduction, deduction, and induction. Complete proof, then, stands on reality with its two legs, abductive and inductive material logical inferences (Nesher 2001, 2002b: II and X, 2007a, 2011, 2016).

It does not seem to me that mathematics depends in any way upon logic. It reasons, of course. But if the mathematician ever hesitates or errs in his reasoning, logic cannot come to his aid. (Pierce, *CP* 4.228, 1902)

However, epistemic logic as the semiotics of our cognitions is the science of reasoning, so mathematicians cannot make their reasoning sound, as though it is without controlling the logic of their operations in confronting mathematical reality.

And to say that mental phenomena are governed by law does not mean merely that they are describable by a general formula; but that there is a living idea, a conscious continuum of feeling, which pervades them, and to which they are docile. (Peirce, *CP* 6.163, 1892)

For pragmaticist epistemology, every human behavior and conduct, perceptual and scientific, is based initially on *logica utens*, as our habitual reasoning is instinctively and practically self-controlled, which evolves into logical ducens, whose rules are formulized and reasoning is rationally self-controlled (Peirce, *EPI*: #8, 141, 1878).

## **4.2. Our Propositional Meanings Proved Clear and Distinct by Proving Their True Representation of Reality**

Philosophical and logical sciences develop together in our experience and allow us to understand their basic contributions to our knowledge and to our conduct in nature. Thus, we prove that epistemic logic is our basic science, representing our confrontation with reality from perceptual operations to all other sciences in proving the truth of their representations (Peirce *EP*II: #17, 256–257, 1903). According to epistemic logic, all knowledge is proved to be a true representation of reality, and so logical knowledge is as well. However, we can prove our cognitions to be either true or false; and if we do not prove them, they remain doubtful. Thus, truth cannot be separated from being proved, which is in contrast to classical formal logic, whose propositions are either true or false independent of being proved (Nesher 2002b: Ch. 5, 2011). Therefore, we can no longer accept the principle of excluded middle and cannot prove the provability of any proposition but only their real values or neither, in which case they remain doubtful (Peirce, *EP*II: #12, 168 1903, #25, 351, 1905; Gödel 1986 [1931]; Heyting 1971 [1956]: 18; Brouwer 1981 [1949]: 5, 92; Kleene 1952: Ch. 13; Weyl 2012: 188–189; Nesher 2011). Hence, the meanings of validity, proof, and truth in epistemic logic differ from their meanings in classical logic (Nesher 2016).

A sign (under which designation I place every kind of thought, and not alone external signs), that is in any respect objectively indeterminate (i.e., whose object is undetermined by the sign itself) is objectively general in so far as it extends to the interpreter the privilege of carrying its determination further. (Peirce, *CP* 5.447, 1905)

The determination of a sign by the interpreter lies in proving the true interpretation and the representation of its object, and this holds for propositions and their component signs as well. The identity of a sign is in making its meaning clear by comprehending its meaning in further interpretation, while the meanings of signs are made clear and distinct by proving the truth of their interpretation in the representation of reality, and this latter accounts for the soundness of the reasoning. However, the validity of these operations is manifested in the coherence of meaning interpretations and the soundness of this reasoning is the proof of their truth in representing external reality. This contrasts with the Cartesian subjective feeling of intuiting clearly and distinctly the truth of propositions, which are without any objective criterion for their meaning and truth (Descartes 1985 [1628]: Rule Three, 1985 [1644]: Part One, n. 43–50; Peirce, *EP*I: #8, 124–142, 1878; CP 5.448, 1905).

The very first lesson that we have the right to demand that logic shall teach us is, how to make our ideas clear; … To know what we think, to be masters of our own meaning, will make a solid foundation for great and weighty thought. (Peirce, *EPI*: #8, 126, 1878)

Hence, we can make the meanings of our ideas clear by valid interpretation, and distinct by proving their truth in representing external reality in sound reasoning (Nesher 2002b: Ch. 3; Gaukroger 1989: 60–71)

**[6] Cognitive Operation of Signs Interpretation and Representation of Reality**

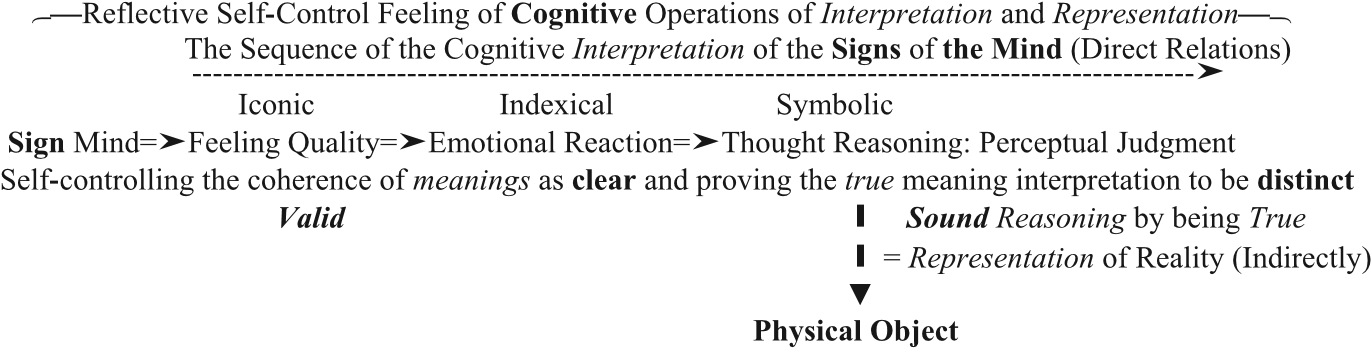


Figure 6: The interpretation of signs to determine their meanings to be clear by their coherency and to prove the truth of their interpretation to be distinct in sound reasoning representing reality.

We have, hitherto, not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear, but they may be ever so clear without being true. (Peirce, *EP*I: #8, 141, 1878)

Meaning is clear by its coherent interpretation and is distinct by being proved a true interpretation by a true representation of reality, such that the true proposition enables our self-controlling conduct in reality.

Logic is the theory of self-controlled, or deliberate, thought; and as such, must appeal to ethics for its principles. It also depends upon phenomenology and upon mathematics. All thought being performed by means of signs, logic may be regarded as the science of the general laws of signs. It has three branches: (1) *Speculative Grammar*, or the general theory of the nature and meanings of signs, whether they be icons, indices, or symbols; (2) *Critic*, which classifies arguments and determines the validity and degree of force of each kind; (3) *Methodeutic*, which studies the methods that ought to be pursued in the investigation, in the exposition, and in the application of truth. Each division depends on that which precedes it. (Perce, *EP*II: #18, 260, 1903)

It is interesting to see that, though Peirce’s semiotics is the basis of epistemic logic, Peirce himself continued to hold a traditional picture of formal logic and pure mathematics by following Kant’s transcendental epistemology. In the realist interpretation of cognitive signs, no complete-absolute determination of their meanings is possible, since all proofs of meaning interpretations are relative to accepted proof conditions, which form the real context in which we operate. However, logical reality cannot be the physical reality that the physical sciences represent, or the cognitive reality that the psychological sciences represent, or any ideal metaphysical reality (Hintikka and Sandu 2006).

Logic does rest on certain facts of experience among which are facts about men, but not upon any theory about the human mind or any theory to explain facts. (Peirce, *CP* 5.110, 1903)

Indeed, this is pragmaticist epistemic logic, the implicit logica utens and explicit logica docens as the basis of all human knowledge, the perceptual and the scientific, including mathematical science. Epistemic logic is, we might say, the Boolean “laws of thought,” representing our cognitive confrontation with reality that enable knowledge and sustains our conduct within it (Nesher 1983a: 244–250, 2002b, 2016).

## **4.3. The Role of Meaning in the Operation of Validity, Proof, and Truth as the Soundness of Epistemic Logic**

We learn the components of epistemic logic from our basic experience, and we naturally begin by reflecting on our basic inference of the implication of the perceptual operation of signs. In formal semantics, if the antecedent is accepted as true, then its implied consequent is also true, and if the antecedent is false, then the entire implication is true. The pragmaticist’s explanation of implication is that the conditional relation is such that we interpret the meaning of the antecedent in the meaning of the consequent by self-controlling their coherency. This is the validity of the interpretation, yet it is not a tautology, which is only a repetition and not an interpretation of content. The connection between the validity of such arguments and the forms of their expression is the meanings involved in the laws of the mind, without which the formalizations remain meaningless.

The last objective criterion of the validity of cognitive meanings is the proof of the truth of their interpretation in representing reality. However, different proof conditions can result in different meanings and different relative truths (Peirce, *EP*I: #4, 56–83, 1869; *EP*II: #15, 208–226, 1903; Nesher 2007b). Hence, by being separated from reality, formal syntax has no theory of meaning based on experience, and formal semantics has no theory of truth based on our confrontation with reality. Although we intuitively understand their meaning and their truth, respectively, we cannot prove their validity and soundness. Hence, we have to look for a logic that can conduct and explain our cognitive confrontation with reality, and we find this in Peircean semiotics, our epistemic logic, as I understand it.

In formal systems, we begin by assuming that the primitive definitions, the axioms, and the rules of inference are true, but in sciences, according to epistemic logic, we do not have to assume these truths, since we can obtain them by proving their truth. However, in epistemic logic, our premises are hypothetical and can only be proved true at the end of our reasoning through the material logic of inductive evaluation applied to the available proved true facts, the perceptual facts themselves, and upon them we prove all our knowledge (Peirce, *EP*I: #8, 124–142, 1878; *EP*II: #25, 350–354, 1905; Nesher 2002b: Chs. 2, 3, and 10).

**[7] Confrontation in Logical Reality Through Coherent Interpreted Meaning of Three Inferences in The Quasi-Proof of the Truth of the Perceptual Judgment:**



Hence, by being separated from reality formal syntax has no theory of meaning based on experience, and formal semantics has no theory of truth based on confrontation in reality; although we intuitively understand their meaning and their truth,

The ultimate purpose of the logician is to make out the theory of how knowledge advanced … so *Methodeutic*, which is the last goal of logical study, is the theory of the advancement of knowledge of all kinds. But his theory is not possible until the logician has first examined all the different elementary modes of getting at truth and especially all the different classes of arguments, and has studied their properties so far as those properties concern [the] power of the arguments as leading to the truth. (Peirce, *EP*II: #17 256, 1903)

These different classes of arguments are the trio sequence of the abductive logic of discovery, the deductive logic of consistency, and the inductive logic of evaluation, which compose the complete proof of truth. Without the methodology of epistemic logic, mathematical hypotheses cannot be proved true or false upon the proved facts of reality. In this way, mathematics depends on the habitual rules of epistemic logic and its rational formulations so as to prove the truth of mathematical theories and thereby make their reasonings sound. However, epistemic logic itself, in confronting its reality, is the methodeutic of all our knowledge (Kerr-Lawson 1997; Nesher 2002b: Ch. 10, 2007c).

**5. Why Couldn’t Peirce Apply his Pragmaticist Semiotics on Logic and Mathematics?**

**5.1. The Problem with the Conceptions of Formal Logic and Pure Mathematic: That Without a Realist Epistemology**

*Two meanings of the term “philosophy”* … mathematical knowledge, which is knowledge of the consequences of arbitrary hypotheses… (Peirce, *EP*II: #27, 372, 1906; Nesher, 2018a)

It is interesting to see that in his discussion of the normative sciences, the theoretical, ethical, and aesthetical, Peirce continues to view mathematics and logic as the Euclidian “arbitrary hypotheses,” or better, as axiomatic systems that are divorced from any reality and from the proof of truth. But though the consequences, or deductive inferences, of these systems cannot prove any truth, Peirce nevertheless speaks of “mathematical knowledge.” I discuss this difficulty in my “‘What Makes Reasoning Sound’ is the Proof of its Truth: A Reconstruction of Peirce’s Semiotics as Epistemic Logic, and Why He Did Not Complete His Realistic Revolution” (*Semiotica* 2018). Though Peirce may have been responsible for the most important philosophical-epistemological revolution of the last two hundred years, he could not resolve the oldest Greek philosophical conception of pure knowledge, which reemerges in the Kantian and neo-Kantian philosophical tradition that I am trying to overcome by developing Peircean semiotics into epistemic logic. This latter puts his formal logic aside and, I suggest, provides an understanding of mathematics as an empirical science and, thus, as the *quantitative* *backbone* of the quantified formulations of scientific theoretical hypotheses and their operations in scientific observations, without which physical and other scientific theories cannot be experimentally evaluated. It is interesting to learn from Russell’s conception of formal logic and of pure mathematics that without a realist epistemology we cannot have logical and mathematical sciences, though we can work with them implicitly by appeal to intuition. Yet we then risk slipping into a sterile scholasticism, as Russell detected in the paradox of Cantorian set theory (Nesher 2012). Pure mathematics consists entirely of assertions to the effect that if such and such a proposition is true of something then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what that something is of which it is supposed to be true. Both these points would belong to applied mathematics. In pure mathematics, we begin from certain rules of inference by which we can infer that if one proposition is true then so is some other proposition. These rules of inference constitute the majority of the principles of formal logic. We then take any hypothesis that seems amusing and deduce its consequences. If your hypothesis is about anything, and not about some one or more of the particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. (Russell 1919 [1901]: 75)

From the above context, we can analyze Russell’s epistemology of pure mathematics, as distinct from applied mathematics. The first proposition suggests the rule of formal inference: if proposition P is true of any x, it is true of some particular a; i.e., (x) (Px → Pa). Thus, pure mathematics is built on formal logic, and it holds vacuously of anything without relation to any mathematical reality, since it is pure and not applied mathematics, as it is in the so-called positive sciences. And since, accordingly, there is no reality that pure mathematics endeavors to represent, we have no objective criterion for the truth of its deduced propositions. Hence “If your hypothesis is about anything, and not about some one or more of the particular things, then our deductions constitute mathematics” and, thus, pure mathematics holds vacuously about everything and actually about nothing. The problem concerns the deductions of formal logic and their role in pure mathematics since we have no objective criterion for validity in pure formal logical inferences. The reason for this is that without perceiving their meanings, we cannot utilize them and cannot have any theories of meaning and truth for formal logic and for pure mathematics (Russell 1919 [1901]: 75– 76; Nesher 2002a, 2007a, 2011, 2012, 2016).

The ultimate purpose of the logician is to make out the theory of how knowledge advanced … so Methodeutic which is the last goal of logical study, is the theory of the advancement of knowledge of all kinds. But his theory is not possible until the logician has first examined all the different elementary modes of getting at truth and especially all the different classes of arguments, and has studied their properties so far as those properties concern [the] power of the arguments as leading to the truth. (Peirce, *EP*II: #17, 256, 1903)

These different classes of arguments are the trio sequence of abductive logic of discovery, deductive logic of consistency, and inductive logic of evaluation, which compose the complete proof of truth. Without the methodology of epistemic logic, the mathematical hypotheses cannot be proved true or false upon the proved facts of the reality. In this way, mathematics depends on the habitual rules of epistemic logic and its rational formulations for proving the truth of mathematical theories in order to make their reasonings sound. However, epistemic logic itself, in confronting its reality, is the methodeutic of all our knowledge (Kerr-Lawson 1997; Nesher 2002b: Ch. 10, 2007).

It is interesting to learn from Russell’s conceptions of formal logic and pure mathematic that without realist epistemology we cannot have logical and mathematical sciences although we can work with them implicitly by intuition; but then we might slip into a sterile scholasticism as Russell detected in the paradox of Cantorian set theory (Nesher 2012).

From the above context, we can analyze Russell’s epistemology of pure mathematics, as distinct from applied mathematics. The first proposition suggests the rule of formal inference: if proposition P is true of any x, it is true of the particular a; i.e., (x) (Px → Pa). Thus, pure mathematics is built on formal logic, and it holds vacuously on anything without relation to any mathematical reality, since it is pure and not applied mathematics, as it is in the so-called positive sciences. And since, accordingly, there is no reality that pure mathematics endeavors to represent, we have no objective criterion for the truth of its deduced propositions. Hence “If your hypothesis is about anything, and not about some one or more of the particular things, then our deductions constitute mathematics” and thus, pure mathematics holds vacuously about everything and actually about nothing. The problem is about formal logic deduction and its role in pure mathematics, since we have no objective criterion for validity in pure formal logical inferences. The reason for this is that without perceiving their meanings, we cannot operate them and cannot have any theories of meaning and truth for formal logic and for pure mathematics (Russell 1919 [1901]: 75– 76; Nesher 2002a, 2007a, 2011, 2012, 2016).

Indeed, Russell comprehends that pure mathematics is based on formal logic, in which “the primitive ideas of logic and its propositions are deduced from the general axioms of logic, such as the syllogism and the other rules of inference.” But then the question is, what are the meanings of the primitive ideas and meanings and truths of the axioms upon which pure mathematics is built? Moreover, how do we know that all the rules of inference of pure logic and pure mathematics are valid (Kline 1980: Ch. 15)?

But today one cannot derive much comfort from the current confusion about what valid mathematics is. This is why Hilbert sought so desperately to restore truth in the sense of objective, unassailable reasoning. As he put it in his paper of 1925 “On the Infinite”: “And where else would reliability and truth be found if even mathematical thinking fails?”

He repeated this concern in a talk he gave at the International Congress in Bologna (1928):

For how would it be above all with the truth of our knowledge and with the existence and progress of science if there were no truth in mathematics? Indeed, there often appears today in professional writings and public lectures skepticism and despondency about knowledge; this is a certain kind of occultism which I regard as damaging …

The future of mathematics has never been of greater promise; the nature of it has never been less clear. The subtle analysis of the obvious has produced a spiral of never ending complications. But mathematicians will continue to struggle with foundational problems. (Kline 1980: 326)

Indeed, if we cannot prove the truth of the meaning interpretations, the validity, and the soundness of the reasonings of all those logical and mathematical operations, then how can we work with them?

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is – insofar as it is thinkable at all – primitive and muddled. (Einstein 1949: 683–684)

Hence, without having epistemological foundations for formal logic and pure mathematics “we never know what we are talking about, nor whether that we are saying is true,” and then, according to the Peircean realist revolution, we cannot understand them as our knowledge and we cannot work with them explicitly as sciences. To overcome “skepticism and despondency” in this regard, let us continue this Peircean realist revolution in epistemology with his methodeutic, the epistemic logic of our knowledge.

**5.2. Why Peirce Could Not Apply His Pragmaticist Semiotics to Logic and Mathematics?** It seems that Peirce developed his semiotics as a realist solution to Kant’s difficulty in the First Critique to explain the ability of the logical judgment to represent phenomenal reality; a difficulty that derives from Kant’s nominalism, namely the impossibility to connect the pure empty concepts with the blind sensual objects. Thus, Peirce’s realist semiotics show that all our abstract cognitions evolve from our basic sensual experience to interpret the signs in our conceptual thoughts of the logical judgment. Hence, as he was educated at Harvard University, were his father taught mathematics, he considered formal logic and pure mathematics as different from the physical and psychological sciences as these were developed in history, from Euclid on to Kantian transcendental epistemology and up to our time. In my epistemological research, and specifically in my recent work on Gödel, I showed that his proof of the incompleteness of mathematics cannot be proved formally, since we cannot prove formally by axiomatic pure logic the relation of mathematics to external reality. I thus showed that axiomatic formal logic is a closed game in which we cannot prove its axioms and cannot prove the truth of its theorems but only infer them formally from their axioms. Interestingly, this empirical explanation can be seen even in Gödel’s late philosophical writings on the foundations of mathematics:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.... (Gödel, 1951: 313; cf. Nesher, 2011, 2012, 2018, 2021)

Epistemic logic, as conceptualized here, is the basic science representing our confrontation with reality, by proving the truth that we actually represent it. The difference between *formal systems* and *realist theories* lies in their different conceptions of proof. *Formal systems* are *hermetically closed games* based on fixed axioms that cannot be proved true, while the *realist theories* can be proved true relative to their proof conditions: the proved true facts of reality and methods of proving their hypotheses. Thus, if *mathematics is to be a science* it cannot be a *pure* *axiomatic closed system* isolated from reality, but an empirical science. Thus, mathematicians could avoid the ambiguity, contradictions, and paradoxes in creating mathematics from baseless axioms.

**5.3. Epistemic Logic and How it Can Explain our Mathematical Knowledge**

Mathematics is the subject in which you don’t know what you’re talking about, and don’t care whether what you say is true. (Bertrand Russell, 1901, reprinted in, 1919: 75)

The epistemic question is about logic and its role in mathematics: What is logic and what is its role in human affairs is the basic epistemological question. Epistemic logic is the basic science representing our confrontation with reality, by proving the truth that we actually represent it. The formal systems are just a *closed game of argumentation* that assumes the *truth* and the *falsity* of the initial *propositions* of the *syllogisms* or *axioms*, and by assuming the *validity* of the inferences, we might reach their conclusions. The difference between *formal systems* and *realist theories* lies in their different proof conditions. *Formal systems* are *hermetically closed games* under their fixed axioms, which cannot be proved true and their formal rules of inference cannot evaluate the truth of their theorematic conclusions. Hence, *axiomatic formal systems are* *complete* and isolated from reality while the *realistic theories* *are* Gödelian *incomplete* but can be proved true relative to their proof conditions: the proved true facts of reality and methods of proving their hypotheses. However, if *mathematics is to be a theoretical science* it cannot be a *pure* *axiomatic closed systems* isolated from reality, but an empirical science. Thus mathematicians can avoid the ambiguity, contradictions, and paradoxes in creating mathematics from unbasted axioms (Byers, 2007; Nesher, 2016, 2018, 2021).

**6. What is Logic and What is its Role in Human Affairs is the Basic Epistemological Question.**

**6.1. Kant’s Conception of Logic Is the Traditional Conception from The Greeks**

In his book *Logic*, Kant summarizes his conception of logic as an a priori pure discipline specifying our rules of thought. This conception affected the following generations of philosophers, logicians, and mathematicians, who somehow accepted aspects of his philosophical system, known as neo-Kantians; the tradition that still dominates philosophy, logic, and mathematics.

If, however, we set aside all knowledge that we can only borrow from *objects*, and reflect simply on the exercise of the understanding in general, then we discover those rules which are absolutely necessary, independently of any particular objects of thought, because without them we cannot think at all. These rules, accordingly, can be discerned *a priori*, that is, *independently of all experience*, because they contain merely the conditions of the use of the understanding in general, whether pure or empirical, without distinction of its objects. Hence, also, it follows that the universal and necessary laws of thought can only be concerned with its *form*, not with otherwise with *matter*. And we can form a conception of the possibility of such science, just of the *universal grammar* which contains nothing beyond the mere form of language, without words, which belongs to the matter of language. This science of the necessary laws of the understanding and the reason generally, or, which is the same thing, of the mere form of thought generally, as we call *logic*. *(*Kant, *Logic*, 1800: 171-172)

According to Kant, the science of logic discovers the *a priori* *necessary* rules of our faculties of understanding and reason, but the rules of other sciences that are about our relations to *particular objects* are *contingently* connected to our particular experience with objects and can thereby change. However, according to Kant’s transcendental epistemology, for the logical rules of our pure cognitions to be *necessary* and valid they must be separated from our sensual experience and must be *formal*, independent of the *matter* of our sensual experience; thus those *pure* rules remain meaningless to us. This Kantian epistemology of logic is, in a nutshell, his essential influence on the philosophy of logic and on the logic that followed him historically, as we can see in Frege, Hilbert, Russell, Carnap, Tarski and others, up until these days (Hintikka, 1973: #VIII). Hence, it makes formal logic sterile, and leaves it Platonist, syntactical, intuitionist, and confronting difficulties, due to its lacking any objective control of its inferences and so-called proofs (Krantz, 2011). Indeed, such logics are closed systems, isolated from our experience in reality, and are mere kinds of arguments that start from axiomatic assumptions to argue for their conclusions without any objective criterion for the *validity* of the inference and the *truth* of their conclusions. (Hintikka, 1996; Nesher, 2002, 2011, 2016, 2017). Indeed, Kant does not have any comprehensive theory of truth to prove the validity of the rules of formal logic and he must accept them as *absolute* and as *necessarily* *independent of all experience*. But without knowing their meanings we cannot think rationally (Kant, *Logic* 1800: 171).

**6.2. The Axiomatic Formal Systems are *Artificial* by Abstraction from Human Cognitive Operations, and are Closed Games that Cannot Explain True Representations of Reality to Direct our Conduct**

Axiomatic formal systems cannot explain and direct human cognitive operations of proving our true representations of reality to guide human conduct. *Formal systems* are by definition *closed games* with rigid rules and axioms that cannot be formally proved true, since the deductive rules of inference cannot evaluate the truth of theorems in relation to reality. The epistemological basis of axiomatic formal systems lies in a certain conception of truth and its acceptance, namely, in the assumption that truth and falsity are ideal and determine whether our sentences are true or false. Hence, every sentence is bivalent, it can be either asserted or un-asserted, and, in accordance with the *principle of the excluded middle*, it can be only true or false. In practice, however, formal logicians do not live in any Platonic heaven, and to discover axioms and rules of inference they use their experiential intuitions, which remain vague, to compensate for their formal rigid rules. Due to the abstraction and sterility of logical formal systems, logicians are divorced from reality, and thus might go astray and face antinomies and paradoxes. The axiomatic formal systems are *artificially* abstracted from human cognitive operations, yet logicians attempt to accommodate their formal systems by intuiting new axioms and new systems of logic without being able to reach reality (Hintikka, 1996: #2).

The difficulty with the validity and truth of formal logic can be overcome only in *epistemic logic*, in which the meanings of the logical components that are essential for proof originate in our basic perceptual experiences of confronting external reality. However, there is an epistemological distinction between the conceptions of interpretation in Peircean semiotics, where interpretation of signs constitutes *meanings* and proved *true* representations of reality, and the formal Tarskian semantic interpretation considered as representing artificial models. So, also, the intuitionist conception of interpretation, considered an inner mental activity of proof is a hermeneutic interpretation isolated from reality (Tarski, 1969; Nesher, 2002: II, V). Accordingly, the completeness of formal systems is only with respect to their assumed true axioms and valid inferences. It does not bear on any representation of external reality unless we pretend that the axioms cover the facts of reality by being identical to the model itself. Hence, we cannot hold the model-theoretic picture, which floats above the world without any known support, without the realistic approach that already belongs to the Gödelian revolution in mathematics, and eventually in logic as well. But then logical and mathematical realities cannot be Platonic entities, à la Gödel, that come from nowhere (Gödel, 1951: 313; Nesher, 2002: X, 2011).

**6.3. Peirce Developed Semiotics as Epistemic Logic from Introspection into our Perceptual Operations by the Complete Trio of Inferences Proving our Perceptual Judgments**

Peirce’s Phaneroscopical inquiry is an essential break from the traditional and contemporary engagement with the difficulty of how we can logically understand our representation of external reality. Indeed, only epistemic logic in its entire *trio* sequence of abduction of discovery, deduction of prediction, and induction of evaluation, can provide the complete proof of the truth of human cognitions, which originate in our pre-rational operations, and to quasi-prove their perceptual judgments (Peirce, *CP*: 5.121–145, 1903).

**[8] Complete Cognitive Operation is the *Trio* Sequence of Abduction, Deduction and Induction**:



=> is the *plausibility connective* suggesting the hypothesis A, when  is the *necessity connective* deducing the abstract object or fact C, and =❥ is the *probability connective* evaluating the relation of the concept or theory A to the new experience of objects or proved facts C. Peirce developed his semiotics into an epistemology of our perceptual confrontation with reality, which is manifest in the duality of the expectation of the iconic feeling sign, the *ego*, and the indexical emotional reaction sign, the *non-ego*. Interpreting our genuine signs in their *coherent* synthesis into the complete proof of the true representation of reality conditions the *validity* of the *meaning* interpretation and the *soundness* of the proofs.

**[9] Epistemic Logic: Confrontation With Logical Reality Through Coherent Interpreted Meanings of Three Inferences in the Quasi-Proof of the Truth of Perceptual Judgment:**

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We can explain how epistemic logic can prove that mathematics can and should be an empirical science that supports and develops all our knowledge of reality and of ourselves.

We find that through our cognitive clashbetween the iconic sign of*Ego and* the indexical sign of *non-Ego*, we first become conscious of the reality that is independent and external to us:

And what do we mean by real? It is a conception which we must have had when we discovered that there was an unreal, an illusion; that is, when we first correct ourselves. (Peirce, *CP*: 5.311, 1868)

This explanation can be considered a philosophical proof of the existence of something external that is independent of the way we initially present it. Further, when we interpret the *coherency* of themeanings of iconic and indexical signs, we can prove our positive knowledge of this external reality (Peirce, *EP*I: #8, 136–137, 1878).

Axiomatic *formal systems* are *complete* and isolated from reality and *realistic theories* are *incomplete* and true relative to their proof conditions. Epistemic logic is a basic and universal science whose rules represent the method of self-control in reality by proving that we truly represent it, and in this way refute Berkeley’s *solipsism* and Kant’s *apriorism*. The basic conceptions of *epistemic logic* hold that every instance of knowledge had been proved to be a true representation of reality, and thus we prove our cognitions to be either *true* or *false*,and if we do not prove them they remain *doubtful*. Therefore, we can no longer accept the *principle of the excluded middle*, and *truth* cannot be separated from being proved, in contrast with the logic of formal systems and other kinds of metaphysical realism and internal realism (Nesher, 2002: III, 2011). Since the validity of logical inferences depends on the coherency of their signs-meanings with respect to the *proof conditions* in which their true interpretations are decided, all inferences are valid by the coherency of their meanings in true interpretation. However, different proof conditions can have different meanings and truths; thus, if **P** does not include the meaning of **C,** then we cannot infer **C,** since the implication **P** ➞ **C** is not valid. With the rules of inference, **Pi** ➞ **Ci**, **Pi ∧ Ci**,and **Pi ∨ Ci**, the epistemological and logical question is how the elimination of the *law of* ***excluded middle*** by the realist theory of ***truth*** can affect *deductive inference* as it operates in Pragmaticist epistemic logic. If the propositions **Pi** and **Ci** are proved true or false or doubtful, what are the conditions of *validity* for the inferences **Pi ➞ Ci**, **Pi ∧ Ci**, **and Pi ∨ Ci**? Thus, **Pi ➞ Ci** is valid when the meaning of the consequent **Ci** is contained in the meaning of its antecedent **Pi**. Further, their *truths* must be proved by their trio of complete proof according to common proof conditions for **Pi** and **Ci**, since if they were proved true on different proof conditions, the truth **Pi** cannot entail the truth of **Ci**, since the complete true meaning interpretation depends on the entire proof of truth. In epistemic logic, the deductive rule of inference ((**Pi ➞ Ci**), **Pi**) ➞**Ci**), **Pi** and **Ci** evaluated in induction ((**Pi** Ab, **Ci** In) =❥Pr. m/n (**Pi** Ab **➞ Ci** in)), when empirically proved true. But this entailment cannot be achieved by the conventional formal semantic *Truth Tables*, since in epistemic logic the *truth* and *falsity* of propositions are proved in confrontation with reality Thus, the formal semantic language, with "if," "suppose," "provable," "unprovable," etc. is meaningless and disallowed (Gödel, 1931; Hintikka, 1996: 46-87; Nesher, 2011, 2016).

**7. The Epistemology of Mathematics: The Conception of Pure Mathematics Isolated from Reality and How it Can be Theoretical Science**

**7.1.** **Euclidean Geometry, Formal Logic, and Pure Mathematics are Epistemically Closed Games**

The problem with Euclidean geometry and the formal mathematics that was created to investigate some structures and properties of reality is that they remained pure sciences with their a priori assumptions and without confrontation with reality (Russell, 1919: Chap. XVIII-204; Nesher, 2017).

Now, the intuition which pure mathematics lays at the foundation of all its cognitions and judgments which appear at once apodictic and necessary are space and time. For mathematics must first present all its concepts in intuition, and pure mathematics in pure intuition; that is, it must construct them. If it proceeded in any other way, it would be impossible to take a single step; for mathematics proceeds, not analytically by dissection of concepts, but synthetically, and if pure intuition be wanting there is nothing in which the matter for synthetical judgments *a priori* can be given. Geometry is based upon the pure intuition of space. Arithmetic achieves it concept of number by the successive addition of units in time,... (Kant, *Prolegomena* 1783:282-283; Hintikka, 1973; schema [4])

Indeed, Kant based his epistemological conception of pure mathematics on his analysis of syllogistic structures and operations as a conception of *axiomatic systems of transcendental logic and mathematics*. The following explains Kant’s epistemology of knowledge in which *pure mathematics* is a closed game isolated from any reality and which cannot prove any truth (Kant, *CPuR*:B316-7; Nesher, 2011, 2012, 2016).

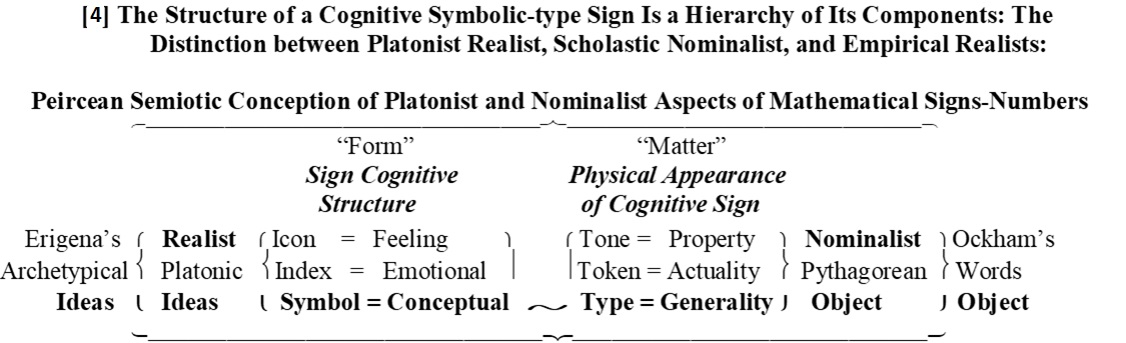
**\*\*\*[10] Kantian Conception of Knowledge Based on Pure Concepts and Empirical Sensations: The Evolvement of Empirical Concepts from Blind Sensual Intuitions and the Empty Pure Concepts, into their Synthesis in Perceptual Judgment, and the Pure Mathematics in Pure Intuition:**



This schema can explain how the synthesis of the indeterminate meaning of the *blind object* with the *empty pure concept* makes the concept meaningful and the object determinate and thus the *empirical object* can be determined by being subsumed under the *empirical concept*. However, the Evolvement of the Empirical Concepts in Perception from the Sensual Intuitions to the Pure Concepts, and with Imagination to their Synthesis in Perceptual Judgment reveals Kant’s difficulty with the epistemology of empirical concepts (Kant, *CPuR*: #24-B150-151). Kant’s transcendental epistemology is based on the *mystical* conception of *schematism*, to bridge between *form* and *matter* without it his philosophical system cannot hold. The component of pure *a priori* knowledge includes the conception of *pure mathematics*, but the formalism cannot work without the empirical matter, the meaning of the form. Since Kant assumed that mathematics is pure science based on transcendental pure intuition, he had difficulties explaining this intuition and in his *Critique of Pure Reason* B-1787, he explains empirically the basic mathematical intuition, by counting fingers or dots.

In thinking merely that union of seven and five, I have by no means already thought the concept of twelve; and no matter how long I dissect my concept of such possible sum, still I shall never find in it that twelve. We must go beyond these concepts and avail our ourselves of the intuition corresponding to one of the two: e.g., our five fingers or (as *Segner* does in his *Arithmetic*) five dots. In this way we must gradually add, the units of the five given in intuition. …. For then it is very evident that, no matter how much we twist and turn our concepts, we can never find the [number of the] sum by merely dissecting our concepts, i.e., without availing ourselves of intuition. (Kant, *CPuR*: B14-15)

The first epistemological difficulty is with numbers, whether they are ideas or objects and this can be seen from the semantic structure of the signs-symbols: The Realist Platonic Ideas on the left and the Nominalist Phenomenal Object on the right side of schema [6]. The epistemological difficulties in mathematics is *what numbers are*, objects of signs or signs of objects, and what is *mathematics* and *proof* in it (Russell, 1901).

**[11]****The Mathematical Reality Upon Which the Pragmaticist Structure of Cognitive Symbolic-Signs Operate**

Historically, Plato conceived of numbers as *ideas* and Pythagoras conceived of them as *objects* but this is an epistemological confusion. Those two aspects of *signs-numbers must go together* since otherwise *they are not signs*, we cannot grasp sign *meaning* without its *appearance* and cannot understand the appearance without its meaning. The sign in Peircean semiotics is the conjunction of “form” and “matter,” or better, the **sign** has two componentsthat cannot exist separately. Moreover, modern mathematicians and philosophers are not clear about whether *numbers* are signs or objects, they take these aspects to be two separated entities such that numbers are both signs and objects. This confusion about the nature of numbers resulted in the difficulties, ambiguities, and paradoxes of set theory, namely, by considering the phenomenal-objective component of the sign-number as the object of its cognitive-idea component (Nesher, 2012). Thus, the number’s phenomenon was assumed to be the object of the number’s idea; that is, it was assumed that the number can be the object of itself. This confusion is the basis of Russell’s paradox in set theory based on the assumption that a number can be a member of its own set, but if a number is a sign, it cannot be an object and of course not an object of itself (Russell, 1901, 1919). Moreover, the formalist epistemology of logical positivism and analytic philosophy, which assume that cognitive signs and language, with their syntactical and semantical aspects, can be represented by other meta-signs and meta-languages, brings with them further difficulties and paradoxes (Byers, 2007). Hence, cognitive signs and languages are not physical objects that can be cognitively represented, we can only interpret their meaning and prove their truth or falsity (Wittgenstein, 1921: 3.33-3.34; Nesher, 1986, 2011, 2012).

**7.2. On the Nature of Mathematics: Mathematical Proofs at a Crossroad from the Pure Formal Game to Empirical Theory**

The number signs cannot be of objects of empirical experience, but are the discovered the signs components of the human empirical operations of counting, grouping, and measuring physical objects. (Nesher, 2011). The *discovery* of the concepts of these operations of enumeration contains natural numbers, and the further *discovering* oftheir expansion through abstractions and generalizations constitutes our mathematical hypotheses, which will be evaluated and proved upon the extended mathematical reality (Krantz, 2011). Hence, by proving the truth of perceptual facts representing mathematical operations we represent mathematical reality.

**[12]** **the Double Layer of Mathematical Operations: (1) Counting Physical Objects, 2)Perceptually Quasi-proving the Truth of Discovering the Numerical Signs and of Operating With Them**

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By understanding that mathematical reality consists of perceptually self-controlled numerical operations on physical objects, we can see how Peirce, and also Gödel, confuse the meaning-content of mathematical signs with Platonist mathematical abstract forms as objects. The arithmetical numbers are neither *physical objects* nor *abstract concepts* but the *conceptual components of our quantitative operations on physical objects* as the mathematical reality, upon it we prove the truth or the falsity of our abstract mathematical hypotheses (Nesher, 2012).

**7.3. Mathematics Is an Empirical Science, Neither Queen Nor Servant of Other Empirical Sciences But Their Quantitative Backbone**

The problem is to explain the difference between mathematical science, the other sciences, and their collaboration, when all are empirical sciences representing different aspects of reality but with basic epistemic logic in developing our knowledge of reality. Thus, in mathematics, we cannot have true theories without proving them upon mathematical reality. Mathematicians essentially develop their theories by discovering hypotheses as formulations of theoretical patterns, typically within physics, but also within all other sciences, and evaluate them upon mathematical reality by applying quantitative operations to predicted physical observations. Thus, physicists and mathematicians have different realities to represent with their theories, and the mathematical theory that is proved true in the measurement of observed physical facts is the condition for the evaluation of physical theories. The truth of mathematical theories enables the experimental proof of the truth and the falsity of theories. In this way, we can understand the Gödelian epistemic intuition about the nature of mathematical theories, yet not by confusing mathematics with other sciences and identifying mathematical reality with physical reality.

When difficulties emerge between a physical picture of reality and the mathematical model of it, such that it becomes impossible to make measurable predictions, the challenge is to inquire why we are unable to evaluate the physical hypothesis experimentally. Hence, mathematics without operationally measuring the predicted and the eventually observed true facts of reality cannot be true and cannot be on a much firmer ground than physics without a testable prediction. Both have to prove their own truths upon their realities.

However mathematical intuition in addition creates the conviction that, if these formulas express observable facts and were obtained by applying mathematics to verified physical laws (or if they express ascertainable mathematical facts), then these facts will be brought out by observation (or computation) (Gödel, 1953/9-III: #16).

How may one understand this hinted explication of the relationship between intuitive mathematical truth representing its own reality and its application to physical theories to enable observable predictions of them? At the end, mathematics is neither the *queen* of science nor its *servant* but it is empirical science that when its hypotheses are consistent by being proved true and thus it can serve the *quantitative* *backbone*—that is, the quantified formulations of scientific theoretical structures and their operations on scientific observations, without which physical and other sciences cannot be evaluated experimentally (Nesher, 2011). This empirical explanation can be seen in Gödel’s late philosophical writings on the foundations of mathematics:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.... This whole consideration incidentally shows that the philosophical implications of the mathematical facts explained do not lie entirely on the side of rationalistic or idealistic philosophy, but that in one respect they favor the empiricist viewpoint. (Gödel, 1951: 313)

Hence, we can know experientially the mathematical facts of the mathematical empirical reality.

**8. The Eventual Failure of Kant in His Three Critiques Due to Peirce’s Realist Semiotic Alternative**

**8.1. In his Transcendental Epistemology Kant Cannot Reach Experience in Reality and Remains a Nominalist**

In the following, I summarize Kant’s failure to explain human knowledge and its eventual practice in human life by listing the epistemological difficulties in Kant’s three transcendentalist critiques and Peirce’s pragmaticist alternative.

**1.** The *Critique of Pure Reason*: Reason cannot be pure if it is to prove its *logical judgments* and, due to the gap between the *form* of *empty* pure concepts and the *matter* of *blind* objects it cannot do so. Therefore, we cannot scientifically know nature. To perform its function, pure reason cannot be the pure presentation of *phenomenal objects* but only *forms* of *empty meaningless words*.

**2.** The *Critique of Practical Reason*: Pure practical reason cannot be practical if moral freedom and its categorical imperatives must be absolute as then they would be unable to direct human moral conduct or be practical in personal and social life by being true to their empirical reality. Thus, *practical reason cannot be practical.*

**3.** The *Critique of Judgment* cannot be objective if reflective judgment is only subjective feeling without being publically proved true or false under its proof conditions as is the Kantian *logical judgment* in perception. Thus, *reflective judgment* cannot be objective judgment that “A is B is proved true aesthetic judgment,” but *only a subjective feeling* as “is B”.

Kant cannot explain the validity and objectivity of the aesthetic judgment of taste, because in Kantian philosophy, fine art is not only a genius’s creation separate from the aesthetic judgment of taste, but in this reflective judgment we cannot even distinguish between artworks created by people and natural objects. However, without the unity of all three inferential components of the creative operation – the abductive discovery of artistic intellectual ideas, the deductive quasi-inference of aesthetic ideas from intellectual ideas by the productive imagination of the artist’s creation of the artwork, and the inductive evaluation for achieving aesthetic judgment and its beauty – we cannot have a complete explanation of harmony in aesthetic creation and an evaluation of artworks.

However, there is no true aesthetic judgment without confronting and representing reality, as in Peircean *epistemology*.The question then is, how can we know whether the artist’s spirit and inner intellectual ideas, “the content, aim, and meaning,” of the created artwork, have actually been interpreted truly by the aesthetic ideas, and how can we evaluate whether “the external, the particular, appears exclusively as a presentation of the inner,” since there are many possible aesthetic modes of presentation of the inner content by the external form (Hegel, 1835: 95-96)? What, in other words, is the truth of artworks and how is beauty connected with it? This remains for pragmaticist epistemology to explain. It can be achieved by reconstructing the Kantian aesthetic, and thus resolve its difficulties (cf. Nesher, 2003, 2004a, 2005a, 2005b, 2007a). Claiming that the aim of art is the self-interest of the human spirit, separated from its function to represent reality, cuts it from any objective criterion of truth and beauty.

The Peircean pragmaticist solution to these Kantian predicaments is to suggest that general and abstract cognitions develop gradually from the initial sensual intuitions through their interpretation in emotional reactions to them, the coherency of which is interpreted in the synthesis of thought and is embodied in proving the truth of our perceptual judgment, which is our basic representation of reality and from which all our general and abstract knowledge develops. In this epistemology, general rationality developed from particular experience, without any need to bridge the Kantian gap between transcendental and the empirical components. Hence, Peircean semiotics is actually the epistemology of knowledge upon which I developed the epistemic logic that allows us to show that the Kantian conception of formal transcendental reason and understanding cannot prove the truths of our hypothetical cognitions (Nesher, 2011, 2016 2018, 2021).

This realist epistemology allows Peirce to solve the predicaments in Kant’s three critiques, with his conception of the three normative sciences, the theoretical, ethical and aesthetical, which represent different modes of reality, and explain how these sciences, when their hypotheses are proved true in their respective proof conditions, can be normative and practical in human life and conduct in natural, social, and engaging lives respectively (Peirce, 1903, Nesher, 2002: X, 2007, 2018a/b).

**8.2. How to Overcome Kantian Epistemology in order to Explain Knowledge of Scientific Theories? The Practical Norms of Theoretical, Ethical, and Aesthetical Sciences in Physical Nature and Human Social Life**

There is no question about the Kantian roots of Peircean pragmaticism. The problem is rather to discover how Peirce developed his empirical realist philosophy with its semiotics as epistemic logic from Kantian transcendental idealism as phenomenology. My thesis is that Peirce proved that we can explain human cognitive behavior empirically without speculating about an *a priori* transcendental domain that is distinct from empirical sensual experience. I will show that, turning the tables on Kant’s Copernican Revolution, Peirce explains how we can exploit Kant’s important insights into human cognitive operations in order to provide an epistemological explanation of our aesthetic, ethical, and scientific knowledge of external reality. This is in contrast to some Peircean scholars who interpret him as a neo-Kantian (e.g., Apel, Putnam, Habermas; cf. Wellmer, 1991: 170ff.; Nesher, 2002b: III, VIII, X, 2003, 2004b).

Indeed, one can find some Kantian elements in Peirce’s discussions of aesthetics, especially in his early writings and in his mature philosophy, in which we can see how he revised and reconstructed his Kantian epistemological foundations. The question here is how, by following Peirce, we can revise Kant’s philosophy of aesthetic artworks to understand aesthetics as a normative science and explain the creation and evaluation of artworks, their beauty and truth. This, as I have suggested, can be done in the framework of Peircean epistemology, which we can reconstruct from his writings.

Peirce analyzes philosophy as consisting of “three grand divisions”: (1) *phenomenology* as a description of our preliminary experience; (2) the three *normative sciences*, *logic*, *ethics*, and *aesthetics*, which comprise the comprehensive epistemic logic of the three main human modes of representing reality: scientific theories, norms of action and moral conduct, and aesthetic epitomes as knowledge of human life-divisions that are comparable to Kant’s three critiques; and (3) *metaphysics*, which according to Peirce is not *a priori* knowledge but, I would say, our most generalized and abstracted comprehension of reality, as distinct from Kant’s “Metaphysics of Experience” (Paton, 1936, I: 258; cf. Peirce, *CP*: 5.121; Nesher, 2002: III, 2010).

**8.3. Kant’s Three Types of Judgment: Theoretical, Practical, and Aesthetic**

When discussing Kant’s conception of *aesthetic judgment*, it is crucial to understand his general conception of *judgment* as the basic operation of human cognition.

I then find that judgment is nothing but a way of bringing given cognitions to the objective unity of apperception, (Kant, *CpuR*: B141-142)

A judgment is the presentation of the unity of the consciousness of several presentations, or the presentation of their relation so far as they make up one concept. (Kant, *Logic* [1800]:#17; cf. Peirce, *TEP*: 19, 191; Longuenesse, 1998: 73-80)

Judgment is a function that is an act of synthesizing a number of presentations into a unity, or one common presentation as a claim (cf. Kant, *CPuR*: B92-94). With this general conception of judgment, we should inquire how judgments differ according to their epistemic domains of operations (cf. Kant, *CPuR*: A85/B117, B143; cf. Peirce, *CP* 2.461-516, 4.2-5; comp. Beck, 1960: 128-129&n5).

Hence all our judgments can be divided, in terms of the order of the higher cognitive powers, into *theoretical*, *aesthetic*, and *practical* ones. But by aesthetic one I mean [here] only aesthetic judgments of reflection, which alone refer to a principle of the power of judgment, as a higher cognitive power... (Kant, *CJ*: First Int. VIII 226'; e.g., *CPuR*: B94, 141; cf. Peirce, *CP* 6.378)

The aesthetic judgments of reflection are *judgments of taste*, which are about the subjective feeling of beauty by itself, not representing any object or property. They are thus distinguished from the *aesthetic judgments of sense,* which are about the agreeable and are interested in the existence of some object. However, in all three kinds of judgments, *theoretical*, *aesthetic*, and *practical*, we consciously reflect on our operations to feel the relationship among their different operated presentations (cognitions). It is a reflection that does not deal with the objects themselves. Rather, its function is to compare and detect *agreement* and *conflict*, or *harmony* and *disharmony*, between the given presentations of our cognitive powers, reason, understanding, and imagination, in order to perform *affirmative* or *negative* judgments in their epistemically different domains (Kant, *Logic* [1800]: #6, *CPuR*: B316-324, *CPrR*: 105-106, 124-125, 160, 60-70; *CJ*: 220').

*Deliberation* (*reflexio*) does not deal with objects themselves in order to obtain concepts from them straightforwardly, but is our state of mind when we first set about to discover the subjective conditions under which [alone] we can arrive at concepts. It is our consciousness of the relation of the given presentations to our various sources of cognition – the consciousness through which alone the relation of these presentations to one another can be determined correctly. (Kant, *CPuR*: B316; cf. B317-324, A85/B117, B143)

The Kantian distinction between transcendental and aesthetic judgments is that we do not have determined rules and concepts in *aesthetic reflective judgment*, and thus we cannot determine our cognitive operations as representations of objects. We only reflect to compare the relationship between the presentations of the imagination and the understanding as components of the power of judgment. We thus feel the pleasure or displeasure that Kant assumes they involve because of the harmony or disharmony between the presentations of these cognitive faculties. Yet the distinction between *transcendental judgment* and *reflective aesthetic judgment* also lies in their respective epistemic structures.

Judgment in general is the ability to think the particular as contained under the universal. If the universal (the rule, principle, law) is given, then judgment, which subsumes the particular under it, is *determinative* even though [in its role] as transcendental judgment it states a priori the conditions that must be met for subsumption under that universal to be possible. But if only the particular is given and judgment has to find the universal for it, then this power is only *reflective.* (Kant, *CJ*: 179)

Hence, both the *theoretical judgment of knowledge of nature* and the *practical judgment of moral law* have transcendental, a priori, pure rules and concepts, whereas the *aesthetic reflective judgment of taste* operates without such rules and concepts and can be explained only by empirical psychology or by analogy with other judgments about the cognitive relationship between the imagination and the understanding (cf. Kant, *CJ*: First Intr.: X, 237'-238', #17, 233-236). Both the *theoretical judgment* (*of knowledge of nature*) and the *moral practical judgment (of morality)* are determinate operations of reason but in different capacities and, therefore, with different logical-epistemic operations (cf. Kant, *CPuR*: A135/B174). The *theoretical judgment* of pure reason must bridge the abyss between the transcendental universals and the sensual particulars, whereas the *moral judgment* of practical reason is entirely within the suprasensible realm. Kant explains the difference between the *theoretical judgment of knowledge of nature* and the *practical judgment of moral law* in terms of the logical-epistemological operational relationship between *a priori* *principles* and *concepts* and *sensible objects* (cf. Kant, *CPrR*: 16, 42; *CJ*: 355-356). On this distinction, Beck writes:

But though formally similar, the two syllogisms are quite different in their epistemic or transcendental function (Beck, 1960: 128-129).

Beck analyzes the different functions of the faculty of judgment in two different domains of human cognition, the theoretical and the practical; however, the aesthetical is also crucial. The differences among these three types of judgments are not only in their epistemic or transcendental functions but also in their formal structures, in the syllogistic procedures by which judgments of these different types should be proved (cf. Kant, *CpuR*: B360-366; Beck, 1960: 154 & n56 & Ch. IX#10). Since every cognition in pragmaticist epistemology, by being proved true, is a representation of both physical and psychical realities, there is no distinction between aesthetic reflective judgment, on the one hand, and theoretical and practical judgments, on the other. In all three types of representations, it is through reflection, whether instinctive, practical, or rational, that we self-consciously reflectively self-control our minds’ operations (cf. Peirce, *CP* 5.119; Nesher, 1990, 2002b: II, III, 2004b).

**9. Peirce on the Basis of Pragmaticism in the Normative Sciences Replacing the Kantian Transcendental Epistemology of the Three Critiques: Are Logic and Mathematics Also Normative Sciences?**

**9.1. The Basis of Pragmaticism Is the Normative Sciences As the Criticism of the Kantian Epistemology of the Three Critiques As Pure and Empty Without Representing Reality and Our Practices Within It.**

Kant (whom I *more* than admire) is nothing but a somewhat confused pragmatist (Peirce, *CP* 5.525, 1905).

As I showed elsewhere, it is reasonable to understand Peirce’s criticism of Kant’s nominalism as deriving from the gap it introduces between his transcendental formalism and sensual materialism, which cannot explain our knowledge of external reality and of ourselves. In order to overcome Kant’s Copernican Revolution, Peirce develops his realist epistemology to show that all our knowledge and practices must arise from our sensual experiences that get interpreted in our perceptual judgments, which constitute our basic facts (Peirce, *EP*II: #27, 1906; Nesher, 2007).

Hence, we can assume that, for Peirce, the role of the normative sciences, Theoretical, Ethical and Aesthetic, is to show how to solve the difficulties of Kant’s three critiques, Pure, Practical and Judgment, which remained epistemologically barren. Thus, pure reason must be alike the theoretical science, which aims to represent reality and, by being normative, to adjust it to living a better life within it. Similarly, the practical reason of ethics can be made practical by transforming absolute ethical freedom, with its ideal categorical imperative, into relative freedom, according to our knowledge of reality and according to the relative power with which we are able to practice our ethical value in reality. However, in aesthetical judgment the pragmaticists must show that the *beauty of artworks is an aesthetic true representation of reality* and that the role of this normative science is to allow that this knowledge contributes to our knowledge of reality and of ourselves and to our ability to elaborate upon the beauty and harmony within ourselves, our society, and nature (Nesher, 2002:X, 2007, 2021, 2022).

#### **9.2. Kant’s Pure Reason, the Impossibility of Knowing Reality of Self and Nature: The Peircean Pragmaticist Alternative**

In his Copernican Revolution, which was intended to overcome Hume’s empiricism, Kant suggested starting with transcendental *a priori* formal concepts to control our material sensual experiential intuitions, yet he did not have any method to combine them. Thus, Kant suggested a bizarre conception of *schematism*, which cannot overcome the gap between the *empty pure concepts* of the transcendental understanding and the *blind objects* of the empirical intuitions. Indeed, Kant admitted his failure to bridge this gap at the end of his inquiries.

**\*[13]**

This schema is meant to explain the synthesis of the meaningless *empty pure concepts* with the indeterminate meaning of the *blind object* so as to make the concept meaningful and the object determinate. Thus, the *empirical objects* can be determined by being subsumed under the *pure concept*, which determines the perceptual judgment by which phenomenal reality is represented. However, Kant’s transcendental epistemology is based on a *mystical* conception of *schematism* that is meant to overcome the gap between *form* and *matter* without which his philosophical system cannot hold.

This schematism of our understanding, i.e., its schematism regarding appearances and their mere form, is the secret art residing in the depth of the human soul, an art whose true stratagems we shall hardly ever divine from nature and lay bare before ourselves. Only this much can we say: The *image* is [here] a product of the productive imagination’s empirical ability. (Kant, *CPuR*: A141/B180-1; cf. A121, B185-187)

Hence, we can see that all of Kant’s efforts in his first Critique to explain our scientific knowledge of nature through our phenomenal experiences cannot work with his transcendental epistemology. Further, I suggest that following Peirce’s mature realist epistemology we must begin from our empirical experience and show how our conceptual knowledge develops from our basic perception and the proof of the truth of our perceptual judgments. Moreover, those judgments are our first cognitive facts upon which we can develop our true scientific cognitions with our *epistemic logic*, which I developed from Peircean pragmaticism and his theory of truth (Nesher, 2002, 2010, 2011, 2016, 2018).

The following is Peirce’s epistemological alternative to the Kantian failure: the proof of the truth of our perceptual judgments. Peirce developed his semiotics into the epistemic logic of our perceptual confrontation with reality, manifested in the duality of the *ego* and *non-ego*, by interpreting our genuine signs as complete proof of the true representation of external reality, conditioning the *validity* of the interpretation and the *soundness* of the proofs.

**[14] The Confrontation in Physical Reality by Coherent Interpretation of Meanings of the Three Inferences in the Quasi-proof of the Truth of Perceptual Judgment Representing Reality**:

**Validity of Meaning Interpretation and Soundness in Proving the Truth of** **Perceptual Judgment**

**Meaning and Validity of Inferences, Coherency, Proving True Perceptual Judgment**

*Hypothesis* *Prediction* *Evaluation* Proof Truth

**Perceptual Signs Inferential Prediction Empirical Evaluation Perceptual Judgment,**

**Ab(C, A➞C)➾A)** + **Dd**((A➞C), A)➞C) + **In**((AAb, CIn) =❥Pr. m/n (AAb➞Cin)) = **Falsity or Truth**

[Initial Sign] **Icon Index Icon, Index Symbol:** **Perceptual Judgment**

⇘ ⇙ ▲

Ego non-Ego ❙

**Truth Conditions** = Duality= Comparison ❙ ⇙ ⇘ ❙

**Truth Condition: *Confrontation in Reality*** ❙

▼ ▼ ❙

**Logical Reality**: *Incoherency Coherency* ❙ ▼ ❙

Hesitation Assurance ➠ **Assertion**: **Subject**

**Internal Proof-Condition: *Confrontation in Reality* Self-Knowledge**

❙ **Representing** Physical Reality

❙ **Object**P by **Description** **(AAb** ➞**Cin)**

▼ **This Is the Proved True Basic Fact**

**Represented** **External Reality**

We find that through our cognitive clash with reality, we first become conscious of the reality external to us: this is our negative knowledge of reality, whereby we cognize the existence of something that contradicts our expectation, yet we still do not have a positive true representation of it (Nesher, 2017).Thus, the evolvement of the empirical concepts in perception from the sensual intuitions and imaginations into their synthesis in thought of perceptual judgment reveals Kant’s difficulty with the transcendental epistemology of empirical concepts, which remain empty verbalizations. This is Kant’s nominalism, as Peirce explains it (Kant, *CPuR*: #24-B150-151; Peirce, *EP*II: #25, 1905). From this realist solution Peirce, in his latter research, developed his conception of normative science to show the practicality of the theoretical sciences in enhancing the human ability to adjust nature and society for the benefit of increasing their freedom and their quality of life (Peirce, *EP*II: #27, 1906; Nesher, 2007a).

**9.3*.* Kant in His *Pure Practical Reason* Cannot Make Pure Morality Practical**

The transcendental ground of Kant’s *Critic of Practical Reason* is the *a priori* assumption of the *fact of pure practical reason*, which appeals to the concept of a fact as a pure valid truth from which he can develop his moral theory, without any need for a *deductive justification* of the system of pure practical reason by means of experiential components, as in the first Critique.

The consciousness of this basic law may be called a fact of reason, because one cannot reason it out from antecedent data of reason – e.g., from the conscious of freedom (for this is not antecedently given to us) – and because, rather, it thrusts itself upon us on its own as a synthetic a priori proposition not based on any intuition, whether pure or empirical (Kant, *CPrR*: 31).

We can understand the status of *fact* as the basic assumption that is accepted without any rational proof from other assumptions, just as scientists use facts as certain true pre-conditions for any scientific enterprise. Yet the question is, how is this fact accepted? However, according to Peircean pragmaticist epistemology, *facts* are not components of external reality but the proved true perceptual judgments and other scientific hypotheses (Nesher, 2000, 2001, 2002). Realistically, we may argue that the moral principle and the moral concept develop in our social experience with the moral behavior of humans in their society, and their combination constitutes the moral law that forms the major assumption for deducing the possible moral act (cf. Kant, *Logic* (1800): #33). Yet, assuming or accepting the reality of the moral law cannot be done circularly as Kant seems to do (cf. Kant, *CPrR*: 42-50; Beck, 1960: X#2). Moreover, even if we assume, à la Kant, the reality of the practical moral law of pure reason, we do not have any judgment that can be synthesized from the pure practical law of reason and from a particular sensible action to obligate moral action in the world (cf. Cassirer, 1938: 73-78).

Hence, the proof, or the quasi-proof, of the truth of moral laws and their actionable application can be achieved only by our empirical knowledge of human nature and of the social behavior of humans in their society (Kant, *Logic* [1800]: #33). The way to solve this predicament is to prove the truth of moral practical laws. Their imperative abstract actions in the sensual world will then be applied by abductive discovery, deductive inference, and inductive evaluation of the truth, and thus the reality, of these laws and their application in the empirical world (cf. Kant, *CPrR*: 29-30). In this manner, we overcome the dichotomy between the supranatural world of freedom and the natural world of determinism and we understand freedom Spinozistically as a person’s internal determination through self-conscious and self-controlled conduct according to the psychological and physical laws of nature (cf. Nesher, 1999).

**[15] the Kantian Eventual Evolvement of Rule of Conduct from the Principle of Ethic, With the Concept of Conduct, Being the Object of the Initial a Priori Moral Concept**

**Pure Practical Reason**

*A Priori* *Apperception*

**The Eventual Connection Between the Normative Moral Rule and Practical Concept of Conduct**

*Transcendental Logic* **I** *Transcendental Aesthetic*

*Analytic**Principle Analytic Concept*  I *Moral Feeling of Desire*

*a priori Fact of Reason* Anticipating the I *the Rule of Conduct*

*Practical Principle* *Concept of Conduct and concept sensual object*

**The Gap**

Between a priori Pure Fact of Practical Reason principle and the *Rule of Conduct*

(Kant, *CPrR*: 89, 101, 102, n.509, n.512) (Kant, *GMM*: 410-11 & note\*)

In the above schema, we can see that transcendental moral principles and concepts cannot have any cognitive relation to the practical moral objects, or conducts, in the sensual domain of practical life. Kant’s problem with apperception is that it cannot consider the empirical person and, thus, cannot explain theeventual connection between the normative moral rule and the practical concept of conduct. The eventual reason for such separation is that in order to avoid the relative experience of *logical judgment* of the sensual experience of what it *Is* we must ensure the validity or absolutism of the *categorical* *imperative* of *moral judgments* to be eternal and *Ought*, independent of our relative sensual experience.

Hence, it is interesting to explain the Kantian conception and the role of *empirical* *apperception* from within a Peircean realist epistemology.The Proof of the Truth of the Normative Moral Rule of Conduct.

**[16] The Kantian Pragmatic Point of View *Elevating the* Experience of Empirical Cognition *into The Unity of Intuition and Rational Discursive* by *Epistemic Logic***

***Epistemic Logic***

~⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯*Empirical* *Apperception⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯*~

Operation of **Imagination**-Reflective Self-Control-Interpretation-Synthesis in Perceptual Judgment

~Operation of **Imagination**-Reflective Self-Control-Interpretation-Synthesis in **Practical Reason**~

**The Proof of the Truth of the Normative Rule of Conduct**

*Transcendental Logic**Transcendental Aesthetic*

*Analytic**Principle Analytic Concept Moral Feeling of Desire*

Person Discover *a priori* Anticipating the Evaluatingand Accepting

with a *will* *Practical Principle* *Ethical Concept of Conduct Proving True Rule of Conduct*

**Abduction (C(A**➞**C)** ➞**AAb) + Deduction((A**➞ **C) A)**➞**CDd)+Induction((AAb, CIn) (AAb**➞ **CIn)**

Situational Concept Hypothetic Principle, Concept Principle, Concept Normative Rule of Conduct

Discovery the ethical habit To predict the *concept* To evaluate and prove the truth of

and the principle of conduct: ***Is*** of expected conduct: ***Ought*** *ethical Rule of Conduct* when the

(Kant, *CPrR*: 89, 101, 102, n.509, n.512). *concept* CIn represents this object: ***Is***

Pure reason can eventually reach the reality it intends to, the sensual intuition of objects, and so can pure moral practice reach the deed in the real world and prove its true representation of the cognitive relation of *Is Ought Is***.** How can the *fact of pure reason* be established and how can we be conscious of the moral law as the fact of reason? Are the moral law and the freedom of rational persons reciprocal, when one assumes the other (Kant, *GMM*: 4:446-448, *CPrR*: 29-31, 42ff.; Allison, 1986)?

The epistemological conclusion must be that in contrast to *theoretical pure reason*, which must be justified by deduction to explain its connection with sensual experience in the sensual world, according to Kant, *practical pure reason* need not justify itself in the *sensual-empirical world*, since it is the *fact of reason* that aspires to *prove itself in us practically* and to determine how the principles of pure reason of morality determine the empirical subject’s moral practicesthrough dynamiclaws of causality (Kant, *CPrR*: 42-3). And yet, this cannot be established in Kant’s transcendental practical reason due to the epistemological *gap* between the *form* of the principle of morality of the *fact of pure reason*and the *matter* of moral deeds in the *world of sense*.

**9.4. Kant and the Impossibility to Evaluate Beauty Rationally**

According to Kant, in the aesthetic mood we reflect only on our perceptual operation, and, like animals, we feel the relation between the cognitive operations of the imagination and of the understanding instinctively, without explicit rules and concepts (Kant, *CJ*: First Introduction. V, 211'). As Kant suggests, our feeling in response to the relation between these faculties can be of pleasure or displeasure, depending on whether we find the relationship between these cognitive faculties to be harmonious or disharmonious. Thus, the suggested form of object (“beautiful”) represents the quasi-object, more specifically our “feeling” it as beautiful.

**[17] Abductive Suggesting the Universal Quasi-Concept for the Given Particular Quasi-Object: Aesthetic Reflective Judgment** (Kant, *CJ*: 287)**:**

⎡Blind Reflective Comparison⎤

Harmony/Disharmony

*Understanding* *Imagination Understanding*,

*Feeling Discovery Inferred Intuitive concept of*

Particular Universal Particular *Reflective* *Judgment*

**Abductive** Quasi-Rule Percept: (**Quasi-Object**, (Form of Object➞Quasi-Object)=>**Indeterminate Concept** *Sensation*IndeterminateConcept❙

Presenting: ❙ Subjective

Beautiful/Ugly 🡻 feeling

**Quasi- Object**

The conceptions of quasi-rule and quasi-object show that we do not cognize them under any principles or concepts, but by our habitual pre-rational cognitions, through instinctive and practical reflection, self-controlling the outcome of such operations, while the *reflective judgment* is only indeterminately rationally.

When we *reflect* (even animals reflect though only instinctively, i.e., in reference not to acquiring a concept, but to—say—determining an inclination), we need a principle just as much as we do when we determine, where the underlying concept of the object prescribes the rule to judgment and so takes the place of the principle. The principle by which we reflect on given objects of nature is this: that for all natural things, *concepts* can be found that are determined empirically. … For if we were not allowed to presuppose this, and did not base our treatment of empirical presentations on this principle, then all our reflections would be performed merely haphazardly and blindly, and hence without our having a basis for expecting that this [reflection] is in agreement with nature. (Kant, *CJ*: V, 211’-212’; cf. V, 211’-216’; Nesher, 2002b: V.5.)

Indeed, we must understand the difference between reflection in general, which is instinctive and practical, and the rational, self-conscious, and self-controlled operation of our cognition. Whereas the former is used in determining an inclination, the latter is used for determining intention, knowledge, and conduct. However, this is different from the reflection involved in aesthetic judgment, which controls only our subjective feelings of pleasure and beauty, without involving any knowledge of objects, as with the logical judgment, and yet it is not “performed merely haphazardly and blindly” (Kant, *CJ*: V, 211’-212’).The artist achieves the beauty of a created artwork when he is satisfied with the harmony that holds between his *intellectual ideas* and the exhibited *aesthetic ideas* in the art. But how can this be achieved and explained? Is it possible in Kant’s conception of the reflective judgment of subjective feeling without being proved true and public under its proof conditions or rather, it cannot be an aesthetic judgment?

[18]



**Reflective**

The problem is to reconstruct the Kantian aesthetic theory in order to show how the creation and evaluation of the beauty of artworks can be *objective* and *true*, rather than just a subjective experience of pleasure or displeasure of the *reflective judgment* (cf. Kant, *CJ*: 241; Nesher, 2021-22). Creating beautiful artworks is the *true interpretation* of the intellectual ideas, of the artists in their exhibition of aesthetic ideas, which, being true, *represent* experienced reality *aesthetically*. According to Kant, an artist’s *reflective judgment* of this creativity is based on instinctive and practical *self-control* of *free play* with ideas of the understanding and the productive imagination in order to attain rational control of its success.

Regarding reflective aesthetic judgments, these are based on the artist's imagination, which, according to Kant, affords a comparison between the aesthetic artwork and the known facts of reality. Such judgments render not only pictorial images but also intense reactions. However, such aesthetic presentation of reality is performed by the emotional images evoked through the contemplation of the aesthetic artwork, which is already infused with the artist’s intellectual ideas that can be elicited from the context of the specific work’s creation. Moreover, by reasoning and discussing, we can come to agree on its beauty as an aesthetic presentation of reality.

Now, for Kant there are two different kinds of judgments, cognitive *logical judgment* and *reflective* *aesthetic judgment*. However, the synthesis in the former case is based on the imaginative reflective self-control of the relation between the imagination and the understanding in presenting an object, while the synthesis in the latter case is based on the reflective self-control of the relation between the understanding and the imagination,but the reflective aesthetic judgment is just the subject’s feeling, without any aesthetic representation of any objects as the criterion of its truth.

However, in Peirce’s theory of perception we can see the prototype of the structure and operation of the human mind confronting reality. According to the particular combination of the basic components of the perceptual operation, one of Kant’s components of the mind predominates: imagination in aesthetic judgment, reasoning-volition in moral judgment, and rational understanding in theoretical judgment. These are the embryos of Kant’s three Critiques and of Peirce’s three normative sciences representing reality: aesthetically, emotionally, and rationally (e.g., Kant, *CPrR*: 33-34, 109-110; Peirce, *MS*: 283, *EP*II: #27; Nesher, 2004b, 2007a, 2008a, 2009, 2017a).

The Peircean alternative to the Kantian deadlock in understanding the aesthetic judgment of artworks is that by discussing the pragmaticist epistemology of creating and evaluating artworks we find that all historical theories of art and artistic and literary movements, with their manifestos, emphasized only some aspects of the artistic methodic operation. And yet art historians view them as the only essential elements of artistic creation and evaluation: either the aesthetic ideas or the intellectual ideas of the artists, either the contents or the forms of their intentions in creating artworks, either the feeling of harmony or the representation of reality, the feeling of the truth of the artwork, the sincerity of the author-artist, and so on (Faulkner and Ziegfeld, 1969:430ff.). The following is the Peircean reconstruction of the Kantian intuition of artistic creation and evaluation of artworks. There are threefold stages of the artistic creation and evaluation of artworks representing reality, based upon the accepted knowledge of our three normative sciences, theoretical, ethical, and aesthetical:



The artist, with his spirit and productive imaginative free play, interprets the generality of *intellectual ideas* into the singularity of *aesthetic ideas* and thus exhibits the intended artwork. By means of such a quasi-deductive inference, the artist exercises, à la Kant, the *reflective* *manner* (*modus aestheticus*) to achieve the harmonious interpretation between the ideas of the understanding and of the imagination and the unity of the aesthetic ideas of the created artwork. This is an elaboration of Kantian aesthetics but replaces Kant’s subjective conception of *harmony* with the Peircean realist confrontation with reality, which can then serve as the objective criterion of truth and beauty. Indeed, such reality is represented by the common-sense knowledge of reality,which is the historical accumulation of our perceptual and scientific knowledge that is available to the artists in their creation and evaluation of their artworks (Kant, *CJ*: 1781-87; Nesher, 1994, 2007a, Cha. 7).

The difficulty is to explain the principal role of art and the aim of the artist, whether it is to imitate nature, to decorate our life, to entertain us, or to represent reality so as to guide our conduct in it, to prompt us to be involved in moral activities and political movements that change reality according to the knowledge and the impetus we gain from the created artworks. However, it is interesting to explore how a piece of artwork can affect our cognitions to help and elaborate our social, moral, and intellectual conduct in reality. Indeed, this is like an interaction between persons, such that the artist, by expressing his intellectual ideas of reality in imaginatively creating the aesthetic ideas, the embodiment of the artwork, to be an aesthetic representation of our reality, allows us to contemplate and enjoy the created aesthetic images. Like Quixote or Karenina, they affect our own spiritual images that are beautiful precisely because they express our understanding of our own life. In other words, the ’purpose of art is to enable us to bring to mind the truth about ourselves, and so to become aware of who we truly are and how we are to behave in life. Art, therefore, is not just for art’s sake, but for knowledge and beauty’s sake, for the sake of a distinctively sensuous form of human self-expression and self-understanding.

**10. CONCLUSION: How Peirce’s Realist Semiotics as Eventual Epistemic Logic Solves the Difficulty that Kant Cannot solve with his Pure Transcendental Empty Concepts to Explain Knowledge of Ourselves and of External Reality**

**10. CONCLUSION: Kant Cannot Transcend the Pure Subject’s Empty Concepts to Explain the Knowledge of Ourselves and of External Reality while Peirce Solved This Difficulty with his Realist Semiotics as Eventual Epistemic Logic**

In the centuries following Kant’s meteoric appearance with his transcendental writings, his Copernican Revolution against Cartesian rationalism and Humean empiricism, which cannot solve the basic problems of epistemology and philosophy. We can understand Kant’s transcendentalism as a criticism of Descartes’ rational idealism, according to which the only thing he assumed to know was his thought, namely, “I think therefore I exist” (*cogito ergo sum*), and thus could not know external reality. This was the basic problem for Kant and most other philosophers who did not develop any theory of truth, as Kant admitted himself (Kant to C. Grave, September 21, 1798, AK 12:257; Russel, 1901).

Hence, in order to overcome the *subjective idealism* of Descartes, Kant suggested that with his *transcendental idealism* he can go beyond self-knowledge, that he can *transcend* subjective self-knowledge to reach the sensual intuition of objects and combine Humean empiricism with Cartesian idealism, yet without knowing *external reality*, i.e., the things in themselves, the *noumena* (Kant, *CPuR,* A367-369). In my inquiry on the relation between representation of ourselves and external reality, I suggested that one cannot prove knowledge of external reality without simultaneously proving knowledge of oneself. This is so because to prove the truth of the first one has to self-control one’s operations even at the level of sensation, emotion, and rational self-control. This is what Kant calls *apperception* and Spinoza calls *reflection*. Thus, like Siamese twins, we must prove the true representation of both of them together (Nesher, 2007b).

Since without perceptual experience, there is no meaning to the concepts of the transcendental subject, and since Kant is a nominalist (as Peirce argued), he cannot know himself in order to know his existence, as Descartes claimed of himself. Thus, he remains a blind fictional rational idealist who clearly cannot transcend his fictional entity even to reach his sensual experience, let alone noumenal reality. Thus, we can understand why even Descartes cannot know himself and why all *rational idealism*, including formal logic and pure mathematics, remain fictions as long as they do not involve sensual experience, and so, too, Humean blind sensual experience. Hence, the solution to this challenge comes from Spinozist realist empiricism and Peircean realist semiotics, as eventual epistemic logic. These can explain how from our sensual experience we abductively discover the perceptual concepts and interpret them deductively in our feelings and emotions and then synthesize them into the meaningful concepts of our perceptual judgments that can be inductively proved to be true representations simultaneously of ourselves and of external reality (Nesher, 2007b).

However, in order to overcome the Descartes’ *subjective idealism* Kant suggested that his *transcendental idealism* can go beyond the transcendental subject’s *pure empty concepts*, that is, he can *transcend* the subjective self to reach the sensual intuitions of objects and thus combine Humean *phenomenal empiricism* with Cartesian *subjective idealism*, yet without knowing *external reality*, i.e., the things in themselves, the *noumena*. Kant calls this knowledge *transcendental realism*(Kant, *CpuR,* A367-369). Indeed, the attempt to combine Cartesian rational idealism with Humean sensual empiricism remained unsolved by Kant due to the *gap* between them. As he admitted, his mysterious *schematism* cannot *transcend* the *empty concepts* of the *transcendental subject* to reach the *blind sensual objects* (Kant to C. Grave, September 21, 1798, AK 12:257; Nesher, 2000, 2022; comp. Hintikka, 1972).

Plausibly, many of the philosophers of the last few centuries were neo-Kantians, in that they accepted at least some components of the Kantian epistemology, like *phenomenalism*, l*ogical positivism, analytic philosophy, ordinary language philosophy, universal grammar*, *Russell* and *Wittgenstein’s philosophical solipsism.* and more*,* without any *theory of truth* as a method to represent reality. Alternative epistemologies can be found in Spinoza’s realist empiricism, with his theory of truth and his conception of freedom as relative to our knowledge of ourselves, nature, and other factors that may restrict our freedom (Nesher, 1994). Indeed, in his later realist pragmaticist work, Peirce developed his *semiotics* as an experience theory of truth, which I further elaborated into *epistemic logic*. He also showed how the epistemology and practicality of the *normative* sciences, *theoretical*, *ethical*, and *aesthetic* can overcome Kant’s deadlock in his three critiques, which aimed but failed to *transcend* the pure rational Cartesian self by appeal to the *pure understanding* of the transcendental subject. He further showed how it is possible to make them experiential judgments in order to represent oneself and phenomenal reality and thus transcend Cartesian solipsism and, moreover, how they can be used to adjust social and physical realities and improve our lives within them. Indeed, the evolution of Peircean pragmaticism in his later work differentiates him from American and other philosophers who call themselves Peircean pragmatists such as William James and John Dewey, though they actually remain neo-Kantians who do not follow the Peircean realist revolution, as I explain in my philosophical inquiries (e.g., Nesher, 2005a/b).

The question of Peirce’s mature epistemology is how Kant’s inability to *transcend* the *pure empty concepts* and connect them with the *sensible blind objects* brought Peirce to develop his *semiotics* as a solution to this challenge and as the seed for *epistemic logic,* which grounds our knowledge of reality in basic perceptual experience, with its sensual and conceptual meanings. The solution could be that he had to give up the nominalistic pure *a priori* *empty components* to overcome the deadlock within Kant’s idealist epistemology, yet Peirce did not overcome the axiomatic closed structures of *forma logic* and pure *mathematics*. This might be a revolution beyond his epistemological horizons, which progresses from the Euclidian to the Kantian and on to our time when we can consider *formal logic* and *pure mathematics* as special, absolute, kinds of knowledge based on intuitions and inference alone (Hintikka, 1997). Indeed, epistemologically, we can understand formal logic and pure mathematics as merely *closed games*, the truth of whose *axioms* and *conclusions* we cannot prove without having any proof of their truths with respect to their realities. Thus, though we find in them *contradictions* and *paradoxes*,they are considered *scholastic disciplines*, which we can nevertheless use, by correct intuition, as the skeletons of our empirical sciences (Russell, 1901, 1902, 1910, 1919; Gödel, 1951, 1953: Nesher, 2011, 2012, 2018, 2021). We must utilize *epistemic logic* to prove their truths or falsity and to show how both *epistemic logic* and *realist* *mathematics* can be empirical *normative sciences*. In this way we may complete the Peircean realist revolution against the Kantian Copernican Revolution (Nesher, 2012, 2018).

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