**Information propagated by longitudinal waves near phase transition**

Matan Mussel

Department of Physics, University of Haifa, Haifa, 3498838, Israel

Center for Biophysics and Quantitative Biology, University of Haifa, Haifa, 3498838, Israel

mmussel@sci.haifa.ac.il

**Abstract**

Longitudinal waves that reversibly cross a phase transition have a nonlinear behavior that resembles neuronal signaling, including a sigmoidal response to stimulation strength and annihilation upon collision. In the present work new theoretical results are reported, demonstrating that longitudinal waves carry more information than is typically considered in neuronal and artificial neural models: (1) the waves propagate both digital and analog data about the stimulus strength, (2) the type of stimulus can be identified by certain but not all observables of the wave, and (3) the collision site between two waves stores information about the stimuli and may be used as a fading memory. Our results unravel a rich encoding of information in a phenomenon that is both common in a plethora of materials and mimics neuronal signaling. Therefore, it may be a useful candidate for materialcomputation.

Modern computing is commonly identified with conventional von Neumann architectures, relying on symbolic Boolean concepts and sequential execution. A broader space of possibilities, however, is gradually unravelling by harnessing physical properties of different systems for information processing [1–3]. A nonexhaustive list includes elastic materials [4], fiber optics [5], excitable chemical media [6], macromolecules [7], and bio-macromolecules [8]. Biological systems have been particularly influential, motivating the exploration of unconventional computing schemes using biological signals. For example, understanding the properties of *action potentials* in excitable cells directly contributed to the successful development of *artificial neural network* models [9].

It was recently demonstrated that longitudinal waves propagating within lipid monolayers share many similar properties with action potentials. These similarities include a sigmoidal response to stimulation strength, annihilation upon collision, similar time and velocity scales (1–100 ms and 0.1–100 m/s, respectively), a resembling characteristic shape, and an electrical potential aspect with a similar voltage scale (1–100 mV)  [10–14]. The sigmoidal response is especially important because it provides a close analogy to the *all-or-nothing* nature of action potentials. The key ingredient is that the solitary waves generate a localized and reversible change of phase within the medium [10,11,15].

The close resemblance between action potentials and longitudinal waves implies that the latter may be exploited for unconventional computing schemes similar to biological and artificial neural activity. An important step in this direction, which is the goal of this letter, is to understand how information about the stimulus is stored in longitudinal waves that reversibly cross the phase transition. This is achieved by investigating a hydrodynamic model that includes the simplest form of phase transition, namely, the van der Waals (vdW) constitutive relation [16,17].

We previously demonstrated that a sigmoidal response of the density to stimulation emerges from the vdW fluid model and clarified the relation between the phase diagram and the nonlinear response [16]. The structure of the phase diagram qualitatively captures the order–disorder transition in lipid membranes and is also suitable for a plethora of other soft materials, including the liquid–vapor transition in fluids composed of a single species, such as water and nitrogen [18]; volume transition in polymer gels [19]; and metamaterials with an unstable inclusion [20]. Accordingly, exploring the information propagated by longitudinal waves that reversibly cross a vdW phase transition is relevant to a broad class of materials.

Unconventional wave-based computation has previously been explored in waves that either superimpose or annihilate upon collision, offering computational algorithms at different levels of sophistication [21–27]. In these works, however, the propagating waves are described using a single variable. For small amplitude longitudinal waves—a common situation in water and air—these computational schemes can be sufficient. The reason is that the oscillatory variations of the density, pressure, and temperature are linearly proportional to one another. However, at the nonlinear limit—as obtained, for instance, using a sufficiently strong stimulation amplitude—the variations of these observables are no longer proportional, and the information about the stimulus is cast into a high-dimensional space of observables [28]. This property makes an acoustic system extremely suitable for material-based *neuromorphic computing* [22], which can potentially lead to reduced energy consumption and enhanced computational performance [29,30]. In this letter we demonstrate how information about the stimulus strength and type are encoded in different aspects of the wave. In addition, we propose that collision sites between waves can be used as locations for short-term memory. Our results emphasize the richness of the phenomenon; consequently, it may be used for material-based computing that mimics and potentially even surpasses neural network dynamics.

*The vdW fluid model.* The compressible fluid model consists of three conservation laws (mass, momentum, and energy) and two constitutive relations. These equations couple the five variables of the system: specific volume, pressure, temperature, energy, and velocity fields—respectively, . For simplicity we focus on one spatial dimension of the medium, resulting in plane-wave solutions. Following the ansatz of Slemrod, the equations in the Lagrangian frame of reference are [17]

with the stress in the fluid, the dilatational viscosity, the capillarity coefficient, the spatial coordinate in the Lagrangian frame, defining the scale of vs. the laboratory coordinate in the Eulerian frame, the coefficient of thermal conductivity, the Boltzmann constant, the mass of the fluid particle, the average attraction between particles, the volume exclusion by the fluid particles, and the specific heat capacity.

 The system of equations (1) is numerically solved using a pseudo-spectral method. We focus on solitary waves stimulated by locally increasing either the pressure, temperature, or energy for a brief duration. A detailed description of the model and stimulation properties is described elsewhere [16]. The parameters and the numerical method are provided in the Supplementary Materials.

*Information about the stimulus strength.* Small-amplitude longitudinal waves cause a parallel incremental variation of the density, pressure, and temperature about their equilibrium values. By setting the equilibrium state close to the phase transition region in the low-density (disordered) phase, the response of the three observables is strongly modified by the phase diagram. At small amplitude of stimulation, , the response is almost linear, as shown in figure 1a, the inset of figure 1b, and figure 1c. Increasing the amplitude of stimulation to larger values, in figure 1, brings the system into the phase transition region. At this range of stimulation amplitude, a substantial increase in the amplitude of the density aspect is obtained because the system is softer in this region. At even stronger stimuli, , the system reaches the condensed phase. Because of the exclusion of volume, the amplitude of the density aspect saturates, while the pressure and temperature aspects do not.



Figure 1: amplitude of (a) density, (b) pressure, and (c) temperature aspects of a solitary wave as a function of local mechanical stimulation strength, as calculated at a distance from the stimulation point. Dashed line in (a) represents the maximum density allowed by the exclusion of volume. Parameter values are provided in the Supplementary Materials.

Accordingly, the response of the density aspect to stimulation strength near phase transition has a sigmoidal shape, while the response of the pressure and temperature, is nonsaturating and resembles a smooth rectified linear unit function (sReLU). Hence, solitary sound waves near phase transition carry both digital-like and analog information about the stimulus strength in different observables. For lipid membranes, additional observables that co-propagate with the wave can be considered, including the medium charge density, electric potential, and mobile particles that interact with the medium [28]. Although not shown here explicitly, we find that the concentration of charged ions near the membrane, pH, and electric potential difference also demonstrate sigmoidal responses to stimulation, while the total energy shows an sReLU response.

*Information about the stimulus type.* We previously demonstrated that using different types of stimulation generate similar responses of the density aspect of sound waves that reversibly cross the phase transition. This was demonstrated by replacing the pressure stimulus with temperature or energy stimuli [16]. Therefore, upon measuring the density aspect, it is challenging to deduce the source of stimulation, as shown in figure 2a for local pressure (solid black curve) and temperature stimuli (dotted-dashed yellow curve). In contrast, upon studying the pressure and temperature aspects, we recognize significantly different responses, as shown in figures 2b and 2c, respectively. The trajectory of the signal in the p-w plane of the vdW phase space is shown in figure 2d. Evidently the temperature stimulus generates a stronger response of the pressure and temperature observables as compared to the pressure stimulus. Thus, these aspects of the wave carry information about the type of stimulus, in addition to its amplitude.



Figure 2: comparison of the (a) density, (b) pressure, and (c) temperature aspect responses at a distance x/L=1 from the stimulation point, upon using either a pressure (solid black curve) or temperature (dotted-dashed yellow curve) stimulus. (d) Trajectory of the wave in the p-w projection of the vdW phase space. Initial state is marked with a black filled circle. Dark-blue and blue curves represent the co-existence and spinodal curves, respectively. Parameter values are provided in the Supplementary Materials.

*Collision site as short-term memory.* Collision sites do not exist in the linear regime, and two pulses penetrate one another. However, the transition from the linear to nonlinear response of the medium is accompanied by partial or complete annihilation, as shown in figure 3a. This behavior is in accord with measurements in lipid monolayers [12]. The annihilation events—as calculated from the vdW fluid model—are associated with a long-lived nonequilibrium state whose spatial size can be five times the width of the wave. The shape of the collision site, however, is sensitive to various properties of the colliding waves, including the distance of the stimuli from one another and their amplitudes. The effect of the latter is exemplified in figure 3b, showing how two waves stimulated at different amplitudes generate an asymmetric collision site.

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Figure 3: (a) Numerical solution of density field as a function of space (x-axis) and time (y-axis) for two pulses in the nonlinear regime. Blue denotes the disordered phase and red the ordered phase. (b) Collision site is asymmetric when pulses have different amplitudes. Parameters are similar to figure 1.

To exemplify how information about the two waves can be extracted from the collision site, we connect the peak amplitude at the collision site with the amplitude of the two stimuli. The maximum values of the density, pressure, and temperature fields at the collision site are calculated in a region enclosed by the two dashed lines shown in figure 3b. The results are plotted in figure 4 as a function of the amplitude of one stimulus while the other is held fixed at (solid black curve), 160 (dotted-dashed dark red curve), and 140 (dashed blue curve), respectively. The peak of density and pressure shows variability depending on the two amplitudes (figures 4a,b), while the peak of temperature is sensitive mainly to the larger amplitude (figure 4c). Thus, in certain situations it is sufficient to read the peak value of a single observable—either density or pressure—to pinpoint the amplitudes of the two stimuli. However, if the amplitude of one stimulus is small, the peak density is multivalued, and the pressure field shows little sensitivity to their amplitudes. In such situations reading the peak value from two observables can be sufficient.



Figure 4: The maximum value of the (a) density, (b) pressure, and (c) temperature aspects as calculated at the collision site (area enclosed by the two dashed lines in figure 3c) as a function of the stimulation amplitude. The second stimulus was held constant at (solid black curve), 160 (dotted-dashed dark red curve), and 140 (dashed blue curve), respectively. Parameters are similar to figure 1.

 *Discussion.* Many properties of longitudinal waves that reversibly cross a vdW-like phase transition are similar to action potentials [10–16,28]. Therefore, it may be possible to harness these waves for *in-materio* computing schemes based on principles of biological or artificial neural network algorithms. But in this letter, we demonstrate that longitudinal waves carry more information than is typically considered in neuronal and artificial neural models. The basic unit of artificial neural models applies an activation function—typically a sigmoidal or a ReLU function—to a weighted sum of the inputs [31]. The sigmoidal response to stimulation was inspired originally by the activation of action potentials in biological neural networks. However, longitudinal waves near phase transition propagate both a sigmoidal response of the density and electrical (in charged medium) aspects, and an sReLU response of the pressure, temperature, and energy aspects. Thus, these waves propagate digital-like and analog information about the stimulus in parallel (figure 1). Algorithms that could exploit this feature have not yet been considered.

It may be possible, for instance, to propose algorithms that under certain circumstances rely mainly on the digital signature while in other conditions rely on the analog data. In lipid membranes, the electric potential aspect becomes visible or hidden depending on the membrane charge density, which can be modified by changing the subphase acidity [28]. This provides a means to reveal or hide the digital aspect. A practical device for this implementation is a lipid monolayer, where it is straightforward to monitor pressure changes using a Wilhelmy plate and electrical changes using a standard surface potential sensor [13].

 The insensitivity of the density aspect to different types of stimulation, shown in figure 2a, stresses the importance of the multi-dimensionality of the signal. By reading the pressure or temperature aspects, we gain knowledge about the source of stimulation not visible from the density or electrical potential aspects. If the similarities between longitudinal waves that cross a phase transition and action potentials originate from mutual physical principles, this observation may have profound implications regarding the information propagated by action potentials. It is well-known that nonelectrical aspects co-propagate with the action potential [32–34]. But it is unknown whether these nonelectrical changes propagate analog information and whether neighboring cells can perceive it—for instance, through changes in surface tension [35]. This may open an entirely new dimension of action potential encoding.

Finally, we propose that long-lasting collision sites could be used as short-term memory indicators [36]. This property can be combined, for instance, with collision-based computing algorithms [25]. Additional information encoded by the collision site—which was not described in detail in this work—includes the distance between stimulation points and the duration of the two stimuli. Another method that can be considered to retrieve information from the fading-memory region is using a third wave that interacts with the collision site.

In conclusion, nonlinear longitudinal waves that cross a phase transition represent a subclass of material response that exists, in principle, in any material under certain conditions. These waves show a rich dynamic behavior that resembles neural activity and casts the information about the stimulus into a high-dimensional space. This could be a useful candidate in exploring different approaches for material computation.

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