**Incorporating Critical Thinking Skills into the Mathematics Classroom in a Unit on Probability and Statistics in Daily Life**

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**Abstract**

The acquisition of critical thinking skills has been acknowledged among education researchers as an important educational goal. This paper presents a teaching experiment in which a learning unit for 10th graders combined explicit teaching of critical thinking with the content of an existing mathematics unit called "Probability and Statistics in Daily Life". The original unit was designed to teach probability and statistics through real-life scenarios at the high-school level. To create the new learning unit described here, the original mathematical content was infused with a progressive sequence of critical thinking skills, so that both the mathematics and the critical thinking competencies developed hierarchically, growing more and more complex as the unit progressed. This paper uses a series of examples from the classroom implementation of the resulting unit (3 out of 15 lessons) to illustrate how the topics were combined and to show the mutual benefits to the teaching of both statistics and critical thinking. The paper then discusses the problem of critical thinking skills transfer and presents some promising elements in that direction that derive from the analysis. Finally, it discusses some educational implications of the effort, the limitations of the unit's first run and possible directions for future research.

**Keywords**

**Introduction**

The term "critical thinking" (CT) has been used in academic circles for less than a century, but evidence of its relevance to education is far older – spanning several forms of civilization and 2500 years of human history, beginning with the Greeks. The universal applicability of these "ancient" skills is more relevant than ever in today's complex, ever-changing reality, which requires independent decision-making on a daily basis. Fostering and developing the ability of students to think critically and to be capable of engaging in inquiry and evaluation based on the rational consideration of the various messages they are exposed to in different areas of life, is, therefore, an important part of their education (Bryan,1987; Glaser; 1972; Perkins, 1992; Regev, 1997; Swartz, 1992). While this need to focus on the promotion of CT skills has long been widely recognized by educators (Ku, 2009; Willingham, 2008), no consensus has been reached on *how is best accomplished*. One question in this on-going debate is whether critical thinking should be taught as a subject in its own right or integrated into a subject already present in the school curriculum. Furthermore, if critical thinking *is* integrated, with which subjects? And when teaching CT skills in conjunction with another topic, should these skills be taught *implicitly*, as a hidden component "immersed" in the primary material? Or should they be an *explicit* part of the learning experience, whereby the students' attention is drawn to their presence as an additional component "infused" into the instructional material?

Those who believe that critical thinking should be taught in conjunction with other areas have suggested using mathematics for this purpose. Mathematics has traditionally been considered by educators to be a branch of knowledge particularly suited to teaching and learning higher-order thinking skills. Mathematics curricula all over the world, including in Israel, identify the acquisition of these skills as one of their goals. The description of mathematics as a discipline suited to teaching critical thinking also appears in the research literature (Elder and Paul, 1994; Paul, 1999; Paul & Elder, 2001; Paul, Elder & Bartell, 1997). Despite this assumption, however, very few empirical studies to date have engaged in the question of whether the study of mathematics at school indeed develops or even requires this mode of thinking, and the answer to this question is far from clear.

The study presented in this paper attempts an approach to teaching critical thinking through mathematics that *explicitly* integrates the topic into a learning unit designed to teach high-school level probability and statistics. This topic is particularly well-suited for the acquisition and practical application of critical thinking skills. The model used was based on the combination of two theoretical elements: the hierarchical model of critical thinking skills presented in Ennis' taxonomy (1985, 1987a, 1987b, 1989); and the "infusion approach" to teaching posited by Swartz (1992). To create the new learning unit whose classroom implementation is presented in this paper, the mathematical content of an existing learning unit entitled "Probability and Statistics in Daily Life" (Lieberman & Tversky, 2001) was (in collaboration with one of the unit's co-creators) "infused" with a hierarchical progression of critical thinking skills according to Ennis' taxonomy (Ennis, 1989). This paper presents, in order of increasing complexity, a series of three examples from the classroom implementation of the unit. These examples illustrate how the two topics develop hierarchically together, and also illustrate how each lesson combines them anew – calling on the students to draw on both their mathematical and their critical thinking skills to solve problems based on daily life.

**Theoretical Background**

*Critical Thinking and Ennis' Taxonomy*

Modern scholarly literature has provided a range of definitions in an attempt to characterize what "critical thinking" is (Dewey, 1933; Ennis, 1987a, 2002; Glaser, 1941; Lipman, 1991; McPeck,1981; Norris,1985; Kennedy, Fisher, & Ennis, 1991; Siegel, 1988; Willingham, 2008). To cite just a few examples, McPeck (1981,1990,1994) defines it as the "skills and dispositions" necessary to "appropriately use reflective skepticism," while Lipman (1991) offers a more extensive description as "thinking that enables judgment, is based on criteria, corrects itself, and is context-sensitive." Willingham (p. 21) goes into yet more detail, defining critical thinking "in layman's terms" as "seeing both sides of an issue, being open to new evidence that disconfirms your ideas, reasoning dispassionately, demanding that claims be backed by evidence, deducing and inferring conclusions from available facts, solving problems, and so forth." This paper draws upon the definition suggested by Ennis (1962, 1963, 1985, 2002), by which critical thinking is "reasonable, reflective thinking that is focused on deciding what to believe and do" (Ennis, 1985, p. 2) and on the hierarchical model of CT skills that goes along with it. Ennis' approach to critical thinking was deemed most appropriate for purposes of this study, for several reasons. First, unlike the other two definitions cited above, Ennis' definition explicitly marks critical thinking as an asset that influences how the person who employs it interacts*with the outside world*. This makes it particularly suited to the learning unit "Probability and Statistics in Daily Life," which also emphasizes the relevance of the cognitive mathematical content it teaches to understanding and making decisions about real-life situations. Second, Ennis (1996a, 1996b, 2002) distinguishes between two elements involved in critical thinking, which he calls "abilities" and "dispositions"; "abilities" refer to the cognitive aspects of critical thinking, knowing *how* to think critically and *what* critical questions to pose (Ennis' list of abilities includes identifying the focus, defining terms, identifying unstated assumptions, judging source credibility), while "dispositions" refer to the socio-cultural aspect – to acquiring a "critical spirit," a notion of *when* or *whether* something ought to be engaged with critically. Dispositions include caring that beliefs be true and that decisions be justified, and taking care to present a position accurately and clearly. Finally, Ennis positions the 28 skills he associates with critical thinking in a hierarchical structure (Ennis,1987b). Nine of these were selected to frame this study and were correlated in their hierarchical progression with that of the mathematical content in the "Probability and Statistics in Daily Life" unit.

*The teaching method: the infusion approach*

Studies in the teaching of critical thinking have shown that that students do not engage in CT naturally or develop it spontaneously (Kuhn, 1999; Van Gelder, 2004). Critical behaviors such as asking the right questions are *learned* behaviors, and the processes and judgments that critical thinking involves require a considerable – and deliberate – mental effort (Browne & Keeley, 2004; Onosko,1992; Tamir, 1993). Innabi and Sheikh (2006) reiterate this point, claiming that students will not think critically unless they are taught to do so, but their position also emphasizes the importance of the *teacher*'s CT skills, because teacher knowledge and understanding of critical thinking will inﬂuence how it is transmitted to their students (Innabi & Sheikh, 2006). Given these general premises on the necessity of teaching CT skills and on the basic requirements expected of teachers who wish to teach them, the most important issue that arises is how CT should be taught. As stated, education researchers agree that teaching critical thinking is important, but opinions vary as to the best method of doing so. A major question emerging from this debate is whether critical thinking and other higher-order thinking skills are general (and thus can and should be taught as an independent course of study) or content-dependent (and therefore must be taught in conjunction with another subject) (Perkins & Salomon 1989 ; Kuhn & Dean, 2004).

**Methodology** *The "Probability and Statistics in Daily Life" learning unit as a basis for teaching CT*

"Probability and Statistics in Daily Life" is a learning unit developed by Lieberman and Tversky (2001) and which has been expanded and modified for the purposes of this study. This unit was selected as a foundation for the teaching experiment reported in this paper because it was designed to teach mathematical content using problems and stories from daily life, and because its rationale already alluded to "elements of critical thinking", citing these among the "issues relevant to daily life" that it hoped to teach (Lieberman & Tversky 1996,2001; Introduction p.3).

The original unit was based on Tversky and Kahneman’s well-known work on decision-making under conditions of uncertainty (Tversky & Kahneman, 1974; Kahneman, Slovic & Tversky, 1982). It covers topics in probability and statistics in hierarchical order, connecting each to daily-life scenarios and decisions. The purpose of the original unit was to turn students into "intelligent" consumers of information by introducing them to modes of thinking beyond the mechanics of mathematical calculation. The use of problems from daily life exposed the students to a variety of additional fields such as medicine, economics and law, illustrating the practical applications of probability and statistics to these fields and showing the students how statistical considerations are inextricably woven into their lives. Furthermore, the unit required them to analyze problems, raise questions and think critically about the numerical data and the information placed before them. By encountering problems that did not necessarily have a single, correct, clear-cut answer, students learned not to be satisfied with arriving at numerical solutions, but to assess the validity of data, and to address the problems before them in a qualitative – and not just a quantitative – manner.

The revised unit was composed of three chapters:

Chapter 1: Concepts in Statistics and Probability – Statistical ConnectionThis chapter is an introduction to the basics of probability and statistics. It opens with a historical account of probability as rooted in gambling theory, including stories about Girolamo Cardano and Pierre de Fermat. The historical context leads into the classical and empirical approach to probability with the law of large numbers and relative frequency. The chapter also includes basic statistical tools such as distribution measurement and measures of central tendency. Finally, students are taught how to organize data into a two dimensional matrix, and how to determine the presence of a statistical connection among the data.

Chapter 2: Statistical connection to causal connectionThis chapter develops from the first by introducing the relationship between statistical connections and causal connections. It emphasizes the importance of understanding the nature of the statistical connection between two variables, and also focuses on the importance of knowing whether a connection is only statistical, or causal as well. After establishing that statistical connections between variables must be found before causal connections can be understood, this chapter asks, "How do we test causal connections?" In this context, the chapter includes a discussion of the difference between controlled, uncontrolled and observational experiments, including explanations of methods that may be employed to establish the validity of causal connections in controlled experiments (for example, the way drug trials employ tools such as the placebo effect, and blind and double blind testing to ascertain connections between cause and effect).

Chapter 3: Making decisions based on the heuristics of availability, anchoring and representativenessAfter the students have been thoroughly versed in the "objective" mathematical approach to statistics and probability, this chapter introduces them to the subjective approach posited by Tversky and Kahneman (1974), and examines the differences between the two. As part of this third section, the students are introduced to the three heuristic principles – availability, anchoring and representativeness – proposed by Tversky and Kahneman to account for subjective judgments made under conditions of uncertainty. The students are then asked to apply these to a series of problems and indicate the heuristic they used to make their decision in each case. The students' experience in applying critical thinking emerges here, because they now have two different methods for arriving at conclusions – the numerical method they had learned earlier, and a subjective one, requiring consideration of the validity of data – and are required to intelligently choose between them.

Like the original "Probability and Statistics" unit, our revised version was also designed to provide students with the statistical tools and probabilistic models required to assess statistics and probability in their day-to-day lives. Like the original unit, the revised unit was structured around the analysis of cases from daily life and sought to draw teacher and students into discussion, investigation and analysis. Each lesson in the new unit, however, carried the *added* component of an *explicitly taught* set of critical thinking skills, based on the hierarchy set out in Ennis' taxonomy. The concepts in critical thinking were introduced gradually as the unit progressed, with each new concept being added to the old, so that older skills were continually reused and reinforced even as new ones were acquired (see Table 1). For some of the lessons, questions that explicitly draw upon critical thinking skills were simply added to the cases and questions of the original unit. In others, the *idea* – the mathematical purpose – from the original case was extracted, while the story or article suggested there was replaced by one that rested on the same mathematical principles, but was more relevant and conducive to CT acquisition.

A typical lesson was thus structured as follows: (1) the students were presented with a text describing a claim or a scenario from daily life and a problem was posed about it; (2) the students discussed the problem as a group and tried to solve it intuitively; (3) the class worked together on relevant mathematical content (e.g. Bayes' Theorem, statistical connection), after which the original problem was revisited by the students, with no clear solution provided by the teacher; and (4) elements of critical thinking were explicitly introduced and applied to the problem. This four-step method was expected to generate critical thinking through the encounter with conflicting information that arises from the students' original intuitive answers, the revised answers suggested by later mathematical calculations, and the questions regarding the validity of these answers once critical thinking was applied (Aizikovitsh-Udi, 2012, Aizikovitsh-Udi & Amit, 2008).

*Setting, population and data collection*

The results presented are from a single subgroup of one class taken from a larger population of six classes [147 students in all ­– three experiment groups (70) and three control groups (77)]. The larger population was used for deriving quantitative results regarding the experiment's efficacy; these are not presented here. The experiment consisted of 15 sessions – 90 minutes each – in the course of the academic year, which served as the probability and statistics section of the mathematics curriculum for that year. The group whose sessions are described here was taught by me in my capacity as the mathematics teacher of these students.

Data was collected from three sources – written products, classroom sessions, and personal interviews. Written products included exams, in-class papers and homework, which were all collected. Classroom sessions were recorded, transcribed and analyzed (with special attention to CT skills), with the teacher maintaining a log on every session. (iii) Personal interviews were conducted with 27 randomly chosen students (four from each of the seven experimental classes) at the end of the first and second semesters; that is, in the middle and at the end of the unit. The purpose of these interviews was to bring to light the changes taking place in student attitudes toward critical thinking in the course of the academic year. Some of the interviews were closed/structured while others were open/semi-structured. All data was processed by way of triangulation from the three sources using qualitative methods, enabling the researcher to follow the thinking patterns of the students.

The interview questions were: (a) What do you think about the importance of critical thinking ability? In which fields/activities is it important? (b) Can you give an example of a situation (from school, everyday life, etc.) where critical thinking is necessary? Have you used it? Did it help you? (c) In your opinion, is it possible to develop/improve critical thinking ability? How? Do you have any suggestions for improving it? (d) In your opinion, is it possible to change dispositions for critical thinking? How? What influences the dispositions? Do you have any suggestions for improving them? (e) Did your studies in the other disciplines improve your ability for critical thinking? If yes, in which courses and in what way? If not at all, why not? and (f) How would you evaluate yourself in the area of critical thinking?

The six interview questions were intended to (1) ascertain the degree of the student’s awareness of the nature of critical thinking (primarily questions a, b, e, f, through the pertinence of the examples and the answers), (2) identify their ideas about possible fields of application for critical thinking (a, b), and in particular to identify suggested fields far from those proposed during the course, as an indicator of internalization and a basis for possible transfer to other contexts, and (3) obtain feedback on their perception of the course itself – what aspect or what particular moment had an impact on their CT skills and dispositions (c, d), in order to assess student awareness of the aims of the course.

**Results**

In what follows, three lessons are presented, taken from three progressive points within the learning unit. For each, the daily-life topic upon which the lesson was based is presented, then the integration of the mathematical content is shown, followed by the integration of the critical thinking skills. In addition to demonstrating how the two topics were tied together, the sequence of lessons shows how both mathematical content and critical thinking skills were constructed hierarchically as the unit progressed; each step built upon the previous one, and students were involved in meaningful activities relating to the aims of the learning unit (development of both CT and mathematical competencies).Each sample also highlights a different element of the program. The first lesson, in which the students are sent outside the classroom to gather data for themselves, highlights the centrality of the practical "daily-life" element in the study unit. As this example shows, each daily-life story or problem places the students in a position where they must draw on both the mathematics and the critical thinking skills to gather the information they need to make their decision. The second lesson is the most detailed of the three examples, and it shows (through a long excerpt and the related analysis), how the working hypothesis of melding the topics was implemented (thus showing its feasibility). It also shows how by that point in the learning unit students had already internalized specific features of CT, and how specific CT expressions entered student language and were appropriately used by them (see Discussion). The third example stresses the four-step format of each lesson, emphasizing the function served by each; it shows how the daily life scenario triggered an intuitive response, which was then enlightened and modified by the mathematics, before being revisited and adjusted a second time by further application of CT. This demonstrates the unit's lessons at their final and most advanced stage, where the students already see that CT can be used not only to support conclusions based in mathematics, but also to look beyond them.

Example: Bayes’ Theorem – The Aspirin CaseThis final example is taken from the third chapter of the learning unit, which factors in the topic of subjective probability. The students were given the following problem:

*Your brother woke up in the middle of the night, crying and complaining he has a stomachache. Your parents are not at home and you don’t know what to do. You give your brother aspirin, but an hour later he wakes up again, suffering from bad nausea and vomiting. The doctor that regularly takes care of your brother is out of town and you consider whether to take your brother to the hospital, which is far from your home. You read from a book about children’s diseases and find out that there are children who suffer from a deficiency in a certain type of enzyme and as a result, 80% of them develop a bad reaction to aspirin, which could lead to paralysis or even death. Thus, giving aspirin to these children is forbidden. On the other hand, the general percentage of cases in which some bad reactions occur after taking aspirin is 10%. We know that 0.5% of children lack this enzyme.* (*Probability Thinking*, p. 30, with slight revisions made by the researchers)

*Should you take your brother to the emergency room? What should you do? Can aspirin consumption be lethal?*

This lesson, toward the end of the learning unit, contains all of the mathematical elements covered by the unit and nearly all the critical thinking skills. It requires a high level of variable identification, and continues the student work on conditional probability through Bayes' theorem. In terms of critical thinking, the students have already acquired a willingness to investigate questions beyond the basic requirements of the problem. Though this lesson does not directly address any of the heuristics posited by Tversky and Kahneman (1974), it nevertheless belongs in the subjective section of the unit because it offers the students an opportunity to make a decision based on intuition – and, having understood what the mathematics in the question entail, to *overrule* or *look beyond them*. The lesson was divided into four parts: (a) First impressions: The students received the text and were asked to decide – yes or no? (b) Group discussion: A class discussion was held to consolidate all of the "intuitive" answers of the students, which they had based on their impression of the numerical data provided in the text, but had not verified through any "real" mathematical calculations. Nearly all of the students chose to take their brother to the hospital. (c) Applying the math: The problem was "solved" mathematically – isolating the variables, defining the conditional probability, figuring out what needed to be found (i.e. are we looking for *P(A/B)* or *P(B/A)*?) by means of Bayes' theorem, and building two two-dimensional matrixes. The conclusion in the classroom was that there was only a 1% chance that the brother would have the enzyme. (d) Thinking critically:In light of the mathematical findings, the class *suspended its initial judgment* (i.e. the decision to take the brother to the hospital) and resumed discussion of the question. The mathematical calculations proved that some of the initial *assumptions* of the students had been incorrect (they had assumed that the chances would be higher, they had assumed that they could not understand the variables fully and had better "play it safe"). Interestingly, even after the calculations had shown the chances of danger to be extremely small, most of the students *still* decided to go to the hospital. As a result, the attempt to resolve the conflict between initial intuitions and mathematical data led to a discussion on *alternatives* – not to the original answer, but to the means of *supporting* that answer. Instead of basing their answer on false assumptions and ignorance, students were able to question the absolute finality and power of the mathematical result, and to consider whether mathematical solutions were always a *sufficient* basis for making daily-life decisions. As one student said, "if it's my brother, then that 1% may as well be 100%," suggesting that "hard" mathematical facts do not always eliminate the influence of other pertinent, deciding factors from consideration.

**Discussion & Conclusions**

As the examples above show, the mathematical content of statistics and probability is well suited to being "infused" with instruction of critical thinking skills. The hierarchical patterns in which both the content and the skills can be taught complement one another structurally, with more elements of critical thinking coming into play as the mathematics becomes progressively more complex. The two instructional goals (statistics and critical thinking) are also mutually beneficial in terms of content; the mathematics provides the students with a means to engage in critical thought and pursue independent challenges and confirmation of the available information. The “Probability in Daily Life” unit provides the mathematics lesson with a practical "real world" context that can help students comprehend the material more concretely than simply with abstract manipulation of numbers on a page. The various critical thinking skills could then be taught explicitly in the context of using mathematics to solve problems in real-world situations. The addition of critical thinking provides an added reflective dimension to the mathematical treatment rendering both the approach and the results more meaningful to students.

*Educational implications*

One of the benefits of the curricular adjustment presented here is that it did not require a complete overhaul of an entire school curriculum, but was implemented at the discretion of the individual teacher; however, as Innabi and Sheikh (2006) stress, teachers who implement such changes must *themselves* be able to understand and use the tools they impart to their students. Teachers who wish to incorporate critical thinking into the regular content of their curricula face a dual challenge: they themselves must not only be comfortable with CT and its relation to the content that they teach, but they must also have an idea of how to pass this knowledge and understanding on to their students.

Van Gelder (2004) offers a series of "lessons" for teachers to keep in mind when teaching critical thinking. One of these is "practice makes perfect" – critical thinking is a set of skills, and skills must be repeated over and over if they are to be mastered. In this context, the hierarchical format of the program presented here is one of its strengths, because the knowledge and skills learned in earlier lessons are continually drawn upon, reused and rehearsed in the later ones. This program also reflects Van Gelder's recommendation that the practice of CT be done *explicitly*, with teachers addressing it openly as a topic in its own right rather than expecting students to come by it independently based solely on the insertion of an implicit "critical emphasis" into their regular school content. Willingham (2008) also notes the importance of practice to attaining "the ability to think critically," but he stresses too that this ability also depends on a familiarity with the domain knowledge about which one is thinking. It is therefore important that this program, although it teaches CT explicitly and discretely from the mathematical content, teaches them *together*, providing the students with a background of contextual domain knowledge within which to practice their critical skills.

*Critical thinking via probability and statistics – the question of transfer*

Transfer – i.e. the ability to take information acquired in one context and apply it to another – is one of the main goals of teaching higher-order thinking skills like critical thinking, yet is difficult to put into practice both within and across disciplines (Bransford et al., 2000, El-Sheikh, 2001). Transfer remains one of the greatest challenges to teaching and learning critical thinking, and though it occurs to some extent, it does not do so nearly as often as educators would like (Van Gelder, Bissett & Cumming, 2004; Zohar, 1996). Swartz divides transfer into several different levels, including "transfer *within* a restricted field of study to new examples within the field, transfer *across* disciplinary boundaries, and transfer *into* the thinking practices in which we engage in our everyday reasoning" (Swartz, 1992). To these categories he also adds an injunction to help students "develop a 'spirit of critical thinking'," by which he means "good thinking attitudes and dispositions that prompt us to use these skills" (p. 37). The issue of transfer is pertinent to the assessment of an infusion approach to teaching critical thinking. When faced with a program that teaches students to use critical thinking skills when addressing problems of statistics and probability, one might ask: "but will they be able to use these skills anywhere else?" Because this study focused on the efficacy of combining CT with statistics and probability (i.e., first and foremost with whether the skills added were being learned at all), it did not perform a specific check for signs that students were transferring what they had learned to contexts external to the mathematics classroom. However, the study *did* yield data upon which to base some tentative suppositions about the manner and extent of the transfer that occurred, and from which some suggestions for future changes and additions to the program may be derived.

A language of critical thinking

One unexpected element that arose from the data and may be seen as a step in the direction of transfer is the acquisition by the students of a *CT vocabulary*. Language is a central component of mathematics and of mathematical education; it both supports the thinking processes and is a medium for teaching mathematics and shaping knowledge. Since the learner's thinking development is determined by language, and since thinking to a large extent consists of language, it is impossible to separate language from learning (Vygotsky,1962; Sfard & Linchevsky,1984; Koenig & Harris, 2005; Zohar, 2000; Zoller,1999). The main use of language in mathematics is in the definition of principles and terms, the expression of mathematical ideas in the form of formulas and speech, and the solving of mathematical problems, in particular geometry and verbal problems (Onosko,1990; Talaska,1992; Tishmann, 2000; Zhang, 2002). As the teacher of this course, however, I brought a new set of ideas into the classroom and introduced these ideas explicitly to the students using a new and specific terminology. The new skills learned by the students in the course were closely associated with the recurring words and phrases that went along with them, particularly because naming the skills we were using required language of a higher register than the students typically used. This connection is evident in the words of one student, who, when asked if the course had improved his critical thinking skills, answered that, "They did improve in a certain sense, but not in things that found practical expression," because "it’s awfully hard to speak such a high language." As we have seen in the excerpt presented and analyzed in Example 2, the adoption of the "high language" associated with the practice of critical thinking is evident in the following student comments during the lessons:

*"First, we should check the information source’s reliability," "The conclusion is not valid because we don’t have all the data," "Despite all the numerical data, I don’t accept the researcher’s conclusion," "We may have found a statistical connection, but we didn’t find a causal connection between the factors, so we can’t determine the direction of the connection."*

The language used by the students in class also shows that they had gained an understanding of the kinds of questions upon which critical thinking relies. Our in-class discussions were punctuated by student remarks such as:

*"Where is the article taken from? Can we see the article for ourselves," "Is the article's source reliable? How can we check it?" "I can’t accept this numerical data," "Who did this research/ survey?" "We need more information."*

The development of the students’ critical thinking language is also demonstrated by comparing the student answer sheets from before and after the learning unit. The basic vocabulary and methods of mathematical thinking were present before the new learning process; the students were able to use the regular methods of approaching a probability problem using a two-dimensional table and Bayes' formula for conditional probability. Similar answer sheets completed by students after undergoing the learning unit demonstrate an extensive addition of verbal explanations to the two-dimensional table, evidence of a new richness in their language and of the added value of critical thinking in informing their work in probability and statistics. Even without analyzing the actual content of what the students wrote in the post-learning-unit answer sheet, the sheer difference in the ratio of words to calculations shows that an additional level of *verbal* thought has been added to the simple calculation of solving a problem.

Further evidence of transfer (or lack thereof) can be gleaned from student interviews. Many of the students claimed that the course had taught them a more critical approach to life that extended beyond the strict confines of statistics and probability. Their adoption of critical thinking was often expressed as having become more "aware," more "cautious" and more "suspicious." One student described having critical thinking as being more "aware of information sources that explain things to me; for example, if someone gives me a newspaper, I will be somewhat skeptical." The reference to awareness is echoed by another student, who said that after the course, "I put things into question more. I pay attention to things. I’m aware of them." The student also added, "It helps me a lot to know whether the one who conducted the research is someone with power (prepared to pay for a certain experiment, apparently, there is something good in the experiment or something bad, one has to check)." A third student defined critical thinking as knowing, "Not to trust everything, to check before one decides. Not to believe every survey. To think about everything," and added, "In the past I didn’t think about studies and surveys and articles in newspapers, in the past I used to believe everything, now I check everything." While these quotes give little indication of how *successfully* these students will apply their newfound critical tools, they do suggest that they now "recognize the importance of good thinking and have the initiative to seek better judgment" – a dispositional aspect of critical thinking that *combines* with cognitive aspects to determine what Ku calls "a person's actual thinking performance (Case, 2005; Ku, 2009, P.71).

While the confident use of words such as "always" and "everything" by the students in reference to their newfound critical faculties suggests that they believe in their ability to generalize their new knowledge and apply it to other contexts, their interviews indicated several limitations in their ability to do so. Most significantly, nearly all students had difficulty citing examples of critical thinking other than those we specifically discussed in class. Our lessons were based largely upon newspaper articles that cite surveys and scientific research and/or endorse products. Accordingly, many of the students said in their interviews that critical thinking is useful "in research and shopping" and in reading the newspaper. One student said, "There is research carried out all the time, and every company wants to show that it is the ideal one and the best, and they do this by influencing the public." Another gave mathematics itself as an example, saying that "in mathematics, when you have all kinds of problems to solve and they teach it to you one way, it is possible to look for another way and not to say right away that if you were taught one way, that’s all there is." In a third example generalized from a specific case discussed in class, this student claimed that critical thinking is important:

*Because if, for example, there is a survey that was done on a certain group and does not apply at all to the total population, we need critique in order to think about such things. An example for such a case is a survey that was commissioned on products of a certain company by the company itself and is conducted on a certain target group, while the identity of the specific group was not published in order to present the products in a better light.*

Another potential obstacle to transfer that arose from the interviews is that some of the students – although the CT skills were taught explicitly – had difficulty separating the CT skills they had learned in the course from the mathematical content. One student claimed that critical thinking should be taught together with mathematics "because it’s related because of the percentages and numbers." Another claimed that, "A student who knows mathematics has critical thinking, because he knows how to think, [while] a student who doesn’t know mathematics can’t think critically." A third student acknowledged that mathematics are not strictly necessary "for the critical thinking itself" and that "one can refute a claim even without mathematical arguments," but when asked to name concepts in critical thinking, he answered, "Statistical connections, identifying variables, defining variables, sets, causal connection, conditional probability" indicating a certain level of confusion of which terms belong to which topic.

Separation of terminology may help clarify the distinction between the skills in the two domains. Such a distinction, as well as overcoming both limitations noted above, might be preferable if students were introduced to applications of critical thinking in *other* contexts as well. This could be done on a smaller scale by including examples from other disciplines (history, for example) within the "Probability and Statistics in Daily Life" unit itself. Alternatively, it could be achieved by "infusing" critical thinking skills into the curricula of other academic disciplines, and thus creating a more "comprehensive critical thinking program" for the students (Resnick, 1987; Swartz, 1992).

One noteworthy element reported by several students to have influenced their ability to transfer critical thinking outside the classroom was an increase in confidence. One student reported that the unit had "increased [her] confidence" in her own doubts – that before the unit she "didn't say it," but that now "I can say that I am not sure, that I can examine other possibilities." Interestingly, this rise in confidence was expressed several times as a willingness to confront the authority of the teacher. One student thought that critical thinking was applicable to any subject "because in any subject there is a teacher who explains, and with any explanation it is possible to ask questions about the topic that I don’t understand or try to correct the teacher when she is making a mistake." This student also thought that his own critical thinking was not very good, "because I am a shy boy and usually don’t ask questions when I don’t understand something." In contrast, another student, who also thought "the ability for critical thinking is really important" because it can be used "to criticize the teacher," claimed that though this was something he previously "never could do in my life," he now thought that "there is a chance it will happen." The subject of confidence was also raised by a third student, who said that he uses the new concepts he had learned (like "I put something into question") only "with people who are close to me," and that "my criticality depends of how comfortable I feel around people that I am with."

**Concluding Remarks**

This paper has reported an investigation into the use of the infusion approach to teach critical thinking skills while also teaching conventional probability and statistics content. The students acquired critical thinking skills that they came to value. These skills were so entangled with the mathematical skills being taught that many students came to see critical thinking as an aspect of mathematical thinking. This "confusion" on the part of some students highlights the profound theoretical question on the relationship between critical thinking and mathematical thinking (statistical thinking, in particular). One strategy for disentangling the two modes of thought may be to focus on the typical expressions of probability, statistics and critical thinking. The development of an identifiable vocabulary of critical thinking has been documented here and is one of the most interesting outcomes of this study because it is a crucial premise for transfer. Despite this, the use of learned skills (whether statistical or critical thinking) in contexts other than those in which the skills were acquired remains uncertain and inconsistent. Nonetheless, analysis of implementation of the infusion approach uncovered elements in simultaneous teaching of statistical and CT skills (like the acquisition of typical expressions) that encourage optimism with respect to transfer, as the explicit teaching of critical thinking becomes more pervasive in mathematics and in other subjects.

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