**INTRODUCTION: On the origins and essays of the following chapters**

Early in my life as a member of a kibbutz in Israel, I became interested in philosophy and found myself particularly attracted to the philosophy of Spinoza. I focused on his philosophy during my studies at Brandeis University where I wrote my thesis: *An Analysis of Spinoza’s Political Philosophy* (1972). At Brandeis, I studied under Karl Popper, and when he invited me to his office to discuss philosophy, I was surprised to discover that he situated himself in the neo-Kantian tradition. He gave me a copy of his work *Of Clouds and Clocks* (1965). At that time, I used to meet regularly with the Spinozist Harry Wolfson at Harvard University to discuss Spinoza’s philosophy, and I became acquainted with Peirce’s philosophy too. From then on, I studied him intensively. During a two-year period I consulted Peirce’s late manuscripts at Harvard’s Houghton Library. I subsequently learned that he had studied Kant very seriously and that in his later life he developed his epistemological realism and revolutionized Kant’s Copernican Revolution. Later, at my academic home the University of Haifa, , I worked on Spinoza’s philosophy and pragmaticist epistemology to study Kant’s reach in philosophy; I was interested in how various philosophers have attempted to overcome his transcendentalism, which lacks a theory of truth, to prove our knowledge of external reality. In the following chapters I draw on my discussions with Karl Popper and consider the neo-Kantian philosophical paradigm, which, with different components of Kant’s epistemology, has had a durable influence for the past two centuries. However, philosophizing is shaped by progressive discovery, analysis, and relative proofs according to the available proof-conditions that allow the researcher to develop new and more comprehensive philosophical systems. The chapters of this book are based on articles that were published between 1976 and 2018. In what follows, I give a short summary of each one along with background information. Together they present the development of my research in the epistemology of science, a branch of philosophy that Einstein deemed fundamental:

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683–684)

**1. A Proposed Criterion of Demarcation Between Science and Metaphysics.** A paper from the International Workshop On The Cognitive Viewpoint: *Cognition and Communication*, Belgium: 335‑343 (1976).

This is one of my early works, written after obtaining my PhD from Brandeis University. I had accepted a post at the University of Haifa, Israel, where I was able to expand on my philosophical research. The normative and the descriptive are two components of our knowledge of the basic sciences, and theory, ethics, and aesthetics all come into play, as Peirce suggested in his mature writings, as well as metaphysics (Nesher 2007a). The role of metaphysics is to give a global picture of our available knowledge of physics, sociology, and psychology, and other disciplines. In representing reality, such a cognitive, meaningful picture can direct us in any particular enterprise and can suggest a better understanding of reality. But the discipline, or project, of metaphysics is not dogmatic; it develops constantly, since philosophers and epistemologists are always ready to challenge their deepest assumptions and respond to new scientific knowledge. Hence, metaphysics can offer new horizons by suggesting a new picture of reality to support the development of our sciences. In this chapter I explain the reasons why I disagree with Popper’s criterion of demarcation between science and metaphysics, which is at the core of his philosophy of science.

**2. Methodological Changes in Spinoza’s Concept of Science.** Published in *Spinoza Studies: Three Hundred Years in Memoriam*. Edited by S. Fuks, The University of Haifa Press (1978).

In my early work and teaching at the University of Haifa I worked on Spinoza’s philosophy and collaborated with philosophers of other universities in Israel to publish several anthologies. This is one of my contributions from that time on Spinoza’s philosophy of science. Spinoza was inspired by the ideas of Descartes and studied Cartesian formal deductive ontology. He became critical of the Cartesian geometrical conception of science based on metaphysical axiomatization and pure deductivism, which starts from formal axioms instead of empirical experience. This is reminiscent of Kantian transcendentalism, which fails to reach sensual experience due to the metaphysical gap between empty formal concepts and the blind matter of sensual intuitions. Spinoza’s realism can solve such epistemological difficulties by leaning on the empirical basis of our knowledge from which we develop scientific hypotheses, deduce their particular results, and compare them with scientific facts.

**3. On Common Notions in Spinoza’s Doctrine of Knowledge and Scientific Philosophy.** In *Baruch De Spinoza: A Collection of Papers on His Thought*. Edited by M. Brinker, M. Dascal and D. Nesher, University Publishing Projects (1979).

Spinoza developed an realist epistemological stance toward science. Following scientists of his time, e.g., Galileo, Kepler, Descartes, Boil, Huygens and others, he considered knowledge to begin from sensual cognition and evolve into scientific hypotheses in order to prove theories. Hence, as a metaphysical realist he constructed a comprehensive picture of Nature, which has an underlying unity with its attributes, the physical and the mental, and which is identified with an infinite plenum he metaphorically called God. Thus, for science to be possible human beings have to measure the components of their experience in order to form hypotheses and theories about the available parts of the natural plenum. To this end, we develop *common notions* of *size*, *time*, and *number* with a view to perceiving components of Nature. The epistemological conclusion is that space and time are neither Newtonian metaphysical entities nor Einsteinian physical entities, as I also explain in Chapter VIII.

**4. Where Would Spinoza Have Stood in the Einstein-Bohr Debate on the Scientific Representation of Physical Reality?** Based on a paper published in*Metadebates on Science: The Blue Book of Einstein Meets Magritte*. Kluwer Academic Publishers (1999).

When I was working on the epistemology of physical sciences I was particularly struck by something I read: Bohr tells us that in his discussion with Einstein at Princeton in 1937, regarding interpretations of modern science, they did not get beyond a humorous debate on “which side Spinoza would have taken if he had lived to see the development of our days” (Bohr 1949, 237). An important element in the Einstein-Bohr controversy about the problems of modern science was a focus on intrinsic philosophical-epistemological issues (e.g., Bohr 1949; Einstein 1949, 683-4). I presented my thoughts on this subject at the international conference “Einstein Meets Magritte” in Brussels when I was a fellow at the University of Pittsburgh’s Center for Philosophy of Science.

The essence of the controversy is this: Einstein conceives of physical reality as being outside of or external to our experience; it is independent and undisturbed measurement, and this can be called *timid realism*. According to Bohr, on the other hand, physical reality is our cognitive experience, including our experimentations, and this can be described as *phenomenal realism*. For Spinoza (if he had lived to see the development of modern science), there is a continuous interaction between the human body, the measuring instrument, and the measured object. Therefore, the measured object is affected by measuring instruments that are operated by the human body as components of reality but independent of human cognition.

Hence, according to Spinoza the body of a human being is a physical reality that encounters the measuring instrument, and when the two operate together they can affect external reality. Thus, when we use instruments in experiments we even create new components in the plenum of physical reality. A solution to this difficulty is presented in this chapter: We can obtain knowledge of these new components through theoretical calculation, and by using our instruments we can create new partials, waves, fields, and more from the matter of the physical plenum.

**5. How Can Our Knowledge of External Reality Be Explained? Controversies About ‘Facts,’ ‘True Propositions,’ and ‘Truth-Conditions’ and the Pragmaticist Solution.** (Manuscript, 2000).

As a fellow at the Center for Philosophy of Science at the University of Pittsburgh (1998-2000), I worked on my book *On Truth and The Representation of Reality* (published in 2002), and simultaneously continued my research on the epistemological realism of Spinoza and Peirce. The philosophical problem with our cognitive representation of external reality essentially involves our conception of the *truth* of our propositions and our conception of *facts* as the verifiers that “make” our propositions and hypotheses true or false. Over the last two centuries, the neo-Kantian tradition has had a significant influence, drawing on certain aspects of Kant’s epistemology and thus operating without any theory of truth. For example, logical empiricism, analytic philosophy, formal semantics, and ordinary language philosophy have assumed and adopted facts, models, sense-data, truth-conditions, and more as substitutions of reality.

Since our cognitive confrontation with external reality is anchored in perception, the central problem is to explain how our perceptual operations with their perceptual judgments represent objects of external reality. Hence, I develop a line of epistemological realism that is based on Peircean semiotics, drawing on epistemic logic to prove the truth of our perceptual judgments. These equate to basic *facts* that represent external reality upon which we can prove the truth or falsity of our scientific hypotheses (see also Chapter XII).

**6. On the Epistemology of Physical and Psychological Sciences: A Pragmaticist Alternative to the Shortcomings of Analytical Philosophy (“Scientism”) and Hermeneutic Phenomenology (“Artism”).** Presented at the 5th International Fellows Conference of the University of Pittsburgh’s Center for Philosophy of Science, held in Poland, Rytro, May 26-31, 2004.

Habermas emphasizes the historically developed conceptual dualism between natural and cultural (or psychological) sciences. Some refer to the latter as social or human sciences, and thus the terminology is not rigid. I prefer the Peircean distinction between *physical* and *psychological sciences*, because the Quinean use of “natural” is physicalist, so that mental life must be seen as supernatural, as opposed to a Spinozistic understanding of nature as all that subsists, its physical and mental aspects alike. The use of “human science” is also misleading because all sciences (physical and psychological) are human, and psychological science deals with the mental behavior of animals as well.

As for the epistemic explanation of the logical structures of the cognitive operations in these different sciences, Weber, for example, understands that these two types of sciences, natural and cultural, are distinct in *principle*. Psychological reality, which our judgments of psychological fact represent, exists before any operation of interpretation even starts, and therefore its existence is independent of its being interpreted and represented. However, one can argue that our knowledge of psychological reality depends on the operation of interpretation, and the meaning of psychological reality thus depends on how it is interpreted.

This would be correct if psychological reality were subject to arbitrary interpretation, such that everyone interpreted it differently without any objective constraints to those interpretations. If its meaning depends on its interpretation, the subject matter of psychological sciences does not exist independently of our representation of it. Moreover, in line with this conception of the relativity of meaning, there cannot be any objective truth about psychological reality, and therefore “anything goes.” In analytical philosophy, science is considered to be based on *a priori* assumptions and formal logic (“scientism”), following Kantian transcendental logic, while hermeneutic phenomenology is based on Kant’s sensual intuition components (“artism”). This is distinct from the Peircean conception of the three normative sciences (theoretical, ethical, and aesthetic) that represent reality in different modes by proving the truths of their respective hypotheses, as I develop in my later inquiries (Nesher 2007a).

**7. The Epistemology of Proving Our “Empirical Basis” or Scientific Hypotheses by the Trio of Abduction, Deduction, and Induction** (Manuscript, 2007)

As a student of Carl Popper at Brandeis University in 1970, I considered him to be part of the neo-Kantian lineage ‒ a philosopher who tried to explain the epistemology of science from a Kantian perspective, in contrast with Hume and other philosophers who assumed that inductive logic was the basis of the objectivity of our scientific knowledge, which ended in skepticism. This led Kant to his Copernican Revolution: he suggested that the logic of science is based on deduction and yet, by deduction alone we cannot prove our hypotheses. Thus, Popper suggests that we can only falsify them when we accept what he calls an “empirical basis” similar to Kant’s “sensual intuition” (Popper 1959). However, since Kant and the neo-Kantians do not have any theory of proof, the empirical basis remains only phenomenal without any relation to reality, and thus we cannot even falsify our theoretical hypotheses.

We can follow Peircean epistemology, using the *trio* of abduction, deduction, and induction as our basic and complete epistemic method to prove the truth of our hypotheses, interpret them deductively, and evaluate their truth by induction. This can be considered as a refutation of Popper’s epistemology.

**8. On the Concepts of Space and Time: Looking for a New Picture of Physical Reality.** (Manuscript, 2010).

I started to read the works of contemporary scientists, including Penrose, Greene, Davies, Moffat, Smolin, Barbour, Woit, Randall, Magueijo, Wilczek, and others, not to mention earlier scientists such as Mach, Poincaré, Planck, Einstein, Heisenberg, Weyl, Wheeler, Bohm, and others. In Lee Smolin’s *The Trouble with Physics* (2006) I found that the problem of understanding the concepts of space and time lies in the grand picture of physical reality, and in the blowing up of dimensions in string theories. I felt that an essential problem of contemporary physics is philosophical and epistemological, that is to say it concerns the basic principles of our picture of physical reality. I therefore believe in the need for a reevaluation of the concepts of *space* and *time* as understood in classical Newtonian terms as *metaphysical entities* and in Einsteinian and modern/contemporary physics as *physical entities*. In this chapter,the suggested alternative picture of physical reality is a *dynamic plenum*, while the Spinozist conceptions of *space, time,* and *number* are adopted as our “common notions,” i.e., methods for measuring its components, as I asserted and elaborated in Chapter III.

**9. The Role of the Productive Imagination in Creating Artworks and Discovering Scientific Hypotheses.** Presented at the 33rd Wittgenstein International Symposium, Kirchberg, Austria, August, 8-14, 2010. Published in *Image and Imagining in Philosophy, Science and the Art*. Edited by E. Nemeth, R. Heinrich and W. Pichler. Vienna: Hölder-Pichler-Tempsky. Kirchberg 2010.

In this article I elaborate on Kant’s conception of artistic *productive imagination* in creating artworks, and I generalize it to explain the scientist’s *intellectual intuition* in the discovery of new hypotheses. Kant explicates *intuition* as the representation of the imagination, and develops the concept of *productive imagination* to explain the genuine creation of fine art.

For the imagination (as a productive cognitive power) is very mighty when it creates, as it were, another nature out of the material that actual nature gives it (Kant 1790, 314).

Kant conceived of *intellectual intuition* as connected with supersensible objects of reason, as distinct from empirical ones. I turn this transcendental concept into a cognitive operation to explain all cognitions experientially. Hence the role of *productive imagination* lies in the artistic creation of new exemplary artworks, and the role of *intellectual intuition*, as productive imagination, lies in the discovery of new scientific points of view.

*I am enough of an artist to draw freely on my imagination. Imagination is more important than knowledge. Knowledge is limited. Imagination circles the world.* (Einstein to Hadamard, 1945)

Within the framework of pragmaticist epistemology I explain that artists and scientists use their productive imaginations differently in their respective enterprises to construct their different modes of representing reality. These two kinds of operation are based directly and indirectly on the perceptual images of empirical objects. To understand the artistic creation of exemplary artworks, and the scientific establishment of new hypotheses, we have to elucidate the roles of the productive imaginations of the artist and the scientist; this can be done by analyzing the different structures of the *aesthetic reflective judgment of taste* and of the *logical reflective judgment of coherence*. I criticize Kant’s narrow conception of *judgment* and put forward pragmaticist epistemic logic as a complete proof of truth.

**10. Gödel On Truth and Proof: Epistemological Proof of Gödel’s Conception of the Realistic Nature of Mathematical Theories and the Impossibility of Proving Their Incompleteness Formally.** Paper presented at the 7th Quadrennial International Fellows Conference of the Center for Philosophy of Science, University of Pittsburgh, at Ataturk University in Mugla, Turkey, June 2011.

In dealing with epistemic logic and mathematical theories I attempt a pragmaticist epistemological proof of Gödel’s understanding of the realist nature of mathematical theories as representations of external reality, but I argue that this cannot be proved formally. In Gödel’s Platonism mathematical theories are taken to describe an ideal reality consisting of eternally true facts. However, grasping such abstractions with mysterious pure intuition is beyond our human capacity. Indeed, *formal logic* remains sterile because it is a formally closed game in respect to mathematical reality, and thus Gödel’s incompleteness theorem is unprovable and cannot hold. In Peircean realist epistemology, empirical theories represent external reality by *epistemic logic*, and mathematics is indeed an empirical science, but the reality it represents involves neither *ideal objects* nor *physical objects* but only operations of *counting* and *measuring* empirical objects which we perceptually prove true as mathematical basic facts.

**11. On the Nature of Mathematics and the Limitations of Peano Arithmetic: The Empirical Epistemology of Mathematics and How Confused Epistemologies Affect the Practices of Mathematicians.** Previously unpublished manuscript, 2012.

This chapter stems from a research paper I wrote in connection with my correspondence with Hilary Putnam. After receiving his feedback on my work on Gödel’s incompleteness theorems, I replied to him: “As to your question about the axioms of Peano arithmetic, it might seem simple, but as I see it, it is the most difficult question in the epistemology and methodology of mathematical sciences.” An essential issue is whether mathematics is an axiomatic closed formal game with more or less rigid rules for inferring theorems or, alternatively, an empirical science representing reality. This is an essential question, as Einstein wrote about the importance of the epistemology of science (Einstein 1949, 683–684).

I am not a mathematician, and I might not correctly understand technical mathematics; however, from my pragmaticist-Peircean point of view, I picture the epistemology of mathematics as a specific empirical science capable of representing reality, with a fundamental role in human knowledge in general and in scientific knowledge in particular.

From my perspective, the Peano axioms can be considered as relatively true vis-à-vis mathematical reality. They do not constitute, however, a complete game with rigid rules of formal inference (which would be considered proofs in formal systems), as Peano probably believed. Indeed, a theoretical representation of reality cannot be constructed by formal inferences alone, and this is why mathematicians practically compensate for their limitations with indefinable intuitions.

However, complete mathematical proofs can work only by means of what I call epistemic logic, which enables one to prove the true propositional facts of mathematical reality; on the basis of these, mathematicians can generate hypotheses in order to evaluate whether or not they are true theoretical representations of mathematical reality (Nesher 2002, X; 2007a, 2011). Axiomatic formal systems, I claim, are epistemologically sterile in respect to external reality, and formal “proofs” are only *inferences* from the unexplainable acceptance of the relevant axioms. Hence, Peano arithmetic contains the standard model of the sequence of natural numbers without even explaining how we know it (and what it means), and this raises several questions: How does Peano know the meaning and truth of this arithmetical model, and how can the theorems inferred from these axioms be true to this model if the model represents arithmetical external reality? Is it only the formal circular completion of the expository game? (Peano 1889, #1)

We can explain our conception of numbers in mathematics as symbols involved in our operations in mathematical reality, but how are we then to understand definitions, axioms, theorems, and proofs in mathematics? If we can show that the origins of numbers lie in our experience in reality, perhaps we can forego apriority and vicious circles and view formal models simply as artificial realities.

According to pragmaticist epistemology, we can understand the meanings of mathematical symbols only when we confront mathematical reality and truly represent it (Nesher 2007b, 2011, 2013). The apparent difficulty is that in our basic perceptual experience of arithmetical operations on physical objects, we discover and use arithmetical numbers as signs representing such operations while at the same time using abstractions and generalizations to consider number signs as if they themselves were the objects of our calculations, and not as they are, i.e., realistic meaningful signs that represent our operations on objects.

However, mathematicians and philosophers in modern history have not clarified whether numbers are ideas, objects, or both; hence, they consider these two aspects as separate entities, such that numbers are ideas as well as objects. This confusion about the nature of numbers, i.e., viewing the phenomenal-objective component of the number sign as the object of its cognitive-idea component, led to the difficulties, ambiguities, and paradoxes of set theory. Thus, if the phenomenon of a number can be the object of that number's idea, then the number can be viewed as the object of itself. This confusion is the basis of Russell’s paradox in set theory, as it assumes that a number can be a member of its own set; but, if a number is a sign, then it cannot be an object and, of course, cannot be its own object (Russell 1901, 1919).

**12. ‘What Makes Reasoning Sound’ Is the Proof of Its Truth: A Reconstruction of Peirce’s Semiotics as Epistemic Logic, and Why He Did Not Complete His Realistic Revolution.** *Semiotica* 2018.

In his philosophical inquiries, Charles S. Peirce endeavored to discover and develop a theory of cognitive signs that would interpret one another to develop a true representation of reality originating in our basic perceptual operations of interpretation. His explanation of how we represent reality was rooted in the quasi-proof of the truth of perceptual judgment. The essential problem was to discover and explain how, by cognitive interpretation of the sequence of perceptual signs, we can represent external physical reality and reflectively represent our mind’s operations involving these signs. In an endeavor to develop his pragmaticist epistemology, Peirce starts from his basic perceptual experience and, through phenomenological introspection, or phaneroscopy, cognizes and explains the sequence involved in interpreting signs, where there is an interplay between iconic and indexical properties. He synthesizes them into symbolic thought with the emergence of perceptual judgments. In the process of interpretation, there is a split between *ego* and *non-ego* whereby expectation is either fulfilled (to represent external reality) or disappointed, thus representing reality negatively (Nesher 2002b, III). Through his introspective phaneroscopy, Peirce shows how, without going outside our cognition, we can represent external reality. With this explanation, Peirce can avoid the pitfalls of Berkeleyan, Humean, and Kantian phenomenologies, as well as of modern analytic philosophy and hermeneutic phenomenology (Marty 1982; Nesher 2002b, VI; 2004a/b).

**13. Epistemic Logic and How It Can Explain Our Mathematical Knowledge.** Presented at the pre-proceedings of the 41st International Wittgenstein Symposium. Published in the Volume of the Symposium: *Philosophy of Logic and Mathematics*, Vol. XXVI 2018, edited by Gabriele M. Mras, Paul Weingartner, and Bernhard Ritter Hrsg. Kirchberg, Austria; August 2018.

Epistemic logic is a basic science that aims to represent reality, by proving that we actually represent it. Formal systems are closed games of argumentation where the truth and the falsity of the initial axiomatic propositions are assumed, and where we may draw conclusions by assuming the validity of their inferences. The difference between formal systems and realist theories lies in their proof-conditions. Formal systems are based on fixed axioms that cannot be proven true, and their formal rules of inference cannot evaluate the truth of their theorematic conclusions about reality. Hence, axiomatic formal systems are complete and isolated from reality, while Gödelian realist theories are incomplete, but they can be proven true with respect to their proof-conditions: the proven true facts of reality and methods of proving their hypotheses. However, if mathematics is to be considered a theoretical science, it cannot be a pure axiomatic closed system isolated from reality; it has to be an empirical science based on our experience of counting, measuring and timing, namely our Spinozian “common notions,” and it can thus form the backbone of the sciences. In this way, mathematicians can avoid ambiguities, contradictions, and paradoxes.

**14. Epistemic Logic: the Cognitive Representation of our Experiential Confrontation in Reality.** Published in *Semiotica* 2020.

What is logic? What is its role in human affairs? These are basic epistemological questions. Epistemic logic is a basic science, a branch of epistemology that is concerned with reasoning about knowledge and with proving that we actually represent reality. Formal systems are *closed games of argumentation* in which the *truth* of the initial *propositions* of the relevant *syllogisms* or *axioms* is assumed, and in which we might reach conclusions just by assuming the *validity* of the inferences made. The difference between *formal systems* and *realist theories* lies in their proof-conditions. *Formal systems* are *hermetically closed games* with axioms that fix their meaning but which cannot be proved true, because their formal rules of inference cannot evaluate the truth of their theorematic conclusions about reality. Hence, *axiomatic formal systems are* *complete* and isolated from reality, whereas Gödelian *realist theories* *are* *incomplete* but can be proved true with respect to their proof-conditions: the proved true facts of reality and methods of proving their hypotheses. However, if mathematics is to be *a theoretical science* it cannot be a *pure* *axiomatic closed system* isolated from reality; it has to be an empirical science, and in this way mathematicians can avoid the ambiguities, contradictions, and paradoxes in creating mathematics from unbased axioms.

**Chapter 1**

**A Proposed Criterion of Demarcation between Science and Metaphysics**

(Popper’s Criterion of Demarcation Is Insufficient)

1. **Introduction**

In this chapter I would like to show that Popper’s criterion of demarcation (D) does not successfully accomplish its goal of distinguishing between science and non-science (Popper [1954] 1968, 38-39).

The ‘testability’ or ‘falsifiability’ criterion (D) is regarded by Popper as a suitable basis for the characterization of empirical science (Popper [1963] 1965, 197, 256).

According to Popper’s criterion the ‘demonstrative’ or formal sciences—incapable as they are of testability or falsifiability—are placed in the same camp as non-science and metaphysics. Thus ‘scientific’ becomes synonymous with ‘empirical’ (Popper [1954] 1968, 34).

But since this leaves the ‘demonstrative’ sciences on the same side as metaphysical systems (Popper [1954] 1968, 37), I would like to suggest a different criterion of demarcation, one that, I hope, will distinguish more properly between science and metaphysics.

1. **Popper’s Criterion of Demarcation**

Popper’s criterion of demarcation is at the center of his philosophy of science and is based on his concept of science, which deviates in principle from that of the inductivists and positivists (Schilpp 1974, 976 ff., 981; Popper [1954] 1968, 34-35).

Popper contends that (D), as a methodological rule, is bound to be vague, quite apart from the essential vagueness of the subject matter it demarcates. As he says, “... the transition between metaphysics and science is not a sharp one.” (Schilpp 1974, 981). I believe, however, that even if (D) must be vague in its application, it ought to be clear with respect to what it intends to demarcate, namely the difference between science and non-science (cf. Popper [1954] 1968, 38-39). Unfortunately (D) seems to me too ‘empirical’ if it keeps the formal science on one side of the demarcation line and ‘empirical’ science on the other.

Popper says: “My criterion of demarcation between the theories or statements of *empirical science* and those that do not belong to it (but perhaps to pseudoscience, logic, and metaphysics) is *testability,* or *falsifiability.*” (Schilpp 1974, 987).

There are some preconditions for dealing with the problem of demarcation. To maintain the analytic and synthetic dichotomy with respect to theoretical systems, one should have *two demarcation lines* in order to distinguish between science and metaphysics. First a distinction can be made between testable and untestable theories on empirical grounds. Second, we may infer from Popper’s view an additional demarcation line with regard to the non-empirical theories: a line between demonstrative and non-demonstrative theories, based on the analytic-synthetic dichotomy to distinguish between analytic or formal sciences and synthetic metaphysics (cf. Popper [1963] 1965, 197-198). After accepting ‘empirical refutation’ as a criterion of demarcation between science and non-science Popper had to demarcate formal science from metaphysics. He therefore postulates “that philosophical theories, or metaphysical theories, will be *irrefutable by definition.”* (ibid.).

Thus metaphysics is characterized as irrefutable empirically and also, being a synthetic theory, unprovable logically. But if metaphysics is also irrefutable logically as Popper claims, it is altogether excluded from any possible criticism, even on being logically inconsistent. If we accept Popper’s position that “logic is the organ of criticism” (OJC. p. 318, 121; C & B), and without deductive logic you undermine the method of critical discussion (Popper 1972, 305), it turns out that Popper can demarcate metaphysics from science by regarding the former—I believe unwillingly—not as meaningless but as irrational.

The claim of Popper that metaphysics cannot be refuted or proved, but that it is, nevertheless, capable of being true or false (C & B, pp. 194-7), seems to go against our intuition about truth and falsity. Popper is right that from irrefutability we cannot infer truth and from unprovability we cannot infer falsity, but despite this he decides that some forms of metaphysics are false and others are true. Since neither the logical nor the empirical procedures can constitute decision procedures for metaphysical theories, what kind of procedure could have been employed by Popper in this connection?

In answer, I would like to take an alternative perspective on metaphysical theories. In my view, metaphysical theories are normative systems not descriptive ones. As normative, they cannot be considered under the category of truth and falsity. (Thus, we cannot say The Ten Commandments are true-or-false.) According to this view, the demarcation line will divide normative from descriptive theories: normative theories are those that are not scientific in the empirical sense but are nevertheless rational. Cases of normative theories are: the propositional calculus of logic, moral, aesthetic, ideological, judicial, and pseudo-scientific theories. The problem of demarcation will be, therefore, to distinguish between scientific (empirical) and normative theories.

1. **A Philosophical Approach to Formal Science: Gödel’s Realism**

As we have already seen, Popper cannot demarcate conclusively between scientific and non-scientific theories by his criterion (D). He needs another criterion (D’) in order to complete this undertaking. This deficiency can be the result of two causes: (1) an erroneous concept of science; (2) a shortcoming in the criterion of demarcation. I believe that both are responsible for this difficulty.

The problem, as I see it, lies in understanding the formal sciences as analytic theories. Popper adopted this position uncritically from the logical positivists, instead of following Kant’s or some other realist’s position in regard to formal science, namely, K. Gödel, E.W. Beth, J.R. Myhul, S. Komer, J. Hintikka, E. Stenius, and R.H. Thompson (as I understand them).

Popper’s view of formal science can be understood from the historical point of view concerning the impact of logical positivism on the philosophical atmosphere in Vienna of the twenties and the early thirties. Model theory was then in its very beginnings—with the works of Gödel, A. Tarski and others—and the full impact of these developments on the philosophy of science had not yet been felt. Yet at the time when Gödel proved his theorem of incompleteness and thereby gave the death blow to Hilbert’s formalist program, Popper already completed his first draft of *The Logic of Scientific Discovery*, with the proposed criterion of demarcation (Schilpp 1974, 62-69).[[1]](#footnote-1) I see the Gödelian turn as a move toward a *realistic* philosophy of formal science, in much the same way as was Popper’s interpretation of the Einsteinian revolution with regard to the philosophy of empirical science (Schilpp 1974, 28-9).

But Popper’s analytic approach to formal science effected the combination of (D & D’). In this approach, he basically took the same position as did Carnap, Hempel, Nagel, Bar-Hillel and others.[[2]](#footnote-2) The interesting point of this position is the combination of an idealistic approach to formal science with a nominalistic approach to empirical science.

I believe that apart from the philosophy of logic and mathematics (which is ‘strictly non-ontological’) and the ‘crude’ empiricism which assumes ‘direct interpretation’ of formal science in empirical experience, there is another possible approach to formal science with regard to its ontology. This approach can be called *realism* and it assumes that there exist certain kinds of *abstract objects,* e.g., sets and numbers, which constitute the subject matter of logic and mathematics. This philosophical approach to formal science is in the Platonic and Kantian tradition, which was developed in modem times by G. Frege, K. Gödel and others in the framework of the Model Theory.[[3]](#footnote-3)

In order to formulate a new criterion of demarcation between science and non-science we need, beside the realistic concept of empirical science, a related concept for formal science. This will enable us to keep formal science in the realm of science by the criterion of demarcation without adding (D’) which is insufficient to distinguish between formal science and metaphysics.

The realistic philosophy generally assumes that there exist two ontologically independent domains of entities: (1) the domain of concepts, propositions (thoughts), and theories and (2) the domain of objects, structures and reality. Popper states his realistic approach for empirical science in the following:

“At that time I looked upon myself as an unorthodox Kantian, and as a realist. I conceded to idealism that our theories are actively produced by our minds rather than impressed upon us by reality, and that they transcend our ‘experience’; yet I stressed that a falsification may be a head-on clash with reality. I also interpreted Kant’s doctrine of the impossibility of knowing things in themselves as corresponding to the forever hypothetical character of our theories.” (Schilpp 1974, 65).

A similar approach in formal science is represented by Gödel’s philosophical interpretation of his revolutionary theorem.

Without going into too many details I would like to present Gödel’s realism as it appears in his “Russell’s Mathematical Logic” (Schilpp [1944] 1951) and “What is Cantor’s Continuum Problem ?” (Gödel 1947), according to my reading.

Gödel assumes that there is a domain of abstract objects—classes, numbers—which are necessary metaphysical presuppositions in order that formal sciences will be reasonably understood (cf. Barker, 1969).

“Classes and concepts may... be conceived as real objects... existing independently of our definitions and constructions. It seems to me that the assumption, of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory theory of mathematics as physical objects are necessary to obtain a satisfactory theory of our sense perception.” (Gödel [1944] 1951, 137).

The main point for our purpose in Gödel’s ‘Incompleteness Theorem’ is that there is a possible sentence S formulated in the notation of a language L which is true in a domain D but neither S nor its negation -S can be deduced from a theory T formulated in L. Therefore, S is undecidable with respect to T and T is incomplete with respect to D. The underlying assumption is that there exists a structured domain D of elementary objects independent of T. To this situation Gödel refers, I believe, in the following:

“It has turned out that (under the assumption that modem mathematics is consistent) the solution of certain arithmetic problems requires the use of assumptions essentially transcending arithmetic, i.e., the domain of the kind of elementary indisputable evidence that may be most fittingly compared with sense perception.” (RAIL. pp. 127-8).

Gödel praises Russell on his ‘realistic attitude’ and his suggested analogy between mathematics and natural science (Gödel [1944] 1951, 521), which manifests itself in Russell’s early writings. Gödel writes:

“He compares the axioms of logic and mathematics with the laws of nature and logical evidence with sense perception, so that the axioms need not necessarily be evident in themselves, but rather their justification lies (exactly as in physics) in the fact that they make it possible for these ‘sense’ perceptions’ to be deduced.” (ibid., 127).

In Model Theory or Formal Semantics a domain D, a non-empty set of objects, is specified in advance or, in Kant’s terms, posited. These objects and their primary properties and relations constitute the values for an interpretation of a given language L. The domain D is considered as a *structural feature* of a reality (world) necessary for the determination of the truth value of a theory T formulated in L. R. H. Thomason (1973) calls this structured domain a *semantic determinant*: “Semantic determinants are things that determine valuation.”

I see this structured domain, which is independent of L and T, as identical with Gödel’s ‘logical evidence’ mentioned above. This structured domain—a set of structures—of formal science, on its elementary objects is represented by a *background theory* (BT), which is effective and simpler than T itself. This is the way, to my understanding, that the subject-matter of formal science is given. The function of the theories Ts of formal science is to give a description of these specified structured domains.

From this analogy of structure between formal science and empirical science, which the realistic approach suggests, it is quite plausible to infer that in both of them we face the same problem of the incompleteness of our theories. In this context I understand Gödel’s comment:

Perhaps also the apparently unsurmountable difficulties which some other mathematical problems have been presenting for many years are due to the fact that the necessary axioms have not yet been found. Of course, under these circumstances mathematics may lose a good deal of its ‘absolute certainty’; but under the influence of the modern criticism of the foundations, this has already happened to a large extent.” (Gödel [1944] 1951, 127).

Gödel developed the idea that a well-determined domain of objects can be established effectively (CJ’. pp. 518-19, 520, 521, 523 n. 26), and with reference to it we elaborate our theories. Some hypotheses about this reality can be proved from a given set of axioms and some others are undecidable “from the axioms in their present form.” (Gödel 1947, 520). Dealing with a specific problem (of Cantor’s continuum conjecture) Gödel faces it in the same approach:

“In respect to Cantor’s conjecture its undecidability from the axioms as known today can only mean that these axioms do not contain a complete description of this reality… not only that the axiomatic system of set theory as known today is incomplete, but also that it can be supplemented without arbitrariness by new axioms...” (ibid.).

The philosophical conclusion of this discussion is that any axiomatic theory of formal science will never exhaust all the possible structures of their abstract objects. In order to theorize (Gödel: ‘describe’) these structures, we will have to develop new theories and replace the old ones by more comprehensive ones. K. Popper reached the same conclusion with respect to empirical science: ‘Explanation is always incomplete... This is why the evolution of physics is likely to be an endless process of correction and better approximation toward an unattainable end.’ (Schilpp 1974, 104).

1. **Science and Non-science, Descriptive and Normative Theories**

Assuming this interpretation of Gödel’s philosophical approach to formal science is correct, we can see that it leads towards a new concept, which differs from the formalistic, logistic, and nominalistic philosophical concepts of formal science. The important outcome here is the resemblance in \*structure between the concept of formal and empirical sciences. Under these assumptions I propose a new criterion of demarcation between theories of science and non-science, assuming they are all consistent theories:

*(D\*).* Scientific theories are incomplete while non-scientific theories, (normative ones), are complete*.* (There are no theories beside descriptive (scientific) and normative ones). (Usually, of course, we think that questions of normative theories are just those that we cannot decide definitely. But this is only in practice. And if this is the case, it means that we still do not have formalized normative theories but only sketches of them.)

The result of my (D♦) is quite different from Popper’s (D & D’) criterion; normative theories are not irrefutable by definition, because if they are inconsistent they are refutable. Being consistent or possibly amended to be such, normative theories are always “true” with respect to their ‘structured domain.’ However, this “truth” says nothing about reality but only about our normative approach to it, the way we determine reality. Normative theories are devoid of any explanatory power with respect to their structures of objects even though most of the time they create the illusion of having this function.

1. **Metaphysical Theories and Their Models of Interpretation**

It is assumed that metaphysics is a specific kind of normative theory, and this kind will concern us here. It seems that for a metaphysical theory there is no object outside its models of interpretation; the set of structures (domain) of the objects of metaphysics and the set of models for it are identical. Otherwise, there will be some objects that are not determined by the metaphysical theory, which are ‘outside the world’.[[4]](#footnote-4)

In light of this characterization, metaphysics can be understood as a normative theory versus the descriptive character of scientific theories. In other words, metaphysical theories are not aimed to describe structures but to be exemplified in models. The efforts that have been made from time to time to verify metaphysical statements (or theories) in the same methods employed for scientific ones, cannot gain any success (cf. Kattsoff 1967, chap. 8; Bunge 1973, 146, 158).

Assuming the above picture of metaphysics, the question arises: Why (according to our linguistic intuition) is Popper mistaken in claiming that metaphysical theories might be both false and irrefutable?

From the assumption that it is impossible for two incompatible theories to be simultaneously true and from the postulation of their irrefutability Popper infers that irrefutability cannot entail truth (C & B, p. 195). But incompatible theories can be simultaneously true with respect to different structures. In different possible worlds incompatible theories can be simultaneously true without generating any contradiction.

A counter argument can be raised to the effect that all metaphysics refer to the *same world,* our actual world (Bunge 1973, 158). (Metaphysics is ‘transcendental logic’ and not formal logic, which should be true in all possible worlds.) A solution to this predicament can be that metaphysical theories are understood as sets of ‘rules of procedure’ which interpret and determine ‘*the same world’* in different shapes according to the different conceptual frameworks and ontological presuppositions of each metaphysical theory. If my argument is correct, it becomes clear why two incompatible metaphysical theories can be both ‘true’; they are true in different models.[[5]](#footnote-5)

The last problem of concern here is how metaphysical theories replace each other, or why we sometimes give up one metaphysics to which we earlier used to adhere? A solution to this problem might be that even if consistent metaphysical theories are irrefutable, nevertheless, as with moral or ideological theories, we accept or reject them if they agree or disagree with our intuitive model of the world (or a categorical part of it). Metaphysical theories rise and fall with their models of interpretation. If, for example, nobody wants to accept Kant’s model of nature, the metaphysics that is connected with this model becomes *useless* and neglected. In short, metaphysical theories appear and disappear with their models.[[6]](#footnote-6)

To conclude the entire discussion: If our analysis and conjecture are correct and if the concept of science (Popperian and Gödelian) is accepted and if the result of our discussion with regard to metaphysics coincides with a reasonable one, then, the new criterion of demarcation (D\*) allows us to draw a better distinction between science and non-science or metaphysics. This can be the case if the concept of theory is maintained accurately.[[7]](#footnote-7)

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**Chapter 2**

**Methodological Changes in Spinoza’s Concept of Science**

From *Treatise on the Emendation of the Intellect*to *Ethics*

1.

The recently published[[8]](#footnote-8) Hebrew translation of Spinoza’s *Treatise on the Emendation of the Intellect* affords us an additional opportunity to discuss this work and its connection to Spinoza’s later philosophy. For my part, this is one of Spinoza’s more interesting works, in light of both its subject matter and its structure and clarity of argument. The *Treatise* discusses logic and the theory of knowledge, as implied by its full title: *Treatise on the Emendation of the Intellect and the Way in which It Is Best Directed toward a True Knowledge of Things*. Though the *Treatise* is surely worthy of a more comprehensive treatment, I shall limit myself to a number of comments, which I shall present hereinafter.

When considering the philosophy of Spinoza, as with the system of any other philosopher, a minimum of three approaches can be employed: the regressive historical approach, which sees Spinoza’s philosophy as a clarification and summation of those philosophical systems which preceded it;[[9]](#footnote-9) the progressive historical approach, which sees Spinoza’s philosophy as a kind of point of origin or master plan for the modern philosophy which developed in its wake;[[10]](#footnote-10) and the critical-analytic approach, which understands that theory and philosophy develop as a means of addressing questions arising from contemporaneous scientific, societal, and political issues. This last approach, exemplified by K. R. Popper, posits that by understanding this context – namely, that of the “Situation Problem” – and employing critical analysis, it becomes possible to understand a particular philosophical system, its internal structure, and the function of its specific constituent concepts and elements.[[11]](#footnote-11) I have tended towards the Popperian approach in this discussion, although I have also from time to time relied on other philosophical systems for the sake of comparison, in order to point to an analogous model or similar solution, thus economizing on the scope of the discussion at hand.

In order to further understanding of the document before us, I offer a hypothesis which, among other things, touches on the question of the relationship between the philosophical system that emerges from the *Treatise* and that of Spinoza’s later writings, particularly *Ethics*. It appears to me that, to a certain extent, this hypothesis contradicts the interpretation by Y. Ben-Shlomo in his introduction to and commentary on the translated text.

I argue that the system of knowledge expressed in the *Treatise on the Emendation of the Intellect* is *rationalistic*, or, more precisely, *conceptualistic*,[[12]](#footnote-12) whereas three connected systems can be identified in *Ethics*: the conceptualistic, the realistic, and the nominalistic, with the structure of the scientific system being realistic (Fregian-Popperian). In order to shed light on my approach and the terms I employ, I will begin with a suggested understanding of the structure of Spinoza’s philosophical system in *Ethics*, and only then move on to a discussion of the philosophical system in the *Treatise* and the link between them.

2.

One of the foremost issues in elucidating Spinoza’s philosophy is the difficulty of explaining how to unify, within a single system, a nominalistic approach and a conceptualistic approach (more commonly called a rationalistic approach). In my view, one should see Spinoza’s system, as expressed in *Ethics*, as a combination of three systems, or three distinct linguistic nexuses.[[13]](#footnote-13)

The *first linguistic nexus* is quotidian language, largely free of scientific and metaphysical terminology,[[14]](#footnote-14) which serves to facilitate contact and communication between human beings. The premise is that a language’s system of objects (physical bodies – inanimate and animate) is finite with respect to any given context. The language’s nouns represent these objects or finite groups of said objects, and its predicates represent these objects’ attributes.[[15]](#footnote-15)

This characteristic of language is nominalistic in the Aristotelian or Goodmanian[[16]](#footnote-16) sense, which is to say that it brooks no independent existence of concepts – of the meaning of predicates – except in the context of the group of objects to which each refers. One might say, albeit reservedly, that concepts in such language are “derived” from its objects, a kind of empiricism untainted by inverse deduction – that is, reduction.[[17]](#footnote-17)

The arguments made for objects and their relationships in such language are based on personal experience and subjective associations and are therefore fundamentally misleading: the degree of knowledge expressed in this language is deduced by the senses, and thus “mutilated” and “confused.” The senses’ perception differs from person to person, or even, if speaking of a single person, from situation to situation. This linguistic nexus is a kind of proto-language[[18]](#footnote-18) which permits constancy and agreement only for the names of objects (individuals and groups), while the concepts characterizing these objects’ attributes are subjective and imagistic. The usage of concepts, of generalities, in this language is inconsistent, misleading, coincidental and confused with respect to the empirical objects they purport to describe. Thus the usage of concepts in this language is termed transcendental, meaning it belongs more to individual’s manner of experience than to the objects causing the experience themselves.

The *second linguistic nexus* is the language of science, which assumes the existence of two independent realms of being: a realm of objects and a realm of ideal entities – the concepts, arguments, and theories of scientific language. This linguistic nexus is called realistic language, as its ontology resembles that of G. Frege and K. Popper, or that of Plato in his later writings.[[19]](#footnote-19) At its core, this is also the approach taken by Kant, who proposed two separate sources for scientific knowledge, which he put in terms of human capabilities – sensibility and understanding – instead of Frege’s formal-semantic form, which emphasizes the ontological propositions: concepts and objects. With Spinoza, the objects of scientific language are partially supplied by quotidian language, which constitutes a prerequisite for the potential development of the scientific linguistic nexus.[[20]](#footnote-20) Without empirical, sensory experience, science itself is impossible, however divided these two nexuses might be from one another. Science comes into being wherever the subjective empirical experience ends and the empirical experiment organized by the intellect-generated conceptual nexus begins. The fundamental concepts of science are “common notions”[[21]](#footnote-21) which provide organization to the experiment. The intellect creates basic measurable scientific concepts, determines their quantification and the relevant units of measure for each concept (such as length, distance, and weight), and implements such quantified concepts within the perceptual realm of the empirical objects.[[22]](#footnote-22) The premise is that the creation of concepts, their logical connections, their theoretical organization, and the mathematical operations between them, are distinct from their implementation within the realm of the experiment.[[23]](#footnote-23)

The distinguishing factor of the scientific linguistic nexus is that its universal notions, namely scientific concepts, have a source, a form (phraseology), and a status different from the status of the universal notions of quotidian language. In the first linguistic nexus (quotidian language), the universal notions’ source is the imagination, and they are indeterminate and subjective in status; in the scientific nexus, however, these notions’ source is the intellect, with a defined phraseology and objective status. These are not “transcendental” or “universal”[[24]](#footnote-24) concepts as one would encounter in the realm of personal experience;[[25]](#footnote-25) rather, they are adequate, that is, true concepts that apply to the empirical realm. They are correct, or in other words, phrased as law and clearly defined. (Spinoza sees the concepts within the system of knowledge as individual beings) (*Treatise* § 101).

What is interesting about Spinoza’s philosophy is that the passage from one linguistic nexus to another brings about changes in the character, status, and validity of terms and sentences.[[26]](#footnote-26) Thus one could justify seeing in them independent linguistic systems of context, or distinct “language games.” The realistic approach to the structure of the scientific nexus says that one should assume as given two distinct realms of beings, namely concepts and objects;[[27]](#footnote-27) the role of science is use these concepts to fashion a theory explaining the behaviors and relationships of the relevant objects. Spinoza calls this theoretical system “universal” or “rational” thought (*cognitio universalis*).[[28]](#footnote-28)

The *third linguistic nexus* of Spinoza’s philosophy is the metaphysical, what he calls knowledge of the third kind, or intuitive knowledge. One cannot truly understand Spinoza’s philosophy without understanding the role played by the deductive method. It is by this method that the rational structure of the world is arranged (*Treatise* §§ 30, 49, 104). The nature and attributes of the intellect are organized in such a way as to understand things and the world itself through a system of deduction (*Treatise* § 108).

A rational method is impossible without deriving assertions from intellectual axioms, a derivation based on the pure strength of the intellect, proceeding in accordance with an understanding of its organization (*Treatise* § 91).[[29]](#footnote-29) If we accept that the deductive method served as a model for the development of Spinoza’s philosophy, then many seemingly incomprehensible passages and expressions will ultimately prove quite sensible, and apparently metaphorical notions will be clarified as being technical notions belonging to the method.[[30]](#footnote-30)

This metaphysical linguistic nexus is rooted in the primary terms and axioms of the intellect, as perceived by the intuition; it is from them, with the aid of certain proper deductive principles, that the whole of the system’s arguments and beings are derived (*Treatise* §§ 44, 49, 140). This nexus is distinct from the two preceding it: the nominalistic and rationalistic are *normative*, meaning their primary norms determine all beings within the nexus, with none existing save for the primary notions of the intellect and those which are derived therefrom. This approach runs in parallel to Platonic philosophy as expressed in Plato’s earlier writings (for example, the doctrine of forms in the *Republic*), in that individual objects exist only insofar as they “take part” of the Forms.[[31]](#footnote-31)

Examples of more modern normative methods, parallel in structure to Spinoza’s metaphysical-normative nexus, include the Hegelian method, which is best summarized, to my understanding, by the assertion that only what is derived from principles of reason can be said to exist; another is the intuitionistic approach in the 20th century philosophy of mathematics (L. E. J. Brouwer), which demands an actual “construction” of arguments and beings from the axioms of the intuition, and argues that only that which is proven according to the principles of the system can be said to exist. Even if we accept this analogy, which asserts that Spinoza’s philosophy is constructed from three linguistic sub-nexuses – metaphysical, scientific, and natural (quotidian, communicative) – the question of the relationship between these nexuses remains open. This question as a whole is quite complex, and it may well be that Spinoza never provided it with an unequivocal answer; thus it continues to serve as a topic for philosophical, logical, and semantic discussion in modern research on the philosophy of science and language, in the question of demarcating between science and metaphysics, and in the question of the relationship between natural and scientific language.

3.

In contradistinction to Y. Ben-Shlomo’s interpretation, according to which the difference between the *Treatise on the Emendation of the Intellect* and *Ethics* lies only in the manner each present their methods,[[32]](#footnote-32) I argue that there exists a fundamental distinction in general philosophical perspective between the two works. This difference is rooted first and foremost in the ontological premises of the system, the place and function of science in the system of human knowledge, and the relationship between metaphysics and science. It may well be true that the system of notions and categories in the *Treatise on the Emendation of the Intellect* is entirely identical to that of *Ethics*, but there are sufficient differences in a few key concepts, fundamental assertions, and areas of emphasis to alter the character and structure of the philosophical method.

From a chronological perspective, the *Treatise on the Emendation of the Intellect* precedes *Ethics*, with its writing demonstrably completed (or set aside) in 1662, while *Ethics* was finished in 1675 (see Letters 6 and 68); this is a substantial temporal gulf, sufficient for an evolution in philosophical approach. But the decisive factor in considering the distinctions between the two methods is not this temporal one; rather, it stems from a substantive internal difference. The character of the *Treatise on the Emendation of the Intellect*, from an ontological-methodological perspective, is still fundamentally Cartesian,[[33]](#footnote-33) meaning that its method of knowledge is deductive-rationalistic (conceptualistic), in fact identical in form to the metaphysical system of knowledge in Spinoza’s later method, except that the entire system of knowledge is included within metaphysics.[[34]](#footnote-34) This system is a normative-deductive one, developing out of its own intuitivist first principles. These principles are “the first elements…the source and origin of Nature” (*Treatise* §§ 75, 76), the Idea of the world – perceived as self-evident ontological and semantic requirements, from which is derived, per specific and correct logical laws and within a specific order, the entire system of knowledge, which is itself identical to the system of the world in terms of structure and content. “True knowledge proceeds from cause to effect” (*Treatise* § 85), meaning from fundamental principles, which are the source and origin of nature, to its individual assertions, in contradistinction to hypothetical knowledge, “when we infer the cause from some effect” (*Treatise* § 19 III),[[35]](#footnote-35) meaning when the individual assertions are taken as given, and we seek out some hypothesis to provide them with an explanation. The purpose of the scientific trial is only to determine the rules of transformation, which themselves determine the status of the individual thing under study as subject to the general system of natural laws which regulate it. Experimental science makes possible empirical experimentation, thus bringing sensory experience under the aegis of the eternal deductive-metaphysical conceptual explanation, “so that at last we may infer from them according to what laws of eternal things it was made.”[[36]](#footnote-36) From this perspective, the system of knowledge in the *Treatise on the Emendation of the Intellect* is normative, not hypothetical.

The procedure Spinoza undertakes in the *Treatise on the Emendation of the Intellect* ends up decisively favoring the metaphysical-normative nexus of science by eliminating all other forms of knowing, which do not promise true knowledge (*Treatise* § 29). Thus, the various modes of knowing are not necessary steps on the journey to complete knowledge; rather, they constitute data points needing further investigation, alternative possibilities disqualified in various ways, thus militating in favor of knowledge of the fourth type in the *Treatise on the Emendation of the Intellect* (the third degree of knowledge in *Ethics*).

In fact, it is the *Method* that proves to be the hero of the *Treatise’s* narrative, rescuing the princess, the *true idea,* from the treacherous castle of *sensory experience* and the witchery of language, bursting through the palisades of *universal knowledge*; this method is none other than the *deductive method*, as applied to the *ontology of the normative approach*.

Interpreters of Spinoza tend to rely on the explicit statement in the *Treatise on the Emendation of the Intellect* that the Method is no more than reflexive knowledge (§ 38). This appears to be an error, plucking the sentence above from its precise context and conferring upon it the force of a universal statement. In section 49, Spinoza lays out the steps of the deductive method:[[37]](#footnote-37)

1. Seeking the first idea, the axioms – ’the universals of nature as a whole’;
2. Determining the rules of deduction – ’the adequate syllogism’
3. Determining the order of deduction – ’the adequate order.’

It is reasonable to assume that reflexive knowledge is first and foremost dedicated to an understanding of the true idea, found within us “as an inborn tool[[38]](#footnote-38)” (*Treatise* § 39), or, in other words, first and foremost dedicated to the realization of the Method’s first part: “Understanding that difference constitutes one part of the Method” (*Treatise* § 39). Alongside this, it may be that reflexive knowledge (perception) is a necessary part of accepting all three of the Method’s fundamental norms. If we identify the Method with the three fundamental structural principles of the system, we see that reflexive knowledge is a meta-theory that expresses the Method, a sort of intuitive confirmation or declaration of principles that makes the system as a whole possible;[[39]](#footnote-39) and, naturally, one must make a distinction between the perception of principles and the principles themselves (*Treatise* § 49).

This but scratches the surface, and I will not embark here on a more detailed analysis of “the universals of nature as a whole.” I will limit myself to suggesting that these might include ontological presuppositions, such as the universal being (nature) and its modes, as well as semantic presuppositions, such as the attributes of said being. These presuppositions can be considered the metaphysical premises of the method as a whole.[[40]](#footnote-40)

We now come to the question of the relationship between metaphysics and science in the context of Spinoza’s philosophy. The answer provided by the *Treatise on the Emendation of the Intellect*, insofar as I understand it, is that science is derived from the adequate logical rules of syllogism within metaphysics. The model of the system of knowledge in the *Treatise* – like that of Descartes – is deductive-mathematical,[[41]](#footnote-41) and by analogy to the Fregian-Russellian logistical treatment of mathematics’ place vis-à-vis logic, we can conclude that science in the *Treatise* is built on the base of metaphysics – and thus, the system of knowledge is scientific metaphysics.[[42]](#footnote-42) In the context of the *Treatise on the Emendation of the Intellect*, metaphysics constitutes a necessary and sufficient precondition for science.

4.

Spinoza’s methodological turnabout occurred against the backdrop of wide-ranging advances in the fields of theoretical science and experimental study. This was a journey from a system of knowledge based on the geometric-mathematical model as Descartes understood it to a more complex philosophical structure, one which included (as a partial domain) the model of natural sciences, as understood (more or less) by Galileo and Kant. The theoretical background to this turnabout is part and parcel of Spinoza’s critique of Cartesianism: it is impossible to deduce from the ontological premises of extension, or from the premises of analytic geometry, either physical theories or empirical phenomena.[[43]](#footnote-43) This critique found its fullest expression in Spinoza’s correspondence (see letters 81 and 83). From a methodological perspective, Spinoza’s altered approach was expressed by his introduction and use of “common notions” as accepted empirical concepts that point to objects’ attributes and become integral elements of scientific theorems.[[44]](#footnote-44) The insertion of “common notions” into the system of scientific knowledge constitutes an acknowledgment that, beyond notions born of pure intellect,[[45]](#footnote-45) science also requires empirical experimentation, and consequently sensory experience as well. This experience should be constructed upon the notions of those attributes that apply to the experiment, are common to all human beings, and are thus objective.[[46]](#footnote-46) Hence the linguistic nexus of quotidian language, from which these terms were drawn, is at least partially legitimized as a precondition for the possibility of science.[[47]](#footnote-47)

Spinoza’s realism is a kind of combination of the demands of British empiricism and French rationalism, one that found a more explicit expression in the philosophical methods of Kant, Frege, Popper, and others, though its foundations are already apparent in Spinoza’s later writings. One of the most significant phenomena of this methodological turnabout is the more prominent role afforded to knowledge of the second type – “universal knowledge” – or, in other words, hypothetical knowledge, which belongs to the realm of positive science. In this new context, science cannot be derived from metaphysics, and instead becomes a relatively independent authority, one which itself influences the construction of metaphysics.[[48]](#footnote-48) Of course, Spinoza continues to pursue his goal of a universal and perfect deductive system of knowledge, deduced from its own intuitively understood first principles. But this goal is retained only as a sort of scientific ideal (*Ethics*, Part V, Prop. 25). In fact, it is clear to Spinoza that the number of conceptual assertions deducible from metaphysical first principles is highly limited. Thus, in his later philosophy, the nature of the connection between metaphysics and science changes, as do the characteristics of the metaphysical system itself. In this stage, comparatively less emphasis is given to the metaphysical-notional premises of science, while more is given to normative-ethical directives and to the practical role played by metaphysics in determining the social, spiritual, and intellectual behavior of man.

At any rate, the main thrust of our present discussion has focused on the conceptual elements contained within metaphysics, or, in other words, on ontological premises and on the categorical-conceptual framework[[49]](#footnote-49) which constitutes a precondition for the system of knowledge. The most significant result of our discussion in this context has been to note the change of the relationship in Spinoza’s philosophy between science – as a hypothetical system of knowledge – and metaphysics as a normative system.

To the extent the analysis I have undertaken here, as well as the description of the three linguistic nexuses constituting the basis of Spinoza’s later philosophy, prove reasonable, then as a result of Spinoza’s realistic conception of the scientific system, we arrive at the following conclusions:

1. Metaphysics constitutes a necessary precondition for the possibility of science: without its system of notions, there can be neither clarification nor explication of scientific concepts, nor any logical connection between them,[[50]](#footnote-50) nor any deductive methodological perfection for science whatsoever.
2. Metaphysics alone is no longer a sufficient precondition for the existence of science; science has additional conditions, which find expression in the existence of “common notions” (among others). These conditions rely on a nexus of natural (quotidian) language which makes possible the empirical experiment.

We can thus conclude that there exist at least two domains of preconditions that enable the hypothetical-descriptive system of scientific knowledge:

1. The metaphysical, normative system, which determines science’s attendant theoretical entities, as well as the rules for connecting them. We can call this system the super-language of science.
2. The system of quotidian language, which determines science’s attendant observational entities. We can call this system a “background language”[[51]](#footnote-51) or “the language of empirical basis.”[[52]](#footnote-52)

If this summary of Spinoza’s method is correct, it is far removed indeed from Descartes’ system of normative-deductive science, as well as from Spinoza’s first declaration of principles – as expressed in the *Treatise on the Emendation of the Intellect* – which remains fundamentally Cartesian.

**Chapter 3**

**On Common Notions in Spinoza’s Doctrine of Knowledge and Scientific Philosophy**

1.1

In the second scholium to proposition 40 of book II of *Ethics*, Spinoza briefly describes the three kinds of knowledge (consciousness) available to human beings. One of them is *scientific knowledge*, which is thus summarized: we create universal notions for ourselves “from the fact that we have common notions and adequate ideas of the properties of things… This I shall call reason and the second kind of knowledge.”[[53]](#footnote-53) In the first scholium to the same proposition, Spinoza argues that “common notions” are “the foundation of our reasoning.” The Latin term translated as “reasoning” or “ratiocinations” is *ratiocinii*, and it seems to refer to structured or organized thought, something quite appropriate to our concept of scientific thought.

1.2

Spinoza understands science as the second kind of knowledge, one that is adequate and true (it is also called “universal knowledge” or “general consciousness,” as in *Ethics* Part V, post. 36, sch. 1). This consciousness arises from universal premises in order to explicate the order of nature as a whole (*Ethics*, Part II, post. 7, sch. 1; post. 29; see also Part I, post. 11), not through the action of sensory experience or imagination (as in knowledge of the first kind), but through the organization of knowledge by the intellect, via the notions it creates for itself in order to understand the affinities, differences, and contradictions between things as it abstractly represents them. Spinoza, as a philosopher, certainly challenged himself with the question of how knowledge in general is possible, and as a philosopher of science, he could also ask how scientific knowledge is possible, or which conditions and assumptions are necessary for the possibility of science. Because we are focused here on consciousness of the second kind, universal knowledge, we shall not deal directly with the characteristics and possibility of intuitive consciousness, or knowledge of the third kind, which we might (albeit somewhat reservedly) call metaphysical consciousness (Letter 27). Spinoza postulated the necessity of this metaphysical consciousness for a scientific understanding of nature, but it is not itself sufficient for this end, nor is it possible to construct and formulate science on the basis of metaphysical premises alone. The question, then, is what else is needed to make scientific consciousness possible. The place of sensory or imaginative experience in the creation of true scientific consciousness is problematic, as such experience is defined at the outset as fundamentally misleading, though it is nevertheless reasonable to assume that there exists some role for the senses in the construction of scientific consciousness, if not by itself and apparently not as its own distinct kind of knowledge. (See, for example, *Ethics*, Part II, Posts. 38 and corollary; 39 and corollary; 150-157; C. De Deugd, 1966; H. H. Joachim, 1901: 172). Hence, in order to arrive at proof of the possibility of scientific consciousness (from a methodological perspective, at least), one should investigate this scientific consciousness itself, and in so doing it will perhaps prove possible to understand its relation to other modes of consciousness.

1.3

One of the key terms for understanding scientific consciousness in the framework of Spinoza’s philosophy (and perhaps for understanding science as a whole) is “common notions,” which serve as the foundation of scientific consciousness and make it possible.

Spinoza discusses the characteristics of common notions (*notiones communes*) in the second book of *Ethics* in propositions 37, 38, 39, 40, 44, 46, and 47, and in the fifth book in propositions 4, 7, 12, and 20 sch. 1 – provided, that is, that our suggestion that “common notions” and “adequate ideas of the properties of things” are related, or even identical, holds true. The same term is referenced in Letter 4 to Oldenburg (1661) and in the *Theologico-Political Treatise* in sections 10, 19, 47, 70, 79, 153, and 222, and potentially in other places I have overlooked.

1.4

Answering the question of what precisely the common notions are is decisively important in understanding Spinoza’s doctrine of scientific consciousness and scientific philosophy, and in understanding his philosophy as a whole. Detailed knowledge of the universalities of nature is contingent on the development of scientific knowledge. Spinoza argues that the more we know singular things, the more we know nature (*Ethics*, Part V, post. 24). Per Spinoza, consciousness of singular things, or individuation, is possible through two methods: a) through consciousness of the first kind, which is “mutilated” and “confused”; b) through consciousness of the third kind, or intuitive knowledge, which is adequate and true. Consciousness of the second kind, or scientific knowledge, is universal, and grants us not knowledge of singular things but rather knowledge of these things’ common attributes, and thus knowledge of groups of singular things according to their common attributes – or, more precisely, knowledge of a singular thing according to its shared attributes with other members of the group to which it belongs. Thus, scientific consciousness does not enable us to truly know a certain man *h*, or a certain atom *a*, or a certain electron *e*, but rather groups of men, atoms, or electrons that share common features. (Cf.: “the common property of proportionals” in *Ethics*, Part II, post. 40, sch. 2). This is the universal character of scientific knowledge. The intuitive metaphysical consciousness of an idea or the fundamental principles of nature indeed allow us to begin to know nature as a perfect whole – but there is no way to deduce the entirety of nature in all its ways and means from metaphysical propositions (see, for example, Letters 10, 32, and 83). The traces of the Cartesian-style schema of metaphysical science that does allow for such deduction, still apparent in the *Treatise on the Emendation of the Intellect*, were finally abandoned by Spinoza in the more sophisticated philosophy of *Ethics*. Intuitive consciousness’ procession from an “adequate idea of certain attributes of God” to a consciousness of the essence of singular things (*Ethics*, Part V, post. 25) is midwifed by scientific inquiry, which formulates the laws governing singular things’ behavior and brings us nearer to an understanding of their essence via intuitive consciousness (*Ethics*, Part II, post. 47, sch. 1; Posts. 4, 7, 20, sch. 1; Posts. 25, 28, 30).

Spinoza’s declarations demonstrate that *common notions* constitute a key factor in understanding the scientific consciousness whose importance to Spinoza’s philosophy we have already noted above. We shall devote the rest of this article to clarifying this complex central concept.

1.5

Any discussion of *common notions* is rendered difficult by the fact that Spinoza did not explicitly address the concept at any great length, nor did he illustrate it with examples when mentioning it directly. This perhaps accounts for most commentators’ tendency to deal primarily with these notions’ general features, and to attempt, in one way or another, to explore their characteristics and their place in Spinoza’s doctrine of knowledge in general terms, without pointing to the notions themselves, or distinguishing between them and other notions, or discerning their function in the creation of scientific knowledge. This is no easy task, and what I suggest here can be considered no more than an initial hypothesis unable to be more fully tested or incontrovertibly verified in the present context.

2

2.1

There exists a problem of whether to identify “common notions” with “adequate ideas of the properties of things.” Some commentators equate these two terms (for example: S. Hampsire, 1951:94 ff.), while other differentiate them (for example: G. H. R. Parkinson, 1954:164 ff.). The Latin original of scholium 2 to proposition 40 of *Ethics* Part II is as follows:

“3. Denique ex eo, quod notiones communes rerumque proprietatum ideas adequatas habemus.”

Differentiating these two concepts can be justified even without giving weight to the distinction made between “*ideas*” and “*notiones*,” considering that Spinoza himself was sometimes seemingly inconsistent in distinguishing his usage of these terms. In the corollary to proposition 38 (*Ethics*, Part II), this distinction is apparently narrowed by use of the word “*sive*”: “From this it follows that there are certain ideas, or notions, common to all men…,” yet the text supports an alternate reading in which, by his use of the inclusive “or,” Spinoza asserts that both “adequate ideas of the properties of things” (or a portion of them) and “common notions” are shared by all human beings.

2.2

Thus we must examine whether there is any justification for distinguishing between “common notions” and “adequate ideas of the properties of things,” and consequently between the “foundations of our reasoning,” imputed to the former (*Ethics,* Part II, post. 40, sch. 1), and the “foundations of Reason,” apparently imputed to the latter. To this end, we will first investigate what these “properties” might be, so that we might finally arrive at a discussion of common notions themselves.

In at least one instance, we are given an example of a *common property*. In proposition 31 of Part II of *Ethics*, Spinoza writes: “…since (in P30) we have demonstrated from this common property of singular things …” (and in proposition 30, “…[a body] is determined to exist and produce an effect from such causes as are also determined by others to exist and produce an effect in a certain and determinate manner, and these again by others, and so to infinity”). Such an idea can be summarized as a law of causal determination (*Ethics*, Part I, Axiom 3), identical in the sense of the property of extension to the laws of motion and rest common to all bodies (*Ethics*, Part II, post. 38, cor.) as indicated in Lemma 2 (see Letter 6).

2.3

Intuitively, it seems possible to equate these “properties” with the physical attributes of objects as specified by the predicates of language. Yet, judging by the examples we have introduced, we can conclude that Spinoza means to refer to universal laws as expressed by the general propositions of metaphysics or of science. But given that the propositions of science are not necessarily understood or shared by all human beings, it is reasonable to assume that the *common properties* of singular things, those ideas shared by all humans, are in fact universal premises which are not empirical-scientific laws, but rather something readily understood and accepted by anyone with empirical experience. Common ideas concerning these common properties are not literally ingrained from birth; they result from the amalgamation of human beings’ mental disposition with their empirical experience (much as Kant expressed it). These properties are evidently the universal laws of nature, held in common by all singular things, or by all modes of God’s attributes, and they constitute the eternal and infinite essence of God, of which the human mind is adequately conscious (*Ethics*, Part II, Posts. 46, 47). In the scholium to post. 47 of Part II, Spinoza declares: “From this we see that God’s infinite essence and his eternity are known to all.” The scholium goes on to clarify that there is a distinction between “adequate ideas of the properties of things” and “common notions”: “But that men do not have so clear a knowledge of God as they do of the common notions comes from the fact that they cannot imagine God, as they can bodies ….” Hence it may be possible to conclude that common notions are connected to our manner of imagining bodies, and they constitute a link between sensory experience and the ideas the intellect creates for itself (cf: C. De Deugd, 1966:152 ff.).

Concerning the question of the manner in which we imagine or distinguish between bodies, Spinoza implies elsewhere (*Ethics*, Part I, post. 15, sch. 1): “…by body we understand any quantity, with length, breadth, and depth, limited by some certain figure.” Later in the same scholium, he says: “If someone should now ask why we are, by nature, so inclined to divide quantity, I shall answer that we conceive quantity in two ways: abstractly, or superficially, as we imagine it, or as substance, which is done by the intellect alone. So if we attend to quantity as it is in the imagination, which we do often and more easily, it will be found to be finite, divisible, and composed of parts; but if we attend to it as it is in the intellect, and conceive it insofar as it is a substance, which happens with great difficulty, then (as we have already sufficiently demonstrated) it will be found to be infinite, unique, and indivisible” (cf. Letter 12).

2.4

These distinctions are central to Spinoza’s philosophy. They separate nature as a universal from its states of energy, the infinite from the finite, the theoretical-universal postulates of science from the singular propositions of observation, and the fundamental principles of metaphysics from experiential perceptions of objects. For Spinoza, this entire dichotomy is linked to the perception of nature as a unity on the one hand, and a multiplicity of modes on the other. To overcome this aforementioned dichotomy is to find solutions to Zeno’s paradoxes, which are based on it (cf.: *The Principles of Descartes’ Philosophy*, Part II, post. 6, sch.).These paradoxes are tied to the problem of continuum and the possibility of its infinite division. Spinoza undertakes this discussion most prominently in Letter 12 (cf: M. Guéroult [1968]; *Principles* above; *Appendix Containing Metaphysical Thoughts*, Part II, Chap. 10; *Ethics*, Part I, post. 15, sch.).[[54]](#footnote-54) Afterwards, Spinoza arrives at a threefold distinction related to the division of the continuum or the perfection of nature: real, modal, and rational (cf.: *Metaphysical Thoughts*, Part II, chap. 5, and onward). After utterly refuting the Cartesian possibility of the existence of several distinct substances, it is impossible to speak of a real separation in nature. Therefore, it is impossible to separate a certain part of nature from the whole. Thus it is also impossible to conduct an experiment on an entirely isolated natural system. The system in which the experiment is conducted can only be subject to a modal differentiation by way of chemical or physical breakdown, or by way of an intellectual experiment which implements a solely rational differentiation. All distinctions applied to a substance, whether it is called eternal, infinite, or total perfection, are rational distinctions alone, and thus one must understand the relationship between a substance and its attributes. It might prove interesting to study the place of the infinite modes in this context. As infinite and eternal, they should be distinguished from a substance by intellect alone, as attributes are. Yet, being modes, they are apparently also differentiated modally, and it may be that this dual differentiation allows them to explain the finite modes’ contingence on a substance (see: Letter 12; *Short Treatise on God, Man and His Well-Being*, Chap. 1, 8, 9). The scholastic terminology in this context only serves to obscure the discussion, giving the impression of a theological discussion even when it is, in fact, ontological, related to the doctrine of knowledge, and ultimately germane to the philosophy of science: the infinite mode of extension is the laws of motion governing the extended material (see: *Short Treatise*, Chap. 1, 9 (1), (2)), and the interesting question in this context is how we know the laws of motion for physical-material bodies. The modal differentiation of components of matter occurs when these components are separated from one another by other natural forces, including by human activity, such as, for example, splitting an atomic nucleus into subatomic particles (cf: *Ethics*, Part I, post. 15, sch. 1; *Short Treatise*, Chap. 1, 2 (21)), or by “separating” them from matter, “the material substance” – extension, through annihilation of the specific mode, which is a certain energetic structure, despite the persistence of the energetic matter itself (see *ibid*). Distinctions of this type as applied to individual modes cannot be made by the intellect alone. For this, we need experience (Letter 10), or, in other words, a conception of the reality of things in relation to a specific time and place (*Ethics*, Part V, post. 29, sch. 1).

3

3.1

The solution I suggest relies on understanding *common notions* as modes of thought we use to perceive bodies through qualitative notions and notions of quantity and duration, but if these perceptions of singular things must serve as the basis for scientific measurement and calculation (Letter 6), then common notions must be related to *adequate ideas of the common properties of singular things.* The nature of this relationship is that both are common, and both apply to the part and the whole alike (*Ethics*, Part II, Posts. 37, 38, 39, 46). Yet the common properties of singular things are eternal, and their ideas should be conceived not with any regard to time, but rather “a certain species of eternity” (*Ethics*, Part II, post. 44). Common notions, on the other hands, perceive reality through abstraction, and as a kind of quantity subject to division (post. 45, sch. 1). Thus, they do not perceive it as an eternal and infinite reality. The question of whether this *abstraction* is the activity of the intellect (Reason) or the imagination (the imagining power) is as significant as it is complex, and Spinoza himself does not directly answer it; indeed, at times it seems that he purposefully leaves it open (Letter 12; *Metaphysical Thoughts*, Part I, chap. 1). I shall try to offer a solution to this problem when I deal with the problem of time. At this point it is sufficient to point out that one can think of measure and time in two ways: as subordinate to the “confused” ideas of imagined knowledge (that is, the imagination), or as subordinate to the adequate ideas of the common properties of singular things, which are the means by which the intellect obtains scientific consciousness of singular things. To wit:

I would think that notions derived from ordinary usage, or which explain Nature, not as it is in itself, but as it is related to human sense perception, ought neither to be counted among the chief kinds, nor to be mixed (not to say confused) with pure notions, which explain Nature as it is in itself. Of the latter kind are motion, rest, and their laws; of the former are visible, invisible, hot, cold, and as I will say at once, also fluid and solid, etc. (Letter 6, IV/28, line 10).

3.2

The issue is that, in order to formulate scientific theories, we need to be conscious of singular things, and such consciousness can be attained only through experience. Consciousness of singular things in another fashion is only possible through intuitive knowledge. But this too, as we have mentioned, is conditioned on universal scientific knowledge. Experiential knowledge might be subordinate to the imagination, and thus it also might be *confused* knowledge of singular things (even though such knowledge is generally sufficient for the purposes of distinguishing and identifying particulars in the context of daily life), or it might be subordinate to notions born of the intellect. In this context, we have two types of experiments: a) “experiments (the readily available and doubtful ones I have adduced), where we don’t know what Nature contributes and what things intervene”; and b) “[experiments] regarding which it is established with certainty what things are contributed” by *evidence* and *calculation* (Letter 13, IV/67, lines 12-16);[[55]](#footnote-55) (Oldenburg: “algebraic calculation,” Letter 33); “for it is by reasoning and calculation that we divide bodies to infinity, and consequently also the Forces required to move them. But we can never ‘confirm’ this by experiments” (Letter 6, IV/29, lines 14-17).

In order to carry out the scientific experiments that make scientific knowledge possible, we must set common notions below adequate ideas of the shared properties of singular things, which are, according to my interpretation, “the Mechanical principles of Philosophy,” the “Laws of Mechanics” to which all changes in bodies are subordinate (Letter 13). These are the laws of motion and rest, also defined as an unmediated infinite mode of the attribute of material extension (*Ethics*, Part I, Prop. 32, Cor. 2; Letter 63).

3.3

It seems that, for Spinoza, a more or less reliable depiction of nature would be as follows: nature is a continuous perfection which can be observed, and conceived of, through various aspects. Two such aspects are known to us as its “attributes,” namely thought and extension. When observing this continuum via the aspect of extension, which expresses the material characteristics of nature, the continuum is an energetic structure in which is made manifest the *law of conservation of energy*, “the ratio of motion to rest,” in Spinoza’s phrasing (Letter 32). But this continuum has energetic states (energetic crystallizations), and one state is differentiated from another by its unique energetic structure. These structures (modes) are to be found in the causal physical relationships between one another, with each influencing the next, undergoing partial change, and even ceasing to exist without the energetic matter itself disappearing, and without changing the fundamental quantity of the energy whose states they constitute (*Short Treatise*, Chap. 1, 2 (22); *Ethics*, Part 1, post. 15, sch. 1; Letter 12). It is perhaps difficult to explain this last assertion of Spinoza’s, but we might draw nearer to its meaning with an allegory: in a chemical reaction, compounds undergo change without changing their constituent atoms or the underlying energetic system, which is considered a closed (perfect) system. In a similar fashion, one can see every degree of matter’s modal disintegration and reconstitution. As Spinoza writes in Letter 50, “…it’s manifest that matter as a whole, considered without limitation, can have no shape, and that shape pertains only to finite and determinate bodies.” Human beings can distinguish separate energetic structures if they know their laws of motion and rest. We suppose that such a universal system of laws exists, and that it is *the mechanical principles of philosophy*, as mentioned previously and as formulated by Spinoza in *Ethics*, Part II, following post. 13; it is difficult to determine how we arrive at this knowledge of the laws of motion, but it is clear that such knowledge does not include these laws’ explicit scientific formulation, which can be grasped only by way of empirical experience and experimentation. To this end, we must try to differentiate the various states of energy – the finite modes of nature. In order to move from qualitative formulations of the infinite and eternal laws of motion and rest to a quantitative treatment of specific states of energy, or finite modes, we must *create* such notions as allow us to deal with finite sizes, or concepts of quantitative units, for otherwise there can be no possibility of measuring and calculating the motion of bodies and the forces acting upon them. Without *evidence and calculation*, science is impossible – or, in Galileo’s words, there is a need for the geometrization of motion.

3.4

Common notions are therefore the notions we create in the intellect in order to move from dealing with a qualitative concept of motion, which is universal and unvarying (infinite and eternal), to dealing with individual units of energy. Hence *common notions* are *quantitative*, notions of space and time of *numerical value* that enable us to measure phenomena. Armed with notions of space, time, and number, one can make the transition from the infinite continuum of nature to its finite modes (cf. *Ethics*, Part V, Prop. 29, *Note*; Letter 12; *Metaphysical Thoughts,* Part I, Chaps. 1, 4, 10; Part 2, Chap. 11).

The notions of space, time, and number are required for measuring individual phenomena and the motion of certain bodies, but they are insufficient for identifying and individuating these phenomena’s characteristics. Such identification can only be carried out by the senses and imagination, but this constitutes only an *identification*, not a description or explanation (for example, an identification of liquid bodies and the various types thereof – Letter 6; cf.: *Principles of Descartes’ Philosophy*, Part II, post. 6, sch. 1). Hence the usage of notions of space (the quantities of bodies – *Ethics*, Part I, Prop. 15, sch. 1) and time (comparing the motion of bodies against the certain cyclical motions of other bodies – *Ethics*, Part II, post. 44, cor. 1, sch.; *Metaphysical Thoughts*, Part I, Chap. 4). In order to deal with characteristics, such usage is dependent on individuation, carried out via the perception of the senses, thus bringing about an *integration of intellectual notions with the activity of the senses*, which constitutes a precondition for the expression of certain singular propositions, such as: “Body *a* crosses space *Si*, over the time *Tj*, under certain conditions *Ck*.” The speed at which the body travels is a *quantitative notion of motion*, expressed in notions of space and time (*Principles*, Part II, post. 6, sch. 1). These singular propositions cannot be directly deduced from the axioms or universal postulates of a scientific theorem, but without them no scientific theorem can be considered complete, nor can *explain* or *predict* the behavior of singular bodies (cf. Letter 33, from Oldenburg). To derive such propositions from the universal postulates of a theorem demands singular propositions that express an experiment’s “initial conditions,” which can be seen as the *fundamental propositions of scientific theories* (cf.: K. R. Popper, 1934, Ch. 1, 7, “empirical basis”). The simple form of “initial conditions” should be: “Body *a* exists in place *p* defined by the coordinates *Xe*, *Yf*, and *Zg*, and at time *t* it began to move in direction (*Xh*, *Yi*, *Zj*) *q*.

3.5

Since Spinoza’s interest is in the “first properties” of bodies (shape, measure, size, motion, and rest – cf: Letter 11, and also Galileo’s view, W. R. Shea: 1972: 100 ff), or, in other words, in features and relationships that are expressible in terms of space, time, and number, and which, from a qualitative standpoint, apply and are common to all bodies; and since these notions apply equally to a specific body whether it is part of another body or is itself composed of parts, the expression of these notions’ laws of motion and rest is adequate, and they cannot be confused in the way propositions expressed in terms of those notions which explain nature as it relates to human senses are (Letter 6). Indeed, in order to express and encourage universal scientific theories, science demands not a *description* of bodies through sensory consciousness, but rather a *identification* of them strictly through *sensory perception*, aided by notions of space and time. These are the conditions for the formulation of singular propositions concerning the behavior of singular bodies, expressed through the notions of space and time, quantitatively measurable, and experientially testable (cf: W. R. Shea, 1972: 105, for a description of Galileo’s scientific method).

But the geometrization of the motion of bodies as it occurs within an abstract physical-geometric model is insufficient. Expressions relying on such a model are only general propositions expressing certain quantitative relationships, whereas a test of such propositions, and even a description of the abstract model, can be carried out only by means of certain individual experiments, such as freefall, motion along a slope, motion along a plane, and so on. Hence singular propositions that determine “initial conditions,” and are expressed with the aid of *common notions*, constitute the fundamental principles of our scientific theories, and we should thus understand Spinoza’s use of the phrase “the foundation of our reasoning” in Part II of *Ethics*. We should understand this “reasoning” as *deductive axiomatic theories*, and further distinguish between “the foundations of reason,” which are the universal axioms of the laws of motion and rest, and “common notions,” which allow us to formulate propositions according to initial conditions, which are “the foundation of our reasoning.” In conclusion, the transition from the infinite and eternal continuum of nature, expressed through the determinate and universal postulates of the laws of motion (“adequate ideas of the properties of things”), to the behavior of singular bodies, expressed through singular propositions, is achieved by use of *notions of space, time, and number*, the “*common notions*,” in whose terms we express the propositions of initial conditions, which are the empirical basis for science.

4

4.1

Until now I have more or less dogmatically presented my hypothesis on the nature of “common notions” as “beings of reason” which serve to define and verify the adequate ideas of the intellect and apply them to our sensory experience. These notions are *measure, time,* and *number*. They are applied to the idea of motion and rest, which is expressed in notions of quantity and duration, as abstractly perceived from eternal and infinite nature (Letter 12). The role of common notions in scientific theories is decisive, because it is only the laws of motion and rest that are relevant to the description of individual objects within the attribute of extension. Such a description cannot be made by means other than these notions, which make possible the mathematical-quantitative science of nature (cf: *Short Treatise*, Chap. 1, 9 (2); Letter 6). Spinoza knew the sciences of his time as they were presented by Kepler, Galileo, Descartes, Boyle, Huygens, and others (see especially the letters and list of books in Spinoza’s estate); his philosophy can be seen, in a broad sense, as a philosophy of science; thus, despite his use of traditional terminology (especially in *Ethics*), one should understand Spinoza’s terms as mainly technical ones expressing concepts belonging to the philosophy of science. In order to understand his philosophy, we must try to find the correct “translation” of these terms within the framework of his entire corpus, and the conditions under which he lived and wrote. In order to address certain issues in my interpretation of the terms “common notions,” I shall use the latter half of this article to briefly touch on the distinction between *ideas* and *notions*, as well as explain *measure*, *time*, and *space* as notions of the intellect or aids to the imagination.

4.2

In order to shore up my assertion regarding the distinction between “adequate ideas of the common properties of things” and “common notions,” I intend to demonstrate (among other things) that, for Spinoza, there exists an important methodological difference between “ideas” and “notions.”

In Chapter 1 of *Metaphysical Thoughts*, under the heading “By what modes of thinking we explain things,” Spinoza writes: “We also have modes of thinking which serve to explain a thing by determining it through comparison to another. The modes of thinking by which we do this are called time, number, and measure, and perhaps there are other besides. Of these, time serves to explain duration, number discrete quantity, and measure continuous quantity.” (A slightly different phrasing of the relationship between quantity and duration and notions of measure and time can be found in Letter 12.) The question is, what are the characteristics and status of these notions that I have equated with “common notions,” and what are the characteristics and status of notions in general in Spinoza’s philosophy? To the extent that we are discussing knowledge (which is our present concern), Spinoza distinguishes between at least three types of modes of thinking:

1. Ideas (*ideas*) – born of the intellect, they represent real things that exist outside of the intellect and are distinct from the ideas themselves: substance, infinite modes, and finite modes.
2. Notions (*notiones*) – “beings of reason,” which, unlike ideas, have no corresponding real objects, and are thus neither true nor false, though they can be *adequate*, meaning clear and distinct. (*Ethics*, Part II, Def. 4; Prop 38.; Letter 60).
3. Perversions of the imagination (*fictione*) – created by a combination of terms indicating images related to sensory experience, when such a connection is made in a way contingent not on reason, but on the mind’s laws of association (which Spinoza, in this context, called “will”). A universal image made through this mode has no corresponding object external to the mind. (*Metaphysical Thoughts*, Part 1,chap. 1, …; *Ethics*, Part I, Prop. 33, Corollary 1; Part II, Prop. 44, sch. 1; Part IV, Preface).

We shall not deal with this third mode of thinking here, mainly because it is not our primary concern. Spinoza himself does not deal with it at length, and the explanation he gives it in *Metaphysical Thoughts* is still Cartesian. Such an issue demands separate study, which necessarily falls outside the framework of this article.

What unites the latter two modes of thinking is that neither has any corresponding external object outside the mind they describe (*Metaphysical Thoughts*, Part I, chap. 1). The difference between them is that the second is *adequate*, meaning clear and distinct, while the third does not allow for the existence of any clear and distinct notion (*ibid* p. 166).

4.3

What is this “being of reason” that I identify with “notions” in distinguishing them from “ideas”? Spinoza’s answer is as follows: “…a Being of reason is nothing but a mode of thinking, which helps us to more easily retain, explain, and imagine the things we have understood” (*Metaphysical Thoughts*, Chap. 1, lines 30-33). (We will discuss the connection between the intellect and the imagination, which seems strange in this context, when he deals with the notion of time. Cf.: *Ethics*, Part II, Prop. 44, sch. 1; Letter 17). One must distinguish between the status of beings of reason and their manner of creation, or, as Spinoza frequently says, the manner in which the “intellect creates notions for itself” (Letter 37). With respect to the status of these beings, it is clear to Spinoza that they are modes of nature belonging to the attribute of thought, but the question is how they should be distinguished from the other modes of thought, particularly ideas. Spinoza thus responds:

Nor do they speak less improperly who say that a being of reason is not a mere nothing. For if anyone looks outside the intellect for what is signified by those words, he will find it to be a mere nothing. But if he means the modes of thinking themselves, they are indeed real beings. For when I ask, what is a species, I seek nothing but the nature of that mode of thinking, which is really a being and distinguished from another mode of thinking. Still, these modes of thinking cannot be called ideas, nor can they be said to be true or false, just as love cannot be called true or false, but [only] good or bad. (*Metaphysical Thoughts*, Chap. 1, lines 10-19).

Spinoza clearly makes a distinction here between notions and descriptive assertions and those that are not descriptive, such as distinctions of adequacy, validity, good and bad, and what is generally considered to belong to the normative realm. Under the heading “Why beings of reason are not Ideas of things and are nevertheless taken to be ideas,” Spinoza asserts: “So it is evident that these modes of thinking are not ideas of things, and can not in any way be classed as ideas. So they also have no object that exists necessarily [cf.: *Ethics*, Part I, Post. 33, sch. 1: “by reason of its essence or by reason of its cause,” in other words, substance or mode] or can exist. Moreover, the reason why these modes of thinking are taken for ideas of things is that they arise from the ideas of real beings so immediately that they are quite easily confused with them by those who do not pay very close attention. So these people also give names to them, as if to signify beings existing outside our mind…” (*Metaphysical Thoughts*, Part I, chap. I; Letter 12).

Support for my assertion regarding the distinction between notions and ideas can be found in Letter 9, which concerns definition. Here Spinoza makes a distinction between definition of the first kind, which explains a thing as it is outside the intellect, and, because it has a specific object in mind, is necessarily true; and a distinction of the second kind, which explains a thing as we conceive it, and thus must be adequate and free of logical contradictions – but it cannot be either true or false.

In several places, Spinoza refers to common notions as arising from our intellect via images of bodies (*Ethics*, Part II, Prop. 47, sch. 1), or, perhaps more precisely: he refers to notions we create in our intellect in order to compare objects drawn from our sensory experience with one another (*Metaphysical Thoughts*, Part I, chap. 1; *Short Treatise*, Part I, chapter II, Dialogue, section 12; Letter 37, and elsewhere).

With the aid of these notions, we can divide bodies into parts, and measure their relative size, weight, and motion (*Metaphysical Thoughts*, Part I, Chap. 4; Letter 12). These notions do not describe anything real in the bodies themselves, but through them one can formulate assertions regarding their energetic structures, as well as the laws of science governing the modes of the attribute of extension.

4.4

The category of *notions* does not just include “common notions,” but other classes of notions as well, which are explained alongside the common notions in the first scholium to proposition 40 of *Ethics*, Part II: “second notions,” “transcendental terms,” and “universal notions.” We shall not deal with the two latter groups here, save to mention that for every notion, there are two types of usage: proper and improper. It is important to understand, first of all, that they are merely notions, and not ideas; and second, the appropriate context for their employment. “Transcendental terms,” to the extent they are fittingly defined and clarified, are appropriately employed in the context of metaphysics. Without such clarification, any usage of such terms in other realms of knowledge is confused. “Universal notions,” to the extent they are defined by the ideas of motion and rest with recourse to the common notions, are appropriately employed in the context of science (that is, knowledge of the second kind). Thus, through nominal definitions, they become scientific notions (cf: *Metaphysical Thoughts*, Part I, chap. 6). “Second notions” are considered logical notions, although I am unaware of any point at which Spinoza explicitly discusses them. In section 10 of the Dialogue in *Short Treatise* Part I, chapter II, Spinoza writes, “the whole is a second notion, which is no thing in Nature, outside human thought.” This same assertion reappears in Part I, Chapter II, section 19 when Spinoza defines “part” and “whole” as beings of the intellect alone. (Compare the discussion in Letter 32 of the relativity of the notions of part and whole in the example of blood and its parts, 161-162). In *Metaphysical Thoughts* Part I, chapter 5, Spinoza deals with a group of notions including “Opposition, Order, Agreement, Difference, Subject, Adjunct, and whatever others are like these.” (Here we once again see Spinoza’s lack of consistency in using the terms “notions” and “ideas,” despite his emphasis on the distinction between them in the discussion of content (p. 117). It seems that one could naturally include the notions “part” and “whole” in this group. Spinoza emphasizes that the source of this group of notions, too, is in the comparison of different objects with one another.

The question is: what is the justification for separating this group from the group of notions I identify with “common notions”? At first glance, one can see that the difference between them is between the qualitative notions of logic and other, mathematically quantifiable, notions belonging to science and nature. Furthermore, we see that in the discussion of “adequate ideas of the common properties of things” and “common notions,” Spinoza has already made use of the notions “part” and “whole,” and it is reasonable to assume that these notions deal with the relationship between notions and assertions, and between notions and objects and phenomena. Hence they should be understood as notions belonging to the discussion of language, and not that of the objects themselves – in other words, they must be logical or semantic notions.

From the entirety of the discussion thus far, I conclude that, in modes of thinking, one must distinguish between *notions* and *ideas*, and thereby also between *common notions* and *adequate ideas of the common properties of things*. Thus one must also distinguish different groups of notions by the contexts in which they are appropriately used, as well as by their function within these contexts.

5

5.1

Though I have argued that the common notions are those of space, time, and number, and that, following Spinoza, *beings of reason* belong more properly to the *intellect*, we must still address the difficult question of how to reconcile these assertions with Spinoza’s statement that “Measure, Time, and Number are nothing but Modes of thinking, or rather, of imagining.” Spinoza’s argument is that the duration and quantity of individual modes can only be perceived by the imagination, and that the notions of space, time, and number are “only aids of the Imagination.” “From what has just been said,” Spinoza concludes, “it is sufficiently evident that neither Number, nor Measure, nor Time (since they are only aids of the imagination) can be infinite. For otherwise Number would not be number, nor Measure measure, nor Time time” (Letter 12, IV/58, lines 16-18, IV/59, line 1; cf.: *Ethics*, Part II, Prop. 44). The solution to this dilemma involves analyzing Spinoza’s conception of the “imagining power” (the imagination), and distinguishing between *imagined movements* as perceived by the *senses* and *real movements* as perceived by the *intellect* (alongside a new interpretation of the sensorily-perceived motion).

5.2

This last problem is related to the development of mathematical-quantitative modern science. This type of science endeavors to explain the motion of bodies by describing the forces acting upon them: measuring their size and direction via the infinite division of bodies through numbers. Thus it also tries to arrive at a calculation and proof of motion as defined by notions of space and time. (Compare, for example, Letter 6). The need for an intellectual description of real motion emerged in modern science with Copernicus, and found its resolution in Galileo’s universal theory of motion (terrestrial and celestial), as part of a struggle and debate with Aristotelian/medieval science, which erroneously explained motion as it is “perceived by the senses.” The problem is connected to the example of a body in freefall from the top to the base of a tower, and to the description of such motion. An observer of the falling object “sees” it falling as if in a “straight” line. This faulty perception was used to refute the Earth’s rotation around the sun (and along its own axis). Galileo argued that because the observer “participates” in the motion of the Earth, they cannot perceive the real motion of the falling object. Such motion can only be perceived by observation from a viewpoint external to the moving Earth. To illustrate this possibility, Galileo described the fall of an object from the mast of a rocking ship, both as viewed from the ship and as viewed from land, a frame of reference external to the ship (Cf: P. K. Feyerabend, 1975).

I mention Galileo’s experiments because I think that Spinoza was confronting this same problem of modern science when he gave precedence to the description and intellectual explanation of real motion over the motion of bodies as “perceived” by the senses. Sensory perception alone shows us a body falling in a straight line, shows us “the sun taking the same course through the sky,” “about 200 feet away from us,” and other such impressions; when we are accustomed to seeing things as associated with one another in the past and present, we imagine that they will remain so associated in the future (*Ethics*, Book II, Post. 35, sch. 1; Post 44, sch. 1). Sensorily perceived motions and objects are contingent on the structure of the senses, our position relative to the thing being perceived, and the mental laws of association that provide us with our impressions of things and phenomena. This being the case, we cannot truly perceive any motion in which we “participate,” nor any motion slower or faster than a certain range of speeds, nor the true dimensions or proximity of any object beyond a certain point or size – and yet, we allow our sensory observation of things’ past behavior to inform our predictions of their future (cf.: Letter 6). In order to explain the motion of bodies, such as an object in freefall from the top to the base of a tower, we must precisely measure, with reference to another body that can, through measurement and numbers, be divided into an infinity of parts, the distance from the object’s origin to the base of the tower and the height of the tower itself; so too must we measure and calculate the various forces acting on the object in motion (the force of Earth’s gravitational pull, the various forces generated by the movement of the Earth itself, etc.) and, from this data and the determinate laws of motion, *prove* the route taken by the real motion of a body. It seems that Spinoza had this in mind when differentiating those notions that explain nature in relation to human senses from those purer notions that explain nature as it truly is.

This, then, is Spinoza’s distinction between the first kind of knowledge, rooted in sensation and imagination, and the second, namely science, based on notions created by the intellect, and on the universal and determinate laws of motion and rest, which can be used to prove the real motion (“Nature as it truly is”) of bodies.

5.3

We have yet to consider the status and role of common notions within the theoretical framework, and scientific explanation, of motion. There can be no quantitative description of bodies’ motion without measure, time, and number. Yet even if these notions are aids to the power of imagining, they cannot be, per our premise, pure notions enabling a true description of nature. The question is whether measure, time, and number should be understood as notions of the power of imagining, or notions of the intellect. In Letter 12, IV/54, line 12, Spinoza explains that we can attend only to the modes’ essence in and of itself, meaning abstractly, without regard to the modes’ associations with the order of nature as a whole. In this case, we cannot conclude from the fact of their existence how long they might continue to exist or when their existence might cease, for such a conclusion is contingent on an infinity of variables that we are incapable of knowing or calculating – that is, it is contingent on the general order of nature. If we could know all these variables, we would be in a state of “superhuman intellect,” which Laplace suggested was an ideal of natural knowledge: “nothing would be uncertain for it, and the future, like the past, would be present to its eyes.” In Spinoza’s terms, one could say that we might know in which constellation or at which stage of theoretical determination the various modes would be created, fade away, or undergo change, without any need for quantitative notions of space, time, and number. (It may be that even this description is too “human,” considering that in *Metaphysical Thoughts* Part II, chap. 7, I/261, lines 20-22, Spinoza asserts that “God has never had a potential intellect, nor does he conclude anything by reasoning”). And so, because we cannot perceive the whole of nature as eternal, we must perceive the individual modes with a certain degree of abstraction from universal nature.

From all this it is clear that when we attend only to the essence of Modes (as very often happens), and not to the order of Nature, we can determine as we please their existence and Duration, conceive it as greater or less, and divide it into parts—without thereby destroying in any way the concept we have of them. But since we can conceive Eternity and Substance only as infinite, they can undergo none of these without our destroying at the same time the concept we have of them. (Letter 12, IV/55, lines 4-11).

We can thus conclude that if we understand the notions of space, time, and number “in their true sense” as notions that define and measure motion, then they do not destroy any notions (or, perhaps more accurately, *ideas*) of the modes that we explain through notions common to the laws of motion (*Descartes’ Philosophy*, Part II, post. 6, sch.; Letter 12, IV/55). In Letter 17, IV/77, lines 15-20, Spinoza teaches us that the imagination can accord to either the constitution of the body (to imaginary images) or to that of the mind (soul).

We see that the imagination is also determined by the constitution of the soul alone; for as we find by experience, it follows the traces of the intellect in everything and links its images and words together in order, as the intellect does its demonstrations, so that we can hardly understand anything of which the imagination does not form some image from a trace.

This description of the imagination as subordinate to the intellect, rather than to the senses or bodily constitution, is highly significant. In this manner, it may be possible to resolve what appears to be a dual meaning arising from Spinoza’s description of common notions in one instance as “modes of the imagination,” apparently related to sensory perception (which is a source of confusion), and elsewhere as “modes of thought,” apparently related to the imagination, as acted upon by (and according to the laws of) the intellect. In this way, we can understand under which conditions the notions of space, time, and number can serve as suitable grounds for the formulation of scientific theories: these conditions exist in relation to the imagination, itself related to the intellect, and allow for the creation of abstract models to suit science’s needs. In order to consider the real motion of a falling body, we must imagine ourselves as if stationed in a frame of reference external to the Earth. That which can be practically accomplished from an observation point atop a rocking ship – namely, modally differentiating the observer from the system-in-motion in which a certain body experiences freefall – could not have been accomplished (at least, not at the time) had the rocking ship instead been the Earth, the sun, or other such bodies. Hence a thought experiment is necessary – or, in other words, a rational distinction between the observer and the moving Earth. Moreover, we could not even consider the model of the ship as a demonstration of the problem without first imagining a frame of reference independent of the Earth. But we remain reliant on the notions of time, space, and number as demonstrated by the intellect-related imagination for any description of modal relationships between modes in nature. Such a description will always be relatively abstract, meaning it will be a model of bodies and their relationships in nature, and thus it will always deal with rational distinctions as well (freefall). Hence, if such notions cannot be understood as “pure,” the possibility of the science of nature, and knowledge in general, is cast into doubt.

6

In *Ethics*, Part V, Prop. 29, *Note*, Spinoza argues that “we conceive things as actual in two ways: either insofar as we conceive them to exist in relation to a certain time and place, or insofar as we conceive them to be contained in God and to follow from the necessity of the divine nature.” It seems to me that he is discussing the two kinds of true knowledge: *physics* and *metaphysics*, upon which is founded *ethics* in its most expansive sense (Letter 24). Likewise, Spinoza indicates that “[The first axiom of Part IV] concerns singular things insofar as t*hey are considered in relation to a certain time and place” (Ethics, Part V, Prop. 37, Note*), meaning that it belongs to physics, and Spinoza takes pains to remind us of this when dealing with metaphysics, making sure to emphasize the difference between them. I conclude from this that Spinoza understood that if scientific knowledge is possible, it because we can describe singular things quantitatively, with the notions of space and time.

Per Spinoza, scientific knowledge is also universal because it disregards and abstracts certain properties of singular things. When it abstracts the images of things as perceived by the senses and the imagination, through their notions (Letter 6), it becomes truer and more objective – but, being abstract, it is deficient and cannot describe the essences of singular things as they might be understood through intuitive knowledge. But because knowledge of a thing’s essence is contingent on the development of scientific knowledge, which itself is contingent on the common notions, our quest to extract knowledge is left to rely on two argumentative methods. The first distinguishes between two kinds of notions of space, time, and number: a) subjective notions related to sensory perception and to the associations made by the imagination, which vary from one person to the next; and b) objective notions common to all who accept their definitions and act within a scientific framework. The most pressing problem with this alternative is that Spinoza defines common notions as being common to all human beings, not merely “to those who have no prejudices” (*Ethics*, Part II, prop. 40, sch. 1). The second alternative, implied in Letter 17, encourages us to distinguish between two (or possibly three) kinds of imagination.

The common notions are in fact acceptable to all. But it is only when they are notions born of an imagination subordinate to the intellect, aiding in the determination and quantification of the universal laws of motion, that they can help us describe the motion of bodies as it truly is in nature. Perhaps it is to this property of the imagination that Spinoza refers in his assertion that if “[the mind’s] faculty of imagining depended only on its own nature, i.e. (by ID7), if the Mind’s faculty of imagining were free,” we would count it among the mind’s virtues, not its vices (*(Ethics*, Part II, prop. 17, sch.). It seems to me that this is the state of affairs Spinoza describes in Letters 12 and 17.

Thus I suggest seeing imagination as a faculty of the mind whose purpose varies according to its reliance on sensory consciousness, rational consciousness, or intuitive consciousness. The imagination has a seemingly significant role in the framework of intuition integral to knowledge of the third kind, but a no less significant one in creating abstractions of reason, which are necessary for scientific knowledge.

Einstein writes about the constructive-speculative character of science and sees conceptual-scientific creation as unlimited (Einstein, 1949: 21); Bunge sees the modern scientist as a “creature that builds and tests models” (Bunge, 1969: 216).

I believe that within this Galilean conceptual framework, Spinoza understood science as universal knowledge, and as an abstraction based on both “adequate ideas of the common properties of things” and “common notions.”

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# Chapter 4

# Where Would Spinoza Have Stood in the Einstein-Bohr Debate on the Scientific Representation of Physical Reality?

## **Introduction: The Quest to Understand the Scientific Representation of Reality**

Bohr tells us that in a discussion with Einstein at Princeton in 1937, regarding the more philosophical aspects of the problems of modern science, they did not get beyond a humorous debate on “which side Spinoza would have taken if he had lived to see the development of our days” (Bohr 1949, 237). It is widely known, though, that an important element in the Einstein-Bohr controversy about modern science was their ongoing debate concerning intrinsic philosophical-epistemological issues (e.g., Bohr 1949; Einstein 1949, 683-4).

Hence, the primary focus of this chapter is on epistemological problems concerning the interpretation of modern science within the framework of evolutionary epistemology. Elsewhere, I interpreted Spinoza’s theory of knowledge as evolutionary epistemology (Nesher 1987a, chap. 1), and I consider his philosophy of nature (as distinct from “natural philosophy”) as the best basis for the philosophy of science. The quest here is to understand science as one of the human cognitive modes of representing reality. I shall analyze the epistemological controversy between Einstein and Bohr in regard to the interpretation of scientific theories and their capacity to represent reality, and I shall then go on to compare their philosophical positions with Spinoza’s views.

I believe that through philosophical discussion of the epistemological problem of the scientific representation of reality we can reach a better understanding of our scientific theories. This is expressed forcefully by Einstein:

The reciprocal relationship of epistemology and science is of a noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683-4; cf. Bohr 1958, 1)

We may understand the philosophical controversy between Einstein and Bohr in light of the present-day debate between those who adhere to metaphysical realism and those who subscribe to phenomenalism (or internal realism), respectively (cf. Rorty 1982, chap. 12; Putnam 1981, 49-54; 1990, chap. 1; Dummett 1982; Devitt 1984). This problem will be analyzed within the framework of Spinoza’s naturalism (as distinct from physicalism) and Peirce’s logic of cognition (semiotics). I believe that Spinoza and Peirce reject both metaphysical realism and internal realism and take a third, pragmaticist perspective, which I call representational realism (cf. Nesher 1986, 1987a). There are various ways to explicate realism and therefore *anti-*realism. In the present chapter, I would like to give a wider definition of realism than Dummett’s and to suggest that the pragmaticist’s conception of realism is grounded in two central tenets that must be accepted together: (1) the ontological tenet, that reality exists independently of its cognitive representation; and (2) the epistemological tenet, that this reality is represented by our cognitive minds: we know it. Dummett accepts (1), but instead of (2) stipulates a stronger condition: “that reality renders each statement in the class determinately true or false, again independently of whether we know, or are even able to discover, its truth-value” (Dummett 1982, 55; cf. Putnam 1981, 49ff.). The question is whether, according to my explication of realism, the “metaphysical realist” and the “internal realist” are realists at all. The metaphysical realist naively accepts the existence of external reality (1), but since the truth of a statement is determined by reality independently of whether we know it or not, it is not clear whether humans can know that there is such an external reality (2). The internal realist, by rejecting the possibility of knowing reality outside our cognitive states (1), must also be skeptical, like Hume, or categorical, like Kant, about the existence of this mysterious transcendental reality (2).

Realism seeks to answer the question whether we (humans) can know that there is reality external to our cognitive experience, and if so, what is its nature? This is the distinction Peirce makes, and it occupies the middle ground between *negative* and *positive* knowledge of reality. The first is grasped in the way we learn that there is something different from us; the second in the way we learn that there is something which corresponds to our cognition. Negative knowledge of reality begins evolving in the child’s awareness of ‘the real’ in cases of error or ignorance, and we are aware of it whenever a fact surprises us or frustrates our expectation (cf. Peirce, 5.233-234, 5.311). In this case, through awareness that what we expected turned out to be an erroneous subjective cognition, we also know that there is something independent refuting our ideas, and “to this we give the name of the *real*” (Peirce, W3:8). Popper, with his celebrated theory of *refutation*, accepted only the negative concept of reality, while in his later writings he tried in vain to show how we should leap from frustrated hypotheses into *eternal truths*.

According to Spinoza and Peirce, we can achieve representational knowledge of external reality only through our causal interaction with such reality. Our positive description of real objects is based on our “natural instinct for truth,” namely, reacting positively to these objects in the perceptual process (cf. Spinoza 1985, TIE 31; Peirce, 7.220 [1903]; Nesher 1989; aboveTh, I:7).

Peirce understands reality as being *independent* of human cognition, since without being forced from without, how can investigators progress toward one and the same conclusion? The question for Peirce is whether we can cognize or positively describe “some external permanency,” or “something upon which our thinking has no effect” (5.384), which is independent of “the thinking part of the mind” (7.338). In what sense is there a reality that is independent of the human thought thatrepresents it? (cf. Peirce, 5.553-5.564). And how does “reality” depend on (connect to) our thought or other cognitions by which it is represented? (cf. Peirce, 1.578, 7.336‑345, 5.405‑410; on the controversy between Einstein and Bohr about the definition of reality, cf. Stagnate 1983, 7.2).

**2. The Epistemological Controversy between Einstein and Bohr about Physical Reality and Its Scientific Representation**

The question of the representation of external physical reality became even more problematic after the discovery of atomic (“quantum”) reality, which revealed the behavior of a new category of objects that deviated from the behavior of the objects described by classical physics (cf. Bohr 1963, 60; 1958, 71-73; 1949, 222-23, 233-34 [1935], 237-38; Bohm and Hiley 1993, chap. 1, 176). Hence, we must take great care when using the concept of *object* for the entities of the atomic domain in order to avoid confusing them with the classical concept of *object* with its classical properties (cf. Bohr 1934 [1927], 53; Einstein 1936, 71). This predicament calls for a new theoretical explanation, with non-classical concepts to refer to the non-classical properties of this revealed atomic reality, or a new philosophical epistemology, or perhaps both (cf. Bohr 1934 [1929], 92ff.; Einstein 1949, 683-84; Heisenberg 1958, 128ff.).

Although nature appears to be a continuous whole, the classical distinctions between discrete objects and between the subject/observer and object observed become obscure and problematic at the quantum level (cf. Bohr 1934 [1927], 53-57, 62-68; 1958, 91-93).

The philosophical controversy concerning our understanding of the wholeness of the atomic (“quantum”) world is presented by two seemingly unsatisfactory opposing positions: Einstein’s “classical (atomic) realism” and Bohr’s phenomenalism (comp. Schrödinger 1935, 158-160). This is indeed a very crude characterization of both epistemological perspectives, but it is essentially correct. In regard to Bohr’s philosophy and epistemology of science, there are many different interpretations, and the question is whether we can ascribe a third position to his worldview, a sort of realism which is neither classical realism nor anti-realism (e.g., False 1985, 21-27; Honker 1987, chaps. 5, 8; Murdoch 1987, 195-199 -245; Faye 1991, chap. VIII).

When faced with these two philosophical positions, we encounter a difficult dilemma: either having to accept the classical separation of objects, even for the atomic domain, in order to maintain external reality and its representation independent of the observer and their measuring instruments, or having to retain non-separability of the wholeness of experimental phenomena and therefore having to renounce the idea of a reality that is independent of cognitive representation. These differing philosophical approaches fueled the controversy between Einstein and Bohr for many years, and their analysis is important for clarifying the epistemological problem of the scientific representation of reality (cf. Einstein 1949, 11-13, 672-674; Bohr 1935, 145-6; 1958, 2; 1963, 1-7).

In his article “Discussion with Einstein on Epistemological Problems in Atomic Physics” (1949), Bohr reiterates their disagreement regarding the understanding of modern science:

Not least through a new discussion with Einstein in Princeton in 1937, where we did not get beyond a humorous contest concerning which side Spinoza would have taken if he had lived to see the development of our days, I was strongly reminded of the importance of utmost caution in all questions of terminology and dialectics. (Bohr 1949, 236-237)

My understanding of Spinoza’s philosophy leads me to the conclusion that he would not have taken either side. This is because Einstein and Bohr are both right and wrong in respect of Spinoza’s conception of reality and its cognitive representation. Spinoza theorized nature as a continuous whole and suggested that the mental and physical domains have such a specific connection that despite the wholeness of nature there is no *modal* causal relation between cognitive minds and physical systems, but only a *real union* of mind and body such that the represented physical reality remains independent of its cognitive representation (cf. Nesher 1987a; aboveAs , chap. I).

Therefore, with respect to Spinoza’s philosophy, Einstein is correct in claiming that the represented reality is *independent* of its cognitive *representation* (Einstein 1949, 81); but he is wrong in his conception of reality as *absolutely* independent of its measurement, which, nevertheless, can be represented *completely* by scientific theory (Einstein et al. 1935). Einstein’s position is expressed clearly in some programmatic paragraphs of the famous paper he wrote with Podolsky and Rosen, in which the three authors presented what has become known as the Einstein-Podolsky-Rosen paradox (EPR). According to their thought experiment:

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. (Einstein et al. 1935, 138 (EPR)

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.T (Einstein et al. 1935, 138 (EPR)

While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.” (Einstein et al. 1935, 141 (EPR); cf. Einstein 1949, 666-72, 681-84; 1950, 276-77; comp. Heisenberg 1927, 66; cf. Fine 1993, 266-271).

These formulations, especially the first, emphasize “this criterion... of recognizing a physical reality,” and others (e.g., Einstein 1936, 1949, 1950) remind us of Peirce’s nominal definition of reality, according to which reality corresponds to the ideal scientific theory that represents it completely (5.407f.). Broadly speaking, we could interpret these conceptions of *complete theory* and *reality* as nominal (ideal) definitions; but if only ideal theory is complete and true, there are no ideal theories; accordingly, the controversy with Bohr becomes vacuous because not only quantum theory, but every scientific theory is incomplete to some degree (cf. Bohr 1935, 148; 1949, 230-235; Einstein 1949, 83, 666-674; Schrödinger,1935, 153; Born 1950 in Einstein 1950, 277). Nevertheless, it seems reasonable to understand that , in arguing for the *incompleteness* of quantum theory, Einstein means that it is indeterministic and unrealistic in respect of its representational function of “individual systems.” According to this interpretation, Einstein believed that quantum theory violates the basic characteristics that are *required* of physical theory, namely *determinism* and *realism*, but that even in atomic physics this kind of *completeness* of theories can be achieved (cf. Einstein 1949, 13, 672-3; 1950, 276-7; Folse 1985, 143ff., 222; Fine 1993, 259(E), 264ff.).

Bohr is also right and wrong regarding Spinoza. He is right in asserting that “no sharp separation can be made between the independent behavior of the objects and their interaction with the measuring instruments which define the reference frame” (Bohr 1949, 224), namely in the physical domain; but he is wrong in that by looking for complete certainty in science, despite the knowledge that we do not have complete control over the relation between the measuring instrument and the measured system, he avoids speaking about the measured system existing independently of our cognitive representation. The result is that he restricts “physical reality” to our *phenomenal* description of the “whole experimental arrangement” and thus endorses a direct interaction between the mind and objects (e.g., Bohr 1949, 233-34 [1935, 146ff.]; 1958, 72-73). Bohr’s phenomenalism (and instrumentalism) is expressed clearly in the following:

In our description of nature, the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience. (Bohr 1934, 18; cf. 1934, 4)

In terms of objective description, or the unambiguous communication of experience (e.g., Bohr 1963, 3), it is indeed more appropriate to use the word phenomenon to refer only to observations obtained under circumstances where their description includes an account of the whole experimental arrangement. In such terminology, the observational problem in quantum physics is deprived of any special intricacy (Bohr 1958, 73; cf. Bohr 1939, 24; comp. Espagnat 1983, 58).

Strictly speaking, the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectations about observations that are obtained under well-defined conditions and specified by classical physical concepts, i.e., common-sense communicable physical concepts (Bohr 1958, 73; 1963, 60; cf. Bohr 1958, 3, 68, 71; 1949, 222-23, 233-34; [1935], 237-38).

Although Bohr considered the effect of the measuring instrument on the measured object (system), and even of the human body on the former, as parts of the experimental perceptual-observational process, he despaired of the possibility of “unambiguously” describing their interactions in the physical realm (cf. Bohr 1958, 39; 1963, 3; Honner 1987, chap. 3.5). To avoid ambiguous (non-objective) language in the description of this uncontrolled relation, he invented the principle of “complementarity.” This allowed him to apply the mathematical formalism of quantum mechanics to different descriptions of separate experimental arrangements without relating the different results to an independent reality of a particle; hence “an unambiguous meaning can be attributed to such an expression as ‘physical reality’” (Bohr 1935, 145, 151; cf. 1958, 1-2; 1963, 2-7; Heisenberg 1958, 179).

According to Bohr, “the interaction between the measuring instruments and the objects forms an integral part of the phenomena” (Bohr 1963, 4). These phenomena are the “experimental arrangements,” and the “evidence about atomic objects obtained by different experimental arrangements exhibits a novel kind of complementary relationship” (Bohr 1963, 4). In quantum physics, evidence of different experimental arrangements “appears contradictory when combination into a single picture is attempted.” Bohr therefore concluded that even if there is such a reality that causally determines our phenomenal experience, we cannot have unambiguous (objective) knowledge of it. Thus, our “purpose is not to disclose the real essence of the phenomena” (Bohr 1934, 18). Yet the contradicting evidence can be accounted for by the notion of *complementarity*, which “exhausts all conceivable knowledge about the object” (Bohr 1963, 4). However, the question remains: What is this object? Is it experientially “in” the phenomena or transcendentally “behind” them? Let us analyze Bohr’s phenomenalism and his problem with the conception of the physical reality of science:

The epistemological problem under discussion may be characterized briefly as follows: For describing our mental activity, we require, on the one hand, an objectively given content to be placed in opposition to a perceiving subject, while, on the other hand, as is already implied in such an assertion, no sharp separation between object and subject can be maintained, since the perceiving subject also belongs to our mental content. (Bohr 1934 [1929], 96)

This discussion is about the “objectivity of phenomena” belonging to the quantum domain, which is different from the perceived classical material objects that are considered in the classical sense. Bohr expressed this epistemological problem in phenomenological terms, which echo Kant’s distinction between cognitive states (*Vorstellungen*) and their content or sum (*Inbegriff*), i.e., their appearance, which is viewed as the object (cf. Kant 1781-87, B236/A191, A105). Bohr follows Kant’s phenomenological holism, in claiming that there is no sharp separation between subject and object, since both the object and the perceiving subject belong to our mental content. Weizsacker draws attention to Bohr’s Kantian position, according to which objects are the “sum” or the “content” of the phenomena themselves:

The fact that classical physics breaks down on the quantum level means that we cannot describe atoms as “little things”. This does not seem to be very far from Mach’s view that we should not invent “things” behind the phenomena. But, Bohr differs from Mach in maintaining that “phenomena” are always “phenomena involving things,” because otherwise the phenomena would not admit of the objectification without which there can be no science of them. For Bohr, the true role of things is that they are not “behind” but “in” the phenomena. (Weizsacker 1980, 185)

This Kantian epistemological and ontological position – which was probably imparted to Bohr by Høffding, his philosophical teacher and friend – preceded the formulation of quantum theory and helped Bohr in its epistemological explanation (cf. Faye 1991, chaps. III, VIII). It is interesting to see whether phenomenology allows Bohr to show the completeness of quantum theory by using the notion of complementarily to explain the different contradictory results of distinct atomic experiments in respect of “the same atomic object” (cf. Folse 1985, 156ff.; Honner 1987, chap. 3.6). In explicating his concept of “phenomenon” and his phenomenological epistemology, Bohr writes:

It is certainly more in accordance with the structure and the interpretation of the quantum mechanical symbolism, as well as with elementary epistemological principles, to reserve the word “phenomenon” for the comprehension of the effects observed under given experimental conditions. These conditions which include the account of the properties and manipulation of all measuring instruments essentially concerned, constitute in fact the only basis for the definition of the concepts by which the phenomenon is described. (Bohr 1939, 24; in Folse 1985, 157-8; cf. Bohr 1958, 73)

The question is whether Bohr’s *atomic object* is the *cognitive content* of different experiments or whether he also had to invoke *external reality* beyond our “objectively” given mental content. Furthermore, with regard to the latter, is this atomic object a Kantian *transcendental object,* which somehow determines the experimental phenomena, while remaining outside our scientific knowledge? Or is it what the pragmaticist would call a *real external object* that is represented by such knowledge (comp. Folse 1985, 156; Murdoch 1987, 10.10)?

In an interesting analysis, Faye shows the development of Bohr’s explication of his concept of “phenomenon” in the course of his philosophical discussion with Einstein, and especially in the context of the EPR thought experiment. Thus, before 1935, Bohr’s definition of “phenomenon” was the atomic object to which different effects of the various experimental arrangements were ascribed as the object’s different properties, which assumes that the phenomenon is the same object independent of the experimental measuring arrangement. Later, he redefined “phenomenon” such that the special effects of different experimental arrangements are understood as “different types of quantum phenomena” (Bohr 1939, 22; 1958, 64, 71-74; 1963, 2-7; cf. Faye 1991, 191-95). Folse interprets Bohr’s position as a kind of realism that combines scientific phenomenalism with metaphysical realism:

Thus Bohr makes it obvious that such classical terms as “position” and “momentum” are “deprived of all meaning” apart from the context of their application to describe particular observation interactions of *phenomenal objects* as they appear in specific observational interaction.... He does not assert that the very notion of such an *independent reality* is itself without meaning or that there is no need to refer to such *atomic objects* in the description of observation as interaction. (Folse 1985, 156; my emphasis)

It is clear, therefore, that the *phenomenal* objects of different contrasting observations cannot constitute “the same object” to which all these different described properties are related (comp. Folse 1985, 164-5). The outcome must be this: ascribing the contrasting properties to a *non-phenomenal physical object* results in the same paradox of using *ambiguous language* to describe this *object*, which, because of this, cannot be the *objective scientific language* of science.

It is interesting to see that Bohr, in leaning on his *complementarity* thesis to avoid the phenomenal paradox of contrasting phenomena related to different experimental arrangements (e.g., particles or waves), had to introduce a new paradox between complementary “contrasting phenomena” and the underlying referred *atomic object,* which functions like the ideal “single picture” of the “atomic system,” or like “a consistent picture of the object under investigation” of classical physics, but without this consistency (cf. Bohr 1963 [1958], 4ff). The phenomenalist solution to this paradox, namely avoiding the ascription of contrasting properties to “the same atomic object,” is to declare this physical object to be *undescribable*. Thus, Bohr’s general epistemological lesson for the scientific enterprise as a whole is to restrict scientific knowledge to phenomenal objects only, and to maintain that *external reality*, though probably causing our experience, nevertheless cannot be the object of science; in Kant’s terms, it is unknown. This reality is at most an “abstract” *metaphysical presupposition*, namely a *transcendental object*. But if contrasting properties can be ascribed to such an object, how do we know that it is “the same atomic object”? (cf. Folse 1985, 244ff.) In this context, it is interesting to see how Folse defends Bohr’s standpoint as though adhering to realism, while admitting that it is basically a Kantian position, which combines “empirical realism” with the “transcendental object” (cf. Kant 1781-87, A370). This is actually achieved by accepting, as Kant did, Hume’s view that…

It is both impossible and unnecessary to expect that natural science justifies its realistic outlook by empirically demonstrating that there is a reality existing independently of experienced phenomena. (Folse 1985, 242-3)

Avoiding this Humean-Kantian position and adopting the Spinozist-Peircean perspective, we can see that *representing* external realities is exactly what science does by proving or justifying its theoretical claims about them (cf. aboveAs, I:6,7, II;4). However, owing to the problem of epistemologically explaining quantum theory, Einstein and Schrödinger disagreed over the intrinsic incompleteness of quantum mechanics. Honner explicates Bohr’s phenomenology and its difficulties with a comprehensive description of physical reality:

Bohr’s notion of objectivity differs from the classical account, however, in that he stresses that our descriptions of nature are not descriptions of independently existing realities, but descriptions of *our encounter with* such realities.... Quantum physicists, restricted to describing experiments in terms of the whole apparatus-system interaction, find themselves in the same position as people trying to describe in detail the totality of their mental activity, “since the perceiving subject also belongs to our mental content” [Bohr 1934, 96]. (Honner 1987, 146, 147; cf. Folse 1985, 154-167)

Thus, for Bohr “*physical reality*” is the “atomic observed phenomenal object.” Moreover, this observed atomic object is itself at best only a constellation of the phenomenal features of the measuring instrument, interpreted as the measured atomic object. Bohr, Einstein, and Spinoza’s different conceptions of physical reality are shown in the following scheme of the experimental arrangement:

**[1] The Experimental Situation: Observation-Measurement and**

**Represented “Physical Reality” for Einstein, Bohr, and Spinoza:**

Person Observing

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  | Measuring  Instrument |  | Phenomena  (Bohr’s Reality) | Mind |  |
| Measured | |  |  | |
| Object | |  |  |  | Human Body | |
| (Einstein’s Objective Reality) | | |  |  |  | |

Therefore, Bohr is essentially a phenomenalist in claiming that scientific knowledge constitutes only “descriptions of *our encounter with* such (independently existing) realities.” Accordingly, a description of human phenomenological experience is the only reality available to humans and this is exactly what Kant dubbed “empirical realism” (cf. Kant 1781-87, A370; comp. above, II:3, III:2; Honner 1987, chap. 5.2). It is interesting to note that in the above scheme Einstein’s conception of physical reality is separated from Bohr’s physical reality, namely what is outside or external to our experience. According to Einstein, we represent physical reality through our sensual encounter with the measuring instrument, but can describe this reality as being independent and undisturbed by its being measured. For Spinoza (if he had lived to see the development of modern science), there is a continuous interaction between the human body, the measuring instrument, and the measured object. Therefore, the measured object cannot be really independent and undisturbed, yet through this phenomenal encounter we can learn and represent, with relative truth, the relatively independent measured atomic and subatomic objects.

**3. Criticism of “Phenomenalist” and “Classical Realist” Epistemologies**

In this context, it is interesting to read Peirce’s criticism of the phenomenalist-positivist position of Comte, Poincaré and Pearson, while suggesting the pragmaticist alternative about scientific theories, verification, and reality:

An explanatory hypothesis, that is to say, a conception which does not limit its purpose to enabling the mind to grasp into one a variety of facts, but which seeks to connect those facts with our general conception of the universe, ought, in one sense to be *verifiable*; that is to say, it ought to be little more than a ligament of numberless possible predictions concerning future experience…. But Comte’s own notion of a *verifiable* hypothesis was that it must not suppose anything that you are not able **directly to observe**. From such a rule it would be fair to infer that he would permit Mr. Schliemann to suppose he was going to find arms and utensils at Hissarlik [according to his hypothesis about the city of Troy], but would forbid him to suppose that they were either made or used by any human being, since no such being could ever be detected by direct perception... Comte, Poincaré, and Karl Pearson take what they consider to be the first impression of sense, and they separate these from all the intellectual part of our knowledge, and arbitrarily call the first *real* and the second *fictions*. (Peirce, 5.597 [1903]; boldface added)

Along the same lines, Schrödinger argues against the doctrine of contemporary quantum mechanics, which holds that “models with determining parts that uniquely determine each other, as do the classical ones, cannot do justice to nature,” and therefore this doctrine requires a rejection of realism (Schrödinger 1935, 153). Schrödinger writes:

Reality resists imitation through a model. So one lets go of naive realism and leans directly on the indubitable proposition that *actually* (for the physicist) after all is said and done there is only observation, measurement. Then all our physical thinking thenceforth has as a sole object the results of measurements, which can in principle be carried out, for we must now explicitly *not* relate our thinking any longer to any other kind of reality or to a model. (Schrödinger 1935,157)

Thus, the framework of *complementarity* as “a ligament” of contrasting phenomena became a license for the *phenomenological* understanding of *physical reality*: “a radical revision of our attitude (‘the customary viewpoint of natural philosophy’) as regards physical reality” (Bohr 1935, 151, 149; cf. 1934, II, 93ff.; 1935, 145-6, 149-151; 1958, v-vi, chap. II; 1963, 1-6! -7; 1949, 209-211). Schrödinger put it wittily: “Bohr wants to complement away [*wegkomplementieren*] all difficulties” (Schrödinger, in Pais 1991, 425; comp. Einstein et al. 1935, 139: the alternative (2); Einstein 1949, 674; Schrödinger 1935, 153-155, -160; Davies and Brown 1986, 20-28; Espagnat 1983, chap. 3; Bohm and Hiley 1993, chap. 2). Bohm and Hiley make the same kind of criticism of Bohr’s phenomenology:

Bohr would never allow the type of language that admitted the independent existence of any kind of quantum object which could be said to be in a certain state. That is to say, he would not regard it as meaningful to talk about, for example, a particle existing between quantum measurements even if the same results were obtained for a given observable in the sequence of such measurements. Rather, as we have seen, he considered the experimental arrangement and the content (meaning) of the result to be a single unanalysable whole. (Bohm and Hiley 1993,18, 23; cf. Bohr 1949, 232-4 (1935); comp. Kant 1781-87, Bxxv-xxvi; cf. Bell 1986, 51; Stapp 1993, 63)

In the same vein, Espagnat criticizes this phenomenalist-operationalist approach, which restricts physical reality to operational statements and their perceived results in order to maintain certain and complete physical knowledge.

Indeed, strictly speaking, nothing is really certain to us except our operations. If science is required to be certain, then it can make only operational statements. However, operational assertions have no meaning except in reference to the community of the operators… If I demand that science be certain, then the notion of any scientific object whatsoever reduces completely and is totally exhausted by the notion of a given set of operations that *we* perform and of the results that *our* mind can perceive. (Espagnat 1983,130-131, cf. pp. 17-19; Schrödinger 1935, 153)

It is worth noting that from Bohr’s (and Heisenberg’s) phenomenalist interpretation of quantum mechanics, which does not distinguish clearly between human knowledge and its object, some physicists have inferred the effect of the former upon the latter (cf. Heisenberg 1927, 83). In this theoretical situation, Stapp went even further to conclude an “idea like” physical world, which is similar to the Leibnizian world of “Monads” (cf. Stapp 1991, chap. 11).

I propose that Einstein and Bohr’s philosophical mistakes, viewed from a Spinozist-pragmaticist point of view, are partly due to the fact that both were somehow influenced by Kantian philosophy, yet they embraced different aspects of it (cf. Einstein 1936, 61-62; 1949, 672-674; Margenau 1949, 249; Espagnat 1983, 63; Faye 1991, xix, 172, 197-211; Kaiser 1992). Einstein believed in the reality of *things in themselves*, which are completely independent of human activity, but which can nevertheless (miraculously) be known (cf. Faye 1991, xix, 180); and Bohr viewed physical reality as identical with experiential phenomena*, namely* dependent on human cognition (cf. Bohr 1934 [1929], 93, 103; Faye 1991, 211; Wheeler 1986, 59; Bohm and Hiley 1993, 16-19; Stapp 1993, 65, 117-119; Redhead 1987, 51). Thus, Einstein and Bohr were involved in “the so-called ‘deep truths,’” in Bohr’s terms, namely “statements in which the opposite also contains deep truth” (Bohr 1949, 240). For example, when Einstein speaks of the “incomplete description” of physical reality provided by quantum theory, he means the reality of things *in themselves*, while Bohr by “complete description” means the formalism of quantum theory as applied to the observed phenomena only (cf. Bohr 1935, 145f., 149-150; 1949, 222-224, 237-8; Schrödinger 1935, 153-54, 157-60; Einstein 1949, 681ff.). On the incompleteness of quantum theory from the realist (not the naive realist) perspective and its apparent completeness from the phenomenological standpoint, Schrödinger writes:

The rejection of realism also imposes obligations. From the standpoint of the classical model the momentary statement content of the ψ-function is far from complete; it comprises only about 50 per cent of a complete description [“the other half then remains completely indeterminate” - p. 132]. From the new standpoint it must be complete for reasons already touched upon at the end of Sect. 6 [p. 157]. It must be impossible to add on to it additional correct statements, without otherwise changing it; else one would not have the right to call meaningless all questions extending beyond it [as Bohr does]. (Schrödinger 1935, 159; comp. Einstein 1949, 668ff.)

Thus, returning to Einstein and Bohr’s philosophical positions in their 1935 controversy about scientific theories, their completeness, and the nature of physical reality, it seems to me that they are both somehow wrong. First, they are wrong because they uncritically rely on a basic “philosophical instinct” to believe in the completeness of scientific theories; but from Spinoza and Peirce’s pragmaticist point of view, there cannot be a complete (certain) description of external reality since human knowledge of it is limited and fallible. Moreover, Einstein’s “criterion of reality” is too strong since there is no *completely independent*, “undisturbed”, reality, and Bohr’s “conception of reality” is too restricted (devised to ensure *complete knowledge*) and thus his “phenomena” cannot describe *external physical reality* (cf. Schrödinger 1935, 153, 155, 157ff.; Espagnat 1983, 65ff.). However, I suggest that when a pragmaticist epistemology is adapted “we are nearing the goal where logical order to a large extent allows us to avoid deep truth,” where contradicting positions are both true and false (Bohr’s wording, 1949, 240). It is true that any representation of the atomic domain depends upon the mind (cognition), but we should not identify, as Kant and Bohr do, the *phenomenal-representation* with the *represented reality* (cf. above, I). To differentiate these two components, representation and physical reality, we should analyze the observational-measurement situation.

In a theory of representation, a distinction should be made between the role of the perceiver-observer who takes measurements and the relation between the measuring instrument and the measured object (system). The measuring instrument is what the experimentalist *perceives* and the measured system is what she or he *observes* through perception of the former. As Schrödinger expresses it:

The systematic arranged interaction of two systems (measured object and measuring instrument) is called measurement on the first system, if a directly-sensible variable feature of the second (pointer position) is always produced within certain error limits when the process is immediately repeated (on the same object, which in the meantime must not be exposed to any additional influences). (Schrödinger 1935, 158)

We can measure an atomic system only mediately when perceiving a medium-sized measuring instrument interacting with this system. If we cannot *perceive* the measuring instrument, we cannot experimentally *measure,* and thus cannot *observe* an atomic system even where an interaction between them occurs (comp. Bohr 1949, 209). However, we should be careful not to confound *perception* with *measuring operation* and understand that even perception and measuring operation are two components of *observation*. Hence, the measuring instrument can operate and affect the measured system without being perceived. Furthermore, we can perceive the measuring instrument when not in operation and therefore it cannot be said to affect the system to be measured. In perception (as part of the observational procedure) the relation of the mind to the measuring instrument is not mediated as such. This, of course, leads to a serious question as to whether “Schrödinger’s cat,” as an instrument measuring the decay of a radioactive substance, can be in an *indeterminate* superposition if it is always to be perceived. It seems that Schrödinger’s intention in his famous thought experiment is to show that according to the doctrine of contemporary quantum theory “an indeterminacy originally restricted to the atomic domain becomes transferred into macroscopic indeterminacy, which can then be *resolved* by direct observation”; but if there is an indeterminacy in the measuring instrument itself (the cat), there cannot be any observation of the atomic system. Hence, if this is the result of the quantum theory, it “prevents us from accepting as valid a ‘blurred model’ [i.e., quantum theory] for representing reality” (Schrödinger 1935, 157). Here Schrödinger distinguishes between the *incomplete* quantum theory, which he metaphorizes as “a blurred or poorly focused photograph,” and the *complicated* quantum reality itself, which he metaphorically describes as “clouds and fog patches” (cf. Schrödinger 1935, 157, 155 [translated by Lockwood 1989, 197]). Lockwood mistakenly ascribes Schrödinger’s second metaphor for a quantum “complete theory,” i.e., “a picture of clouds and fog patches,” to the incomplete quantum theory of wave functions, which according to Schrödinger is only “a blurred or poorly focused photograph” (cf. Schrödinger 1935, 153, 157-58 and Lockwood 1989, 197). Therefore, the macroscopic cat as a measuring instrument, and its condition (the state of being alive or dead) as the pointer, cannot be indeterminate, or uncertain, but always “*within certain error limits*” (Schrödinger 1935, 158).

But serious misgivings arise if one notices that the uncertainty affects macroscopically tangible and visible things, for which the term ‘blurring’ seems simply wrong. (Schrödinger 1935, 156)

Here, as elsewhere, Schrödinger uses “blurring” for an *incomplete theory*, and not for a *cloudy and foggy reality*, as we have already seen above; but “macroscopically tangible and visible things” are neither blurred nor “cloudy and foggy.” Thus, the apparent paradox is that if the measuring instrument is an indeterminate quantum system, then its “pointer” (e.g., the cat’s condition) cannot have a well-defined position; but if its well-defined position is a necessary condition for the experimental evaluation of quantum theory, then quantum theory, which predicts the pointer’s position imprecisely, is impossible. Yet the pointer can have a well-defined position and quantum theory is possible, and so on. The way out of this paradox is to show that the measuring instrument operates at the “quasi-classical” level and has “quasi-local” parts such that the prediction of their position by quantum theory is “*always produced within certain error limits*” (cf. Schrödinger 1935, 156-58; Bohr 1963, 5; Einstein 1949, 670ff.). This is actually the argument of Bohm and Hiley:

The immediate experience in this world is that which is described by what is called common sense... Within the domain of such experience it may be said that this [the overall atomic - e.g., 161, 178] world is *manifest*... it is what can be held in the hand, the eye, and, of course, scientific instruments. Its [the quasi-classic manifestation’s] basic characteristic is that it contains certain relatively stable structures that make the holding possible. These structures must not only be relatively stable, but also essentially local... Without such a [sub-]world we would not be able to make sense of our observations of matter... (Bohm and Hiley, 1993:176)

The distinction between the quasi-classical sub-world and the atomic (quantum) overall-world is the ontological counterpart of the epistemological distinction I made between perception and observation in the measurement procedure. In perception there is an immediate, direct, and familiar relation between a person and a medium-sized (“quasi-classical”) measuring instrument, “that which is described by what is called common sense,” while in observation the representation of the observed system is theoretical, mediated by the measuring instrument (cf. Espagnat 1983, 11.1; cf. above, II:3, 4[8]). Bohm and Hiley’s enterprise here is to avoid a seemingly unbridgeable dichotomy between the common-sense classical world and the scientific atomic world, and they suggest including the quasi-classical world as a limited case of the atomic world, thus permitting an explanation of perception and communication (Bohm and Hiley 1993, chap. 8).

…the reception of a small number of quanta gives only the vaguest sense of optical stimulation. Meaningful perception requires a large number of quanta and therefore, along the lines we have already explained, this will imply an essentially classical behavior. … Rather we are simply calling attention to the observed fact that meaningful sense perception and communication has to go through the [quasi-]classical level in which the effects of this wave function can be essentially left out of account... [T]he overall quantum “world” can manifest itself in the more limited [quasi-]classical “sub-world.” (Bohm and Hiley 1993, 178)

Thus, for observation to be materialized, the measuring instrument must be a “quasi-classical” object that can be perceived; in this case, the interaction between the human body and the perceived measuring instrument is also a “quasi-classical” relation, namely its components are “quasi-separated” and “quasi-localized”. This leaves the measuring instrument essentially independent of the human body of the perceiving person, and therefore observation is possible (cf. Bohm and Hiley 1993, 8.6). But even if we take into account the effect of the perceiver-observer as a physical body upon the measuring instrument, and through this upon the measured system (due to physical non-separability), the same does not apply to the representational function of “the thinking part of the mind.”

**4. The Relative Independence of Reality and Cognitive Mind**

The properties of the atomic system after measurement are not altogether independent of the measurement interference but, according to my analysis, they are independent of their cognitive representation owing to this measurement (cf. Nesher 1994b, #IV). However, regardless of the degree to which the properties of the human body are entangled in the properties of the perceived measuring instrument, the *epistemological* distinction must be between the representing mind on the one hand and the measuring instrument on the other. Moreover, the same argument is even stronger for the representation of an observed atomic system, which *cannot be physically distinguished completely* from the measuring instrument and the body of the observer; together they constitute an extended continuous natural system whose components are only modally (relatively) separated (the essential non-separability of the physical domain). Therefore, any possible division between the three of them is only relative in respect of their *modes of being*. Their modal interaction causes modifications, new real magnitudes, or even new particles with specific properties, since the idealization of atomic systems cannot be like the classical idealized objects (separated and localized) (cf. Bohr 1949, 201-2; 1958, 71; 1963, 2; Bohm and Hiley 1993, chap. 8, 386-90). And yet, the measuring process is determined, like any natural process, according to the laws of nature, and it cannot “disturb” or “interrupt” these laws (cf. Spinoza, EIII Pref). Schrödinger rightly states that,

...in the realism point of view, observation is a natural process like any other and cannot *per se* bring about an interruption of the orderly flow of natural events. (Schrödinger 1935, 158; cf. p. 160#10)

This is the case since observation qua measurement is itself a natural process, yet, as such it actually changes the flow of certain specific events, though according to the laws of nature and without interrupting them. Now, some interpreters of quantum theory suggest that due to the intervention of consciousness the observation-measuring operation “creates” new realities that prior to such intervention were not in the interacting modes of being. That is to say, consciousness brings about an interruption of the orderly flow of natural events (cf. Davies and Brown 1986, 20-26; comp. Bell 1986, 54ff.; Bohm and Hiley 1993, 16-19, 218). But if the physical observation-measuring process is separated from the mental process of perception, and if our minds have no causal effect upon the represented physical objects, then why should this atomic process be understood differently from other natural physical processes? It seems to me that the problem with Bohr’s approach to quantum theory is that he continued to use the classical concepts of *particles* and *waves* and thus had to adapt a Kantian phenomenological approach in developing his philosophical epistemology of science in order to unambiguously describe the resulting experimental phenomena. Weizsacker expressed this clearly:

Niels Bohr is the only physicist in our time who – as far as I know, without having been influenced by Kant – proceeded from a fundamental insight similar to Kant... Only in this framework will physicists be able to do justice to Bohr’s doctrine of the indispensability of classical concepts. (Weizsacker 1980, 342f., 345; cf. Bohr 1934, 8, 17, 53, 77, 94; 1958, 26, 39, 72, 88; 1963, 3, 12, 24; comp. Beller 1993, 245ff.)

The Spinozist alternative considers nature with its natural processes as a *non-homogeneous continuum* with different degrees and intensities of bulk *density* and fields of *thinness* in which the interactions of processes change the structural patterns of these processes. At the quantum level of this continuum, the interaction of such processes exposes either their bulk, appearing as particles, or their thinness, appearing as waves, because we still lack more adequate concepts than these classical ones. This is the *relative* (“*modal*”) *separation* of modes or processes of nature, but not a real or absolute separation. It is similar to the “undivided universe” of Bohm and Hiley (e.g., 1993, 352, chap. 15.12), but different from Bohr’s “wholeness,” which is restricted to the phenomenal experience alone (e.g., Bohr 1958, 71-2; 1963, 2, 4; cf. Murdoch 1987, 91).

Therefore, natural processes involve a certain “creation” of new, but only relatively new, realities from more basic structures that are “potentially” contained in the interaction of more comprehensive physical systems (cf. Bohm and Hiley 1993, 18f., 3.2, 218); this creation is actually a relative separation of the elements from the system that potentially contains them, and this occurs according to the laws of nature governing all natural processes. The specific structures of individual things (containing more basic forms and embedded in less basic ones) are what Spinoza calls “essences” of things (systems) (cf. E IID2, A1”-L7DS). It seems that this Spinozist “picture” of physical reality calls for new and non-classical concepts of physical processes, and it might be that with this conceptual revolution the philosophical interpretation would be realistic, but without requiring either Bohr’s phenomenological detachment from unobserved reality or Einstein’s absolutely separated and undisturbed external physical reality (cf. Folse 1985, chap. 7). However, in its phenomenological interpretation, quantum theory does not venture to explain these changes and creations of new processes, since it is forbidden to go beyond the descriptions of the results of measurements to represent the real systems that are involved in the experimentation (cf. Schrödinger 1935; Bohm and Hiley 1993, 2.2).

The cognitive perception-observation of the operation of the physical process of measurement adds nothing physically to it, but only represents its results (cf. Schrödinger 1935, 160; Bohm and Hiley 1993, chaps. 6, 8). Thus, the *cognitive mind* does not change (“collapse”) the wave states of the measured system through the perception of the measuring instrument. This is the case, since the perceiver-observer *perceptually represents* the measuring instrument *after* the measurement process has terminated and *theoretically represents* the measured system (object) only mediately, when the perceptual process is completed (cf. Bohm and Hiley 1993, 6.1).

**5. Reality and Representational Realism**

We need three components to explain human representation of reality:

A. The *relative interdependency* between the *cognitive mind* and the *reality* represented, which is necessary for the natural process of representation.

B. The *relative independence of reality* from the cognitive mind in order to explain how reality can force itself on human cognition.

C. The *relative independence of the mind* from external physical reality to enable us to explain cognitive errors (i.e., being only relatively true) in representation.

To explain the relation between minds and external reality, we should lean on what I call “representational realism” (cf. Nesher 1986, 1987a, above, I:3, 7).

The following explanation of the *representational situation* and *indirect representation* of reality is based on Spinoza and Peirce’s conceptions of nature, its physical and psychical components, and their laws.

[2] The Representational Situation of Mind and Reality:

**N A T U R E**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Person Perceiving | | |  | |
| Real Object | Cognitive Mental Causality in Mind | |  | |
| Indirect Recantation  of the Physical Object | Sign | E D L  Human  Mind | L  Union  of  Mind- Body | |
|  |  |  |  | |
| Physical Object | {oR+bP } | Human Body |  | |

Physical Causal Interactions of Bodies

In the representational situation, the actual *connection* between the causal process of perceptual sign interpretation, **Sign→E→D→L** (**E**, **D** and **L** are, respectively, Peirce’s emotional, dynamical and logical interpretants), and the causal interaction between the **Human Body** and the **Physical (real) Object**, is the *ontological union* (**U)** of the perceiver’s mind and body. This whole structure (given in bold signs and lines) makes possible the indirect representation (RpI) of the external object by the representing Signs. The interaction between the two bodies modifies them, and the modification in the human body {**oR+bP**} is the combined effect of both the Real (physical) Object, OR, and the Person’s (human) Body, **BP**. The entire situation of the indirect representation as mediated by the mind-body union can be schematized as follows:

[3] The Indirect Representation - RpI (Sign, Object):

RpI((Sign→E→D→L) + [U(Sign, {oR+bP})] + Physical Object)

(comp. above, I:8, III: [2]).

The explanation of the relation between the mind and the real object, represented in respect of the three required components above, may be stated as follows:

1. The two interacting objects, the physical object and the human body, are relatively (modally) independent of each other since they are not absolute parts of one another. (These are the relationships between all modally separated sub-systems of nature, or “modes” in Spinoza’s philosophy; cf. EIIP13S-Postulates.)

2. For perception to be possible there must be a physical interaction between the relatively independent human body (with its brain [and eyes]) and the physical object.

3. Modification {oR+bP} occurs in the human body as well as in the Physical Object, but differently (because they are different objects), and modification in the human body is *united* with mental modification in the mind. However, the human mind’s perceptual process is *connected* with the real (physical) object only through this *union* with “its own” body and the causal interaction of these two objects.

4. Therefore, mental modification, i.e., the percept and its interpretation in signs, is dependent on the reality of the interacting physical objects because the latter are relatively independent of mental processes.

5. Since representation of the external physical object is based on this combined modification **{oR+bP}**, it can only ever be an *indirect representation*, and therefore it will always be an *incomplete representation* (comp. Bohm and Hiley 1993, chap. 15.13).

6. Human *cognitive representation* of physical objects is based on the modification of the human body **{oR+bP}** by the external object, and on the correlative mental modification: the percept. But the later cognitive interpretation of the percept proceeds in *abstraction and generalization*, where this process is relatively autonomous of the percept and evolves according to the background knowledge and laws of the mind to represent the external object.

7. If reality were dependent on human cognition, then human minds would enjoy “their inward freedom which determines their experiential cognition” (Peirce, 2.138) and could not be in *conflict* with their cognitive modes.

8. Since *reality* is relatively independent of the *cognitive mind*, and the mind is relatively independent of reality, the relative falsity and truth of human cognitive representations are explainable.

From the representational realist point of view, we understand that objects are not created by our minds in their being represented, and in this sense there is an independent reality. However, in our scientific experimentations, physical reality is affected by our bodies which operate the relevant measuring instruments but not by our minds which represent physical reality. Of course, if we take our representations to be the only reality, then without any proof of their truth they are only our cognitions, that is to say our experiential sensual intuitions. Yet we must emphasize the distinction between our representational knowledge, which is dependent on our minds, and the represented reality, which *is independent* of our cognitive operations. However, our metaphysical knowledge of reality is the epistemological extrapolation of our experiential and scientific knowledge by intuition, imagination, and reasoning; our comprehensive knowledge of the infinite plenum of Nature needs to be elaborated.

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**Chapter 5**

**How Can Our Knowledge of External Reality Be Explained? Controversies About “Facts,” “True Propositions,” “Truth-Conditions” and the Pragmatist Solution**

In all these systems [Berkeley, Descartes, Malebranche, Leibniz (and so on)], however, there was felt to be something fantastic, and only philosophers with a long training in absurdity could succeed in believing them*.* (Russell 1940, 116*)*

But the impasse that arises over those basic features of the concept of truth which it must have if there is to be such practice as deductive reasoning at all really seems to be insurmountable: here we have reached the outmost limits of philosophical space. (Dummett 1991, 183)

...we often derive from observation strong intimations of truth, without being able to specify what were the circumstances we had observed which conveyed those intimations. (Peirce 1994, 7.46)

For the existence of a natural instinct for truth is, after all, the sheet-anchor of science. From the instinctive, we pass to reasoned, marks of truth in the hypothesis. Of course, if we know any positive facts which render a given hypothesis objectively probable, they recommend it for inductive testing. (Peirce 1994, 7.220)

**1. Introduction: The Problem With Our Representation of External Reality: The Concepts of “Facts,” “Truth-Conditions,” and the “Truth” of Our Propositions**

**1.1. The Philosophical Problem with Our Cognitive Representation of External Reality**

The philosophical problem with our cognitive representation of external reality essentially involves our conception of the *truth* of our propositions and our conception of *facts* as the verifiers that “make” our propositions and theories true or false. Therefore, the key questions are: What are these facts that have so central a place in our theory of truth? Can they be identified with the *truth-conditions* of our perceptual judgments, propositions, and theories?

Since our cognitive confrontation with external reality is anchored in perception, the central issue is to explain how our perceptual operations, with their perceptual judgments, represent objects of external reality. Russell understood that this is the main problem of the philosophy of knowledge, but he could not explain it because he concurred that we cannot disengage ourselves from our perceptual experience, yet at the same time we cannot claim to be imprisoned by it, because we nevertheless represent external reality and interact with the world.

The problem of perception as the source of our physical knowledge seemed to be very perplexing. Such arguments [about the nature of our perceptual experience] make it clear that what we directly experience cannot be the external objects with which physics deals, and yet it is only what we directly experience that gives us reason to believe in the world of physics. That is to say, I am considering the doctrine that there is no valid reason either to assert or to deny anything except my own experiences. I do not think this [solipsistic] theory can be refuted, but I also do not think that anybody can sincerely believe it. (Russell 1959, 78)

Russell thought that solipsism, as an epistemological theory, could not be refuted because he did not inquire into the cognitive process of perception and thus could not explain how from within this experiential operation we can prove our knowledge of external reality. Therefore, he had to assume knowledge of external reality (Russell 1914, III). The problem of what counts as a fact that verifies or falsifies a proposition is crucial in answering questions about the true representation of external reality. For if we cannot move beyond the confines of our experience, facts must be elements of this experience; but if this is the case, how can they be the objective verifiers of our propositions that confirm the truth of our representations of external reality?

Before we can start investigating the nature of “facts,” “true propositions,” and “truth-conditions,” and before we can explain how it is possible to have knowledge of external reality, we have to clarify our epistemological terminology and hence our conceptions of “language,” “proposition,” and “thought.” In the tradition of logical empiricism, analytic philosophy, and ordinary language philosophy, the conception of language is based on the spoken and written word, on physical sounds and syntax, and on the semantic content of words and propositions, which corresponds to the meaning-entities expressed or referred to by sentences or utterances. We can see that this tradition, which evolved from Kant’s distinction between cognitive form and matter, and the responses of Frege and Russell, is still very much alive, for example, in the writings of Quine (1960, #40, 42), Dummett (1978, 116-117, 442, 458; 1993, chap. 4), Horwich (1990, 17-18, 89-109), Kirkham (1992, 54-58), Johnson (1992, #6), Resnik (1997, 15-19) and Soames (1999, chap. 1).

The reason for the separation between *sentences* and *utterances* on one hand, and *propositions* and *thoughts* on the other, stems from the availability of these different terms in ordinary language usage that somehow shapes our grammatical intuition. However, from the philosophical perspective we can see how formal logicians and formal semanticists have elaborated these distinctions by formalizing languages such that “we refer exclusively to the form of the expressions involved” (Tarski 1944). Accordingly, in a formalized language with a specific structure, sentences are treated as classes of physical things with their forms, and not with their meanings, unlike languages that “may depend not only on their form, but sometimes on other, non-linguistic [i.e., formal] factors” (Tarski 1944, 53,, 57; cf. Tarski 1936, 402-403; comp. Field 1972, 87). This is a syntactic approach that attributes language with a specific independent structure, which consists of classes of physical objects and which must be investigated separately from human cognition, since it is considered independent of mental processes.

From a pragmaticist point of view, language is considered as a social behavioral phenomenon. Hence Carnap’s distinct categorizations of *syntax*, *semantics*, and *pragmatics* can only have limited use when we abstract certain components from the real work of our languages of cognition. Indeed, when overgeneralized, it is a harmful distinction (cf. Carnap 1942). The separation of sentences and utterances from their meaning or contents is evoked in Soames’ discussion of truth bearers:

What kinds of things are statements and propositions? Since statements are things that can be said, one might be tempted to identify them with two linguistic candidates: sentences and utterances or, in different terminology, sentence types and sentence tokens. Sentence types are abstract objects related to but not identical with the particular sounds or marks on paper (tokens) produced by the speakers. (Soames 1999, 14)

Moreover, we can see that for Soames sentence types and sentence tokens are different objects that exist independently of each other. Yet, the actual sounds or marks on paper (*tokens*) with their specific perceived qualities (*tones*) without the general regulating rules (*types*) cannot be parts of language but only physical objects. Moreover, since we cannot perceive the general regulating rules without their actual tokens and tones, these so-called types cannot be known by us. This terminology of tone, token, and type can be found in C.S. Peirce’s whittlings (Nesher 1990, 6-10; 1997a, chap. I). Therefore, if we separate them as different entities, they cease to be components of language as human verbal behavior. Soames explains their separation: there are infinitely many sentences in English that will never be uttered and thus we must conclude that some sentences are not utterances.

However, since one cannot utter sentences without uttering them, one cannot show which sentences will never be uttered and, vice versa, only when they are uttered can they be called sentences. The conception of language as classes of physical objects is therefore not meaningful human language but at most an “ideal mechanism” in which no item can be identified. This must be the case, since how can we know that two people, x and y, utter (or write) “the same sentence,” that is to say, how can we be sure that “Sx = Sy,” as Soames asks? For it is not “the sentence” but the utterances that can be identified as different: “Ux ≠ Uy,” and if only abstract sentences exist, we cannot know if they are the same because we cannot compare entities that we do not perceive (cf. Soames 1999, 16-19). This is the same predicament that Frege faced with his conception of “thoughts” as abstract or ideal entities that we can only ever grasp through our subjective sensible language and cannot show that what we have grasped are Platonic ideals (cf. Frege [1918] 1968, 101-105; Nesher 2000b, 4; 2000d, II.1).

To show the way out of this predicament I would like to follow Peirce’s theory of cognition (semiotics) and suggest that language is manifest as human conduct and cannot be independent of human behavior. In analyzing the structure of non-verbal and verbal languages as cognitive systems, we cannot separate the elements of signs that, according to Peirce’s analysis, are composed of *tone*, *token*, and *type*, as if they are independent linguistic entities, as some philosophers, including Soames, suggest. In line with Peirce, we can conceive of them as inseparable aspects of cognition in their representational and communicational functions.

**1.2. Understanding How Language Works: Representational and Communicational Functions**

Every sign, whether iconic, indexical, or symbolic in its representational function, has three components: *tone*, the property of the sign itself; *token*, the actuality of the sign; and *type*, its generality. Without these three components the sign cannot be a sign, it cannot be perceived, nor related to its object, nor have the permanency of being interpreted and communicated (Peirce, 4.535-544, 8.346ff., 8.363-4, W3:62ff.; comp. 8.334; cf. Nesher 1985; 1990, 6-10). The structure of the sign allows it to represent an object, which may be physical, psychical, or derivatively imaginative.

[1] The Internal Structure of the Sign [Thirdness]:

\_\_\_\_\_\_\_\_\_ SIGN \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

| *Tone* = Property (perceived quality) | | *Token* = Actuality (indexical relation) | | *Type* = Generality (syntactical form) |

|\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_|

Signs are experiential forms of cognition, and it can be shown that these cognitive representations signify external physical and psychical objects and at the same time interpret other cognitive representations in the formation of symbols, concepts, and propositions. We can say that the two functions of signs, interpretation and representation, are like Siamese twins which are not identical but cannot live without each other; they function as two different referential relations (cf. Nesher 1982, 1983, 1985, 1990, 1994b, 1997a). To understand the process of the formation of symbols-concepts and propositions we should analyze its primordial development in perceptual experience and in the formation of perceptual judgment. However, in the first place it is important to show why the analytic tradition that conceptualizes propositions as abstract meanings – a tradition that evolved from Platonic Idealism, Kantian concepts and Fregean thoughts – cannot explain our cognitive representation of reality (Nesher 2002, X #5).

Quine observed that philosophers, in order to avoid the relativity of using sentences, turned to the concept of *proposition*:

This is no doubt one reason why philosophers have liked to posit supplementary abstract entities – *propositions* – as surrogate truth vehicles. This done, they speak of the sentence as expressing now one proposition and now another for this man and that, while allowing each such proposition itself to remain steadfastly true or false without respect to persons. This posit is not altogether the philosopher’s doing. Ordinary language has it ‘that’ clauses, and such clauses (with ‘that’ as conjunction, not as relative or demonstrative pronoun) function grammatically as a singular term (except when preceded by ‘such’), thus evidently purporting to designate something. Their purported objects are what the philosopher takes up and calls, subject to certain refinements, propositions... Since a prominent use of the ‘that’ clauses is as grammatical objects of the so-called verbs of propositional attitudes, we found ourselves taking propositions in particular as the things that people believe, affirm, wish, etc. (Quine 1960, 192; cf. Quine 1992, #32)

However, it is problematic for Quine to identify propositions, which are abstract entities and cannot be perceived as physical objects like apples or rivers. Therefore, he must find physical objects like sentences to identify the propositions that they purport to express or refer to. This is the Platonic and Fregean problem of how we can “grasp” Platonic *ideas* or Fregean *thoughts*, or Quine’s *propositions*, as abstract entities and how linguistic sentences come to possess the specific structures to express these abstract entities (Frege [1918] 1979; Dummett 1993, chap.10). However, a sentence without meaning cannot help us to identify propositions, and if it has meaning then we do not need propositions to give it meaning, unless one distinguishes between personal meanings and eternal meanings (though there is no explanation of how we can express eternal meanings with personally meaningful expressions). Yet, Quine still considers propositions to be the truth bearers of eternal truths, for they give meaning to eternal sentences. However, the question remains: what are the eternal truths of propositions? Would this knowledge come from one’s metaphysical realist intuition about the world and its representation (Quine 1995, 67)? If we cannot “grasp” propositions themselves and compare them with worldly things, we cannot know their truth, and if we would like to justify or prove the truth of a proposition we can do so only with the help of sentences (if one accepts this terminology). Yet, if sentences “expressing now one proposition and now another for this man and that” cannot “remain steadfastly true or false without respect to persons,” then we cannot use them as vehicles of eternal truths (Quine 1960, #42; comp. Dummett 1993, chap.10).

Philosophers of the Platonic, Kantian and Fregean traditions search for steadfast meaning and absolute truth, anchoring these in the unchanged and absolute meaning entities that are beyond human experience, because every human experience is personal and relative to circumstances. This is also the case with the concept of propositions as eternal and absolute meaning entities, and the question is, how can we express or refer to abstract propositions that we cannot experience?

However, if we want to understand how language works in its representational and communicational functions, we should not be captive to the surface grammar, as are some ordinary language philosophers. We have to inquire into the nature of our cognitive representations and the propositions we make use of as the basic unities of human language. Wittgenstein in his letter to Russell wrote: “I am very sorry that my objection to your theory of judgment paralyzes you. I think it can only be removed by a correct theory of proposition.” (Wittgenstein, letter dated 22.7.1913). How, then, was Wittgenstein’s theory of propositions, put forward in the *Tractatus*, meant to have offered a better theory of judgment?

I.3. Understanding Our Perceptual Judgments as Basic Experiential Propositions

We can view a proposition as a structure that evolves hierarchically in the experiential process of the perceptual operation that converts pre-verbal sensorimotor signs into a verbal proposition and the initial perceptual judgment. This propositional structure essentially includes the previous phases of the perceptual sign operations of feeling and reaction as the meaning or content of the evolving propositional thought. To avoid complicated Peircean terminology, I will use colloquial philosophical terminology:

**[2] The Evolving Structures of Basic Sign Components of the Proposition:**

I n t e r p r e t a t i o n

From Pre-Verbal Sensorimotor Signs ======> To Proposition [Perceptual Judgment]

|  |  |
| --- | --- |
| Feeling *➔*  Reaction *➔* | Thought |
| Describing Referring | Asserting |
| (Non-verbally) (Ostensively) | (Verbally) |
| Predicating Subject + Predicate | Subject + Predicate (syntactic structure) |
| **Iconic** (Property P)  **Indexical** (*→* is P) | **Symbolic** (refers to object by law) |
| Sign as Quality: *Tone* Sign as Object: *Token* | Signs’ general verbal expression: *Type* |
| Predicate | Subject + Predicate (*→* is P) |

Replica  **Iconic Indexical** (refers to object by act) **The Proposition** **Meaning-**Predicate (term, is P)

Sign as Quality Sign as Object: *Token* Replica **Content** **Iconic** (description of the object)

Sign as Quality: Tone

I n d i r e c t *❙*  Relation of *❙*

Representation *▼*

[OR] Real Object

(cf. Peirce, 2.254-265, 2.274-308, 1.521-544, 8.346-379; Nesher 1997a, #I)

Here the signs that eventually represent the real object include: the **iconic** sign that presents the feeling of the property P of the eventual object; the **indexical** sign that presents the reaction to the feeling of the property P of this object ([this] *→* is the object presented in P); and the **symbolic** sign that presents the thought (referring to the object by following rules), and this is the synthesis of the previous sign-phases of the perceptual operation. With this analysis of the hierarchical evolution of propositions it can be shown that it is only by abstraction that we can separate the type as the form of the proposition from its meaning or content. The *form* and the *content* cannot be separated if they are to remain cognitive signs that can be perceived, conceived, and operated on in the process of representing reality.

With this understanding of our perceptual judgments as the basic experiential propositions we avoid the Kantian dichotomy between *empty concepts* and *blind intuitions* (Kant [1781-1787] 1929, A50–51/B74–76; cf. Nesher 2000c, #2). Thus, the dichotomy between rational and pre-rational operations does not hold since every rational self-conscious and self-controlled operation is itself based hierarchically on pre-verbal pre-rational empirical meaning-contents (comp. McDowell 1994). Peirce formulates this evolving hierarchical unity such that “every symbol must have, organically attached to it, its Indices of Reactions and its Icons of Qualities” and these pre-verbal experiential qualities and reactions are factors that operate in any verbal rational operation (Peirce 1994, 5.119). Moreover, since symbolic words are “empty” without their empirical meanings or contents, we cannot operate rationally with meaningless formal symbolism as Quine envisages (Quine 1960,; cf. Wittgenstein [1921] 1961, ##6.1- 6.12; Russell 1914, 66-67). As opposed to the Kantian “gap” between transcendental concepts and empirical intuitions, or the Fregean “gap” between absolute thoughts and subjective experience, the pragmaticist denies that these two sources of human knowledge are unbridgeable, and argues that concepts are symbols that evolve in a cognitive operation from our pre-conceptual empirical experience and intuition (Peirce 1994, 5.180-195, 2.227-273, 5.212, 7.615ff.; Nesher 1985, 202-204; 1990, 24-27; 1994b, II; 1999a, II.3, III; 1999b, ##2, 3; 2000c). In this way, Peirce understands the nature and function of propositions as cognitive signs to which truth belongs exclusively, though the way we cognitively represent reality is still to be explained.

Truth belongs exclusively to propositions. A proposition has a subject (or a set of subjects) and a predicate. The subject is a sign; the predicate is a sign; and the proposition is a sign that the predicate is a sign of that of which the subject is a sign [cf. 2.310ff.]. If it be so, it is true. But what does this correspondence or reference of the sign, to its object, consist in? (Peirce 1994, 5.553).

From this perspective, our verbal-symbolic language and its propositions are seen to interpret and generalize human experience; on this basis we can start our discussion on *facts*, *true propositions*, *truth-conditions*, and eventually *proof-conditions*.

**2. Controversies About “Facts” And “True Propositions”**

**2.1. The Debate between Austin and Strawson about the Nature of *Truth***

In the debate between Austin and Strawson about the nature of *truth* and our use of the predicate “true” they discussed the question of what counts as “true statements” (or propositions) and “facts” or “states of affairs.” Austin wanted to explain what a true statement is by its correspondence with facts or states of affairs “in the world” and based on his description of the ordinary usage of the expressions “facts” or “the fact that”. In analyzing their function in our linguistic behavior, he prefers the expression “states of affairs” for things and events “in the world.” However, he could not explain how we identify states of affairs in the world and how we can know that they exist independently of our language and of our nonverbal relation to them. According to Austin:

It takes two to make a truth. Hence (obviously) there can be no criterion of truth in the sense of some feature detectable in the statement itself which will reveal whether it is true or false. (Austin 1950, 23n13)

Austin does not have a clear conception of what facts or states of affairs are and how we can identify them and therefore he cannot explain the relation between language and the world, since “It takes two to make a truth” (cf. Kirkham’s analysis of Austin’s theory of truth). The two kinds of conventions, *descriptive* and *demonstrative*, relating to the correlations of our language with *historic* situation or states of affairs in the world cannot help Austin (cf. Austin 1950, 24-25, 27-28; comp. Strawson 1950, 32-33). This is so since, even if these conventions are themselves based on our intuitive relation to the world, for Austin the relations of comparison between language and the states of affairs in the world remain a mysterious assumption that we know and represent states of affairs truly (cf. Austin 1950, 28n23, n24).

‘Fact that’ is a phrase designed for use in situations where the distinctions between a true statement and the state of affairs about which it is a truth is neglected. (Austin 1950, 23-24)

Indeed, when we assume that something is a true statement or proposition we can infer from the basis of the truth of this proposition another true proposition, and so on, but the question is: What makes the initial proposition true? For this we have to show how we can be sure initially, in the perceptual experience, of the states of affairs which our perceptual judgments are about. However, if we cannot detach ourselves from our cognitive skins to compare our cognitive representations with the external world, we may try to detect some feature “in the statement itself which will reveal whether it is true or false” (Austin 1950, 23n13; comp. Nesher 1997a; 1998a; 1998b).

**[3] “Correspondence” of True Statements with a Fact (The Correspondence Position):**

**True Proposition: Subject** *✤* **Predicate ➔** True Proposition *➔* True Proposition *➔* ∙ ∙ ∙ ∙ ∙

*❙*  *❙*

“demonstrative convention” “descriptive convention”

*❙*  *❙*

Unexplained “Corresponds with the fact” or “fitting the fact”

*∇ ∇*

**Fact [or] State of Affairs** [“illusion or mythology” – Strawson 1992, 89]

The World

Thus, Austin claims that “if it is admitted (*if*) that the rather boring yet satisfactory relations between words and world which has here been discussed does genuinely occur, why should the phrase ‘is true’ not be our way of describing it?” (Austin 1950, 31). Yet this correspondence is only an assumption on which we lean to describe the use of ‘is true’ without explaining whether and how this relation exists and operates. What Austin does is only a philosophical analysis of the grammatical usages, of what may and may not be done with the words ‘is true’, ‘truth, ‘facts’, ‘corresponds with the facts’ and other ordinary language expressions, without going farther to any epistemological analysis of our relation with the external world (Austin 1954, 156ff.). Austin and Strawson disagree over whether ‘facts’ are “things genuinely in the world” and whether ‘true’ propositions correspond with the facts (cf. Strawson 1950; 1992; Austin 1950; 1954).

According to his grammatical intuitions of our ordinary linguistic behavior, Strawson suggested that we usually identify facts with what true statements state, such that to state a true statement is to state a fact, and the expressions “It is a fact that S” and “It is true that S” overlap (Strawson 1950, 38). He rejected what he called Austin’s “purified version of the correspondence theory of truth”:

But neither Mr. Austin’s account of the two terms of the truth-conferring relation [words and world], nor his account of the relations itself, seems to me satisfactory. The correspondence theory requires, not purification, but elimination. (Strawson 1950, 32)

However, we can see that Strawson also had difficulties with the two correlated terms, “statement” and “fact,” in his attempt to explain our use of the expressions “is true” and “truth.” Strawson’s conception of the term “statement” is basically grammatical, not epistemological, for he holds that the verbal expression, the grammatical form, can be separated from its performance (which is an event) with its nonverbal meaning, and still retain meaning and truth value (Strawson 1950, 33-35, 1). This is a reflection of Kantian concepts which are empty when separated from our sensual intuitions and is due to the “gap” in Kantian and neo-Kantian epistemology.

The conditions of our having general concepts of the objectively real, of objects in nature, which were not concepts of spatio-temporal things at all would be our enjoyment of a certain kind of experience–an experience in which space and time either played no part at all or at least were totally unrelated to our wholly empirical awareness of the numerical difference of different particular instances of one and the same such concept. It is here that the empiricist principle exerts its power. For this supposed description of a kind of experience remains for us quite empty, a mere form of words, without empirical significance; and the notion of general concepts of empirical objects which were not concepts of the spatial or the temporal remains equally empty. (It will be seen that I here echo Kant’s doctrine of the form of sensibility.) (Strawson 1992, 56)

This empiricist explanation of our experience in the world as verbalizations with empty concepts and empty objects without any epistemological explanation of our experience and life in the world echoes the Kantian dichotomy between *empty concepts* and *blind intuitions* which retains the gap between language and reality (Nesher 2018).

**2.2. Fregean Dichotomy Between Thought and Language**

The Kantian gap between transcendental concepts and sensuous intuition, also expressed in the Fregean dichotomy between thought and language, exists because language is seen in this view as the autonomous realm of “the general structural principles of all our thinking” while thoughts are the meanings or content of linguistic expressions: “Of course, for this structure to have content we must also learn the reference of individual names and the sense of individual predicates,” such that thoughts are the meanings projected through the formal structure of language to refer to objects and properties in reality (Strawson 1992, 56, 97-99; comp. Frege 1892; Wittgenstein [1921] 1961). This separation between concept and its exemplification, or between type and token, constitutes Strawson’s basic conception of a statement as a linguistic structure separated from its actual performance, in his debate with Austin (cf. Strawson 1950, 33-35, 36; 1966, 47-51). This is the first term of “the truth-conferring relation” between language and the world. What, then, is Strawson’s second term? This relates to the worldly components that Austin calls “things,” “events,” “states of affairs,” or “facts”, which are supposed to satisfy this truth-conferring relation. In criticizing Austin’s theory of correspondence to worldly facts, Strawson writes:

That (person, thing, etc.) to which the referring part of the statement refers, and which the describing part of the statement fits or fails to fit, is that which the statement is *about*. It is evident that there is nothing else in the world for the statement itself to be related to either in some further way of its own or in either of the different ways in which these different parts of the statement are related to what the statement is about. And it is evident that the demand that there should be such relatum is logically absurd: a logically fundamental type-mistake. But the demand for something in the world *which makes the statement true* (Mr. Austin’s phrase), or *to which the statement corresponds when it is true*, is just this demand. And the answering theory that to say that a statement is true is to say that a speech-episode is conventionally related in a certain way to such relatum reproduces the type-error embodied in this demand. For while we certainly say that a statement corresponds to (fits, is borne out by, agrees with) the facts, as a variant on saying that it is true, we *never* say that a statement corresponds to the thing, person, etc., it is about. What “makes the statement” that the cat has mange “true” is not the cat, but the conditions of that cat, i.e., the fact that the cat has mange. The only plausible candidate for the position of what (in the world) makes the statement true is the fact it states; but the fact it states is not something in the world. (Strawson 1950, 36-37)

Thus, according to Strawson, the statement is *about* an object or event and it *states* a fact; however, the fact is not something in the world but precisely what the statement *states*.

Mr. Austin seems to ignore the complete difference of type between, e.g., “fact” and “thing”; to talk as if “fact” were just a very general word (with, unfortunately, some misleading features) for “event,” “thing,” etc., instead of being (as it is) both wholly different from these, and yet the only possible candidate for the desired non-linguistic correlate of “statement.” Roughly: the thing, person, etc., referred to is the material correlate of the referring part of the statement; the quality or property the referent is said to “possess” is the *pseudo*-material correlate of its describing part; and the fact to which the statement “corresponds” is the *pseudo*-material correlate of the statement as a whole. (Strawson 1950, 37)

It seems that Strawson’s basic motivation is to follow the behavior of the word “fact” in ordinary language (Strawson 1950, 37). Yet the solution to accept the definition of a fact as “the pseudo-material correlate of the statement as a whole” is a sort of Kantian phenomenon, or better still, the sum of the cognitive components of the linguistic expression “statement”; because without the pre-verbal (linguistic) components, like *iconic feelings* and *indexical emotions*, statements are just physical objects or events without any meaning and, therefore, not elements of language at all (Nesher 1999). However, if “facts are what statements (when true) state,” namely the meaning or contents of statements, then according to our analysis of propositions (or statements) as linguistic performances, these meanings or contents equate to components of statements and not something independent of them that can be their verifiers. These implicit conditions prompt Strawson to declare: “Of course, statements and facts fit. They were made for each other” (Strawson 1950, 38-39). But Strawson does not identify a true statement with the fact it *states* since for him a statement is only the linguistic syntactic structure without its pre-verbal meaning or content, and thus true statements and facts are different things (cf. Strawson 1950, 35-43). What, then, are the correlates of meaningful statements in the world that make such statements or propositions true? In order to answer this, we should show that propositions or statements represent external segments of reality and that when we prove them to be true, we prove their correspondence with these realities. Therefore, we cannot understand the truth of propositions or statements unless they have correlates in reality. Elsewhere I have explained that the truth of a proposition is an element of the interpretation of its meaning (Nesher 1989, 1994b, 1997a, 1998a).

**2.3. Statements and Their Demonstrative (or Indexical) Relation to Reality**

Thus, in spite of rejecting the historical performance component of statements and their demonstrative (or indexical) relation to reality in Austin, Strawson nevertheless suggests a performative-coherent theory of truth in which the expression “is true” operates as an appraisal for statements and not as a confirmation of the correspondence of statements with external facts in the world. However, Strawson did not explain the conditions for using the grammatical predicate “is true.” In other words, he did not answer the question: How do we know when our propositions are true? (Strawson 1950, 1992; Austin 1950, 1954).

[4] Identifying Facts with what True Propositions Express or State:

True Proposition1: ‘Snow is White’ = “It *is True* that ‘Snow is White’” *➔*True Proposition2 *➔*∙

*❙* *Stating* (Strawson 1950, 38)

*▼*

Fact: (“It is a Fact that ‘Snow is White’”)

∙ (Strawson 1950, 38)

∙ Unexplained Use of “is True” (“too narrow a conception of truth”)

**[External World]** (Strawson1992, 89)

The problem for Austin and Strawson is how to understand the relation of a statement to fact: “But what could fit more perfectly the fact that it is raining than the statement that it is raining? Of course, statements and facts fit. They were made for each other.” (Strawson 1950, 38-39). Here we have a kind of tautology, or simply a repetition, where the true statement seems to be identified with a fact, and the problem of correspondence between “statements” and “facts in the world” just disappears because, according to Strawson, facts are abstract “pseudo-entities” (Strawson 1950, 41). Thus, Strawson merely assumed that there are true propositions that can be considered to be stating facts, but there are no “facts in the world” external to our experience with true statements or propositions. This claim seems problematic, if not nonsensical, for Austin: “But in answer to this: surely it is not sense either to ask whether the statement that S fits the fact that S or to state that it either does or does not” (Austin 1954, 160).

This is so because if S does not fit the fact “that S,” “the fact that S” is not the fact that S, and “fitting” becomes an empty concept of the relation of statements to “pseudo-entities” (cf. Putnam on Austin and “direct realism” or “common-sense realism”, 1994b, 1999). Wittgenstein expressed this puzzled feeling, reflecting Strawson’s position: How do we illusorily ‘catch’ external reality in language itself?

What makes us think that a thought, or a proposition we think, contains the reality? It’s that we’re all ready to pass from it to the reality, and we feel this transition as something already potentially contained in it (when, that is, we reflect on it), because we say “that word meant him.” We feel this transition as something just as legitimate as a permitted move in a game. It isn’t only that we see it [the thought] as an extraordinary way of producing pictures and signs, we actually feel as if by means of it we had caught reality in our net. (Wittgenstein 1974, #105)

This can be seen in the way Frege, Ramsey, and Tarski thought about the relation of our linguistic behavior to worldly facts or states of affairs when they formulated the equation “p is true = p” or the equivalence of the form (T): “X is true iff p” (cf. Nesher 2000d; Davidson 1990, 302-305 on facts).

It seems that Strawson, in retrospect, reacts to that highly problematic notion of sheer correspondence and coherence theories of truth when he acknowledges:

First, then, truth. With the aid of our simple example [“If someone *says* that John is bald, what he *says* is true if and only if John is bald”], I imagine someone finding it a merit of a simple scheme or formula we began with that it incorporated a twofold reference – to a saying or belief on the one hand, and to that in the world which the statement or belief was about on the other – and hence envisaging truth as a kind of word-to-world correspondence best understood in semantical terms. My first point is that incautious commitments to this interpretation may involve the risk of either adopting too narrow conception of truth on one hand [Strawson’s coherence without representation, 1950] or of falling into what has been plausibly represented as illusion or mythology on the other [Austin’s unexplained correspondence, 1950]. (Strawson 1992, 89; cf. Nesher 1987b, 1997a, 2000c)

This is to say that the apparent dilemma between the Scylla of the coherence theory of truth and the Charybdis of the correspondence theory of truth are both unsatisfactory alternatives, and the question is how to combine the reflective reference to what one says or cognizes with the representational reference to real objects as two aspects of our cognitive representation of reality and the basis of the theory of truth according to Spinoza and Peirce (Nesher 1994a, 158; 1997a). However, this combination cannot be a mechanical sum of the correspondence and coherence theories of truth, since there are contradictions between some of their basic conceptions, especially their concepts of fact. Austin and Strawson are both currect in their criticism of each other’s positions since, according to my analysis of cognitive behavior, facts are neither Austin’s external entities in the world nor Strawson’s abstract entities that are expressed through true statements; they are instead true propositions (or true statements) that we quasi-prove by our perceptual judgments and epistemic logic as true representations of reality, as I will show below. Strawson and Austin are both wrong in their conceptions of truth, facts, and truth-conditions, since they did not analyze the cognitive structure of propositions and the ways in which they can be proved true or false. Moreover, they cannot show how with our cognitive processes we can truly represent worldly objects, events, facts, and states of affairs. They both reject the conception of fact as a true statement or proposition: Austin because he wants facts to be entities “in the world” and Strawson because he conceives of a statement as a grammatical structure without the meaning or content which the statement expresses as fact (Austin 1950, 1954; Strawson 1950, 38-43; 1966, 1992). Both were led to offer explanations about the truth-conditions of our basic propositions, how we verify or falsify these propositions, and the nature of facts, whether they are true propositions or independent realities or something else? However, for the purposes of this inquiry we cannot be satisfied with a philosophical analysis of our ordinary language usage and the role of grammatical intuition, as Austin and Strawson focused on in their controversy; rather, we must epistemically analyze our basic cognitive operations and behavior. As late Strawson writes:

But the point to be stressed now is the ongoing and continuous character of the individual’s exposure to the world. At any moment, we may say, our current experience (our current observation) *forces on us* at that moment. This may, and generally will, involve no strain; and as already implied, what our current experience does force on us in the way of belief depends on the character of the pre-existent system. But the necessity of this kind of accommodation to current experience is a necessity which is always with us; and always with us, from the time when we could first be credited with beliefs at all; so that, from that time onwards, all subsequent states of our belief system are the outcome of the ongoing process of accommodation to the unceasing pressures of experience. (Strawson 1992, 95-96)

First, we must conclude that any pre-existent system of beliefs is itself dependent on our ongoing experience, which forces us to accommodate to the external world. But the pressure of our experience on our belief system itself depends on the pressure of the external world that is somehow represented by our belief system. Neither Strawson nor Austin solved the problem with our conception of experience as a cognitive process through which we confront the external world. Both leave unanswered the question of how we represent the world through our experience. This is the problem of “truth,” “facts,” and “true propositions.”

**3. Can Brandom Combine Two Incorrect Theories of Truth To Form A Correct One?**

**3.1. The Coherence and Correspondence Conceptions of Truth**

Recently, Brandom discussed the coherence and correspondence conceptions of truth and surprisingly held both Strawsonian and Austinean positions, to the effect that facts are identical with true propositions and true propositions correspond to independent facts. In other words, he merged the coherent performative and correspondence theories. But since neither can explain the concept of *truth* and our use of “is true” Brandom also failed to account for them (Brandom 1994, 327-333). We are faced with two predicaments: If facts are true “claims” or assertive propositions, what verifies them, and if facts are the independent verifiers of propositions, what is the nature of these facts, are they representational cognitive entities or are they “in the world”?

According to the usage endorsed here, facts are just true claims. That is, phenomenalistically, to call something a fact is just to take it to be true. ‘Claims’ here has the semantic sense of what is claim*ed*, rather than the pragmatic sense of the claim*ing* of it – a matter of content, not of force or deontic attitude. Thus to say that facts are just true claims does not commit one to treating the facts as somehow dependent on our claims; it does not, for instance, have the consequence that had there never been any claimers, there would have been no facts.(There are no possible situations in which there would have been no facts. A situation or set of circumstances just is one sort of set of facts.)… talk of facts as what *makes* claims true is confused if it is thought of as relating two distinct things – a true claim and the fact in virtue of which it is true – in such a way that the former might be *explained* by appeal to the latter [i.e., “P is true = P”]. (Brandom 1994, 327-328)

The concept of *claim* is not clear enough in this context and the question remains as to whether claims are cognitive facts or whether facts are the verifiers of cognitive claims? For Brandom, a fact is, in a phenomenalistic sense, “something” that we believe “to be true.” But what is this something and why believe it to be true? According to Brandom, this something is “what is claim*ed*, rather than the pragmatic sense of the claim*ing* of it – a matter of content, not of force or deontic attitude.” But the content of a claim or an assertive proposition is a cognitive component of the proposition itself and therefore cannot be independent of the claim/proposition. In Brandom’s view, there can be facts without claimers (persons who state these claims) or the claiming facts. This implies, then, that the contents of claims or propositions exist independently of linguistic performance, and these claim*ed* contents or facts are something like Frege’s conception of thoughts that exist independently of humans. However, if nobody claims them, how can we know them? How do they obtain their truth values, and what are their verifiers? We should remember that “talk of facts as what *makes* claims true is confused if it is thought of as relating two distinct things” (Brandom 1994, 328). So far we may observe elements of a Strawsonian position in which statements and facts fit together and are made for each other, since the statement and its content are actually two components of the same entity. But Brandom continues:

True claims do correspond to facts, and understanding claims does require grasp of what the facts must be for those claims to be true... If ‘claim’ is understood as what is claimed, true claimable contents just *are* facts; the relation of ‘correspondence’ is just that of identity [cf. Strawson 1950]. For that reason, grasp of such contents can be identified with grasp of what the facts must be for them [the contents] to be true. But the basic question... – only an alternative way to express it... This is trivially, grasp of what (claimable contents) must be true, what the facts must be, if the claiming is to be a true-claiming – a claiming of a true claimable content… By contrast [with traditional semantics] (as Chapter 3 shows) it is possible to explain the practical significance of acts of claiming, and so to approach the propositional contents they express, without appealing to the notions of truth conditions or fact. The use of expressions like ‘true’ and ‘fact’ can then (as this chapter shows) be explained in terms of these same social practices of giving and asking for reasons. (Brandom 1994, 330; cf. Nesher 1993a)

Now we see that Brandom deserts the Strawsonian position for a moment and embraces the Austinean correspondence theory of truth when he says that “True claims do correspond to facts, and understanding claims does require grasp of what the facts must be for these claims to be true.” However, the correspondence between claims and the verifiers that make them true is simply the identification of claims with their contents, and again we face the question of “what the facts must be” for these claims to be true. It seems that the contents of claims must be true for the claims to be true. Yet, this does not amount to identifying two entities or components, i.e., claims and facts, but rather identifying “grasp of the contents” of the claims with “grasp of what the facts must be for the contents to be true,” (but not for the claims to be true). Thus, we have to grasp the facts that must make the contents of claims true. We self-consciously search for the facts that we must access to make the contents of our claims true. At this stage Brandom claims that “it is possible to explain the practical significance of acts of claiming, and so to approach the propositional contents they express, without appealing to the notions of truth-conditions or fact.” To put it differently, we can explain the use of the expressions “truth” and “fact” in terms of the practical significance of acts of claiming, the “social practice of giving and asking for reasons” but this seems to evoke nothing more than the Wittgensteinian language-games that are separated from external reality.

**3.2. Explaining, Not Just Describing, the Social Practice of Language-Games**

The question that Brandom has to solve and that Wittgenstein did not solve is how to explain, not just describe, the social practice of language-games or, in Wittgenstein’s terms, the evolution of our “inherited background.” This equates to the development of our basic social practices and beliefs, which according to my analysis and argumentation elsewhere cannot be done without confrontation with the external world (Nesher 2002b, 2002c, 2016, 2018).

But the worry may remain that a semantic idiom that identifies facts with true claims (via the identification of taking to be a fact with taking to be true, that is, with acknowledging a doxastic commitment) must inevitably “lose the world” – trading its solidity for a froth of words. A threatening idealism of linguistic practice seems to be implicit in such identification. (Brandom 1994, 330-331)

I would argue that we do indeed “lose the world” in Wittgenstein’s transition from Tractarian metaphysical realism to phenomenalism, “the idealism of linguistic practice” in *Philosophical Investigations* and *On Certainty*. Wittgenstein could not offer a pragmaticist explanation of our confrontation with external reality; his philosophical enterprise describes human linguistic practice without explaining it. Yet it seems that Brandom offers a way out of the idealism of linguistic practice.

But this is a misplaced concern. What must not be lost is an appreciation of the way in which our discursive practice is empirically and practically *constrained*. It is not up to us which claims are true (that is, what the facts are)... The nonlinguistic facts could be largely what they are, even if our discursive practices were quite different (or absent entirely), for what claims are true does not depend on anyone’s claiming of them. But our discursive practices could not be what they are if the nonlinguistic facts were different. (Brandom 1994, 331)

Brandom’s suggestion is to change our understanding of the relation between our social discursive practices and reality.

Facts are (the contents of) true claims and thoughts. As Wittgenstein says: “When we say, and *mean*, that such-and-such is the case, we–and our meaning–do not stop anywhere short of the fact; but we mean: this-is-so.” (1958, #95). What is lost is only the bifurcation that makes knowledge seem to require the bridging of a *gap* that opens up between sayable and thinkable contents – thought of as existing self-contained on their side of the epistemic crevasse – and the worldly facts, existing on their side. ... The world is everything that is the case, (Brandom 1994, 333)

Brandom interprets Wittgenstein Tractarian position as holding that facts are not independent entities in the world but constructions of our own that represent objects and their properties and relations.

But the author of these words [“The world is all that is the case” (Wittgenstein 1921, 1] hastened to point out, those facts are structured and interconnected by the objects they are facts about; they are articulated by the properties and relations the obtaining of which is what we state when we state a fact (claim when we make a claim). To make a claim is to say that things are thus and so – that is, to talk about objects, and to say how they are propertied and related. Propositional contents (and hence facts) cannot be properly understood without understanding their representational dimension – what it means for them to be *about* objects and their properties and relations. (Brandom 1994, 333)

Three questions arise from concerning the above claims: (1) Does Brandom interpret Wittgenstein correctly or is he just reading his own position into Wittgenstein’s *Tractatus*? (2) Does the Tractarian account of what it is to represent reality hold weight? (3) Can a viable theory of representation and truth emerge from Brandom’s perspective? I will deal later with the formal semantic conceptions of “representation,” “true propositions,” “facts,” and “truth-conditions” that Wittgenstein proposes in the *Tractatus*, but first some short remarks about Brandom’s reading of this philosophical work.

1. According to Wittgenstein, facts (or states of affairs) are combinations of objects when objects are unalterable and eternal metaphysical things, properties, and relations in *logical space* (Wittgenstein [1921] 1961, #1-2.011, 2.0122-2.0124, 2.0141-2.0231, 2.0233-2.04). Wittgenstein does not say in the *Tractatus* why “facts are structured and interconnected” of objects, but they are not so “by the objects.” The only explanation of the structure of possible states of affairs or possible facts is that the *logical forms* of objects determine the ways in which they are combined to form actual facts in the world (Wittgenstein [1921] 1961, ##2.0142.06). Indeed, Wittgenstein as a formal semanticist analyzes the structure of facts, but we as “users” of language do not “state a fact,” to use the terms of *Philosophical Investigations*, and if one can find subjects in the *Tractatus* at most they can state propositions (Wittgenstein [1921] 1961, ##5.5421, 5.631-5.633). Moreover, Wittgenstein posits that metaphysical objects in *logical space* are *unalterable* and *subsistent*. As a formal semanticist, he can “talk about objects” and their *logical forms*, whereas propositions only *describe* or *depict* facts in the world; names only *refer* to objects, they cannot *describe* or *represent* them because they are simple and have only abstract “formal properties” (Wittgenstein [1921] 1961, ##2.024-2.0272, 3.221). Only by the configuration of objects (e.g., *Socrates*, *wise*) are “material properties” produced, and they can be represented by means of propositions (i.e., “Socrates is wise”) (Wittgenstein [1921] 1961, ##2.02-2.0231, 3.1-3.26). Facts are not the *contents* of propositions because if they were they would not be able to verify or falsify them, and the entire Tractarian theory of truth and falsity relates to worldly facts (Wittgenstein [1921] 1961, 2.222, 2.225). Facts in the world are independent of the propositions that describe them. The content of a proposition in the *Tractatus* is its *sense*: through a method of projection, the “metaphysical subject” thinks of propositional facts and projects the *referring relations of their names* onto metaphysical objects. Therefore, the content of a proposition is its *sense* together with its *referential relations*, and this echoes Frege (Wittgenstein [1921] 1961, ##3.11-3.13, 3.33-3.331).
2. The problem with the Tractarian theory of representation and truth (a formal semantic explication) is that it does not provide an accurate model for our cognitive representation of reality, since the metaphysical subject who interprets language is located outside the world, like the Cartesian God, but human beings live in the world and cannot compare propositions and facts from outside their cognitive “skins,” to use Davidson’s metaphor (Davidson 1983, 312; cf. Nesher 1998a, #3).
3. The problem for Brandom, and for other proponents of the identity theory (who say that true propositions are facts) is that they hesitate over two important questions: Are facts to be understood as phenomenal entities, or must a true proposition be restricted by something external to them as “facts in the world.” In this discussion Brandom wants to hold both of Wittgenstein’s positions: the metaphysical realism of the *Tractatus* and the phenomenalism of *Philosophical Investigations*. Remember that in the *Tractatus* “The world is all that is the case” and that what is the case – a fact – is “the existence of states of affairs” (Wittgenstein [1921] 1961, #2). Therefore, what is the *case* is identical with *fact*. Here facts differ from true propositions which are not just “propositional facts,” i.e., syntactic physical structures, because their meanings-contents are not factual elements and cannot be described by a metalanguage, and for propositional facts Wittgenstein does not need a metalanguage (Wittgenstein [1921] 1961, ##4.12-4.1212, 3.33-3.333, 5.25; cf. Black 1964, 218; Nesher 1987, #3; Ishiguro 1981). Propositions gain sense in formal semantic terms by the method of projection whereby the “metaphysical subject” projects thoughts onto propositional facts to obtain their referential meaning, as if touching reality with the feelers and reaching out to metaphysical objects in logical space (Wittgenstein [1921] 1961, #2.1515). Only such meaningful propositions can describe facts in the world, and only the propositional facts, the sentences as logical syntactic structures, can be described as physical facts in the world (Wittgenstein [1921] 1961, #3-3.33).

**3.3. What Are the *Truth-Conditions* of Our Basic Propositions and What Are *Facts*?**

However, in Wittgenstein’s later philosophy facts are “the given,” accepted conventions of our *forms of life*, “the inherited background against which I distinguish between true and false”. Thus, facts are either identical with accepted basic *empirical propositions* which are able to justify the truth value of other propositions, or they are expressed by true propositions as their contents. But this follows the tradition of Kantian phenomenalism in which reality is our experience represented grammatically; the empirical contents of our propositions, which embody our common phenomenal world (Wittgenstein 1974*,* #1; 1974, VIII #109-112; 1958, ##211, 226-227, 230, 241-242, 428-429, 471, 497, 607 ; 1972, ##94-98; Kant [1781-1787] 1929, A191/B236, A492/B520; cf. Pears 1988, chap. II; Nesher 1993, VII.1; 1999c, II). Yet one cannot combine the metaphysical realism of Tractarian formal semantics with the later phenomenology of Wittgenstein, which is a kind of internal realism or anti-realism, as Brandom strives for.

Elsewhere, I have explained why metaphysical realism and phenomenalism cannot explain knowledge of reality (cf. Nesher 1997a, 1999c). Later I discuss other problems, such as *what constitutes the contents of our propositions that can be “grasped,” and how do we “grasp” facts*? Yet if *true claims* or *true propositions* are *facts*, or express *facts* as their contents, what makes them true? We cannot proceed without answering the questions: What are the *truth conditions* of our basic propositions, and what are *facts*?

**4. Wittgenstein’s *Tractatus* and His Conceptions Of “Facts,” “True Propositions,” and “Truth Conditions,” and Why the Traditional Correspondence Theory of Truth Cannot Work**

**4.1 Wittgenstein Distinguishes Between Reality and the World to Explain the Meaning and Truth of Tractarian Propositions: Why can’t the Correspondence Theory of Truth Work?**

I construe Wittgenstein’s enterprise in his *Tractatus Logico-Philosophicus* as a typical example of the formal semanticist’s attempt to explain the experienced *world* on the basis of *model theory* and an imagined external *reality*, as opposed to realist epistemology which intends to prove the true representation of *external reality* (Wittgenstein [1921] 1961, 2.12; Peirce 1906; Nesher 2002, X, 2017). Wittgenstein did so by developing the correspondence theory between linguistic expressions (with their sensual contents, or pictures of *facts* pertaining to the phenomenal world), and he proposed that *objects* combine to form all possible states of affairs in reality. Thus, Wittgenstein elaborated the formal semantic tradition of Frege and Russell, which would also be followed by Tarski, Carnap, and others. However, the Tractarian formal semantic system is also embedded in a comprehensive Kantian transcendental metaphysics according to which the metaphysical subject infers from its thoughts the states of affairs that are to be the existing worldly facts which are necessary to explain how language presents our *phenomenal* world: the picture’s elements assume feelers in order to ‘touch’ the Kantian *noumenal* objects of external reality (Wittgenstein [1921] 1961, 2.1515).

The picture that emerges from the meaning of the metaphysical subject’s thought and the pictorial structures of language are facts (existing states of affairs in the world) whereas objects of reality are only hypothetical components of all possible states of affairs used to explain our true presentation of the world.

2.1 We picture facts to ourselves.

2.12 A picture is a model of reality.

4. A thought is a proposition with a sense.

Apicture is an aesthetic interpretation of the sensual contents of a propositional thought, which is able to present reality; but first the contents of the thought must be interpreted as facts of the phenomenal world, since thoughts without the sensual structure of propositions cannot have the perceptual structure of sensual facts. A picture-fact in the world also serves as the model of reality. Hence, we can imagine the model as the pictorial presentation of something independent from us, even though it is something our imagination has created from the propositional thought.

However, in epistemological terms, can we think without conceptual language, and how can language distort a thought, since according to Wittgenstein all philosophy is “critique of language”? ([1921] 1961, 4.0031). Accordingly, philosophical problems can only be resolved by exposing and dispelling such confusion, an activity that aims at “the logical clarification of thoughts” ([1921] 1961, 4.112). However, there is no internal criterion to compare thought and language, and at best it remains a subjective feeling; this is what the formalist calls psychologism ([1921] 1961, 4.116). Indeed, thoughts cannot come from nowhere but rather evolve from our empirical experience, and we can make them clear and distinct only by proving them to be true representations of reality, namely, if we replace Kantian transcendentalism, pure formal semantics, and intuitionism with the realistic epistemology of Spinoza and Peirce (Nesher 2002-2020).

Let us learn, with the help of Wittgenstein’s *Tractatus*, whether language can indeed represent reality and whether we can identify the *truth conditions* on which propositions might be verified as true, or falsified, in respect to facts in the world or objects in reality. Yet the cardinal question is whether the *Tractarian* and other neo-Kantian systems can explain our representation of *external reality* (Nesher 1998a, #2; 2002, X; 2017). In the *Tractatus* Wittgenstein constructed his logico-philosophical semantics, a system that attempts to explain, using the concepts of *meaning* and *truth*, how the thoughts of the metaphysical subject, expressed in the subject’s language, represent *facts* in the *world* by way of *feeling* the *objects* of external *reality*. This led Wittgenstein to formulate his *picture theory* of language, whereby entities in *reality* are independent of our thoughts and language, while language corresponds to states of affairs in the world and therefore represents facts. The idea of a *picture* being a *model* that represents facts and hints at objects of external reality is probably the best analogy that could be taken from our experience to show how one thing, a picture or painting, can present another thing, or fail to do so, by being true or false about combinations of *objects* in reality (Wittgenstein, letter dated 22.7. 1913; [1921] 1961, 4.016, 4.031-4.0311; comp. Frege [1918] 1968, 86-87).

In our everyday common experience we perceptually compare a picture, or its pictorial structure, with the structure of the subject-matter being pictured or depicted. According to Wittgenstein, we can extend pictorial representation to linguistic expression by formal analysis of the “logical forms” that determine the material structures of the propositional signs and the “logical forms” of the inferred worldly facts, and we compare them by projecting the “pictorial forms” of the propositions onto these facts, as we do intuitively when comparing a picture with what is pictured (Wittgenstein [1921] 1961, 4.01-4.463; 5.62; cf. Hacker 1981; Pears 1987, chaps. 5, 6). Wittgenstein’s picture theory of presentation is a formal semantics device to show the correspondence of our propositions, when understood as propositional signs, with the thoughts projected onto them through the one-to-one correspondence between the elements of the proposition and the components that compose the fact that verifies it (cf. Hintikka 1986, 92-94). Yet, first Wittgenstein had to explain how elementary propositions are meaningful, and he was obliged to solve Russell’s (and his own) predicament surrounding false propositions and how they can be meaningful without representing facts (Russell 1910, chap. VII; 1914, chap. III; Pears 1987, 115-121; Monk 1990, #4). To this end, Wittgenstein made the distinction between our *world* as the actual empirical combinations of *facts* and *reality* consisting of all possible configurations of eternal *objects* in *logical space* forming all possible states of affairs in reality. Thus, while names are connected referentially to eternal objects in the logical space of reality, the meaning of a name depends on the sense of the proposition in which it is embedded. But how can we know that such correspondence holds between our thoughts in language and external reality? To explain this relation, the Cartesian, Humean, and Kantian predicament should be overcome: How can one detect whether linguistic expressions convey truths or falsehoods about external reality? According to Wittgenstein ([1921] 1961, 2.224), it is impossible to tell from the picture alone whether it truly or falsely represents objects in reality. Moreover, the picture represents a possible state of affairs in logical space regardless of whether or not it is true.

2.21 A picture agrees with reality or fails to agree; it is correct or incorrect, true or false.

2.22 What a picture represents [embodies] it represents independently of its truth or falsity; by means of its pictorial form.

Thus, false propositions do not correspond to “positive facts” that exist but to logically possible, non-factual, “negative facts” ([1921] 1961, 2.06). The presentation of possible states of affairs in logical space precedes the description of the fact (existent or non-existent) and thus false propositions are meaningful through their presentation of states of affairs in reality regardless of their truth or falsity. In this way Wittgenstein solves the problem of the relation between meaning and truth such that propositions can be meaningful independently of their truth values. However, propositions are true pictures of the world and are thus models of existing states of affairsin reality. What makes a proposition meaningful is whether or not it is possible in logical space, and only if it is logically possible can it be said to have any truth –value; put differently, logical possibility is the condition for a proposition to be evaluable as true or false.

Therefore, in order to understand Wittgenstein’s theory of presentation with its concepts of “propositional facts,” “propositions,” “meanings,” “objects,” “facts,” “truth" and “truth conditions” we should understand the distinctions he makes between: *reality*, which consists of the sum of all possible combinations of metaphysical objects in logical space; all possible *states of affairs*; and the *world* as the totality of the actual combinations of objects, all the actual positive *facts*. Metaphysical objects in logical space are simple, unalterable, subsistent, and eternal entities that constitute the substance of the *world*; these metaphysical assumptions enable Wittgenstein to construct his theory of representation (Wittgenstein [1921] 1961, 2-2013ff.).

2.021 Objects make up the substance of the world. That is why they cannot be composite.

Metaphysical objects are assumed in order to explain the possibility of facts and their representation. They are simple basic components of *reality* and their forms or internal properties, their valences, are their possibilities to combine with other objects into facts. Reality is all possible combinations of objects in states of affairs

2.04 The totality of existing states of affairs is the world.

2.06 The existence and non-existence of states of affairs is reality.

(We also call the existence of states of affairs a positive fact, and their non-existence a negative fact.)

Hence, reality is the sum of all possible configurations of objects, like Kant’s *supersensible nature*, the *noumenal domain*, or an *intelligible* *order of things* which we cannot know directly; they are objects as they are, or *things in themselves*, which constitute all possible states of affairs in reality as well as positive facts, i.e., actual states of affairs in the world. However, among all possible states of affairs in reality, only segments are actual facts, which change in the dynamic world, and we can detect such changes. Thus, the world is the totality of existing states of affairs.

In this distinction between *reality* and *world* we can understand the difference between the *meanings* of the propositions which are determined by their referential relations to objects of *possible states of affairs* in the *logical space* of *reality*. The *truth* and the *falsity* of propositions are determined by their comparison with *actual states of affairs*, *worldly facts*, which they describe or fail to describe.

5.634. This is connected with the fact that no part of our experience is at the same time a priori.

Whatever we see could be other than it is.

Whatever we can describe at all could be other than it is

* 1. Reality is compared with propositions.
  2. A proposition can be true or false only in virtue of being a picture of reality.

The question is how are we to read the above propositions? Can we compare propositions with reality, and if so, how? Moreover, if the truth and falsity of propositions are pictures of reality, how can this help us to see the world rightly? Hence, reality is composed of all possible combinations of states of affairs, and the thoughts of the metaphysical subject are true pictures of all facts in the world, which are, so to speak, the phenomenal experience of the subject.

**4.2 Propositional Signs and Propositional Facts**

Wittgenstein distinguishes between *propositional signs* or *propositional facts*, which are the logico-syntactic structures of language, and *propositions* that are propositional-facts with meanings, when their elements – names – have senses and referential relations to reality (Wittgenstein [1921] 1961, 3144-3203). There are two *meaning* requirements for a propositional fact to qualify as a propositional picture that represents reality: First, the *pictorial form* of the propositional fact must have an isomorphic relationship with the *logical form* of the possible state of affairs if the former is to represent the latter.

2.16 If a fact is to be a picture, it must have something in common with what it depicts.

2.161 There must be something identical in the picture and what it depicts, to enable the one to be a picture of the other at all.

2.17 What a picture must have in common with reality, in order to be able to depict it – correctly or incorrectly – in the way it does, is its pictorial form.

The pictorial form of a picture has the potential to represent possible states of affairs in reality, existing or non-existing, but how can we know the pictorial forms of possible states of affairs in reality?

2.171 A picture can depict any reality whose form it has. . ..

2.18 What any picture, of whatever form, must have in common with reality, in order to be able to depict it – correctly or incorrectly – in the way it does, is logical form, i.e., the form of reality.

2.181 A picture whose pictorial form is logical form is called a logical picture.

2.2 A picture has logico-pictorial form in common with what it depicts.

How, though, can we have knowledge of the logical form of reality such that we can represent the logico-pictorial form of possible states of affairs? Wittgenstein’s conception of logic in the *Tractatus* is similar to Kant’s conception of transcendental logic in his endeavor to bridge the gap between the transcendental subject and *noumenal objects* through empirical experience (Nesher 2007, 2020).

But having *logico-pictorial form* is just a precondition for a propositional fact to reach reality. The representation is not a factual property or relation; *someone* has to project thought onto the logical picture, to enable it to represent, or give meaning to, the propositional fact by connecting it with objects of a possible state of affairs in logical space. (Compare Putnam’s story about the ant walking on the sand and “drawing” a picture of Churchill, and the question of whether these physical signs on the sand can represent Churchill without someone interpreting them as such; Putnam 1981, 1-2). Thus, *someone*, i.e., the metaphysical subject, projects meaning onto the factual syntactical components of a propositional sign to connect them with objects in logical space. The second requirement for the propositional fact to be classified as a propositional picture that represents a state of affairs is that it must have a *pictorial relation* to this situation in the logical space of *reality*.

2.11 A picture presents a situation in logical space, the existence and non-existence of states of affairs.

2.12 A picture is a model of reality.

2.131 In a picture the elements of the picture are the representatives of objects.

2.15 The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way.

Let us call this connection of its elements the structure of the picture, and let us call the possibility of this structure the pictorial form of the picture.

3.21 The configuration of the objects in a situation corresponds to the configuration of simple signs in the propositional sign.

The pictorial form is the logical form of the picture, which makes it possible for the proposition to represent states of affairs. But only by having a thought with sense projected onto its propositional structure can it obtain its referential relation to objects of a possible situation in reality.

2.1511 That is how a picture is attached to reality; it reaches right out to it.

2.1512 It is laid against reality like a measure.

This *someone* that projects thoughts onto propositional facts to transform them into propositions and to reach out to reality is the metaphysical subject that lays the picture against reality as a measure of some of its states of affairs.

2.15121 Only the end-points of the graduating lines actually *touch* the object that is to be measured.

2.1513 So a picture, conceived in this way, also includes the pictorial relationship, which makes it into picture.

2.1514 The pictorial relationship consists of the correlations of the picture’s elements with things.

2.1515 These correlations are, as it were, the feelers of the picture’s elements, with which the picture touches reality.

A sentence or propositional fact (or propositional sign) alone cannot constitute a picture. It is the propositional fact with a sense and thus with pictorial relations to the objects of a state of affairs in reality that together constitute a picture. This is the difference between bare facts and propositions; bare facts cannot be “about” other facts because they lack sense and pictorial relationships to any states of affairs, whereas propositions have correlations with the picture and can touch reality. In his discussion on the *Tractatus*, Dummett misunderstands this distinction between the propositional fact as a syntactically bare fact and the proposition, which is a picture representing a bare fact. Thus, Dummett misses the gist of Wittgenstein’s pictorial theory of representation. There cannot be, as Dummett suggests, “a fact about the properties or relations of certain objects.” (Dummett 1981, 37-38).

2.201 A picture depicts reality by representing a possibility of existence and non-existence of states of affairs.

### 2.202 A picture represents [*stellt ein* = [situate](https://m.interglot.com/en/de/situate)s] a possible situation [state of affairs] in logical space.

# 2.22 What a picture represents [*stellt dar* = depicts] it represents independently of its truth or falsity, by means of pictorial form.

2.221 What a picture represents is its sense.

As we can see, there is a problem with how to interpret the verb form *stellt* in 2.202 and 2.22: This could be rendered as represents, as in the translation of D.F. Pears and B.F. McGuinness (1961)‚ or as *depicts* or *situates*. According to my interpretation of Wittgenstein, the *Tractatus* is a special case of a formal semantics system with components of Kantian transcendental assumptions. All the components of the system are deduced from a priori assumptions, and therefore there is no *representation* of something separated from the system but only *presentation* in the sense of theorematic pictures in the formal semantic system itself.

Hence, by presenting its sense, the propositional picture hints at the possible state of affairs in reality. But depicting *actual* facts in the *world* differs frompresenting *possible* states of affairs in *reality* because, as we have seen, such a picture can present a possible state of affairs independently of its truth or falsity. We can know the meaning of a proposition, understand the proposition, without knowing its truth value. The question is not whether we can understand a proposition without knowing its truth value but whether we can understand a proposition without knowing how to verify it? For example, can we understand the proposition “There is purple grass” without knowing how to verify it? In Wittgenstein’s *Tractatus* we can distinguish between the *meaning-conditions* for understanding the propositions consisting of the sense-referential relations of propositions to *reality*, to objects in logical space, and to the *truth-conditions* that determine their truth values, i.e., whether or not they are true or false in relation to existing facts (Wittgenstein [1921] 1961, 4.05-4.1). Indeed, how are the truth values of propositions determined? The truth value of a complex proposition depends on the truth-function of the elementary propositions. But what are the truth-conditions of elementary propositions? (Russell 1910, chap.VII; Wittgenstein 1974, #105). Accordingly:

**5. A proposition is a truth-function of elementary propositions. (An elementary proposition is a truth-function of itself).**

Wittgenstein argues that elementary propositions have truth-conditions that are not themselves propositions, that is to say they are not linguistic. In the *Tractatus* *facts in the world* are necessary conditions for determining the truth or falsity of elementary propositions. In this way we can understand Wittgenstein’s statement that “An elementary proposition is a truth-function of itself.” Can we compare the special status of Tractarian elementary propositions in relation to states of affairs with the special status of perceptual judgments through which we confront external reality? All other propositions and theories are, in different ways, truth-functions of these basic propositions and can only represent reality through them (Nesher 1997a, 1998a, 1999c, 2002, X).

**5.3 The *Metaphysical Subject* Representing Reality Pictorially, Describing Worldly Facts, and Frege’s Conception of Logic and Mathematics**

How can the *truth-conditions* of elementary propositions be known in order to determine whether or not they truly present the world? In the metaphysics of Wittgenstein’s *Tractatus* the thinking metaphysical subject alone can use Tractarian descriptive language and exploit meaning to depict worldly facts by elementary propositions, thus picturing reality (Wittgenstein [1921] 1961, 4-4.01, 4.461-4.463).

4. A thought is a proposition with a sense.

The difficulty lies in explaining whether the metaphysical subject, the philosopher, and Wittgenstein himself, who is located outside the empirical-psychological world, can truly present actual facts existing in the world and picture objects of possible states of affairs in reality, or in Kant’s supersensible reality? The metaphysical subject can be likened to the Cartesian God located beyond the world, unlike humble human beings, and has separate access to propositional facts and to bare facts that enable him to present their logical form, their multiplicity. Thus, the metaphysical subject can project a new sense onto the propositional fact and its component names in order to connect it with the objects of the states of affairs in reality that it intends to picture.

4.027 It belongs to the essence of a proposition [sign] that it should be able to communicate a *new* sense to us.

The propositional sign is a fact in the world, and through the projection of new thought onto this sign it becomes a proposition with sense (cf. Wittgenstein [1921] 1961, 3.12). However, the logical form of the propositional fact is not enough for a pictorial representation because with one logical form, e.g., R(a, b), one can represent different facts with different material structures, e.g., “This chair is standing on the table” and “This book is lying on the table”. Sometimes a proposition with a particular material structure, e.g., “The bank is crowded” or “Green is green” can represent with different senses different facts that have the same logical form but with different material structures. Only with a specific sense projected as the content of the pictorial form – the logical syntax – can the sign present an object and a propositional sign a possible state of affairs in reality (Wittgenstein [1921] 1961, 2.18-2.22).

3 A logical picture of facts is a thought.

3.01 The totality of true thoughts is the picture of the world.

3.326 In order to recognize a symbol by its sign we must observe how it is used with a sense.

3.327 A sign does not determine a logical form unless it is taken together with its logico-syntactical employment (cf. 3.328, 3.33).

In other words, the logical expression of a sign in pictorial form occurs by means of its logico-syntax. The propositional sign can pictorially present states of affairs only when the metaphysical subject projects sense (by using thought) onto this sign to picture objects of possible states of affairs in reality.

3.11 We use the perceptible sign of a proposition (spoken or written, etc.) as a projection of a possible situation. The method of projection is to think of the sense of the proposition.

3.12 I call the sign with which we express a thought a propositional sign. – And a proposition is a propositional sign in its projective relation to the world. (cf. 3.13-3.1431).

3.5 A propositional sign, applied and thought out, is a thought.

3.21 The configuration of objects in a situation corresponds to the configuration of simple signs in the propositional sign.

However, in picturing or describing facts in the world, there is a difference between picturing reality and presenting states of affairs in the world..

4.01 A proposition is a picture of reality.

A proposition is a model of reality as we imagine it.

4.021 A proposition is a picture of reality: for if I understand a proposition, I know the situation that it represents [dargestellte ~ shown]. And I understand the proposition without having had its sense explained to me.

4.0311 One name stands for one thing, another for another thing, and they are combined with one another. In this way the whole group – like a *tableau vivant* [living picture] – presents a state of affairs.

The above propositions from the *Tractatus* are at the core of Wittgenstein’s picture theory of presentation that comprises his theory of meaning and truth, which goes against some of the basic elements of Russell and Frege’s theories of representation. The epistemological difficulty is to answer the question of how we understand the meaning of the proposition without it being explained to us. The realist-Pragmaticist explanation is that we learn language and propositions through our experience such that the preverbal components of a proposition are the iconic feeling and the indexical reaction to it in their synthesis interpreting in the conceptual meaning of a proposition interpreted in the symbolic thought. In his later philosophy Wittgenstein will explain that it is by learning the language in experience that this happens, as the Peircean semiotic makes clear (Peirce 1906, II: #26; Nesher 2001a. Thus, these preconceptual components are what Kant calls *aesthetic intuitions* of the phenomenal subject, and this can be considered in Wittgenstein as the imaginative or pictorial component of the proposition, namely, the experiential contents of the propositions (Kant [1781-1787] 1929, *A141, Prolegomena ##34-35*). Thus, realistically, propositions are meaningful when they confront reality as their preconditions of being true or false, and their truth and falsity are not the referential objects of sentences; instead, they prove the truth of perceptual judgments as facts representing reality (Nesher 2002, X; cf. Wittgenstein [1921] 1961, 4.064). The factuality of a state of affairs comes from the proof-conditions of the hypothetical propositions, which by being proved true represent the situation. The Wittgensteinian metaphysical subject determines the truth of the elementary proposition when he or she detects that the presented state of affairs is an existing fact in the world. This cannot be done from the structure of the form and the content of the proposition but only from outside the world from a nonhuman perspective on the structure of reality and the factual world.

5.631 There is no such thing as the subject that thinks or entertains ideas.

5.633 Where *in* the world is a metaphysical subject to be found?

You will say that this is exactly like the case of the eye and the visual field. … and nothing *in the visual field* allows you to infer that it is seen by an eye,

5.64 Here it can be seen that solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension, and there remains the reality coordinated with it.

5.641 Thus there really is a sense in which philosophy can talk about the self in a non-psychological way. What brings the self into philosophy is the fact that ‘the world is my world’. The philosophical self is not the human being, not the human body, or the human soul, with which psychology deals, but rather, the metaphysical subject, the limit of the world—not a part of it.

**[5] Wittgenstein’s Conceptions of the Meaning of Propositions Picturing “My World” and Personal Solipsism ([1921] 1961, 5.64-641)**

**“The philosophical self”** “non-psychological I” (5.641) “A logical picture of facts is a thought” (3).

The **Eye** **⁄ MS \**=*Metaphysical Subject* with its Projected Thoughts (Senses) to **World** and **Reality** (5.6331) ⁄ [***Thought*]**\ (3) by the *Metaphysical* *Logic* (5.633-5.641) or *Transcendental Logic* (6.13),

W⎛human body and soul⎞ = “*Pictorial Form*” (2.15-151) = “Representational Form” (2.173-4).

O⎥ / / \ ⎥ = The Structure Possibility of Pictorial Structure. (4.01-4.463; 5.62)

R**⎨P = RL (a \* b)** ⎥ = “*Pictorial Structure* “names and relations with *meaning* and *sense* of

L⎥ | | | |the **Actual** States-of-Affairs: **Facts** in **The World ≈ Propositional Facts** (1.)

D⎥ ▼ ▼ ▼ ⎥ = *Truth-Conditions* of Propositions (4.45) A picture is a *Model* of Reality (2.12).

⎥ ⎛ **fact**1 **fact**2 **fact**3⎞ | “Pictorial Relationship” (2.1513) “pictorial form” (2.22)"sign is a fact”(3.14)

⎥ |[Fact in The World]|⎥Meaning-Conditions of Propositional Signs. (Kant, 1929, B75/A51)

⎝⎝ (Model of Reality⎠ ⎠ = “Logical Space”-“things and relations” (1.12)

“The facts in the Logical Space are the world” (1.13)

The philosophical self is the metaphysical subject that through *thought* and utilizing a method of formal semantics creates its world, but itself remains outside the empirical world,which the philosopher can think about and pictorially explain by thought, which is the content of the language of formal semantics. Indeed, it is crucial to explain that “A proposition is a picture of reality. A proposition is a model of reality as we imagine it” ([1921] 1961, 4.01). Put differently, one can imagine or intend to know reality from its picture, which is inferred from the metaphysical subject’s true thought to present the world and eventually depict reality.

Wittgenstein interprets Kantian transcendental metaphysics with its transcendental subject and incorporates it into his own formal semantic language in which all its components are of the human cognitive mind except the metaphysical subject and the reality of all objects, or reality outside human cognition. Wittgenstein considers the experiential world as belonging to the psychological domain of the human being, the empirical human body, and human soul. For Kant, the human phenomenal subject has *sensual perception* and *aesthetic intuition* (Kant [1781-1787] 1929, B137, B296). Indeed, all philosophical systems that do not have a theory of truth to prove our knowledge of reality are actually solipsist: Russel, Frege, Wittgenstein, Davidson and more besides, even including Spinoza and Peirce.

…. the world as the totality of *possible* states of affairs (*TLP* 5.6), and the world as the totality of *obtaining* states of affairs, or the totality of facts (*TLP* 1–1.21, 2.04)—each of which corresponds to a different version of solipsism. The first version, sometimes called traditional solipsism, corresponds to the world as the totality of facts. (Hessell 2018, 128)

Most commonly, this reality-constituting relation is taken as experience. The traditional solipsist says that only their own experience is real, or that their present experience exhaustively constitutes reality. It is this kind of solipsism that, I will argue, is addressed in Wittgenstein’s discussion of solipsism. (Hessell 2018, 128)

However, in the *Tractatus* an epistemological distinction is made between the phenomenal world of experience including the psychological person or better, the empirical human being, and the reality of the *metaphysical subject* and all existing *metaphysical objects* in reality when both domains are *noumenal* entities, à la Kant.

‘By this “I,” or “He,” or “It,” who or which thinks, nothing more is represented than a **transcendental subject of thought** = x, which is cognized only by means of the thoughts that are its predicates.’ (Kant [1781-1787] 1929, B404/A346)

Frege’s conception of *thought*, inspired by Platonic idealism, provides the starting point for formal semantic epistemology to explain our knowledge of reality, like Kant’s pure empty concepts of the understanding vehicled by the transcendental subject. Thus, Frege separates the logical thought from psychological thinking, but the difficulty lies in identifying where the meaning, or sense, of such *Platonic thoughts* comes from. How are axiomatic true thoughts and formal logic related to psychological thoughts with their senses and references?

In order to avoid this misunderstanding and to prevent the blurring of the boundary between psychology and logic, I assign to logic the task of discovering the laws of truth, not of assertion or thought. The meaning of the word “true” is explained by the laws of truth. (Frege [1918-19] 1968, 290)

This *thought*, and the way it is used by the person that accepts and operates it, is an enigmatic entity with assumed meaning, accompanied by the true axioms of thought together with the logical laws of truth; through formal deduction, theorems are concluded from it. But the logician has to grasp and understand the logical laws of truth and this might be the psychological person who, according to Frege, can contaminate the absolute truth of pure formal logic (Frege [1918-19] 1968, 289-290).

I became aware of the need for a Begriffsschrift when I was looking for the fundamental principles or axioms upon which the whole of mathematics rests.

Only after this question is answered can it be hoped to trace successfully the springs of knowledge upon which this science thrives. (Frege 1984, 235, op. 362)

As Schema 6 shows, in the same way that Frege relies on axioms and formal logic to deduce the entire structure of a mathematic structure, he uses the images of object and concept to present the model of reality; eventually, by checking its coherency, he can infer the truth of the axiom in model-theoretic semantics: “…the structure of the sentence can serve as a picture of the structure of the thought” (Frege [1923-26] 2019, 1 [36]).

**[6] Thought, Apprehended and Operated – Frege’s Conceptions of Meaning and True Model**

The Basic Function: **Sense F**(term [Name/Predicate]) = **Reference** [Object, Concept])

Thought

∇ ∇

(**Sense**N)+(**Sense**P) = the Senses of N and P ⎞

(Platonic Entity) | | = Logical Inference ⎬ Logic of the **Third Realm**

∇ ∇ ⎥

**Proposition** = (N \* P) = Name and Predicate ⎬ Meaning: **Sense and Reference** a picture | | ⎥ of the structure of the thought

| | = the References of N and P ⎥ (The Context of the Meaning)

▼ ▼ ⎬ **References** of the **Sentence**

{**Object**, **Concept** [Property]}=Truth-conditions ⎠ Evaluation of the **Truth of the Thought**

[Pictorial Model as Reality] Dummett 1978, xlii, 7, 179, 185, 379] by the **Logicians Satisficing the Theory** by the **Coherent Structure** of the **Pictorial Model** of the **Axiomatic Thought**

The concept of a model is a device in *formal semantics* to solve the Kantian epistemic gap between a priori pure reason (with its transcendental logic), the pure empty concepts of the transcendental subject, and the sensual intuition of the phenomenal subject with its blind objects. Thus, since formal semantics and pure logic does not offer a theory of truth, the conception of *model* steps into the place of external reality, its role being to prove the consistency of the pure axioms of pure mathematics in order to continue to work with them (Kant [1781-1787] 1929,B19-20, B137, B296; Frege [1918] 1968, 301-2; Wittgenstein [1921] 1961, 2.12, 4.01, 5.62, 6.53; Brown 1999, chaps. 3, 9; Nesher 2002, 2011, 2012, 2020; Textor 2018). The Platonic thinker looks for the immanent truth of a thought by interpreting its sense in sentences and its inference in theimmanent object and concept as its value in the pictorial model as reality, but can logic prove the truth of the pure semantic *thought*?

It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the earth for the first time, a form of words can be found in which it will be understood by someone else to whom the thought is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought. (Frege [1923–26] 2019, 1 [36])

It can be suggested that the role of Frege’s formal logic in formal semantics is not to prove the truth of a thought but rather, like Kant’s transcendental logic, to justify deductively the axiomatic thought in exhibiting the picture, as meaningful content, of its coherence as the criterion for its truth. Hence, since both Kant and Frege do not adhere to a theory of truth in respect to an external reality and so also of the pure formal semantics as a closed-game, the truth of the Kantian transcendental concepts and Fregean Platonic thoughts is through the consistency of the pure axioms in the model-picture (Nesher 2011, 2016, 2018).

According to Frege, empirical epistemology is subjective and psychological, whereas Platonist epistemology is objective; he rejects the empirical conception of truth as correspondence between propositions and states of affairs and suggests that truth and falsity are immanent in objective Platonist thoughts. However, Frege’s epistemological predicament is that his criterion of common understanding of *thought* must rely on the subjective feeling of the interpreters. For the realist pragmatist, the proof of the truth of a proposition is an objective *fact* based on common proof-conditions which, even if personal judgments differ, can be evaluated in practice as objective, but outside the unexplainable Platonic *third realm* (Nesher 2002-2020).

So the result seems to be: thoughts are neither things of the outer world nor ideas. A third realm must be recognized. What belongs to this corresponds with ideas, in that it cannot be perceived by the senses, but with things, in that it needs no bearer to the contents of whose conscious to belong. Thus the thought, for example, which we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. (Frege [1918-19] 1968, 302; comp. Popper 1972, chap. 4)

For Frege the epistemological problem is that to express the Pythagorean theorem in our thoughts as an idea, we have to cognize this theorem in our conscious mind, and thus it is part of our cognition. Moreover, when Pythagoras himself formulated this theorem, his idea belonged to the realm of ideas too. The influence of Frege’s epistemology is evident in Popper’s Platonic conception of objective knowledge in his theory of three worlds. Since Popper cannot explain absolute truth (which he identifies with reality), he cannot show how to prove the truth of our scientific hypotheses which belong to the physical world (World One) and the world relating to the mind and mental states (World Two). According to Popper’s philosophy, we can only falsify or refute scientific hypotheses, we cannot verify them. However, falsifiability depends on an unexplainable “empirical basis” which cannot be objective since we cannot verify it, and this can only be an ideal model. Frege faced a similar difficulty with his absolute pure logic of formal semantics which was meant to serve his conception of absolute knowledge but remained *empty* of meanings with *blind* references (Frege [1918] 1968, 301-2; Popper 1972, chap. 3; Nesher 2000, 2001a, 2001b, 2005, 2007; Eder 2019).

If there is not a third option between *blind* sensual empiricism and *empty* logical rationalism (the basic components of Kantianism), then can we rely on the pragmaticist’s epistemic logic to overcome the Kantian gap between rationalism and empiricism? (Ricketts 1996; Nesher 2016-2020; schema [6]). Frege’s epistemology is Platonistic because he wants to explain truth as absolute and thus it cannot belong to the mental realm, which he takes to be subjective, as opposed to his conception of logic as axiomatic-obsolete truth, and hence related to the Kantian *noumena-supersensible transcendental subject* (Kant [1781-1787] 1929, A538).

Thoughts are by no means unreal but their reality is of quite a different kind from that of things. And their effect is brought about by an act of the thinker without which they would be ineffective, at least as far as we can see. They can be true without being apprehended by a thinker and are not wholly unreal even then, at least if they could be apprehended and by this means be brought into operation. (Frege [1918-19] 1968, 311)

However, this raises the question of how thoughts could be apprehended by a thinker if they are formal and pure by virtue of the assumed content or meaning of the Platonic thought. Indeed, we can compare the role of Fregean logic to that of Kantian transcendental logic insofar as Kant justifies, or deduces, the a priori concepts of the transcendental subject’s understanding and of pure cognition of reason in order to present experiential objects by logical judgments. Thus, Fregean Platonic thought is the content of his transcendental logic, capable of presenting abstract objects and concepts (Kant [1781-1787] 1929, A57-9/B82-4, A62–3/B87; Nesher 2005, 2016, 2018; Achourioti and Van Lambalgen 2011; Frege [1918] 1968). This is similar to Kant’s explanation:

The part of transcendental logic that expounds the elements of the pure cognition of the understanding and the principles without which no object can be thought at all, is the transcendental analytic, and at the same time a logic of truth. For no cognition can contradict it without at the same time losing all content, i.e. all relation to any object, hence all truth. (Kant [1781-1787] 1929, A62–3/B87)

However, due to Frege’s assumption of the gap between logic and psychology, we cannot grasp the senses of Platonic thoughts since they cannot evolve from any experience, but if a human person could grasp platonic thoughts, the latter would be contaminated by psychology and cannot be absolutely true but only relative to the perceiver, unlike Peircean empirical realism and his epistemic logic (Peirce 1998; Nesher 2018).

Hence, Frege’s basic difficulty is that the content of Platonic *thoughts* must be grasped from some perspective so as to be expressed in propositions. Moreover, in formal semantics, following formal deductive logic, without any experiential relation to *reality*, thoughts are of *empty pure concepts* which cannot be understood and verified as true. Indeed, we cannot interpret the meanings of a thought and prove its formal abstract truth or falsity without appealing to our experience in reality. This issue is connected with how one can grasp the *sense* of a name and the *nominatum* of the expressed sentence as components of the *thought*, since we cannot grasp and understand the *thought* without our subjective, cognitive experience in the context of reality (Frege 1892, [1918] 1968; Nesher 2002, X, 2007, 2016, 2018).

Frege’s problem of *empty* thoughts devoid of meaning and lacking *truth* is tied up with the concept of unknown objects, and this reminds us of Kant’s difficulty in bridging the gap between *empty pure concepts* and *blind objects*; even the device of unknown *Schematism*, “the secret art residing in the depth of the human soul, an art whose true stratagems we shall hardly ever divine from nature and lay bare before ourselves,” cannot solve this epistemic difficulty, and under these conditions transcendental logic cannot operate to bridge this gap (Kant [1781-1787] 1929, A141/B180-1). Moreover, *formal semantics* with its formal logic remains a *closed game* that cannot be affected explicitly by our cognitive experience in reality and thus cannot prove the truth of its axioms and the relation of deduced theorems to reality. It follows that it cannot explain knowledge of ourselves and of external reality. Indeed, formal semantics is an aspect of Kantian transcendental epistemology, its formal logic and mathematics. Neo-Kantian philosophies have developed aspects of Kantianism but have also failed to explain our knowledge of reality, including human reality, although Spinoza and Peirce’s philosophies seem to be exceptions (Nesher 2002-2020). Frege holds that judgment is intersubjective and objective and that judgments based on subjective experience cannot be rationally objective and true, for they are no more than feelings and intuitions. Yet, if this is so, how can subjects know objective truth without experiencing the meanings of propositions or of their thoughts in real life? Indeed, the role of the realist’s *epistemic logic* is to show that every proof of the truth of hypothetical propositions and theories is based on accepted proof-conditions, the methodology of proof and proven true basic facts. Moreover, we can show that individuals’ proof-conditions can be the same or can intersect or overlap such that common proofs of the truth of our perceptual judgments and scientific hypotheses are accessible (Nesher 2002, X; 2018, 2020). It seems that to overcome Frege’s Platonic deadlock Wittgenstein appealed to the Kantian *transcendental subject of thought* = x, the eye outside the world, to solve Frege’s problem with Platonic *thought* and its *sense* and *nominatum* (Wittgenstein [1921] 1961, 5.632-5.64). Moreover, it seems that Wittgenstein turned to Kantian transcendental epistemology to accept the deep distinction between the *phenomenal world*, as my world, and the *noumenal-suprasensual* *reality* that we cannot know and nevertheless use to explain our knowledge of nature, self-knowledge, and morality (Nesher 2007). Thus, Wittgenstein’s epistemological approach in the *Tractatus* can be construed as proposing a metaphysically assumed noumenal supersensible reality, which can be used to explain personal life in the world, for it allows for the assumption of noumenal entities without having to (or being able to) prove our knowledge of them. This can be viewed as a type of idealist epistemology, developed earlier by Kant in his transcendental philosophy. Neo-Kantians of the nineteenth and twentieth centuries accepted and elaborated different aspects of Kant’s metaphysical system, in the fields of phenomenology, analytic philosophy, logical empiricism, neo-pragmatism, and more, from Frege to Husserl, Carnap, Tarski, Popper, Quine, Davidson, Putnam, and Hintikka. However, neo-Kantianism has not succeeded in providing a theory of truth to explain how we represent and have knowledge of reality (Nesher 2002-2020). Hence, formal semantics and other trends in philosophy have developed the concept of *model of reality* using different names which, it could be said, are parallel to Kantian *sensual intuition*, replacing reality in order to develop theories of knowledge of it. We can also find this epistemological conception in Wittgenstein’s *Tractatus*, under the influence of his teachers, Frege and Russell (among others).

4.01 A proposition is a picture of reality.

.A proposition is a model of reality as we imagine it.

4.121 Propositions cannot represent logical form: it is mirrored in them.

What finds its reflection in language, language cannot represent.

What expresses *itself* in language *we* cannot express by means of language.

Propositions show the logical form of reality. They display it.

The concept of *model* in *formal semantics* is a substitution for proving the truth of linguistic expressions, based on the assumption that through intuitions propositions show the logical form of reality as though we know what reality is and can formalize it in the formal model. This is basically a Kantian device to introducean unknown *noumenal* reality in order to explain the difference between *reality* and *phenomenality*, combining the *blind objects* of sensual intuition in empirical experience with the *empty pure concepts* of the understanding of the transcendental subject. Both are synthesized to account for our *logical judgments on which* our basic knowledge is based (Kant [1781-1787] 1929, B75/A51; Nesher 2005). Hence, we can generalize epistemologically that formal semanticists, like Wittgenstein in his *Tractatus*, present a *model* without proving that it is factual truth, and it therefore remains only a fiction (or substitution of reality). However, just as Kantian sensual intuition plays a necessary role in the way that logical judgments present phenomenal objects, the role of the model is fundamental in Wittgenstein’s *Tractatus* and Frege’s philosophy of logic (as well as Hilbert’s).

Wittgenstein as a person with a soul and a body is a component of the subjectively experienced world and thus he is a psychological subject with a physical body living in the world and in reality. As such, he identifies the world as being his world, and since he cannot go outside it he is a *solipsist*, as distinct from the philosopher, the metaphysical subject who relates to the world through axiomatic truths and metaphysical logic, presenting facts with its language. This can be compared to the epistemology of the realist for whom subjects come to have knowledge of external reality when they prove the truth of their cognitive representations (Wittgenstein [1921] 1961, 5.63-5.6414; Nesher 2002, X; 2007b). However, Wittgenstein as a metaphysical subject and formal semanticist has no experience of the world; through formal logic and deduction he constructs a picture of the world as a model of reality which he cannot know.

3.2. In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the object of the thought.

4. A thought is a proposition [sign] with a sense.

1.1 The world is the totality of facts, not of things.

1.11 The world is determined by the facts, and by their being *all* the facts.

1.13 The facts in logical space are the world.

Wittgenstein’s Kantian-Fregean model of reality and the world in the *Tractatus* intends to show how thoughts aim to reach reality, “the totality of existing states of affairs”, which we can think about but cannot know, for the knowable, as a system, can only be seen whole from the outside ([1921] 1961, 2.04, 5.6331). Wittgenstein, in developing his *Tractatus* followed Frege, his teacher, to distinguish epistemologically between themetaphysical subject, the non-psychological “I” (who exercises objective thought and pure logic), and the subjectivity of the human soul and its body in the empirical world, which are concerns of psychology. But in order to avoid Frege’s problem with Platonic *thought* that cannot explain our knowledge of the *phenomenal world*, he claims that “the world is my world”, which is not the case with *noumenal reality*. Wittgenstein suggests that thought is indeed objective and as such belongs to the metaphysical subject, the philosopher, yet he does not explain how this microcosm has access to the meanings of thought without being an experiential person engaged in the phenomenal world ([1921] 1961, 5.641).

However, in order to present and comprehend the world Wittgenstein needs to engage experientially with it; he therefore needs to combine the logical and the psychological to bring two selves together, but according to his Tractarian epistemology he cannot do this. Thus, he must lean on his formal semantics and logic as closed games detached from experience in the world and in reality. And yet, Wittgenstein uses the Kantian conception of the transcendental subject to explain *metaphysical thoughts*, at the core of his formal semantics, and to deduce knowledge of the *phenomenal world* and the intuitive feeling of *noumenal reality,* which he can eventually represent through his semantic model, although he can only think about and picture it, in the absence of any theory of truth to prove our knowledge of it. The *thought* of the metaphysical subject is a *noumenon* as distinct from the human subject (with a body, soul, and will), which constitutes a *phenomenon* in the world. Thus, the metaphysical subject is to be noted outside of the empirical world of space and time and outside of the causality of nature, like the principle of morality or freedom of will. It is like Kant’s transcendental subject operating pure understanding or conscious of the fact of pure reason. The empirical human being, on the other hand, is a subject in the world and submitted to the causal laws of nature (Wittgenstein [1921] 1961, 3.-3.13, 5.631-634). The role of the *transcendental subject* in developing cognitive engagement with the empirical world through Kantian logical judgments and the role of the *phenomenal subject* in the presentation of *phenomenal objects* of the *world* are shown in the schema below, which outlines Kant’s epistemology. As he admitted: “The project on which I am now working… must be completed, or else a gap will remain in the critical philosophy” (Kant to C. Grave, September 21, 1798, AK 12:257).

To see the effect of Kantian philosophy on Wittgenstein’s *Tractatus* we can look at Kant’s three critiques: only the first is a science of knowledge based on perceptual logical judgments, as distinct from moral and aesthetic judgments with their emphasis on the roles of categorical imperatives and the reflective judgment of the subject.

6.421 It is clear that ethics cannot be put into words. Ethics is transcendental. (Ethics and aesthetics are one and the same.)



It can be seen in the above schema that Kant’s conception of knowledge does not rely on any theory of truth but makes an artificial effort to bridge the gap between the understanding of the transcendental subject, who uses a priori empty, pure concepts, and the phenomenal subject who has sensual intuition of blind objects via mysterious, unknown schemata (Kant [1781-1787] 1929*,* A141/B180-1; A121, B185-187; [1800] 1992, 67n76).

We can compare Kant’s transcendental subject with Wittgenstein’s metaphysical subject because of their epistemic roles in presenting the experiential phenomenal world.

5.641Thus there really is a sense in which philosophy can talk about the self in a non-psychological way. What brings the self into philosophy is the fact that ‘the world is my world’. The philosophical self is not the human being, not the human body, or the human soul, with which psychology deals, but rather, the metaphysical subject, the limit of the world—not a part of it.

This is the epistemological solution Wittgenstein provides to overcome the problem with the Fregean-Platonist conception of *thought* that assumes that axioms express absolute truth. He thus constructs a formal semantic system to infer the logical structure of language, linguistic conduct, and knowledge of reality. But such a solution fails to bridge the *gap* between the transcendental-philosophical subject and the sensual empirical subject and thus it remains epistemologically unsolvable.

4.112 Philosophy aims at the logical clarification of thoughts…

Without philosophy thoughts are, as it were, cloudy and indistinct: its task is to make them clear and to give them sharp boundaries.

The philosophical self clarifies thoughts by logically deducing facts. As a formal and coherent model, the “logical clarification of thoughts” is a systematic operation based on formal semantics and therefore on the consistency and truth of basic axioms.

**[8] Wittgenstein’s Tractarian Conceptions of Meaning and Truth: Meaning-Conditions and Truth-Conditions of Propositions and Solipsism (5.64-641) A Picture is a Model of Reality (2.12, 4.01):**

Presenting the World and Intuiting Noumenal Reality

⎛ **“The philosophical self”** ⎞ “non-psychological I” (5.641) “A logical picture of facts is a thought” (3).

**| ⁄ MS \**=*Metaphysical Subject* Projecting Thoughts (Senses) onto the **World** and **Reality**

|\_\_ \_ ⁄ [***Thought*]**\ (3) | by the *Metaphysical* *Logic* (5.633-5.641) or *Transcendental Logic* (6.13),

| W⎛human body and soul⎞ | “*Pictorial Form*” (2.15-151) = “Representational Form” (2.173-4).

R|O ⎥ / / \ ⎥ ⎥ = The Structural Possibility of Pictorial Structure (4.01-4.463; 5.62).

⎥ R**⎨P = RL (a \* b)** ⎥ = “*Pictorial Structure* “names and relations with *meaning* and *sense* of

A⎥ L⎥ | | | ⎥ the **Actual** States of Affairs: **Facts** in **The World ≈ Propositional Facts** (1.)

⎥ D⎥ ▼ ▼ ▼ ⎥ = *Truth-Conditions* of Propositions (4.45) A picture is a *Model* of Reality (2.12).

L⎥ ⎥ **fact**1 **fact**2 **fact**3 ⎥ = “Pictorial Relationship” (2.1513) “pictorial form” (2.22)"sign is a fact”(3.14)

⎥ ⎥ [Fact in The World] ⎥↑=

I **⎨** ⎝ (Model of Reality) ⎠ ⎥ of Propositional Signs. (4.01m **4.462**) (Kant [1781-1787]

1929, B75/A51)

⎥ ∇ ∇ ∇ ⎥ = The *Feelers* of the Picture Cannot Represent **Reality: *Solipsism*** (5.62, 6.53)

T **|** | | | ⎥ = ***Eventual*** *Truth*: “Reality is compared with propositions” (4.05)

**|** | | | ⎥ = ***Formal Semantic Meaning*:**no proof of True Representation of Reality.

Y**|** **Relation**, **Object2 Object3 |**= **Possible** Configurations of imagined Objects: Unknown Assumed Reality

**|**All Possible States of Affairs| = “Logical Space”-“things and relations” (1.12) *Meaning-Conditions:*

**|** of *Objects as Themselves* **| (**4.41, 4.461-4.463).

⎠ ***Noumenal Reality*** ⎝**-** Internal Presentation vs. Eventual Representation. (2.06, 2.11, 2.151)

Hence, we can conclude that Wittgenstein’s Tractarian epistemology combines Kantian transcendentalism (and the *transcendental subject*) with formal semantics (a closed game). The deductive inferences of formal logic cannot prove the true representation of reality, they can only assume the metaphysical subject’s axiomatic thoughts of eternal truths which present the phenomenal world of facts by using formal logic (deduction) and the theorematic picture model. Its dynamic evolution can be explained by the discovery of eventual possible states of affairs of *objects in themselves* in reality. Generally, this combination influenced the thinking of philosophers and mathematicians such as Frege and Popper, as well as different aspects of neo-Kantian philosophies that have since emerged (e.g., Russell, Husserl, Carnap, Tarski, Quine, Davidson, Putnam, Hintikka and more; cf. Nesher 2002, 2011, 2016, 2018). Indeed, *the philosophical self*, Wittgenstein in the case of the *Tractatus*, through thought can solve the Fregean predicament about knowledge of the meaning of the Platonic thought, since Wittgenstein owns his world and, as a human being with a body and soul, implicitly experiences and knows it. This can explain the sense and the nominatum of linguistic propositions in presenting facts of the world, but with such an operation he closes the Fregean gap between *logic* and *psychology*.

In the *Tractatus* the metaphysical subject, the philosopher, deduces theorematic facts of the world, “my world”, from the axiomatic content of his thoughts. The picture of his thought (and thus of the world) is the *logical picture* of the philosopher’s thought, the *schema* of the *relation between the axioms*, and as such a Kantian reflective interpretation produced by the imagination and association with a sense impression or intuition: *Schematism* [6](Kant [1781-1787] 1929, A141). Thus, according to Kant’s distinction between phenomenal and noumenal reality (which the metaphysical subject cannot represent and cannot know but only imagine):

3. A logical picture of facts is a thought.

3.04 If a thought were correct a priori, it would be a thought whose possibility ensured its truth.

3.05 A priori knowledge that a thought was true would be possible only if its truth were recognizable from the thought itself (without anything to compare it with).

3.5 A propositional sign, applied and thought out, is a thought.

4 A thought is a proposition with a sense.

4.01 A proposition is a picture of reality.

A proposition is a model of reality as we imagine it.

The epistemological difficulty here is that Wittgenstein’s formal semantic methodology is a *closed game* and even this sort of combination with Kantian transcendental idealism cannot explain the true content of the metaphysical philosopher’s thought which Kant assumes to be the transcendental subject’s understanding using a priori pure concepts and categories, which are connected to the *phenomenal subject’s* sensual intuition by pure reason within the framework of transcendental logic (Nesher 2011, 2018). However, Wittgenstein after Frege separated the metaphysical subject from the psychological person in order to make sure that the pure absolute logic of the objective true thought is not contaminated by the subjective human relative experience, as it is in Popper, Putnam, and others. From this perspective, absolute truth is reality itself. Kant and neo-Kantians make this assumption because they lack any theory of truth. In formal semantics, the correspondence between human thought and facts of reality is viewed as being reliant on our *sense data*, as in the Wittgensteinian picture; alternatively, a correspondence is posited between our phenomenal experience and Kantian noumena, not external reality (Nesher 2002-2020). The question is whether Wittgenstein’s metaphysical subject using formal semantic philosophical language can only feel or imagine reality pictorially, on the basis of transcendental logic; or can the metaphysical subject also represent reality truly? After all, the Kantian transcendental subject only seems to intuit *noumenal reality* in order to complete the Copernican Revolution, rather than represent it truly (in accordance with the approach of transcendental idealism).

**[9] Kant’s Conception of Sensual and Supersensual Domains**: [1781-1787] 1929, A848/B876, [1724-1804] 1998, 4:461;

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⎛ Transcendental Subject **I** ⎞ (cf. A343–4/B401–02, A*355*ff.)

⎛ Noumenal ⎨- - - - -Pure Apperception A355 - - - - - - | ⎞

| ⎝ Human Pure Soul-Substance: A348 ⎠ Apperception?|

| ⎛ ---------------------?---------------------------⎞ |

| | ⇓ | |

| | Empirical Apperception | |

Worlds ⎨ | ⇓ ⎬ Anthropology ⎬ One World?

| | Human Empirical Sensuality | | P. Kitcher, 2017:607

| | Empirical Psychology | |

Empirical ⎨ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - | |

⎝ ⎝ Physical Nature ⎠ ⎠

⎝The Metaphysically Assumed Noumenal Supersensible Reality ⎠

Thus, Tracterian formal semantic epistemology is only a closed game in which the truth of the metaphysical thought remains unexplainable and its theorems are left hanging in the formal air without explicitly reaching the experiential world and external reality (Wittgenstein [1921]1961, 2.15121, 2.1513, 2.1514, 2.1515).

We can see that the formal world only metaphorically feels the phenomenal world and the picture of the thought can only imaginatively feel the external, or perhaps noumenal, reality.

2.18 What any picture, of whatever form, must have in common with reality, in order to be able to depict it – correctly or incorrectly – in any way at all, is its logical form, i.e., the form of reality.

2.181 A picture whose pictorial form is logical form is called a logical picture.

2.2 A picture has logico-pictorial form in common with what it depicts.

3.327 A sign does not determine a logical form unless it is taken together with its logico-syntactical employment (cf. 3.328, 3.33).

The world is assumed to be a component of reality but the human subject can only intuit it and cannot prove true knowledge of it in the absence of a theory of truth. Thus, only a *model* of reality can be suggested, which is based on intuition of the world. It is difficult to answer the question of how the *metaphysical subject* through *thought* can imagine *propositional facts* to be a *model* of reality. It seems that the *metaphysical subject* may be similar to the Kantian *transcendental subject* in terms of the *“transcendental subject of thought* *= x*, which is cognized only by means of the thoughts that are its predicates” (Kant [1781-1787] 1929, B404/A346). Alternatively, the formal semanticist would say that in the *Tractatus* unexplainable Fregean *Platonic thoughts* are replaced by *metaphysical logic* to deduce and justify a presentation of the world that pictures the structure of reality (because propositions share the logical form of reality). The dilemma is whether it is the psychological self or rather the philosopher that stands outside the world and ‘sees’ it? Yet how can a point of view observe itself? Wittgenstein says that “A proposition is a model of reality as we imagine it” (Wittgenstein 1961, 4.01). The suggestion is that whatever it is that brings the world into focus is in the world.

Thus, we can use Kant’s conception of *blind objects* of *sensual intuition* as the basis of a *formal semantic* ***model*** *of reality* to explain ***propositional facts*** which picture reality by feeling the possibility of existence or non-existence of states of affairs. And yet, this is accepted without knowing reality; we rely only on the intuitive imagination that results from our experience in the world. Indeed, like Kant, Wittgenstein does not propose any theory of truth to prove knowledge but, as in formal semantics, he can assume the truth of some propositions in order to show, by axioms and the rules of formal logic, that other propositions are also true (Kant[1781-1787] 1929, A57-59/B82-83; Wittgenstein [1921] 1961, 2.201-2.221, 5.101-5.121, 6.13; Nesher 2002, X; 2011, 2016, 2018; Eder 2019).

Crucial questions for Wittgenstein in the *Tractatus* are: What is reality and what is its role in Kantian *formal semantics*? What is being assumed about reality in our epistemological explanations? Let us say that the metaphysical subject’s logic of the axiomatic true thought picturing all the possible combinations of its content and this is the deducing and eventually picturing all possible states of affairs being the senses of the logical picturing reality. However, whatever we know is made up of world facts, and by picturing more and more of the senses of metaphysical thoughts we are able to feel more of the logical “possibility of existence and non-existence of states of affairs” of the unknown *noumenal* reality. We can suggest that when we depict reality, “independently of its truth or falsity, by means of pictorial form” it is only a logical picture and that true existing states of affairs are already components of our world; we can extend our knowledge by making scientific inquiries and by accepting more states of affairs as true facts in our world (Wittgenstein [1921] 1961, 2.201 2.22).

Hence, true facts can be accepted into our world and then logically and pictorially we come to know more and more of reality, thus extending our own world in which we live. Thus, by using his common-sense intuition Wittgenstein overcomes the closed game of his *formal semantics* and *transcendental logic* to explain our world better than Frege and other formalists. In the *Tractatus* Wittgenstein identifies the world with our thoughts that picture this mysterious reality; for logic is a reflection of the world ([1921] 1961, 6.13).

5.64 Here it can be seen that solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension, and there remains the reality coordinated with it.

In the *Tractatus* Wittgenstein suggests that the meanings of *propositional signs* are achieved through their relations to propositional *facts*, or *phenomena* in the world and imagining that their truth can be understood in terms of possible states of affairs of *objects in themselves* of reality. But this cannot be achieved since the latter are *noumena* which must remain unknowable to us; the metaphysical subject can only think about these (Wittgenstein [1921] 1961, 2.0121).

Indeed, we can elaborate and show that without having an epistemic conception of the proof of truth to represent external reality we remain solipsists, which is evident in Kant’s transcendental epistemology in his effort to overcome the rationalist and empiricist shortcomings in proving the truth of our hypotheses through sensual experience and scientific hypotheses. It is clear that according to Kant we intuit but cannot know supersensual transcendental subjects and objects in themselves, as both are components of noumenal reality, which our inner phenomenal experience only aims to represent. However, the inner sensual content of the proposition cannot come from nowhere; it results from our sensual experience, which contradicts the formal semanticists’ requirement that to be objective it must be separated from the psychological-cognitive domain. But for Wittgenstein it seems that the content of our cognition is reality, and this holds for all neo-Kantian philosophies in different respects, e.g. Frege, Russell, Husserl, Carnap, Tarski, Popper, Quine, Davidson, Putnam, Hintikka, and so on, to include all other philosophers who epistemologically, in different modes, remain solipsists, excluding Spinoza and Peirce (Nesher 2002, I, II, X; 2018, 2020).

Kantian transcendental epistemology relies on reason, understanding and a priori assumptions without proving their truths, in order to explain our sensual intuition about the phenomenal world. But this means that we are not able to represent reality itself, *things in themselves*, due to the lack of any theory of truth (Kant [1781-1787] 1929*,* A57-59/B82-83, [1788] 1996, 49-50-51). Wittgenstein’s views, with his metaphysical *formal semantics*, shares similarities with Kant’s transcendental epistemology. Therefore, he is also not in a position to prove knowledge of phenomena, facts, states of affairs, or the truth of perceptual judgments. In Kant’s epistemology the relation between *empty pure concepts* and *blind sensual objects* falls under *formal semantics* which he tags *schematism*: This is a formal schema, a device to overcome the gap in his transcendental epistemology, but it is a vague explanation which also resides in Wittgenstein’s *Tractatus*. Such formal semantic procedures are intended to connect thoughts of the experiential world with imagined reality. However, this is what brought Wittgenstein to admit to *solipsism*, like Russell and eventually all neo-Kantians and formal semanticists with their conception of a *formal model* or of *sense-data* as constructions of reality (Russell 1914, III; Wittgenstein 1921, 4.01, 5.641).

This schematism of our understanding, i.e., its schematism regarding appearances and their mere form, is the secret art residing in the depth of the human soul, an art whose true stratagems we shall hardly ever divine from nature and lay bare before ourselves. Only this much can we say: The image is [here] a product of the productive imagination’s empirical ability. (Kant [1781-1787] 1929, A141/B180-1; cf. A121, B185-187)

Hence, Kant’s solution to the epistemological *gap* between a priori formal assumptions and *a posteriori* experiential matter is either deduction or schematism and thefact of pure practical reason (morality), but can they really solve this epistemological difficulty? As Kant mentioned in later life in his letter to C. Grave:

The project on which I am now working … must be completed, or else a gap will remain in the critical philosophy. (Kant to C. Grave, September 21, 1798b, AK 12:257)

The gap between logical and real essence of a thing is bridged in transcendental logic by the schematism which formalizes existence itself in its spatial and temporal aspects. The forms of space and time become necessary elements of thought. (Kant [1800] 1974, 67n76)

In respect to the relation of the transcendental understanding of pure intuitions to space and time, and the link between pure concepts and sensual intuition of blind objects, the difficulty lies in explaining how empty pure concepts, i.e., meaninglessness, can be applied to the blind object when they lack clear meanings. Hence, Kant’s solution to this epistemological *gap* between a priori formal assumptions and *a posteriori* experiential matter is either deduction or schematism and thefact of pure practical Reason, whether or not they can bridge this epistemological difficulty.

The principle by which we reflect on given objects of nature is this: that for all natural things *concepts* can be found that are determined empirically. This means that we can always presuppose nature’s products to have a form that is possible in terms of universal laws which we can cognize. For if we were not allowed to presuppose this, and did not base our treatment of empirical presentations on this principle, then all our reflections would be performed merely haphazardly and blindly, and hence without our having a basis for expecting that this [reflection] is in agreement with nature. (Kant [1790] 1914, V: 211’-212’)

We can understand Wittgenstein’s epistemology in the *Tractatus* as a synthesis of formal semantics and Kantian transcendental idealism, thus lacking any theory of Truth and unable to provide any representation of reality, which leads to *solipsism*, as with Frege, Russell and all his contemporaries (Wittgenstein [1921] 1961, 5.6-5.641; Frege [1918] 1968, 524; Nesher 2002, 2018).

It is interesting to compare Wittgenstein’s conception and use of *logic* in connecting the *metaphysical subject*’s thoughts to the world and reality with Kant’s conception of *transcendental logic* in connecting transcendental reason and understanding to sensual experience as a sort of *epistemic logic*, as Wittgenstein expressed it: “6.13… logic is transcendental” (Kant [1781-1787] 1929, A52/B76, [1788] 1996, 90; 1800, 18, 67n76; Nesher 2020, 2.2. [6]; comp. Peirce on physical and psychological facts, 1994, 1.265).

2.04 The totality of existing states of affairs is the world.

2.06 The existence and non-existence of states of affairs is reality.

(We also call the existence of states of affairs a positive fact, and their non-existence a negative fact.)

2.063 The sum-total of reality (Wirklichkeit=actuality) is the world.

Once again, a misinterpretation of the German concept introduces incoherency into Wittgenstein’s *Tractatus*. The difficulty with proposition2.063 is the relation between reality which is, so to speak, the infinite universe, and the world, our world, which is the part of reality we cognize, experience, and discover, gaining ever more knowledge of facts as true states of affairs. How can it be, then, that “The sum-total of reality is the world” since the world is only composed of true existing states of affairs and not the “non-existence of states of affairs,” namely those that are unknowable to us. The solution to this paradox is that in 2.063 the word *reality* should be replaced with the term *actuality*, which is another and perhaps more appropriate rendering of the German*Wirklichkeit*; actuality, then, stands for all existing states of affairs (equivalent to the world).

Reality is all possible states of affairs containing known objects of the world and true facts. When we prove or come to know new and true states of affairs, we interpret them as facts of the world and this we do through our experience and through the sciences. In this way, by proving the truth of new hypotheses and theories (which is possible if we have a theory of truth) we gain knowledge of more facts about our own world (“the world is my world” [5.641]) and thus extend our knowledge of reality. We, as empirical subjects, do not need the Platonic true thought, which we can only know miraculously, since we can prove the truth of our experiential hypotheses (Nesher 2002, X; 2017).

Indeed, to reiterate, Kant does not have any theory of truth and he remains bound by our phenomenal world without being able to explain knowledge of reality, despite his attempt to connect the formal rationalism of the transcendental subject with the material empiricism of the phenomenal subject (who has sensual intuitions), on the basis of a priori assumptions of pure transcendental logic and formal concepts. Wittgenstein, like all neo-Kantians, does not have any theory of truth either and remains bound by the a priori assumption of the metaphysical subject’s Platonist thought; this leaves meanings unexplained, and the metaphysical subject is unable to infer his own world on the basis of formal semantics. He thus remains with pictures of reality (a model) without being able to know reality itself, though he assumes that his knowledge of reality is extended by his experience in his world (Wittgenstein [1921] 1961,2.063-2.15).

The problem is to explain how the truth of our propositions is determined and, therefore, how truth-conditions and facts can be identified and verify these propositions in order to represent reality. To this end, we should ensure that we properly understand Wittgenstein’s conception of meaning and truth in the *Tractatus*. To accept formal logic and mathematics as pure knowledge of the third realm is to separate them from human empirical experience, making them a formal closed game that logicians and mathematicians can play, and yet they cannot avoid paradoxes and dead ends. Hence, logical and mathematical knowledge must be empirical, like other sciences, but backed by a different epistemology. Mathematics can be seen as a human activity which has empirical reality, , which we can use to prove mathematical hypotheses as distinct from traditional axioms. Thus, we do not need any pictorial model for the proofs of mathematical science, similar to Gödel’s realism but without Platonism (Nesher 2011).

**5. What are the Truth-Conditions and Proof-Conditions of Elementary Propositions?**

**5.1. Is it Possible to Make Comparisons between Propositions and Facts of the World?**

It seems that to understand the meaning and sense of an elementary proposition and its pictorial relation to facts, there must be knowledge of its truth-conditions (comp. Wittgenstein [1921] 1961, 4.022-4.024). But this is a sort of common-sense experience that differs from Wittgenstein’s epistemology in the *Tractatus* in which the metaphysical thought includes the axiomatic truth of its propositions of language and the axioms of formal logic, whose elementary propositions have a priori truth values ([1921] 1961, 5). There can be confusion about the relation between the content of the elementary proposition (its sense and reference) and its truth-conditions, for there are also relations of representation to the *actuality* of states of affairs, or positive facts in the world. In the *Tractatus* the elementary propositions are axioms of metaphysical thought, whereas from the perspective of common sense it seems that it is only through the above comparison with *actuality* that their truth values are determined.

However, it seems that truth-conditions of propositions are fulfilled when the pictures of these propositions represent actual states of affairs, i.e., existing facts, or when they depict states of affairs containing objects in reality, as one can interpret Wittgenstein’s propositions in the *Tractatus* ([1921] 1961, 2.22, 2.222, 2.223).

2.21 A picture agrees with reality or fails to agree; it is correct or incorrect, true or false.

2.22 What a picture represents **[*stellt dar* = depicts~**embodies] it represents [darstellt~present] independently of its truth or falsity; by means of its pictorial form.

To understand this epistemological difficulty, we have to explain the relation between metaphysical language and the world in regard to Wittgenstein’s conception of the linguistic picture. The translation of *stellt dar* is problematic; it would be more accurate, perhaps, to refer to depiction rather than representation; likewise,a more faithful rendering of *darstellt* would be presents (not represents), since the conception of picture is a sort of interpretation of propositions in pictorial images. Those images are either world *facts* or intended *models*; they present states of affairs involving *objects* in reality when those pictures can be considered as the deduction of the last theorems of the axiomatic thought following the metaphysical logic of formal semantic epistemology.

According to Wittgenstein, on the basis of understanding the sense of a proposition alone, it is possible to know its *meaning* and its *truth* or *falsity* in the logical space of reality; but is it also possible to know its *truth* in picturing worldly facts? There is a difference between propositions as logical images with “pictorial form” constituting a *model* of reality ([1921] 1961, 2.17, 2.22) and pictorial presentations of existing facts which the metaphysical subject, as a formal semanticist, logically projects onto the factual world in which we humble humans live (Wittgenstein [1921] 1961, 2.1513, 2.22, 5.63-5.541). Indeed, true propositions in their capacity as pictorial expressions of facts, are the truth-functions of elementary propositions, which are the truth-functions of themselves ([1921] 1961, 5).

Propositions and their relations to these conditions respect their structural match such that the *form* and *content* of the presenting proposition depict or picture these facts ([1921] 1961, 2.1513). Therefore, facts are pictured by true propositions and pertain to the world. With regard to the eternal unchangeable objects in logical space, there are two possible kinds of configurations: *formal logic* pictures true and false states of affairs of objects in the *model* of reality, while *transcendental logic* pictures the true facts of the world ([1921] 1961,:5.633-5.641, 6.13).

A metaphysical subject projects axiomatic thought-language formally onto the world of facts and deduces the logical space of all possible states of affairs of reality, positive and negative. However, the difficulty lies in answering whether or not some of these hypothetical negative states of affairs can become positive and known, thus extending one’s pictorial knowledge of facts in the world. But such acquired knowledge must come from the metaphysical philosopher’s axiomatic thought; how, then, can the metaphysical subject extend axiomatic thought in order to picture more and more facts of the world, in the way that a scientific discovery proves the truth of new hypotheses and transmits them from the category of all possible states of affairs in reality to existence in the world? Yet, in order to combine new facts they must be compatible with the meaning and truth of the metaphysical thought and be derived from pure *transcendental logic* as new true facts in the world ([1921] 1961, 6.3211-6.3432).

6.34 All such propositions, including the principle of sufficient reason, the laws of continuity in nature and of least effort in nature, etc.—all these are a priori *knowledge* of the possibility of a logical form.

6.343 Mechanics is an attempt to construct according to a simple plan all the *true* propositions that we need for the description of the world.

6.3431 The laws of physics with all their logical apparatus, still speak, however indirectly, about the objects [*Gegenstände* ~*subject matter*(facts)] of the world.

The epistemological problem in explaining our knowledge of physical nature is that for Wittgenstein, in his formal semantics, metaphysics is a deductive closed game which cannot explain the discovery of new scientific hypotheses to be added and incorporated into his closed game of metaphysical thought. Indeed, the first epistemological difficulty is to answer the question of how the metaphysical subject discovers the axiomatic laws of physics with all their logical apparatus from the abstract model of reality without the cognitive and psychological elements of human experience (which were excluded in Fregean formalism). This difficulty is also evident in Popper’s deductive logic of knowledge; to explain how we reach scientific hypotheses, Popper suggested that we deduce last theorems and make comparisons with the available “empirical basis” to either confirm or falsify them. However, without a theory of truth to prove these scientific hypotheses, which cannot hold when confronted with the deductive inferences of formal semantics, one cannot prove the truth of the “empirical basis” upon which Popper’s epistemology is based. Indeed, this also holds for Wittgenstein and all neo-Kantians, who lack any theory of truth and therefore cannot prove any theory, with the consequence that the axioms of the metaphysical thought are only assumed. Thus, it can be said that the proposition that “Mechanics is an attempt to construct according to a simple plan all the *true* propositions that we need for the description of the world” (6.343) cannot hold in the epistemology of the *Tractatus*.

Hence, we can conclude that the difference between the logic of thought, which presents all possible situations in logical space, and the thought of true elementary propositions, which are truth-functions of themselves, is that the former pictures all possible states of affairs of objects in reality and the latter pictures only all the true facts in the world ([1921] 1961, 2.18 -2.201-2.221, 4-4.431, 5). All the logical possibilities of the configurations of all objects in reality do not depend on *their inner forms*; some configurations are metaphysically impossible and their descriptions are *non-sensical*. Metaphysical possibilities, on the other hand, are all the configurations of all objects *according to their inner forms*, namely, with inner restrictions operating on their possible configurations into actual facts: these are all possible logical propositions which may be *true or false*. But when objects are considered without their internal properties, i.e., their logical forms, they are formal things exemplifying all truth-possibilities of abstract reality ([1921] 1961, 4.431).

The metaphysical thought of the philosopher in the *Tractatus* includes *axiomatic formal logic* and the *formal true language* of elementary propositions, like Kant’s transcendental logic of pure reason and the transcendental subject’s understanding using a priori formal concepts and categories. Some expressions in the *Tractatus* are problematic:

4 A thought is a proposition with a sense.

4.01 A proposition is a picture of reality. A proposition is a model of reality as we imagine it.

4.3 Truth-possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs.

4.31 We can represent truth-possibilities by schemata of the following kind (‘T’ means ‘true’, ‘F’ means ‘false’; the rows of ‘T’s and ‘F’s under the rows of elementary propositions symbolize their truth-possibilities…

4.41 Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

4.411 It immediately strikes one as probable that the introduction of elementary propositions provides the basis for understanding all other kinds of propositions. Indeed, the understanding of general propositions *palpably* depends on the understanding of elementary propositions.

4.431 The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses thetruth-conditions of a proposition.

A proposition is the expression of its truth-conditions.

(Thus Frege was quite right to use them as a starting point when he explained the signs of his conceptual notation. But the explanation of the concept of truth that Frege gives is mistaken: if ‘the true’ and ‘the false’ were really objects, and were the arguments in *~p* etc., then Frege’s method of determining the sense of ‘*~p*’ would leave it absolutely undetermined.)

Indeed, the difficulty is in the interpretation of proposition 4.3: Do the elementary propositions mean possibilities of existence and non-existence of states of affairs? Are they true or false depending on the schemata of formal logic and not true by definition, as proposition 5 (“An elementary proposition is a truth-function of itself”) seems to suggest? Are we not to understand the “truth-possibilities” of elementary propositions as “truth conditions”? Yet if elementary propositions do not express axiomatic truths (by virtue of the metaphysical thought), how can formal semantics picture actual facts?

Agreement between elementary propositions and their*truth-conditions* means that theelementary propositions areaxiomatic truths of the metaphysical thought through which their pictures are expressed. In other words, the truth-conditions of elementary propositions are axiomatically included in their meanings. Reality is the model, the picture of logical space containing all possible proof-conditions and the thought structure of the meaningful language of elementary propositions which express their own truth and picture the world of facts. Hence, according to Wittgenstein, Frege’s mistake was to assume that truth-conditions are really objects separated from propositions, but in the epistemology of his formal semantics they are expressions of truth-conditions inferred or deduced from the propositions themselves.

4.463 The truth-conditions of a proposition determine the range that it leaves open to the facts.

Hence, propositions picture facts of the world and we can say that the word is the phenomenal picture projected by the metaphysical thought of the philosopher, who also projects models of noumenal objects in states of affairs of reality. This is a departure from Kantian transcendental philosophy in which the *sensual intuition* of the phenomenal subject plays an important role in the construction of knowledge; however, it echoes Kant in the use of conceptions of a priori *space* and *time* which the transcendental subject needs in order to picture the *blind objects* of phenomenal reality (Schema [7]).

5 A proposition is a truth-function of elementary propositions. (An elementary proposition is a truth-function of itself).

5.01 Elementary propositions are the truth-arguments of propositions.

5.101 I will give the name *truth-grounds* of a proposition to those truth-possibilities of its truth-arguments that make it true. (cf. 2ff.).

In the preface to the *Tractatus* Wittgenstein asserts that the limit of knowledge is the limit of language in its true pictorial presentation of reality.

2.11 A picture presents a situation in logical space, the existence and non-existence of states of affairs.

2.12 A picture is a model of reality.

2.131 In a picture the elements of the picture are the representatives of objects.

2.14 What constitutes a picture is that its elements are related to one another in a determinate way.

2.15 The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way.

Let us call this connection of its elements the structure of the picture, and let us call the possibility of this structure the pictorial form of the picture.

3.21 The configuration of the objects in a situation corresponds to the configuration of simple signs in the propositional sign.

1.13 The facts in logical space are the world.

The sum total of true states of affairs (all that is the case) is the world. Facts exist in what Wittgenstein calls “logical space” (1.13). Logical space is effectively the realm of everything that is logically possible. True or false, everything in logical space is possible. Only if the proposition is *true* does the picture present a fact which gives it its complete meaning. The meanings of propositions provide pictures of all possible states of affairs in reality; the meanings of thought present facts in the world of our language.According to Wittgenstein, the infinite possibilities of logical space of thought is reality and its limited logical space of language in its picture of facts is the limit of the world.

The metaphysical thought contains logical space which contains all possibilities of picturing the meanings of true and false states of affairs of objects, and this is reality. All existing true facts are the world. Thus, metaphysical thought pictures meanings for all possible states of affairs in logical space, or reality, whether true or false, and includes the truth of its language in picturing existing facts in the world. However, according to the realism of the pragmaticist, neither the transcendental epistemology of *formal semantics* (based on the axiomatic truth of elementary propositions), nor its other component, the phenomenal subject’s *sensual intuition* (like the “empirical basis” of *logical Empiricism)*, can solve the epistemological problem of representing reality that is external to our cognition (Nesher 2002-2020).

**5.2. The Truth-Conditions of the Elementary Propositions in the *Tractatus***

The central epistemological problem here is to explain what philosophers mean by truth-conditions and to define their role in acquiring knowledge of reality.

4.431 The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses thetruth-conditions of a proposition.

A proposition is the expression of its truth-conditions.

The truth of the elementary proposition is axiomatic; elementary propositions express theirtruth-conditions as axiomatically true. Thus, as opposed to *logical empiricism* in which truth-conditions are external to linguistic propositions, in Wittgenstein’s formal semantics *truth-conditions* are inherent to elementary propositions. The metaphysical thought of *language* is the epistemological source of true linguistic propositions and of the truth of *formal logical* rules of inferences. Thus, metaphysical thought interprets the truth of propositions as facts in the world; and reality is pictured by the metaphysical logic of the truth and falsity of eventual propositions in logical space.

4.463 The truth-conditions of a proposition determine the range that it leaves open to the facts.

(A proposition, a picture, or a model is, in the negative sense, like a solid body that restricts the freedom of movement of others, and, in the positive sense, like a space bounded by solid substance in which there is room for a body.

In this essential epistemic inquiry there are two basic epistemological philosophies, the *formal semantics* initiated by Frege and Wittgenstein, and the *logical Empiricism* developed by A.J. Ayer, Herbert Feigl, Philipp Frank, Hans Hahn, Carl Hempel, Karl Menger, Richard von Mises, Ernest Nagel, Karl Popper, W.V.O. Quine, Frank Ramsay, Hans Reichenbach, Alfred Tarski, Friedrich Waismann, and others.

Neither *formal semantics*, with its *empty concepts*, nor *logical* e*mpiricism*, with its *sensual intuition*, can prove our representation of external reality in terms of Kantian *noumenal* reality. It seems that, like Kant, they do not have any theory of truth and therefore lean on the conception of *truth-conditions* to show the axiomatic truth of the thought of the transcendentalsubject (in line with formal semantics), while sense data or models are accepted as an access to eventual reality.

We can conclude that the truth-conditions of elementary propositions are truth functions of themselves, and truth-conditions of compound propositions are truth-conditions of elementary propositions; but how can elementary propositions be truth functions of themselves? This is the same question that we have already raised in a different context; if the truth of perceptual judgments are the truth-conditions of other more general and abstract propositions and theoretical hypotheses, then what are the truth-conditions of those perceptual judgments themselves?

We can elaborate on the difficulty of formal semantics; the basic axiomatic truths of metaphysical thought and the rules of formal logic are defined and accepted as being the basis of the deductive inference of the last theorems, or facts of our world, while all other possibilities are false. However, in the formal semantics of the *Tractatus,* truth-conditions are the axiomatic truths of elementary propositions and not the empirical facts of the world as they are viewed in *logical empiricism* with assumed sense data (or models), Popper’s “empirical bases”, and more, being seen as substitutes of reality.

Indeed, theaxiomatic true elementary propositions of formal semantics, which are like the different fictions of *logical empiricism*, are based on *formal logic*, which is a closed game detached from any external reality. However, in the epistemology of pragmaticist realism suspicious *truth-conditions* are replaced by *proof-conditions* as components of the empirical theory of truth (Nesher 2002, 2018, 2020).

The metaphysical subject’s thought is also the axiomatic source of all possible states of affairs, which Wittgenstein calls reality. This thought is the source of the meaning of language picturing all logical possibilities of all possible states of affairs of objects as projections of thought, and yet, unlike the empiricists, Wittgenstein’s a priori formal semantics provide the source of meaning in pictorial reality and the truth of linguistic propositions picturing the facts of the subjective world. Thus, as Wittgenstein says later: “A picture held us captive. And we could not get outside it, for it lay in our language and language seemed to repeat it to us inexorably” (Wittgenstein 1958, #115).

Hence, for Wittgenstein in the *Tractatus* formal semantics are based on the axiomatic thought from which the metaphysical subject logically deduces pictures as the last theorems of a closed game which compose the world without being able to reach reality. In the pictorial world we are captive as solipsists. If the pictures of formal logic present all possible states of affairs of imagined reality, then they must picture the true facts of the world that give it complete meaning, but only when the proposition is true can its picture express its complete meaning ([1921] 1961, 4.41, 4.461-4.463; 4.431, 4.442, 4.45-3.461, 4.463).

The essential problem is to explain the global epistemology of Wittgenstein in the *Tractatus*. I interpret it as a work of original axiomatic formal semantics, influenced by both the Kantian assumption of the transcendental subject and the Fregean Platonist conception of the axiomatic pure thought separated from empirical-psychological experience (e.g., Kant [1788] 1996, 114; cf. [1781-1787] 1929, A369, A444-51/B472-79; Nesher 2007a). Hence, Wittgenstein’s formal semantics constitute an axiomatic closed game in which the metaphysical subject’s thought has a full-fledged formal logical and linguistic structure, similar to the role of Kant’s *transcendental logic* of pure reason. Moreover, I see a parallel with the fact of pure reason in the Critique of Practical Reason emanating its last verdicts separated from any experience of reality. Thus, in the *Tractatus* we have a model of reality of all possible meaningful states of affairs and true facts of the world, which are only the inferred pictorial theorems of the axiomatic system of full-fledged thought. Hence, Wittgenstein can imagine and think about reality, just like Kant and his unknown *noumenal* reality, but cannot know it. Thus, axiomatic thoughts, theorematic propositional signs with senses, are true facts, which picture the eventual theorematic model of reality.

How, then, can we know axiomatic truths without proving them? Indeed, in formal logic and formal mathematics e.g., Wittgenstein’s formalism after Kant, the components of the system are not proven but only inferred formally from unproved axioms, as distinct from the pragmaticist’s *epistemic logic* which allows us to prove the truth or falsity of our hypothetical thoughts and without it they remain doubtful. In the realist’s theory of truth, on the basis of *epistemic logic*, we can eliminate the formal logical principles of *bivalence* and the *excluded* *middle* because we cannot axiomatically assume *truth* or *falsity*, and without proving them we are confronted with the third option of *doubtfulness* and *skepticism* (Nesher 2002, II, III, V, X; 2011, 2018a, 2018b, 2020).

According to Wittgenstein, the philosophical subject infers from the axiomatic truths of the metaphysical thought the factual world and the truth and falsity of the formal logic picturing all possible states of affairs in the intuited reality. Hence, the axiomatic truth-conditions of the elementary propositions in the *Tractatus* enable us to formally deduce the pictorial presentation which explains the truth-conditions of all the compound propositions of our knowledge (comp. Hintikka 1986, 87ff.). On this question Hacker writes:

The keystone of the conception of meaning which dominates the *Tractatus* is the notion of truth-conditions. The sense of any sentence consists in the conditions under which it is true and the conditions under which it is false. “The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses the truth-conditions of a proposition” (*Tractatus*, 4.431). In the case of a fully analyzed elementary proposition, its sense is a function of its constituent expressions, i.e., their meanings and logico-syntactical arrangement determine the conditions under which the atomic sentence is true. (Hacker 1981, 88)

The truth-conditions of propositions are determined by the truth of their elementary propositions, and their truths are embedded in the elementary propositions themselves and do not depend on any extra conceptual propositions, facts, or objects. Hence, there are no conditions under which the atomic proposition is true, since it belongs to the metaphysical thought and is axiomatically true by itself.

In the tradition of phenomenal empiricism propositions have different truth-conditions from logical propositions because their names refer to objects with internal forms and properties, and their truth or falsity depends only on facts in the world, their configurations with objects, which verify or falsify them. In the case of formal logical propositions, they do not present facts in the world but are all true with respect to all possible states of affairs in logical space. Therefore, the question remains: What are the truth conditions of the empirical propositions? Hacker suggests that “The sense of any sentence consists in the conditions under which it is true and the conditions under which it is false” (Hacker 1981, 88). Hence, it seems that we need to know the worldly facts which provide the truth-conditions of propositions in order to know whether they are true or false. But we should know the verifying facts separately through sensual perceptions and not through propositions; otherwise Wittgenstein and other formal semanticists can survey or axiomatically picture facts as abstract structures in the world.

In regular empirical experience, how can we find the relevant proposition for the actual facts or the relevant fact for the actual propositions? In the operation of representing reality we do not start from the elementary propositions but from the fact that we want to represent by such propositions, so that if they correspond to this fact then we say they are true. The question is whether *the facts* or *the proposition relating to the fact* constitutes the *truth-conditions* for an elementary proposition (cf. Hintikka 1986, 95).

In short, if Dummett’s Verificationist account of *what constitutes understanding* is right, then either truth is a useless metaphysical abstraction or else there is nothing to the claim that *truth* is *a bivalent property*, the claim that characterizes “two-valued” logic. (It is thus that Dummett is led to the radical claim that a sound philosophy of language requires the revision of classical logic itself. (Putnam 1999, 51)\A pictorial relationship, presenting the configuration of the pictorial structure, enables the picture to touch reality with pictorial feelers, graduating to metaphysical objects to touch reality (cf. [1921] 1961, 2.1515).

**5.3. The Construction of the Tractarian System in Formal Semantics**

According to my analysis of the structure of Wittgenstein’s *Tractatus* we should distinguish tbetween the role of Wittgenstein, the formal semanticist, and the function of his “metaphysical subject.” Wittgenstein constructed this formal semantic system with a *philosophical language* that the “metaphysical subject” cannot understand and use since the only language he understands is the *descriptive language* of natural science. This language can only represent states of affairs in reality and in the world and has nothing to do with logico-philosophical semantics, the status of metaphysical objects, and the metaphysical subject himself (Wittgenstein [1921] 1961, ##5.62, 6.53; cf. 1953, #97; comp. Pears 1987, 172-173n61; Hodges 1990, chaps. 3, 4).

The function of the metaphysical subject is to understand the senses of propositions because they are his own thoughts projected into propositional signs, and their truth values are evaluated in order to present the facts of the world and to picture possible states of affairs in reality. This raises the question: How can the metaphysical subject, in his endeavor to present reality, grasp metaphysical objects, with their various forms and combinations, and transpose them to possible states of affairs of reality? Moreover, how can worldly elementary propositions be truth-functions of themselves and of the propositional facts and bare facts, which are the truth-conditions of the compound proposition?

The metaphysical subject who projects his thoughts as the senses of his propositions understands them because they are his own thoughts. But what are the contents of these thoughts and where do they come from? They cannot come from nowhere, and the metaphysical subject cannot find thoughts among worldly facts. Since experience cannot take place in the *Tractatus*, the meaning and contents of the metaphysical subject’s thoughts must be constructed by the formal semanticist. Only the formal semanticist can initiate these thoughts, though not from abstract constructions but from his own metaphysical axiomatic thought, since he does not have any experience with objects in the real world and cannot use any natural language from outside the *Tractatus*. Such experiential meaning and content cannot be introduced into the “mind” of the metaphysical subject.

Thought is surrounded by a halo. Its essence, logic, presents an order, in fact the a priori order of the world: that is, the order of *possibilities*, which must be common to both world and thought. But this order, it seems, must be *utterly simple*. It is *prior* to all experience, must run through all experience; no empirical cloudiness or uncertainty can be allowed to affect it—It must rather be of the purest crystal. But this crystal does not appear as an abstraction; but as something concrete, indeed, as the most concrete. as it were the *hardest* thing there is (*Tractatus Logico-Philosophicous*. No. 5.5563). (Wittgenstein, 1953, #97)

This also holds for knowledge of the forms of objects in logical space, otherwise how can one come to have knowledge of metaphysical objects that cannot be experienced? Since there is no experience in the *Tractatus* of the forms of objects, like Kant’s a priori *space* and *time* in intuitions of the sensual content of blind objects, it must come intuitively from the formal semanticist’s experience of objects and facts in the real world. Therefore, unlike the configurations of metaphysical objects which correspond to pictorial worldly facts, material properties emerge only through perceptible worldly facts, through formal internal properties; the forms of metaphysical objects can be constructed, and according to these forms they are combined into facts (Wittgenstein [1921] 1961, 2.0231). But in the *Tractatus* no one can perceive the material properties of facts because empirical persons cannot be found in its world and the metaphysical subject who is located outside of this world cannot experientially perceive things inside the world (Wittgenstein [1921] 1961, 3.11, 5.631-5.641). One may suggest that the metaphysical subject “grasps” facts intellectually and formally but this is a capacity of the formal semanticist (Wittgenstein 1953, #97). Therefore, all this knowledge, especially perception of the material properties of propositional facts and other facts, must come, once again, from the formal semanticist and his experience in the real world (Wittgenstein [1921] 1961, 5.552, 5.631-5.641).

It would be a mistake to understand Wittgenstein’s theory of representation as an account of our experiential contact with reality. Wittgenstein as a formal semanticist constructs the Tractarian system in order to show and explain how language can represent reality (cf. Wittgenstein [1921] 1961,: 5.61). In this system the metaphysical subject relates his language to the metaphysical objects in logical space, while both are situated outside the empirical world (cf. Wittgenstein [1921] 1961, 5.631, 5.641, 3.11-3.12, 2.0121-2.014, 2.02-2.021, 2.023-2.024, 2.026). These two domains are essential for understanding Wittgenstein’s formal semantic model and how language, through relations with both of these metaphysical domains, operates meaningfully in representing possible states of affairs and describing actual ones, i.e., facts in the empirical world.

5.5561 Empirical reality is limited by the totality of objects. The limit also makes itself manifest in the totality of elementary propositions.

This is a kind of Kantian scheme in which the thinking transcendental subject (Kant [1781-1787] 1929, A346) and the transcendental objects as the things in themselves (Kant [1781-1787] 1929, A109) are necessary assumptions for explaining human experience of phenomenal objects (Wittgenstein’s facts) of an empirical nature (Pears 1987, 90-91). The discussion on the model as a fiction of reality is also based on these assumptions (Eder 2019).

The question of the global picture of the *Tractatus* is about the “division of labor” between the *formal semanticist* and the *metaphysical subject* or its philosophy in this representational enterprise. Wittgenstein, the formal semanticist, constructs the Tractarian system as a model to explain how cognition, expressed by the metaphysical subject’s thought, represents things in external reality (see Frege [1918-19] 1968, E; 1956, 301ff.). And yet, the enigma is about the source of the content and meaning of the Platonic thought which can be analyzed by the distinction between Peirce’s epistemic logic and Frege’s pure logic which rejects psychology **(**Ricketts 1996).

The “metaphysical subject” in the *Tractatus* operates these presentations but it cannot be the formal semanticist that constructs this system, since he can only use the descriptive language of the *Tractatus*, the propositions of natural science. This is the language constructed to describe facts, but the metaphysical subject cannot use the formal semantic language in which the *Tractatus* system is constructed. What formal semantic language constructs, the language of the *Tractatus* can only show (Wittgenstein [1921] 1961, 4.12-4.1212). Therefore, we cannot identify the formal semanticist with the metaphysical subject that can only use the language that presents facts. Yet Wittgenstein can speak the two languages of the *Tractatus*: the philosophical language and the language of natural science. Thus, we can think about the metaphysical subject as the formal semanticist in applying the logico-philosophical system (Wittgenstein [1921] 1961, 6.522-6.54). Moreover, Wittgenstein knows and uses a third language, the common-sense natural language, with which he expresses his experience and reason, and with this he can formulate the formal system of the *Tractatus*. Altogether, then, we have an *evolutionary hierarchy of languages*, for Wittgenstein formulates a logico-philosophical language using natural language to construct his formal semantic system. The third language is an essential inner component of the constructed system, the only language that the metaphysical subject can understand and use, but Wittgenstein mistakenly concluded that by virtue of this language he can eliminate philosophy by rejecting its language (cf. 1975, #1; 1953, #97).

**6. Kantian Epistemology in Wittgenstein’s *Tractatus* and Neo-Kantian Approaches**

**6.1. Does the Tractarian Metaphysical Subject Have Knowledge of Noumenal Objects in Reality?**

How can the Tractarian presentational model work? The logico-philosophical system is already constructed by the formal semanticist; the metaphysical subject has only to operate it. From outside the world the metaphysical subject with “intellectual intuition” “grasps” and identifies the logical syntax of propositions picturing facts (“the world as I find it”), and by projecting thoughts about the sense of these names connects them with the metaphysical objects of possible states of affairs in logical space to give them complete meanings: sense and reference. With the same intuition the metaphysical subject compares these propositions with possible states of affairs in logical space to detect their common logical forms. Then, by understanding the projected thought as the sense of the proposition, the possible fact can be ‘seen’ and represented by this meaningful proposition. Now the metaphysical subject needs to know whether the possible state of affairs being represented is also an actual fact in the world. A decision is to be made about whether a proposition that represents reality is also true in respect to these worldly facts.

However, in Wittgenstein’s system of formal semantics the logical forms and internal properties of metaphysical objects, or the formal structures of objects, are pictured, unlike the Kantian unknown transcendental object, the noumenal things-in-themselves (Nesher 1999a). On the basis of the logical forms and internal properties of objects we can know how such objects configure the structures of worldly facts. This knowledge is essential to understand the actual properties and structures of the facts along with the propositional facts as configurations of their object-components. The function of the metaphysical subject is essential in the projection of its thoughts onto the structures of the propositional facts as their senses. Thus we understand the senses of propositional names, the simple components of any propositions, and through these names we are able to pictorially reach metaphysical objects. These projections of sense capture the pictorial relation to reality and complete the meaning of the propositions: the thought-senses of their names with their application to objects in reality (Wittgenstein [1921] 1961, 3.5, 4).

However, in order to present or describe (phenomenal) facts as actual combinations of eternal (noumenal) metaphysical objects it is necessary to know the logical structure of facts and the logical structure of propositional facts to ensure their common logical form such that their isomorphism enables the proposition to become a pictorial presentation of the fact.

4.022 A proposition *shows* its sense.

A proposition *shows* how things stands *if* it is true. And it *says that* they do so stand.

4.024 To understand a proposition means to know what is the case if it is true.

(One can understand it, therefore, without knowing whether it is true.)

But how can we know that the meaningful proposition that presents reality is also true or false? We cannot recognize this from the proposition alone, and therefore we must have perceptual experience of actual facts. However, this cannot be a human experience since we might be mistaken about facts and about our representation of them.

5.634 . . . Whatever we see could be other than it is.

Whatever we can describe at all could also be other than it is.

We as worldly human beings cannot know facts for certain. Therefore, who is it that experiences actual facts and how do they become known? Indeed, as Wittgenstein puts it:

5.641 …The philosophical self is not the human being, not the human body, or the human soul, with which psychology deals, but rather the metaphysical subject, the limit of the world – not a part of it ().

The *metaphysical subject* must be like the Cartesian God that compares language with reality, though the real creator is rather the formal semanticist that constructs the abstract structures of the Tractarian system: the metaphysical objects in logical space and their possible states of affairs and facts in the abstract world, including their logical forms and the logical syntax of propositional facts (cf. Wittgenstein [1921] 1961, 5.123, 6.432).

**6.2. Wittgenstein’s Pictorial Theory of Representation: Reasons Why It Cannot Explain Human Representation of External Reality**

In Wittgenstein’s *Tractatus* the truth of linguistic axioms is not truth-conditions but the assumed axiomatic truth of elementary propositions as *truth-functions* of themselves. The difficulty with *formal semantics* and *logical positivism* is that they deal only with formal and natural languages and not with human cognition. However, in their conceptions of *truth-conditions*, they have to assume a model or sense data to serve as substitutes for facts or reality, without any proof, since as neo-Kantians they do not have any theory of truth to prove the truth of our cognitive representation of reality. Hence, they cannot compare our language with reality, which they cannot know; then it is either deduced from axioms or from the conception of the language itself, as a picture or a model of reality, without knowing it and without proof.

4.463 The truth-conditions of the proposition determine the range that it leaves open to the facts.

(A proposition, a picture, or a model is, in the negative sense, like a solid body that restricts the freedom of movement of others, and, in the positive sense, like a space bounded by solid substance in which there is room for a body.)

Frege claims that we cannot define truth since we cannot compare ideas or thoughts with external reality. We can compare an idea with another idea, or a thoughtwith another thought, but not with realities that are of a different category. Wittgenstein’s solution in the *Tractatus* is his pictorial theory of representation according to which the metaphysical subject compares the logical form of a propositional fact with that of a possible state of affairs such that if they have the same logical form he can project his thoughts to present this specific possible state of affairs in logical space. The metaphysical subject must then find out whether these states of affairs are pictorial facts, and if so, whether the propositions are true, otherwise they are false (as components of all possible states of affairs which equates to reality). This is the case because *truth-conditions* of propositions in the *Tractatus* are pictorial-actual states of affairs, or *facts of the world*. For Wittgenstein, facts are pictures of the propositions presenting them and determine the truth value of the latter. Thus, facts in the *Tractatus* are not the same as those found in Frege’s semantics, where “A fact is a thought that is true” (Frege [1918] 1968, 101) without any objective criterion for its truth, and without even the *meaning or content* of a proposition, as expressed in Strawson’s proposition “The fact that it is raining” or in Austin’s paraphrase “The fact that S” (Strawson 1950, 38-39; Austin 1954, 160).

The entire system of the *Tractatus* is constructed by Wittgenstein, the formal semanticist, to include the metaphysical subject with its mythical role of projecting thoughts onto propositional facts to picture eventual reality. The question is whether this logico-philosophical semantic theory can explain how humans represent external reality? Wittgenstein developed in his *Tractatus* a paradigm of formal semantics to theorize how humans cognitively represent external reality. The intention of formal semanticists is to create a formal model in which they replace everyday subject matter by suitable abstractly characterized idealizations, *chosen to preserve those features of the original subject that are relevant to the study at hand*. Here they need abstract substitutes for *thought*, for *reality*, and for the thought’s *representational relations* to reality. For though they substitute *language*, or more precisely, a formalized version of parts of everyday language, and for *reality* they substitute something called a *structure*, which is a collection of things or objects suitable for being correlated, as meanings, to various expressions in the language. For the *representational relation of thought to reality* they substitute *interpretation*, or *projection* in the *Tractatus*, that is, a function assigning to certain expressions of the language certain objects in the structure as their meanings or references under the interpretation. Furthermore, some expressions (sentences, propositions) of the language become true under a specified set of interpretations in the pictorial structure, e.g., the correspondence of elementary propositions to worldly facts in the *Tractatus* (Lyndon 1966, 1-5; Carnap 1942-43, #7; Wittgenstein [1921] 1961, 3.2-3.21, 4.063).

The question is whether, if at all, this kind of idealization preserves and can explain how humans with their cognition represent reality. I claim that even with such Fregean, Wittgensteinian, Tarskian, or Carnapian formal semantics, in an enterprise that aims to create an objective scientific semantics, these abstract idealizations cannot preserve the essential relations of mind representing reality (Wittgenstein [1921] 1961, preface; Tarski 1936, 403, 407; comp. Nesher 1996). The interpretative relations between linguistic expressions and entities of the abstract structure which are actually assigned to them by formal semanticists from outside these idealized domains, assume a God’s eye view, or the view of the metaphysical subject, but in the natural situation we cannot afford this because we cannot get outside of our cognitive skins, our thoughts and cognition in general (Hume 1739, 67; Kant [1781-1787] 1929, B127; Davidson 1986a, 312; Putnam 1990, 17). Therefore, this type of formal semantic idealization overlooks relevant features of the original subject matter, failing to preserve in the model the genuine relationship between thought and reality; thus, it only creates an illusion of a theoretical solution, as Wittgenstein realized later (Wittgenstein 1975, 1,1938:13-17; 1953, x, ##23, 97, 114; Ramsey 1923; Monk 1990, 272-274; comp. Putnam 1994a, 315).

This illusion eventually leads to a *metaphysical realist’s* position, dogmatically assuming, without philosophical explanation, the existence of external objects and facts to which linguistic expressions relate, as we can also see in the *Tractatus*. Thus, the formal-semanticist-observer, whether Frege, Wittgenstein, or Tarski, namely the “metaphysical subject”, only relates these linguistic “pictures” of facts, of the real states of affairs. This prominently creates an illusory situation in Wittgenstein’s *Tractatus*. However, language is a type of cognitive performance in which we cannot assume this external position since, as Hume argues, we have no external point of view independent of our cognition. Actually, *we are the pictures*, the cognitive (and linguistic) pictures, and we cannot *compare* these from outside our cognition with external states of affairs (Wittgenstein [1921] 1961, ##2.223, 4.05-4.06; comp. Peirce 5.283, 5.310-317). According to the formal semantic model, to compare propositions with worldly facts in order to detect their truth or falsity, we must have access to facts from outside our cognition, separately from our perceptual experience and propositions (Eder 2019).

6**.3. Can Humans Get Outside of Their Thoughts and Compare the Experiential Meaning or Contents Expressed in Propositions with Reality?**

In real situations persons cannot get outside of their thoughts nor the experiential meaning or contents expressed in propositions to compare them with reality, which is external to their cognitive thoughts, as Frege already argued, following Kant and modern traditional philosophy (Nesher 1999c). The question is how can we make this kind of comparison if, according to Descartes, Spinoza, Hume, Berkeley, Kant, and others, we cannot go beyond our cognitive “skins”, as Davidson expresses it (Davidson 1983, 312; comp. Nesher 1998b).

So, if we cannot dogmatically solve Hume’s predicament about our knowledge of external reality it seems that we must accept, as Kant did, a kind of phenomenalism (Nesher 1994c, I.1, II.2; 1997a, III; 1999c, I.2, II.2). If we cannot know facts and their “logical forms” since they are outside our cognition, and if we cannot know from our cognition alone that our perception and language represent something external, then it seems that we can compare only ideas with ideas, without representing any real objects or facts. It appears that we have to confront an insurmountable predicament if we cannot go outside of our cognition to compare it with reality: Without going outside of our cognition, we cannot know if we are representing external reality, and therefore we cannot have knowledge of reality. However, Frege intends to solve the epistemological question of knowledge of external reality by transcending the human mind and accepting Platonic ideas, but the question remains: What are the meanings of Platonic thought?

The case is similar to that of Kantian transcendental epistemology in which the transcendental subject’s understanding is separated from human empirical experience and must start with pure empty concepts to meet blind objects in order to develop human knowledge. Indeed, without starting initially from human meaningful experience, Frege remains stuck with empty Platonic thoughts in his *formal semantics*, which are somehow made to work with the artificial model of truth conditions, separated from reality. To overcome the problem of Frege’s empty Platonic thought, Wittgenstein in his *Tractatus* starts withthe metaphysical subject’s thought as the philosopher that creates the conception of formal semantics, and yet it is a meaningful one to make sense of his enterprise, but without reaching reality to prove its truth (Eder 2019).

However, since humans cannot adopt the metaphysical subject’s perspective from outside the world, the problem is to explain how, without leaving our cognitive skins, we can nevertheless know that we are confronting external reality through our perceptual experience and representing it truly with our perceptual judgments and the propositions that can be inferred from them.

**7. The Spell of Frege and Ramsey’s “P is True = P” and Tarski’s (T) “X is True = P”: “Grasping” Meaning as Truth-Conditions**

**7.1 Frege’s Conception of “Truth-Conditions”**

Dummett, on the sense of sentences according to Frege:

True, Frege took sense to be immaterial and to exist independently of our grasping it; but this does not exhaust his conception of the sense of a sentence: he said a great deal more about what such a sense consists in, above all, that, in grasping the sense of a sentence, including a mathematical sentence, what we grasp is the condition for that sentence to be true. (Dummett 1981, 34)

But what might Frege mean by saying that in grasping the sense of a sentence what we grasp is the condition for that sentence to be true? When we “grasp” the sense of the sentence we grasp its meaning or content and yet this includes not only the sense of the sentence but also its reference, because without the reference there is nothing (no ‘something’) that can verify the sentence, such that we can decide whether the referential object falls under the referential concept of the sentence. However, according to my analysis, the meaning or content of a sentence (or better, of the proposition that includes the meaning or content) includes the iconic sense and the indexical reference, and their relations of agreement and disagreement are their truth-conditions. Following this interpretation, we can understand Frege’s intuition that in grasping the meaning (sense and reference) of a sentence we also grasp its truth-conditions. Yet the question is whether or not this is a true generalization. Is it possible to understand or grasp the meaning of propositions and theories without grasping their truth-conditions? For example, we can understand the meaning of the equations of string theory without understanding the truth-conditions that will verify or falsify it. Therefore, we cannot identify the meaning or contents of all of our propositions with their truth-conditions. In order to understand why the meanings or contents of propositions do not always contain their truth-conditions, we have to analyze the structure of our thoughts and identify the source of our concept of truth underlying them, i.e., the real assertive force of our perceptual judgments. Why can we not feel this assertive force in general behind abstract propositions? (cf. Nesher 2000c).

**[10] Interpretation and Verification – Frege’s Conception of Truth-conditions:**

**The Basic Function: Sense F(term [N/P]) = Reference [Object, Concept])**

**Thought**

∇ ∇

(**Sense**N)+(**Sense**P) = the Senses of N and P ⎞

| | ⎥

**Sentence** = (N \* P) = Name and Predicate ⎬ Meaning = **Sense and Reference**

| | ⎥ **Logical Deduction** of Coherence Picture **Model**

| | = the References of N and P ⎥ of **Reality**

▼ ▼ ⎥

{**Object**, **Concept** [Property]}=Thought Picture ⎠

Frege’s truth-conditions: The Object either “falls under” or “does not fall under” the concept (property), i.e., the verification procedure of the proposition:

FU/ ¬FU(Object, Property) = Possible Truth Values of **Sentence** = True/False.

The only explanation of “truth-conditions” can be found in formal semantics.

How can we understand the limitations of formal semantics and how can we explain our knowledge of “meaning/contents” and “truth-conditions”?

[11] How Can We “Grasp” “Contents” and “Truth-conditions”

“P is true = P [is a fact]”

|

**| “Grasping”?**  (Dummett on Frege’s Concept of “Grasp”)

▼

“Contents” and “Truth-conditions”

What are the truth-conditions of **(T) “X is true iff P”**? If “X is true iff P [is true]” what are the truth-conditions of P: ‘Snow is white’? When do we experience the “assertive force of the word ‘truth’”?

A third possibility is the following: it might be held (although Tarski himself never, as far as I know, suggests this) that we understand the word “truth” by knowing that a predicate is coextensive with “true” just in case all instances of the above [Tarski’s (T)] scheme are *assertable*. This idea would try to combine… the idea that we understand our language by mastering or “internalizing” assertability conditions, rather than truth conditions in the realist sense, with the idea that the meaning of the word “true” is somehow fixed by the Convention T. (Putnam 1994a, 319-320; my emphasis)

**7.2. Defining “Facts” And “True Propositions” Without Circularity: Can We “Grasp” Their Truth-Conditions?**

“What is a fact? A fact is a thought that is true.” (Frege [1918] 1968, 531). How, though, can “facts” and “true propositions” be defined in a way that is not circular? In other words, how can we avoid resorting to the explanation that “True propositions correspond to facts and facts are what true propositions correspond to”? The way out of this predicament is to accept that “facts” are “true propositions” because by proving that propositions are true they are facts that represent external reality and thus we know reality, which we cannot know otherwise (Nesher 2002, X; 2018).

But “facts” cannot be what “true propositions” represent since before they do so they cannot be true. Let us assume that “facts” are chunks of reality. But then we can only identify them through experiential propositions like P: ‘Snow is white’ is true, à la Tarski. Indeed, what do true propositions represent, if not the reality on which their truth is based? However, if facts are not true propositions but chunks of external reality, how can we know this external reality before representing it, since we can only cognize our perceptual intuitions, like Kant’s *sensual intuitions*. We can only come to know such facts through the propositions that represent them and, therefore, facts cannot be their verifiers. Hence, we face a challenging dilemma: either the *fact* is the *picture* of the *linguistic proposition* or the *object* of our *perceptual judgments*.

An alternative epistemology to *formal semantics* and *logical empiricism* is pragmaticist realism with epistemic logic, which explains how, from the internal structure of our perceptual cognition, we can detect by self-intuition whether or not it succeeds in representing external reality according to the relations of our cognitive components (coherent or incoherent). This inner cognitive operation provides the *proof-conditions* of our perceptual judgment so that the perceptual judgment “Snow is white” is true when we prove it using our inner *proof-conditions* of such experience. What, then, is the function of these proven facts in respect to the truth of these propositions if they cannot be their verifiers? Here “facts” remain either empty words or they can be viewed as being identical with true propositions. But since we want facts to be elements of our proof operations of propositions, they themselves must be the basic true propositions upon which we can base the proof of other scientific hypotheses.

With this analysis we reach the conclusion that facts as verifiers of our true propositions are not what true propositions represent, nor are they chunks of external reality. Thus, facts as verifiers are not represented by our true propositions: facts as verifiers and chunks of external reality are separated. But in order for facts to count as verifiers of other propositions or scientific hypotheses, they themselves should be true representations of external reality. The reason for this separation between external reality and the verifiers of propositions and hypotheses is that we must abandon the metaphysical-realist, formal-semantic, and logical-empiricist arguments that reality verifies propositions and instead accept truth as our proof of our propositions and theories.

The correct objection to correspondence theories is not, then, that they make truth something to which humans can never legitimately aspire; the real objection is rather that such theories fail to provide entities to which truth vehicles (whether we take these to be statements, sentences or utterances) can be said to correspond. If this is right, and I am convinced it is, we ought also to question the popular assumption that sentences, or their spoken tokens, or sentence-like entities or configurations in our brains, can properly be called “representations,” since there is nothing for them to represent. If we give up facts as entities that make sentences true, we ought to give up representations at the same time, for the legitimacy of each depends on the legitimacy of the other. (Davidson 1990, 304)

According to the Strawsonian-Davidsonian position, facts are not real entities that can make our propositions true. However, there are objections to this, from Austin to Kirkham, and the suggestion is that we cannot identify facts with true propositions.

The evidence for the claim that ‘facts’ is just another name for ‘true sentences’ is supposed to be this: we cannot individuate and identify any particular fact save by using the very same words that we use to individuate and identify its corresponding sentence.

There are, however, good reasons for resisting this line of thought: (1) Facts can enter into causal relations in a way that true sentences cannot. There is a sense of ‘cause’ in which the fact that the memo was derogatory caused Ralph to lose his job, but in that same sense of ‘cause’ the true sentence ‘The memo was derogatory’ cannot cause Ralph to lose his job. (2) One of the constituents of the fact that the memo was derogatory is a certain memo, but no memo (distinct from the *word* ‘memo’) can be a constituent of the true sentence ‘The memo was derogatory.’ (3) It should be no surprise that we cannot specify a given fact save by means of the sentence to which the fact corresponds, because it cannot be otherwise. (Kirkham 1992, 138). [Colloquial use of “fact”]

What are these ‘facts’? Are they precisely what true sentences (or propositions) correspond to (or express)? Here we should distinguish between what true propositions represent and what they express. It seems that what propositions “express”, or what we “grasp” when we understand true propositions, are their meaning or contents. But if true propositions express facts and meanings or contents are the essential components of propositions, then facts are the essential contents of true propositions. How are we to conduct a philosophical analysis of the concept of facts that will expose, in the most effective way, our ordinary language use of the term “fact”? Or perhaps we should depart from the philosophy of such ordinary language and develop a realist epistemology of knowledge to explain how we can have knowledge of external reality through the representation of our cognitive language (cf. Kirkham 1992,138-139).

However, if facts are true propositions, what proves them to be true? Otherwise, how can we identify them? Yet what are the proof-conditions that render them true? The way out of this predicament is to identify proof-conditions within our cognitive performances and their accessible components.

We declare the recognition of truth in the form of an indicative sentence. We do not have to use the word “true” for this. And even when we do use it, *the real assertive force* lies not in it but in the form of the indicative sentence, and where this loses its assertive force the word “truth” cannot put it back again. (Frege [1918] 1968, 514. Emphasis added)

Indeed, we can feel *the real assertive force* of our indicative sentence and strong intimations of truth, as Peirce also argues:

...we often derive from observation strong intimations of truth, without being able to specify what were the circumstances we had observed which conveyed those intimations. (Peirce, 7.46; cf. 7.48, 7.77, 1.635, 1.14, 5.571)

The question is: What are the verifiers of our *assertive force*, and what can make our basic propositions true? How do we prove them without accepting them as the basic given truth that we cannot prove but only accept as “empirical premises” and “axioms” of our system of knowledge? By this *real assertive force*, or *strong intimations of truth*, we can grasp proof-conditions. Hence, we must look for the proof-conditions of our basic *true propositions* as our *basic facts*, such that they are no longer the mysterious ‘given on which we build our entire knowledge without any explanation as to why we accept them, like the turtle on which the elephant of knowledge stands.

What, then, are the proof-conditions of our perceptual judgments as basic true propositions, and what are the verifiers of our fact-verifiers? What makes our basic propositions true? How can we prove them without accepting them as basic given truths that we cannot prove but only adopt as our “empirical premises” and “axioms” of our system of knowledge? Moreover, why is it that proof-conditions cannot be elements of external reality? With regard to these strong intimations of truth (that we are unable to specify) what are the circumstances and conditions for this feeling that we are inclined to call “proof-conditions”? Some philosophers suggest that propositions represent their [possible] truth-conditions and that if their truth-conditions are obtained then these propositions are true. But if these truth-conditions are chunks of external reality, how can we have knowledge of them, and how do we know that the relevant propositions represent them, as in Wittgenstein’s *Tractatus*? Since we do not have direct access to external truth-conditions, the only way to obtain knowledge of external reality is to have accessible cognitive internal proof-conditions upon which we can prove our propositions as true representations of reality. Therefore, proof-conditions must be accessible cognitive internal conditions that we can identify and not truth-conditions as assumed elements of external reality to which we do not have any direct access and cannot represent.

**7.3. Why Can We Not Cognize the Truth-Conditions of ‘p’ by Logical Analysis?**

Some philosophers suggest that we cannot explicate the conception of “truth-conditions” because if we accept Frege and Ramsey’s identity of “p is true = p” (or Tarski’s (T) equivalence) we cannot further analyze the perceptual judgment P, e.g., “Snow is white” to find out what truth-conditions make it true. Hence, they conclude that the term “truth-condition” is unexplainable, along with the explanation of meaning based on this (Baker and Hacker 1983).

[12] The “equivalence of the form (T)”: (T) ‘X is true iff p’

where the letter ‘p’ can be replaced by a descriptive sentence of object language and the letter ‘X’ by its name, e.g., *‘“Snow is white” is true iff snow is white.’* We can detect the enterprise to understand ordinary language within the framework of the formal logic of language syntax that is based on axiomatic definitions and formal inferences, thus enclosing natural language in a rigid formal system, in line with logical positivism. Tarski endeavors to characterize language mathematically to reach the concept of truth and relies on logical consequence to syntactically formalize the sentences of the language.

However, our intuitive conception of truth comes from our *strong intimations of truth* in perceptual experience and from the *real assertive force* of our perceptual judgments in natural languages. Yet, we cannot formalize in logic or formal semantics what we have no idea of. Epistemologically, the logical enterprise must be based on the empirical states of affairs that concern us, and equivalence (T) is also based on our common perceptual experience, which Aristotle defined and Tarski tried to formalize: First of all, it must be “materially adequate” (Tarski 1944, 52-53, 55). It seems that Tarski is mistaken in suggesting that the “logical relation” between “X is true” and “p” is equivalence (1944, 55). “X is true” has no independent truth value; its truth value depends on the truth of “p” because the epistemic relation of “p” to “empirical state of affairs” is described by “p” (Tarski 1944, 56). The “logical relation” should capture the epistemological relation that is probably expressed better by “X is true *depends on* p [is true]” or “p [is true] *entails* X is true” when the truth of “p” is accepted intuitively as dependent on non-linguistic factors (“p” has been quasi-proven perceptually). As Tarski writes:

We shall call any such equivalence (with ‘*p*’ replaced by any sentence of the language to which the word “*true*” refers, and ‘*X*’ replaced by a name of this sentence) an “*equivalence of the form* (T).” (Tarski 1944, 55)

Such a partial definition of truth is possible only if we already know implicitly that “Snow is white” or that “Grass is green.” If “p” (for all propositions of the form “X is true iff p”) is separated from our perceptual experience and lacks meaning or content, we cannot define “truth” because we do not understand the meaning of “is true” in this schema. It is not by accident that Tarski uses the perceptual judgment “Snow is white” because it grasps our intuition of “is true” and “truth.”

Now at last we are able to put into a precise form the conditions under which we will consider the usage and the definition of the term “*true*” “as adequate from the material point of view [not formal]: we wish to use the term “*true*” in such a way that all equivalences of the form (T) can be asserted, and *we shall call a definition of truth adequate” if all these equivalences follow from it*. (Tarski 1944, 55)

In this context I understand that equivalences “can be asserted” because we accept them as true. (Compare, Frege [1918] 1968, 514, “the real assertive force”; cf. Wittgenstein [1921] 1961, 4.442; Dummett 1993, 16).

Wittgenstein asks and answers his own question:

Does “‘p’ is true” state anything about the sign ‘p’ then? “Yes, it says that ‘p’ agrees with reality”. (Wittgenstein 1974)

This relation of *agreement* cannot be explained by analyzing the perceptual experience that results in the perceptual judgments “Snow is white” and “Grass is green” since we cannot know their truth values through contemplation but only by proving their truth using the realist’s epistemic logic and not using the axiomatic system of formal logic which is a closed game demarcated from reality.

**8. How Can The Revolution of Knowledge Be Explained When New Ideas Do Not Cohere With the Main Body of Knowledge?**

**8.1. Traditional Phenomenal, Coherent and Performative Theories of Truth: Reasons Why They Cannot Work**

According to Wittgenstein in his series of notes *On Certainty* (1969/1972), the empirical propositions of our “inherited background” are our truth-conditions for understanding our empirical propositions as practical knowledge:

But I did not get my picture of the world by satisfying myself of its correctness; nor do I have it because I am satisfied of its correctness. No: it is the inherited background against which I distinguish between true and false. (Wittgenstein 1972, #94)

The propositions describing this world-picture might be part of a kind of mythology. And their role is like that of rules of a game; and the game can be learned purely practically, without learning any explicit rules. (Wittgenstein 1972, #95)

Indeed, Wittgenstein can only describe but cannot explain how our language-games develop and change because he cannot detect any confrontation with external reality:

It might be imagined that some propositions, of the form of empirical propositions, were hardened and functioned as channels for such empirical propositions as were not hardened but fluid; and that this relation altered with time, in that fluid propositions hardened, and hard ones became fluid. (Wittgenstein 1972, #96)

The mythology may change back into a state of flux; the river-bed of thoughts may shift. But I distinguish between the movement of the waters on the river-bed and the shift of the bed itself; though there is no sharp division of the one from the other. (Wittgenstein 1972, #97)

This raises thee question of how coherence can be a test of “objective truth” without confrontation with external reality, because without it we can only have complete relativism with many coherent theories and no objective truth (cf. Davidson 1986b, 331; Nesher 2002, VI).

We have been trying to see it this way: a person has all his beliefs about the world - that is, all his beliefs. How can he tell if they are true, or apt to be true? Only we have been assuming, by connecting his beliefs to the world, confronting certain of his beliefs with the deliverances of the sense one by one [Schlick], or perhaps confronting the totality of his beliefs with the tribunal of *experience*. No such confrontation makes sense, for *of course we cannot get outside our skins to find out what is causing the internal happenings of which we are aware*. (Davidson 1986a, 312. Italics added)

According to Dummett, we know our assertions but not their Truth, and the epistemological question is whether we can know the truth-conditions of a proposition or only their justification conditions?

A realist maintains a wide gap between the objective correctness of assertion–its truth–and its subjective justification, the evidence possessed by the speaker. For the anti-realist, the gap is narrower: the question for him is whether he can make it sufficiently wide to admit a notion of objective truth which is not lost when our evidence decays and is not acquired for the first time when our information is obtained. Truth, so understood, would have to be explained as consisting in an objective possibility, for a suitably placed observer, of verifying the statement. (Dummett 1991, 338)

The problem with Dummett’s understanding of truth is that he needs the distinction between absolute truth and the individual speaker’s justification, as he still holds the metaphysical realist’s conception of truth because *without it, we shall relapse into a version of solipsism*, or complete relativism.

The term ‘evidence’ is ambiguous in this context: it might be taken to mean either ‘evidence (whether known to us or not)’ or ‘evidence in our possession’. The notion of a possible future world history, consistent with the present evidence, can be construed according to either interpretation of ‘evidence’. When it is construed in accordance with the first interpretation, we must admit a distinction between truth and actual grounds for assertion, as opposed to what would be a ground if we knew of it. Even when it is the second interpretation of ‘evidence’ that governs our conception of a possible future world history, *it will be necessary to admit a distinction between the truth of a statement and the individual speaker’s being justified in making it; without it, we shall relapse into a version of solipsism*. (Dummett 1991, 169-170; emphasis added)

This of course raises the discussion about eliminating the concept of truth from its central role in the philosophy of language and theories of meaning.

The evident remedy is to replace truth, as the central notion of the meaning-theory, by some notion that can be wholly accounted for in terms of the use a speaker actually makes of the sentences of the language. (Dummett 1991, 317)

Indeed, what Dummett is suggesting is a sort of ordinary language philosophy. Perhaps he is also proposing that Wittgenstein’s metaphysical subject, operating from outside the pictured world and his model of external reality, should be replaced by the humble common-sense person who lives inside this world but cannot know it. This is akin to the Kantian phenomenal subject who can at most sensually intuit phenomenal blind objects as distinct from the transcendental subject with pure reason and understanding of pure concepts. This is so since Kant, Wittgenstein, Davidson, and others do not have any theory of truth to prove the truth of human cognition and knowledge of reality.

**8.2. The Pragmaticist Theory of Knowledge of External Reality: The Truth-Conditions and Proof-Conditions of Perceptual Judgments**

The pragmaticist suggests a methodology of proof whereby our perceptual judgments, when grounded in basic facts, are the*proof-conditions*for the proof of all other hypothetical cognitions; but the proof-conditions of proved basic facts themselves stem from our reflective inner self whose cognitive operations indicate our confrontations in reality. Moreover, we can explain and show that particular proof-conditions can be the same or they can intersect or overlap such that we can have common proofs of the truth of our perceptual judgments and scientific hypotheses (Nesher 2002, X; 2018, 2020).

I present here a crude scheme of the *perceptual process of interpretation* as the hierarchical development of the *empirical content* of the perceptual judgment proposition in its representation of the external object. *Empirical matter* and *symbolic form* are not two different sources of our judgments, à la Kant. The iconic sign enters as the *replica* of the indexical sign and both signs become *replicas* of the perceptual judgment symbol in its *representational capacity*. We “grasp” the content as our empirical experience with reality. The *duality* of these components, iconic and indexical, are the *truth-conditions* of our perceptual judgments and *the source of the truth* of the perceptual judgment:our *first* and *simple truth*:

**[13] Perceptual Process of Interpreting Signs: Developing the Empirical Content of the Representational Symbol**

Relations of Interpretation

Seeing: a green piece of paper; reacting: here is; asserting: ‘Here is a green piece of paper’

Percept *➔* Iconic sign *➔* Indexical sign ➔ Symbolic sign: perceptual judgment

Duality Synthesis Truth

What is the meaning or content of a proposition and how are its proof-conditions embedded in its content? According to Frege, Ramsey, Tarski, and many other philosophers, the equation “p is true = p” or (T) ‘“X is true” iff p’ are true for all propositions when the truth-conditions are axiomatic, like the elementary propositions in Wittgenstein, the metaphysical subject’s thought. Yet, according to the above analysis of the perceptual operation, it is only in perceptual judgments that proof-conditions are embedded in the operations of their meaning or content. The reason for this is that the proof-conditions of other non-perceptual propositions and theories are true perceptual judgment propositions, in their capacity as basic facts, but once proven true they are available as proof-conditions for other hypotheses to be proven either true or false (Nesher 2002, X; 2018, 2020). I would like to argue that the pragmaticist’s conception of truth as the result of these proof operations holds for our first and simple truths of perceptual judgments. On the basis of these all our reasoning can be proven, and this also holds for complex truths of other reasoning and scientific hypotheses. Consequently, what should be shown is that our perceptual operations, proven true, are not just “given” facts but must also be mainly “veridical,” as Davidson suggests. For this, we have to show that the basic structure of our perceptual process is a quasi-proof procedure due to being proven at the lower level of self-control. Though distinct from the proof structures of our scientific reasoning, they have the same resulting truths and representational functions.

**[14] The Structure of Perception: The Instinctive Quasi-Proof of Perceptual Judgment**

The Instinctive Quasi-proof of the Perceptual Judgment

Percept *➔* Iconic-*feeling* *➔* Indexical-*reaction* *➔* Symbolic-*thought*: Perceptual Judgment

**Duality** *▲*

**Truth-Conditions** = {tension concordance} *❙*

*▼ ▼* *❙*

**Confrontation** *❙*

Hesitation*/*Doubtfulness ≈ Assurance *➠* Assertion I

*▼*

External Reality

The feeling of *duality* that appears forcefully beyond our self-control, followed by our instinctive *comparison* of its components, presents the *confrontation* between mind and reality.

**8.3. “Our Senses as Reasoning Machines”: – The Three Rules of Cognition According to Pierce**

Peirce explains this feeling of duality, of action paired with reaction, as a sign of *something* that by being beyond our control exists independently of us, and therefore we call it *external reality*. We cannot reach this external reality without confronting it, although we *detect* and *react* to external reality from inside our skins through this feeling of duality in our perceptual processes (cf. Davidson 1983, 312; Nesher 2002, #VI). Peirce developed this epistemology of perceptual proof in “Our Senses as Reasoning Machines” where he gives three rules of cognition to explain the ways in which our instinctive proof mechanism operates like a reasoning machine of human cognition that enables us to gain true perceptual knowledge of reality (Peirce 1900).

According to the pragmaticist’s conception of proof, there are different kinds of truths whose basic structures are the same sequence of *three logical inferences*: abduction, deduction and induction (Peirce’s “trio”). They differ according to the levels of self-consciousness and self-control engaged with their components. The self-control we have over our regular perceptual operations is basically instinctive, and with this control we intentionally regulate our perceptual judgments.

[15] The Three Logical Inferences: Abduction, Deduction, and Induction (Peirce’s “Trio”)

[15.1] Abductive Cognizance: Ab(C, A *➞*C) ==>AAB

[15.2] Deductive Expectation: Dd((A*➞*C)AB, AAB) *➞*CDd)

[15.3] Inductive Evaluation: In ((AAb, CIn) ≈>PRm/n(AAb *➞*CIn))

The confrontation with physical reality through the coherent interpretation of meanings of the three inferences is the *proof-condition* of the quasi-proof of the truth of perceptual judgment representing reality, as an alternative to the phenomenalist *truth-conditions* of empirical positivism.

**[16] The Structure of Perception and the Operation of True Representation of Reality:**



Peirce developed his semiotics into a branch of epistemic logic to deal with our perceptual confrontation with reality, manifested in the duality of the ego and the non-ego; within this structure our genuine signs are interpreted as complete proof of the true representation of external reality, conditioning the validity of the interpretation and the soundness of the proof. The *truth-conditions* of the *perceptual judgments* are the *duality* in which the *instinctive comparison* of the interaction between the *icon* and *index* is *evaluated* in the stage of *induction*. The positive evaluation of our perceptual experience is affirmed by our *feeling of assurance* of having veritably *represented* external reality, and we assert our perceptual judgments as true on the basis of such a feeling. Peirce calls this basic “mechanism” “the natural instinct for truth”; he thus epistemically connects the *matter* and the *form* of our cognitive experience, as distinct from Kantian transcendentalism and branches of neo-Kantianism that use components of his epistemology.

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**Chapter 6**

**On The Epistemology of Physical and Psychological Sciences: A Pragmaticist Alternative** **to the Shortcomings of** **Analytical Philosophy (“Scientism”) and** **Hermeneutic Phenomenology (“Artism”)**

**1. Introduction: The Destructive Dilemma between the “Scientism” of Analytic Philosophy and the “Artism” of Phenomenological Hermeneutics**

What we perhaps intuitively recognize and Gadamer makes explicit, is the idea that in social science empirical ‘objects’ do not exist independent of our description of them. Their meaning as objects depends on how we describe them, and these descriptions are embedded in the living traditions of which we are part. (How 1995, 54)

What explanations can the epistemology of science provide? Do natural and human sciences share the same epistemological basis and epistemic logic vis-à-vis our cognitive operations, or is there an unbridgeable epistemic gap between them (cf. Habermas [1970] 1988, chap. 1; 2003, chap. 1I)? If the former applies, then how can we understand the epistemology of science in general? Would we accept the logical positivist’s position and use formal semantics as a tool for understanding human knowledge and its subject matter, an approach that we can call “scientism” (e.g., Bernstein 1983, part 4; Margolis 2003,1-18, chap. 4, passim); or rather should we adopt an ‘artistic’ conception, leaning on phenomenology and philosophical hermeneutics to offer historical interpretations that disclose the subject and its world (e.g., Gadamer 1960, Part I: I.3, II.2, Part II; Palmer 1969, intr., passim; Bernstein 1983; Habermas 2003, intr., chap. 1; cf. Habermas 1970, chaps. 10-12; and Grunbaum 1984, chap. 2 with his argument against Habermas and the hermeneutic interpretational method)? I would call the latter approach “artism.”

Habermas emphasizes the historically developed conceptual dualism between the natural and cultural (or psychological) sciences. With regard to the terminology used, some prefer to talk about the distinction between natural sciences and social or human sciences. I favor the Peircean terms *physical* and *psychological sciences*, since the Quinean use of *natural* is physicalist and suggests that mental life must be seen as supernatural (as distinct from a Spinozistic understanding of Nature as all that subsists, its physical and mental aspects alike). The use of the term human science is also misleading because all sciences are human, whether physical or psychological, and the psychological sciences deal with the mental behavior of animals as well (comp. Kant [1781-1787] 1929; [1788] 1996; [1790] 1914).

As for the role of epistemic logic in explaining cognitive operations in these different sciences, Weber posits that these two types of sciences, natural and cultural, are distinct in *principle*.

The focus of attention on reality under the guidance of values [established by the cultural sciences] which lend it significance and the selection and ordering of the phenomena which are thus affected in the light of their cultural significance is entirely different from the analysis of reality [conducted by the natural sciences] in terms of laws and general concepts. Neither of these two types of the analysis of reality has any necessary logical relationship with the other. They can coincide in individual instances but it would be most disastrous if their occasional coincidence caused us to think that they were not distinct in *principle*. (Weber 1949, 77; cf. 80)

This raises the question of what exactly is meant by “distinct in *principle*.” Does this refer to something about the methodologies and modes of objectivity used by these two types of science and the function of truth in their “analysis of reality” (comp. Weber 1949, 49ff. and 110)? Habermas emphasizes the seemingly unbridgeable gap between these two epistemic or methodological perspectives, which, as mentioned previously, are known as “scientism” and “artism.” The latter is named after the model of interpretation of artworks that is taken to be the paradigm of human knowledge in hermeneutics (cf. Palmer 1969; Bernstein 1983; Margolis 2003).

Scientistic consciousness obscures fundamental and persistent differences in the methodological approaches of the sciences. The positivistic self-understanding prevalent among scientists has adopted the thesis of the unity of sciences; from the positivist perspective, the dualism of science, which was considered to be grounded in the logic of scientific inquiry [e.g., Dilthey], shrinks to a distinction between levels of development... On the other hand, the historical-hermeneutic sciences, which appropriate and analyze meaningful cultural entities handed down by tradition, continue uninterrupted along the paths they have been following since the nineteenth century. There is no serious indication that their methods can be integrated into the model of the strict empirical sciences. (Habermas [1970) 1988, 1)

Can the epistemic gap between these two forms of science be perceived simply in terms of a historical development? Yet, it is important to remember that both philosophical traditions are extreme approaches, and we can find a middle ground that shows the common character of these two sciences that are only seemingly distinct in *principle*.

This continuing dualism, which we take for granted in the *practice* of science, is no longer discussed in terms of the *logic* of sciences. Instead of being addressed at the level of the philosophy of science, it simply finds expression in the coexistence of two distinct frames of reference. Depending upon the type of science with which it is concerned, the philosophy of science takes the form either of general methodology of the empirical sciences or of the general hermeneutics of the cultural and historical sciences (Habermas [1970] 1988, 2; cf. Kuhn 1991, 17-18).

It seems it would be a mistake to distinguish between the general methodology of the empirical sciences and the “general hermeneutics of the cultural and historical sciences,” as if the latter were not empirical sciences. Moreover, it appears that all human knowledge is empirically based on our sensual-experiential confrontation with reality, including philosophical-metaphysical and artistic knowledge. This is probably in line with the Kantian tradition regarding the philosophy of art which separates the creation and evaluation of artworks from our cognitive representation of reality (cf. Kant [1790] 1914, #49[Ak. 314]; Gadamer 1960, 87; Stolnitz 1961; Bernstein 1983, 118ff.; comp. Nesher 2003).

I would argue that formal semantics and phenomenological hermeneutics are two alternative perspectives with different conceptions of meaning, truth, interpretation, and representation, and both are one-sided and lack the logic of an epistemological explanation of our confrontation with and representation of reality. The former is a stipulated abstract *vertical* semantical relation between formal language and the abstract structure of objects which cannot work as a model for real scientific inquiry; the latter consists of horizontal relations between different stages of interpreting linguistic behavior phenomenologically without any confrontation with external reality (cf. Nesher 2002, chaps. III, V, IX, X, 2003). Habermas claims that these philosophical conceptions account for two different aspects of human relations: the objective relation to external reality and the subjective relation to one another, as two correct theories that must be connected.

The vertical view of the objective world is interconnected with the horizontal relationship among members of an intersubjectively shared lifeworld. The objectivity of the world and the intersubjectivity of communication mutually refer to one another. (Habermas 2003, 16)

Thus, Habermas suggests a complementary relation between the two traditions, which are based on different intuitions with respect to the physical and psychological sciences.

The self-critical development of the hermeneutic approach into transcendental or, as I prefer to say, formal pragmatics would not have been possible without responding to the stimulating suggestion and insights of the analytic tradition. In my view, the traditions of hermeneutics and analytic philosophy today are complementary rather than competing. (Habermas 2003, 52)

However, if neither of these philosophical approaches is able to explain how we represent our physical and psychological realities (our knowledge and existence in the world), how can they be combined into a correct epistemological explanation which, according to Habermas, can be classified as “formal pragmatics” (Habermas 2003, 52)? Yet if “formal pragmatics” is to explain the communicative function of language in the abstract formal semantic model of the analytic tradition (regarding our conception of our representation of reality), then, I believe, it cannot yield a fruitful epistemological theory of cognitive behavior. Yet, I claim that this can be achieved by Peircean pragmaticism, as shown in my pragmaticist theory of truth and representation of reality (cf. Nesher 2002, 2003; comp. Habermas 2003, #5). However, formal pragmatics remains, according to Habermas, transcendental, making rational presuppositions without being able to explain how our communal language (with its meanings, contents, and propositional truths) evolves from the pre-verbal cognitive languages of our minds. This must be the case if formal pragmatics remains phenomenological, because it then fails to explain our experiential confrontation with the real world, with external reality, beyond an individual’s inner lifeworld.

Habermas views the two philosophical perspectives as a duality between “interpretivism and positivism” (Habermas 1970, vii). Thus, Habermas criticizes both sides of this dualism.

Thus, both analytic and hermeneutic philosophy, while approaching language from opposite starting points, confine themselves to its semantic aspects: to the relation of sentence and fact, on the one hand, and to the conceptual articulation of the world inscribed in language as a whole, on the other. The two sides use different means: the tools of logic, on the one hand, and the methods of content-oriented linguistics, on the other. Still the abstraction is the same in both, the holistic approach of formal semantics. Both treat the pragmatics of speech as derivative; they certainly do not expect the structural features of speech to make an essential contribution to the rationality of communication. (Habermas 2003, 62)

How can the pragmatics of speech be substantiated within the linguistic turn of philosophy in general and particularly within the philosophy of science? This is a Carnapian conception of pragmatics, a form of auxiliary support for the syntactic and semantic abstract model of language. However, is this possible within the holistic approach of formal semantics to human knowledge?

I would argue that between the difficulties of the so-called “objectivism” of logical positivism and the “subjectivism” of hermeneutics, there is a third path for the epistemology of science and knowledge in general. This solution is a Peircean pragmaticist conception of interpretation that is based on true representation of reality and vice versa. Yet agreement about our true representation of reality cannot start from any rational consensus, nor even from a pre-rational consensus, but only from an individual’s perceptual confrontation with external reality and true representation of it, since our true perceptual judgments are the origin of all knowledge of reality. Without our perceptual operations we cannot obtain any knowledge of reality, including knowledge of each other’s behavior, which is a prerequisite for any communication about such reality. Therefore, to analyze and explain the intentional judgment, which is the basis of any theory of communication, we need first to analyze and explain the relation between our perceptual and intentional judgments as the grounds for the distinction between physical and psychological facts, for this is key to understanding the epistemology of the physical and psychological sciences. However, the truth of our cognitive representation of reality is always relative to our perceptual quasi-proof-conditions and to the proof-conditions of our propositions and theories that consist of the truth-conditions and methods of proof (cf. Nesher 2002, V, X; comp. Habermas 1981, 277).

In my inquiries into meaning, truth, and the representation of reality I have dealt especially with perceptual judgments, which represent physical reality and constitute the basis of all knowledge and life in the world (Nesher 2002). Now it is time to extend my theory of truth and representation of reality by introducing what I call propositions of intentional judgment that represent psychological reality. In developing an epistemological theory of communicative understanding and activity the problem is to explain the process of human socialization (the ways in which human beings in their shared lifeworld develop and operate) through the prism of perceptual operations and judgments (comp. Habermas 1981, 308-309). This explanation must start from the *first person*’s basic perceptual representation of physical objects with the reflecting self-conscious and self-controlled feeling of their intentional quasi-proof of perceptual judgments. In such an operation this first person can also perceive an interlocutor’s human body as a physical object and understand this *second person’s* subjective intentional cognitive behavior from their own experience in the same environment.

In this work I will deal only with the epistemology of science and the epistemic logic of the structure of cognitive operations without entering into the methodological differences between the physical and psychological sciences.

**2. The Formal Semantic Conception of Interpretation and Referential Representation of Physical Reality and its Shortcomings in Analytic Philosophy**

Analytic philosophers with their formal semantics, from the Aristotelian tradition to the present day, have not suggested an epistemology for understanding and explaining the truth (or falsity) of our propositional representation of (or intention to represent) reality. According to the logical-grammatical understanding of truth à la Frege, Russell, Wittgenstein (in the *Tractatus*), Tarski, Carnap, and others, only elementary perceptual judgments and observational propositions can be true by virtue of the common logical forms of perceptual propositions and common-sense states of affairs, because scientific theories do not represent reality directly. Scientific theories can only be supported by these basic true propositions. In the same vein, according to Popper and Quine, only an “empirical basis” and “observational sentences” are true, while the theories that are built on them are sooner or later refuted (cf. Popper 1934, 1963; Quine 1960, #6; 1992, chap. V; 1995, 67).

Frege’s classical scheme of the truth of basic sentences with their senses and references can be summarized as follows:

**[1] The “Vertical” Conception of Referential Interpretation in Analytic Philosophy:**

**Thought**: Sense = {**Sensen** + **Sensep**}

I I

Sentence: (**Name** \* Predicate)

I I **Referential Interpretation** by **Sense** of

Referring = I = I **Name** and **Predicate** in **Object** and **Concept**

♥ ♥

**Reality**: Reference = [**Object**, **Concept** (Property)] = Truth-Conditions

Falls Under or Does Not Fall Under (Object, Concept) ÷ True or False

Within the framework presented in schema 1, the names and predicates of sentences refer with their senses to referents, which can be objects or concepts (properties), and their truth-conditions are the relations between the reference, object and concept (property). The thought expressed in the sentence is true or false according to whether or not the object to which the proper name (subject) refers falls under the concept to which the predicate refers. We can find similar formulations in Russell’s logical analysis of sentences and in Wittgenstein’s conception of elementary sentences that represent possible states of affairs by having common logical forms. The basic difference between Frege and Wittgenstein’s semantic models is that Frege holds a Platonic metaphysics in which objects are abstract components of thoughts, while Wittgenstein adheres to a sort of Kantian metaphysics in which objects are abstract metaphysical assumptions that combine all possible states of affairs independent of language (with its meaning).

Yet how can we know that the elements of our sentences refer to objects in the real world if our perceptual immediate references are at best only Humean ideas, sensual feelings, sense data, or Fregean objects and concepts of abstract Platonic thoughts, and not concrete connections to the real empirical world?

The study of our cognitive representation of external reality in formal semantics is presented concisely by Lyndon:

The formal study of any subject drawn from daily experience begins by replacing the everyday subject matter by suitable abstractly characterized idealizations*, chosen to preserve those features of the original subject that are relevant to the study at hand*. Here we need abstract substitutes for thought, for reality, and for the connection between thought and reality. For *thought* we substitute *language*, or more precisely, a formalized version of parts of everyday language. It can be argued that all the purely formal aspects of thought are adequately reflected in such a language. For **reality** we substitute something called a *structure*, which is hardly more than a collection of things suitable for being correlated, as meanings, to various expressions in the language. For the *connection between thought and reality* we substitute *interpretation*, that is, a function assigning to certain expressions in the language, as their meanings under the interpretation, certain objects in the structure. (Lyndon 1966, 2. Emphasis added)

The basic question is whether, if at all, this kind of idealization preserves and explains those features of the original subject of the connection between thought and reality (i.e., one’s cognitive representation of external reality), which are essential to any theory of truth. It can be argued that these abstractly characterized idealizations *cannot* preserve the essential relations of mind representing reality, even with Tarskian formal semantics, an enterprise that aims to create an objective, scientific semantics. The assumption that formal language and abstract structures are idealized objective domains, where the former stands in an interpretive relation to the latter, creates only an illusion of relevance to the connection between thought and reality. The formal semanticist assigns formal language interpretations to the entities of the abstract structure without taking into account that this is done from outside these idealized domains. Thus, in this operation the formal semanticist assumes God’s point of view, from outside these two domains, but in the natural situation no human being can get outside his or her cognitive “skin” (cf. Davidson 1986, 312; Putnam 1990, 17). The interpretation of language from outside, which is clearly impossible in cognitive behavior, leads eventually to a naive realist’s position, namely that we can apparently assume, in a Fregean way, without philosophical or scientific justification, that linguistic entities exist and are themselves connected to their objects, such that the observer only asserts their referential relations (cf. Putnam 1990, 11-18). This and other logical and formal semantic formulations of abstract models for linguistic meaning, and for the true representation of reality, are just the heroic attempts of logicians and formal semanticists to solve the problems of the meaning and truth of our propositions, but these cannot help while they continue to hold onto their basic Cartesian assumptions about assigning meaning and truth to human language from God’s perspective (cf. Russell 1914, chap. II; comp. Putnam 1990, chap. 1; Nesher 1998a, II; 2002, III, V, IX, X; Margolis 2003, chap. 4).

However, the formal semanticist’s conception of representation is based on their intuition of representing physical or common-sense objects in perceptual experience, and from such a one-sided perspective they cannot explain how we represent psychological reality, unless they suggest translating it into physical language representing the motion behavior of physical objects (e.g., Carnap 1932; Dennett 1991, 4.2; cf. Wittgenstein 1953, #593; comp. Ayer 1956, 209-214).

The way out of these shortcomings is to be shown in a Spinozist-Peircean dynamic-evolutionary conception of cognition in which an idea is not seen through the Cartesian lens as “something mute, like a picture on a tablet” but rather perceived as “a mode of thinking, viz., the very act of understanding” (Spinoza [1677] 1985, IIP43S). Such a dynamic vision is based on the essential nature of cognition as a continuous interpretation of ideas that represent objects in external reality (Nesher 2002, I, II, III, V). However, since we cannot detach ourselves from our cognitive skins, our perceptual cognition in the interpretive perceptual operation just anaphorically represents a referential relation between the sentential elements of our perceptual judgments and our preceding sensual feelings and reactions to external objects (and here I am referring to our pre-verbal dynamic perceptual signs whose operational synthesis ends in our perceptual judgment sentences). We cannot assume, like the formal semanticists in their Cartesian tradition, the perspective of God from which to compare our ideas with external realities to which we have no direct access. In my pragmaticist theory of truth I show how, without leaving our cognitive skins, we can nevertheless represent objects of external physical reality. We do so by employing a quasi-proof of the truth of our perceptual judgments in the operation of our cognitive interpretation (Nesher 2002, II, VI). However, in such an interpretation we also reflectively represent psychological reality, as I will attempt to show in the following section.

**3.** **The Conception of ‘Being’ In Phenomenological Hermeneutics: Shortcomings, Subjectivity, and Physical Reality**

The contending alternative to the formal semantic conception of meaning and truth in analytic philosophy is that offered by phenomenological hermeneutics, where meaning is conceived of as linguistic content and truth as disclosedness of the essence of being in an interpretative procedure. The conception of disclosedness relates etymologically to the Greek word *aletheia*, which literally means ‘disclosure’ or ‘unconcealment’. Thus, the essence of a being is unconcealed in the meaning or content of the interpretative operation, such that its appearance is a true interpretation of its essence. However, this truth is not the correspondence of a proposition to a state of affairs but the “truth of being” that is exposed in the appearance of the essence of being (Heidegger [1930] 1976; Macomber 1967, 93-140; Palmer 1969, 142-144; Okrent 1988, 236-253). In this respect the true interpretation of the subject (the being) is the representation of their essence. The question is how can this subject know, and how can we know, that this mental and linguistic interpretation of the subject’s inner being is a true interpretation and therefore a true representation (through reflection) of their psychological reality, i.e., their inner essence? It seems that in the horizontal operation of interpretation there can be indefinite possible interpretations, so does this mean that a decision has to be made about which one is the true representation of the subject’s inner essence? Can a person really know him or herself, and can we truly know the psychological reality, or essence, of our interlocutors? The general scheme of the horizontal phenomenological interpretation of psychological reality is shown below:

**[2]** **The “Horizontal” Phenomenological Interpretation of Psychological Reality**:

Interpretation of the Sign by Action/Language

> --------------------------------------------------→

**Sign**→Feeling→Emotion→Action/Language

>---------------------------------------------------→

Indicating the Truth of Action/Language to the Sign

In schema 2 the cognitive *sign*, or the person’s essence, is interpreted, so to speak, through personal feeling, emotion, and action or linguistic behavior, and action and language are the interpretation of the reality of this *sign*. (To simplify my discussion here, I use the Peircean triadic interpretants of feeling, emotion, and logic as the sequence of cognitive interpretation). This *sign* is an independent psychological inner essence which is represented in its final interpretation, either through an action or through language (propositional expressions). According to Heidegger, the truth or essence of a being is unconcealed in the operation of interpreting the sign: *Sign*→Feeling→Emotion. Thus, feeling and emotion direct the proposition to say or express this truth. Truth does not originally reside in the proposition (Heidegger [1930] 1978, 122-124). We can compare this operation to a deductive proof based on a true set of axioms, leading to the truth of the conclusion which takes the form of an action or a linguistic proposition. Now let as assume that the essence of a person (*Design*), taken to be an axiom, is a true essence; how do we know that its appearance, its disclosedness or unconcealment is the truth of the essence of the being? This is reminiscent of Leibniz’s example of Julius Caesar: we can only guess his true essence from his behavior but never truly know it; only God can know this because he conceives Caesar’s essence directly. So how can human beings know the truth of their interpretation of inner essence?

The problem with the conception of truth (or the true representation of psychological reality) in phenomenological hermeneutics is that in the uncovering (*aletheia*) of being, whether we are referring to a human being or an entity such as an artwork, there is no criterion to distinguish between the truth and the falsity of this disclosure. Indeed, from a phenomenological point of view, there are no external constraints to evaluate different interpretations and, therefore, there is no theory of truth that can be applied to the interpretive results, since a confrontation with external reality seems impossible (Gadamer 1980, 111-112). Therefore, one can only intuitively accept, probably by virtue of a strong feeling, that the “true friend,” the “true gold” and the “true artwork” show themselves in the hermeneutic operation of interpretation. This is similar to the feeling of which Descartes, Hume, and other philosophers speak in referring to our perceptual experience and self-evident ideas, notably Frege’s feeling of the real assertive force that determines the truth of indicative sentences (Frege 1918; Nesher 2002, V, IX, X). Nevertheless, this common-sense feeling about our perceptual judgments, intentional judgments, and aesthetic judgments can only be the starting point of philosophical analysis; it cannot replace any philosophical explanation as to why we feel such assurance of their truth (comp. Hume on aesthetic sentiment as a criterion for aesthetic beauty versus the factual force of the truth of perceptual judgments in “Of the Standard of Taste” [1757]). Thus, the theory of disclosure or unconcealment of the true essence of being cannot be proven..

The general question remains: How can a person discover their personal nature, or essence, in order to be able to interpret this truly? Moreover, if there is an objective way of truly interpreting and representing one’s essence or being, it must have some external restriction to differentiate between true and false interpretation. This could be through confrontation with physical reality (one’s environment) and external to the interpretive operation involved in the cognitive representation. This supports a quasi-proof of the truth of the representation of one’s personal essence. The reason for this is that without *confrontation* with external reality (not phenomenal reality) we cannot apply any theory of truth to our cognitive *representation*, either of physical reality or of psychological reality, including the truth of the creation and evaluation of artworks. This phenomenalist hermeneutic misunderstanding of our initial perceptual confrontation contributes to the development of the hermeneutic circle as a dialectic device but it does not offer any epistemological solution to explain our cognitive interpretations. Even the endless regression to historical tradition cannot solve it without confrontation with external reality (cf. Gadamer 1960, 189-191; [1984] 1989, 22; Bernstein 1983, 131-139; Wachterhauser 2002, 53-54, n4; comp. the circularity of Frege’s compositional thesis and Wittgenstein’s difficulty with “ostensive definition” in his *Philosophical Investigations*; cf. Nesher 1988). The pragmatist solution to the predicament of the “hermeneutic circle” is that our cognitive understanding starts with cognitive elements, or individual parts, which and by the rule of habit of our perceptual experience operations and by our reflective self-control of it we develop our understanding by we synthesize into perceptual judgments, through force of habit and reflective self-control, thereby developing our understanding of our perceptual experience (Nesher 2002, II, III, V). These cognitive operations are the quasi-proofs of our propositional judgments, the facts upon which we continue to base our interpretation and its validation or proof. To understand hermeneutics as a conception or theory of interpretation we have to look at Heidegger’s phenomenological hermeneutics and Gadamer’s philosophical hermeneutics.

How can we know the relation of interpretation of *human conduct* to its *essence*? And when art is taken in hermeneutics as the paradigm for truth and human knowledge, the question is about the relation of the artistic artwork to the art’s essence itself. Does its appearance express truly its essence in the interpretation or not?

Whence does the presentative statement receive the directive to conform to the object and to accord by way of correctness [truth]? Why is this accord involved in determining the essence of truth? How can something like the accomplishment of a pre-given directedness occur? And how can the initiation into an accord occur? Only if this pre-giving has already entered freely into an open region for something opened up which prevails there and which binds every presenting. To free oneself for a binding directedness is possible only *by being free* for what is opened up in an open region. Such being free points to the heretofore uncomprehended essence of freedom. The openness of comportment as the inner condition of the possibility of correctness is grounded in freedom. The *essence of truth is freedom*. (Heidegger [1949] 1978, 123)

The main questions are “Whence does the presentative statement receive the directive to conform to the object and to accord by way of correctness [truth]” and “Why is this accord involved in determining the essence of truth?” According to Heidegger, “The *essence of truth is freedom,*” which we can say is the freedom of being open to true interpretation. “To free oneself for a binding directedness is possible only *by being free* for what is opened up in an open region.” However, this is only a vague intuitive conception of freedom that cannot explain how “a binding directedness is possible” (Nesher 1999).

The following is a schematic presentation of Heidegger’s conception of how entities and beings show themselves to be true:

**[3]** **Heidegger’s Conception of How Beings and Entities Show Themselves**:

Interpretation

**Being** [Essence] → Appearance: *Showing itself* *by being free* as **true**

Hence, when the appearance is the true interpretation of being, this is the disclosure of the truth grounded in freedom. We can understand this idea of self-disclosure, as opposed to referring to something else, as the way in which the being or its essence, the being of a being, appears in an empirical or experiential situation. It is similar to the working of artworks in the way they disclose the truth of their essence: the paradigm of knowledge is rooted in the interpretation of the artwork, and authors or creators manifest themselves in their artworks (Heidegger 1965, 278). Here the problem of explaining interpretation comes to the fore as an existential development in Heidegger’s terms or as the cognitive operation in my pragmaticist terms. It is enlightening to see the difference between Heidegger’s conception of interpretation and the analytic philosopher’s conception of reference which Heidegger criticizes. On Heidegger’s phenomenological-hermeneutic shortcomings Habermas writes:

Philosophical hermeneutics fails to appreciate the cognitive function of language in its own right and the specific significance of the propositional structure of declarative sentences. As a result, Heidegger rules out any interaction between linguistic knowledge [*Sprachwissen*] and empirical knowledge [*Weltwissen*]. He does not even consider the possibility that what words in a language mean, on the one hand, and the result of learning processes within the world, on the other, can mutually affect one another, because he gives unlimited primacy to the semantics of linguistic worldviews over the pragmatics of communication. (Habermas 2003, 67-68)

By *the cognitive function of language* and *empirical knowledge* Habermas understands a referential relation to the world, a pragmatic operation in the world, and thus emphasizes the connection between the interpretative and referential functions of language. However, what Heidegger emphasizes is not “the propositional structure of declarative sentences” as perceptual judgments representing physical objects but rather intentional judgments representing the essence of the subjective being which is essential to understanding speech acts and Habermas’ conception of communication. Yet, Habermas’ solution with the pragmatics of communication still lacks an analysis of the perceptual operation which is our basic confrontation with the world, or external reality, as distinct from one’s “lifeworld” or the Wittgensteinian “form of life” (Nesher 1988; 2002, II, III, V, VII). The reason for the primacy of our perceptual sensory-motoric confrontation with reality is that in order to be able to communicate with others we (*first persons*) initially have to perceive them as physical objects (*third persons*) and only secondly as communicative agents (*second persons*) or as our interlocutors (cf. Nesher 1988, IV; 2002, 179, 291, 398, X.10; Habermas 1981, II: V.1, VI, e.g., 119).

Habermas assumes that through communication “different interpretive perspectives come closer to one another horizontally,” according to his conception of rational consensus in the activity of communicating. He then asks how this consensus enables us to represent physical facts vertically:

But the fact that different interpretive perspectives come closer to one another horizontally, as it were, does not yet explain how we can grasp facts in the vertical dimension of reference to the objective world, and how controversy about statements of fact can yield knowledge. The absence of a convincing analysis of the representational function of language, that is, of the conditions of reference and propositional truth, continue to be the Achilles’ heel of the entire hermeneutic tradition. (Habermas 2003, 61; cf. Nesher 1997, 1999, 2002; 2003, chaps. II, III, V, X)

The problem lies in the initial separation of the “horizontal” interpretation from the “vertical” reference, or better, the representation of the physical “objective world.” Thus, we need to explain how “different interpretive perspectives come closer to one another horizontally.” This question is tied up with our confrontation with the “objective world.” Without this confrontation, no consensus, common meaning, or truth can be achieved, let alone the social consensus on normative rules of behavior (cf. Habermas 2003, chap. 5; comp. Thagard 2000, chap. 7). The problem with the modern phenomenological hermeneutic understanding of truth and representation, and its criticism of the scientistic-mechanic correspondence conception of truth in analytic philosophy, is that the analytic philosopher’s conception of science is taken to be the methodology of all empirical sciences. This rejection of “scientism” essentially throws the traditional work of *science* out with the positivist’s bathwater, turning to *art* as the essential model of human knowledge in general and from a scientific perspective in particular. I suggest calling this philosophical approach “artism” (comp. Nesher 2003). This controversy between positivist analytical philosophers and hermeneutic phenomenologists has developed into the methodological and epistemological dichotomy between physical and psychological sciences in the form of a quarrel between “scientism” and “artism” (e.g., Grunbaum vs. Habermas, Grunbaum, 1984.) Habermas’ analysis of this epistemological dichotomy offers a historical account of this situation:

According to Dilthey, the historical human sciences of the nineteenth century were supposed to differ from the natural sciences in virtue of developing the traditional art of textual interpretation into a method of understanding meaning [*Sinnverstehen*]. Their goal is not the nomological explanation of empirical events but the understanding of meaning embodied in all kinds of symbolic expressions, cultural traditions, and social institutions. Heidegger takes this allegedly scientific operation of *verstehen*, or understanding, out of its methodological context and radicalizes it to constitute a fundamental feature of human existence. The original task of human beings is to understand their world, and themselves in this world: “In every understanding of the world, existence is understood with it, and vice versa.” (Heidegger [1927] 1980, 194). Heidegger replaces the phenomenological model of describing perceptions of objects by the hermeneutic model of interpreting texts, but retains the basic outline of Husserl’s “transcendental phenomenology”: “the meaning of phenomenological description as method lies in interpretation.” (Heidegger [1927] 1980, 61). The perspective of the observer perceiving objects is replaced by the perspective of the interpreter trying to make sense of what people’s utterances and their form of life mean. Such a phenomenology with a hermeneutic twist is, however, not primarily concerned with the manifest content of an utterance, but with the tacit contextual features of its performance. (Habermas 2003, 65-66)

It is important to note that the interpretation of the content of utterances and the tacit contextual features of its performance might be a true interpretation of a phenomenal lifeworld and of the human beings in this world but this can only be explained through a confrontation with the physical world and its representation. One may prove the truth of one’s interpretation of the cognitive meaning or content but without proof it remains only the cognitive content of pre-linguistic and linguistic performance and not a true representation of one’s psychological reality and the external world. The problem faced by hermeneutic phenomenologists like Heidegger and Gadamer along with Kantian pragmatists like Karl-Otto Apel, and even formal pragmatists such as Habermas, is that they cannot show how reality can be represented either in the interpretation of human behavior or in their communicative activity. Thus, they have to assume a priori transcendental or empirically transcendental presuppositions, so to speak, in order to explain human behavior without explaining the cognitive evolution of these presuppositions and how our cognitive behavior is possible by truly representing reality (e.g., Habermas 2003, 21-22). Therefore, they actually remain enclosed within a phenomenal lifeworld or within the Wittgensteinian form of life without being able to develop a theory of truth and without explaining our representation of the world or reality and thus the very possibility of a phenomenal lifeworld and its evolution (cf. Nesher 1988,; 2002, III; comp. Wittgenstein 1969, ##94ff.; Habermas 2003, 23).

**4. The Double Function of Representation: Descriptive Representation of Physical Reality and Interpretative Representation of Psychical Reality as Two Modes of Representation**

How, then, can we overcome the shortcomings of the analytic philosophers’ formal semantic conception of meaning and truth assigned “vertically” and of the phenomenological hermeneutic conception of meaning and truth as the interpretative operation explained “horizontally”? An alternative is the pragmaticist conception that combines interpretation and description to represent both physical reality, with its *objects* and events, and psychological reality, containing the cognitive **signs** of persons (comp. Habermas 2003, 16).

**[4] The Interpretational and Representational Relations of Human Cognition**:

The Sequence of the Cognitive *Interpretation* of the Sign (Direct Relations)

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**Sign**M→Feeling Interpretant→Emotional Interpretant→Logical Interpretant: Perceptual JudgmentI *Representation* I (Indirect elation) ♥

(comp. Habermas 2003, 21). **Physical Object**P

The difficulty with the ideal formal semantic model is that it aims to explain how our cognition represents a reality that is independent of our cognitive operations; in this case, how is the connection between thought and reality viable? For if the abstract objects of the model stand for our phenomenal objects, and our perceptual experience, then we are left waiting for an explanation as to how the representational relations of the phenomenal objects to the real objects actually materialize. In the vertical relation of the ideal model, elements of the abstract propositional thoughts are assigned to abstract objects to define their meaning and truth. Yet the formal semantic approach cannot explain how phenomenal objects that stand for abstract objects represent the real objects of external reality (Nesher 2002, V). This is in addition to the difficulty of explaining how our cognitive thoughts represent phenomenal objects, unless one adopts Fregean semantics to equate objects and their properties with the abstract entities of thoughts themselves (Nesher 2002, X: 5-7). To overcome these enormous difficulties, the formal semanticists must give up their pictorial abstract model with its vertical relations, as in Wittgenstein’s *Tractatus*, and turn to an epistemic model such as Peirce’s conception of semiosis as a dynamic interpretation of cognition that can explain how our perceptual operations and perceptual judgments represent physical reality (cf. Nesher 2002, II, X.4).

Now, the problem for Heidegger and others who adhere to phenomenological hermeneutics is to answer the question of how we can be certain that self-disclosure is the true appearance of the being of a being or entity, i.e., its essence? Is it not possible that a feeling of coherence or a feeling of its truth can be invoked in the uncovering, as with Kant’s subjective feeling of beauty? But if there is no known objective criterion for this feeling of truth about an appearance, or for the inferred conception of the truth of an artwork, then we cannot know that “truth happens in the work-being of the [art] work” (Heidegger 1965, 271; cf. Heidegger 1943, 187-138; 1959, 111-138; Tolstoy 1898, 141).

However, the free directedness of the operation of interpretation can only work by self-controlling the interpretation, and this can only be done through confrontation with an external constraint, a real independent object, which in the phenomenological interpretation does not exist at all (Nesher 2002, III). Indeed, Kant aspired to evaluate artworks aesthetically in the free play between understanding and imagination through the harmony or disharmony between them. Unfortunately, he could not show how we can reflectively judge their relation without having any external restriction for making such an evaluation (Nesher 2003). Therefore, in order to grasp the true interpretation of an essence, by free self-control, the interpretive operation must also represent the external physical object related to this interpretation (Nesher 2002, II, III, V, VII, X).

The way out of these predicaments of understanding our cognitive representation of physical and psychological realities is to explain that these are two aspects of our cognition that go together like intertwined Siamese twins, two essential components of our cognitive representation of reality. This is crucial, since the person’s self-controlling of the operation of interpretation is possible only through confrontation with external reality, and this is the truth condition of our cognitive interpretational-proof and of the conclusion derived. This is so because otherwise everything might seem to be correct or incorrect, and this undermines the conception of the subjective being’s *interpretation* and the anaphoric referential *description* of the pre-propositional cognitive sign that stands for a real object, e.g., “This is a white round stone.”

**[5] The Interpretational and Descriptive Relations of Human Cognition**:

Anaphoric Relation of the Description of the Representation of Physical Reality

**// Interpretation** of Meaning for Representation of Psychical Reality

>-------------------**//** ----------------------------------------------------------------------------------→

♥

Subjec**t Sign**=> [Feeling Interpretant=>Emotional Interpretant**]** =>Logical Interpretant [Proposition]

Relations of Causation ------------------------------→ I = **Reference/**Representation

I (Indirect Relation)

♥ [Observation]

(comp. Habermas 2003, 21) **Object**

In my pragmaticist diagrams the initial sign is the cause of the sequence of feeling, emotional, and logical interpretations, and their interpretation of the initial sign is quite similar to Heidegger’s conception of the interpretation of the subjective being, though Peirce developed this semiotic progression at the end of the nineteenth century. However, it should be understood that the interpretational operation cannot take place as a cognitive expression in a vacuum and that the cognitive interpretation cannot represent inner psychological reality without true representation of external physical reality.

**[6]** **Siamese Twins: Interpretation of the Mind’s Sign and Representation of Physical Object**:

**Sign**M =>F =>E =>A/L: Feeling, Emotions and Action/Language *interprets* **Sign**M

\ | /

True to **Object**P: Feeling, Emotions and Action/Language *representing* **Object**P

\ | /

♥

**Object**P (comp. Habermas 2003, 21, 23, 28).

The operation *Sign*M =>F =>E =>A/L, F and E involves the pre-verbal feeling and the emotional interpretations of the mind’s sign (*Sign*M). A is the action of interpreting this cognitive sign, while L is the language or proposition that interprets the entire operation. The interpretation of cognitive signs and the representation of real objects are Siamese twins since when we prove the truth of the relation of A/L to the physical object (*Object*P) we do this only through the interpretation of the *Sign*M by =>F =>E =>A/L. When we prove the truth of the interpretation of the *Sign*M we are able to do so only via the true representation of the physical *Object*P (Nesher 2002, II).

**5. Coherence and Correspondence Conceptions of Truth: Two Aspects of Scientific Proofs of Physical and Psychological Theories**

However, with our cognitive operations we can also represent our cognitive minds, the signs of the mind’s pre-verbal language of feelings and emotions, but we can represent them only through the description of real objects represented by our cognitive signs. Moreover, in the same cognitive framework we can either prove the truth that our cognitive signs represent the physical object or prove the truth of our cognitive signs representing the psychological subject. Thus, in the same cognitive structure we prove that our intended action and language represent the original **Sign**M, i.e., **psychological reality**, while at the same time we prove that they also truly represent the **real obect (Object**P ), as shown in the following scheme:

**[7] A General Scheme for the Interpretational Proof of the True Representation of Physical or Psychological Reality**:

Proof the **Truth** of A/L Representing **Psychological Reality** **Sign**M

>---------------------------------------------------------------------------------→

**Sign**M =>F =>E =>Language/Action*:* *Representing* Sign in *Describing* Object

\ | / and *Representing* Object in *Interpreting* Sign

True to **Object**P: Proof of the **Truth** of A/L Representing the **Physical Reality** **Object**P

\ | /

♥

**Real Object**P

In this general scheme for the interpretational proof of the true representation of physical or psychological reality we can see the interdependence of these two aspects of cognitive representation. We can extend the scheme of proof or quasi-proof to show the real structure of the threefold components: the abductive logic of discovery, the deductive logic of computation and the inductive logic of evaluation. In what follows I show the structure of this complete proof of the truth of **(AAb →CIn)** [A/L] with regard to the initial cognitive **Sign**M: **Ab(C, A →C) =>A)** and to the **Real Object**P: the true representation of reality. Thus, when we prove the truth of the interpretation **(AAb →CIn)** of the **Sign**M we can do so only by truly representing the intentional **subject**’s **psychological reality**, the **Sign**M (cf. Nesher 2002, II).

**[8]** **The Structure of the Complete Proof of the True Representation of Reality**:

**Representing Psychological Reality Ab(C, →C)=>A) by Interpreting/Proving that (AAb →CIn) Is True**

>­­­=========­­­========================================🡺

**Ab(C, A →C)=>A)** + Dd((A →C), A) =>C) + In((AAb, CIn) **≈>**PRm/n(AAb →CIn))

[Initial Sign] **Icon** [Feeling], **Index** [Emotional], **Icon**, **Index Symbol** [Logical]

Mental Reality Truth Conditions = Duality = Comparison ♠

Incoherency/Coherency //

⇓ ⇓ // Hesitation ⇒ Doubtfulness, Assurance 🡺 Assertion

[Confrontation with Reality]

I  **Representing** Physical Reality

I **Object**P by **Description** **(AAb →CIn)**:

I “This [CIn] is a stone [AAb]”

♥

**Physical Reality**: **Object**P

We interpret cognitive behavior but we cannot describe its cognitive meanings since, as Wittgenstein explained in his *Tractatus*, there cannot be any metalanguage Ln that describes the meaning of the object language or any other language Ln-1. There can only be a description of the physical structure of this language. But Wittgenstein did not distinguish between those aspects of language that we use to describe physical objects and events and those we employ in interpreting intentional pre-verbal and verbal behavior (our own and that of others), namely the distinction between our perceptual judgments about physical reality and our intentional judgments about psychological reality. Therefore, only *descriptive* *meaning* (iconic and indexical) representing *physical objects* and *events* can be interpreted by perceptual judgments (symbols) and only *intentional meaning* (feelings and emotions) representing *psychological operations* can be interpreted by intentional judgments (thought) and psychological fact judgments (or symbols) (comp. Wittgenstein’s *Tractatus*; Nesher 1987a, 2002). Thus, if we would like to describe the object language using a metalanguage, what we describe are only the physical features of language. However, when we would like to explain to somebody the meaning of a certain linguistic expression, we can only interpret it by employing other expressions with which the person is more familiar. What we interpret are our cognitive signs and in our perceptual operative confrontation with physical reality we interpret our pre-verbal perceptual cognition by our perceptual judgment propositions. However, when we quasi-prove the truth of our perceptual judgments, in a perceptual operation, they *describe* and thus *represent* the physical object that we perceive, e.g., “This is a stone,” “This is a red stone,” “This red stone is heavy,” or “This red stone weighs ten kilos.” It is important to note that the physical properties represented by our perceptual judgments are not just the Cartesian “primary qualities” of size, shape, position, and motion that we can measure in scientific experimentation; they also include the “secondary properties” of color, softness, hardness, heaviness, and lightness, because without these sensual cognitive signs we cannot identify physical objects through ostensive definition and *describe* them in perceptual judgments (cf. Wilson 1993).

Hence, we can see that through the *coherence* of the different sign-stages of the perceptual operation we quasi-prove the truth of our intentional judgment representing our psychological reality. However, this is the case only when we simultaneously quasi-prove the truth of our perceptual judgment as a **correspondence** with the represented object in physical reality (compare Spinoza’s relations of “adequacy” and correspondence in *Ethics* IID4; cf. Nesher 1994; 2002, I, II, VII; comp. Thagard 2000, chaps. 2, 4).

**6. The Distinction between Physical Facts and Psychological Facts**

Since we are dealing with the distinction between cognitive representations of physical reality and psychological reality, the question of representation must start with the basic facts upon which we build our physical and psychological theories. In my book O*n Truth and the Representation of Reality* (Nesher 2002) I developed a pragmaticist conception of facts as true propositions that we prove in our cognitive operation, while basic facts are our perceptual judgments that we quasi-prove in our perceptual confrontation with reality (Nesher 2002, X). Thus, the crucial questions are: “What is a physical fact?” and “What is a psychological fact?” Furthermore, what is the difference between them? Physical and psychological facts are conceptually distinct because they represent two different realities, physical and psychological respectively. In my pragmaticist theory of truth the emphasis was on perceptual judgments representing physical reality. In this framework, I basically worked with physical facts, without mentioning the problems of communicative propositions that Austin and Searle addressed within the framework of the speech act theory and Habermas in his theory of communicative action. Yet in their discussion of the truth of speech act propositions they basically accept the formal semantic model of meaning and truth, altogether avoiding the epistemological function of the truth of the speech act in communicative propositions according to the performative theory of truth (Austin 1950, 1954; Searle 1969, 137, 153-154; Strawson 1950, 1965; Habermas 1981, I:273-279, 307ff.; 1998, chaps. 4, 5; Brandom 1994, 327-333; comp. Nesher 1982; 1990, 10-20; 2002, X.2-3). The truth of communicative propositions like, “Please open the window” belongs to the propositions of psychological facts, somehow connected to the hermeneutic phenomenalist’s conception of truth as the disclosure of psychological reality in a communicative situation through a speech act (Heidegger 1959; Gadamer 1960; Habermas 1981, I:277-279; 1971, 74-98; Nesher 2003).

I would argue that the conceptual distinction between physical and psychological facts is between facts representing physical and psychological realities respectively, since according to my pragmaticist theory of truth, facts are neither “in the world” nor “the content of perceptual judgments” but rather the true *perceptual* and *intentional* judgments that are quasi-proved in cognitive operations as true representations of reality (cf. Nesher 2002, chap. X; comp. Peirce, 5.115‑116, 5.151-157, 1.383, 2.141-143).

But how do physical facts differ from psychological ones? According to my pragmaticist theory of truth, facts are propositions that have been proved true and are either the basic facts of our quasi-proved perceptual judgments or propositions and theories that have been proved true upon the basis of such facts. However, it is only when our perceptual operations are controlled rationally (namely, when we scientifically control the conditions of such perceptual operations) that we can call them scientific observations. Hence, under these rationally controlled conditions every normal person will basically observe the same reality and will judge the same basic scientific fact. The basic scientific facts of observation are proved in a complete perceptual quasi-proof, as in the following scheme:

**[9]** **The Structure of the Complete Proof of the True Representation of Reality**:

**Ab(C, A →C)=>AAb)** + Dd((A →C), A) =>C) + In((AAb, CIn) **≈**>PRm/n**(AAb →CIn)**)

Thus, in the cognitive operation the result of the abductive discovery inference, **Ab(C, A → C)=>AAb)**is the suggested concept or a theory **AAb** to explain the inductive **CIn** which is either a percept or a set of facts according to the level of our cognitive operation which in turn is either perceptual or scientific generalization and abstraction. In the deductive operation, Dd((A →C), A) =>**C**Dd), we infer an ideal-abstract fact CDd that we expect to be perceived or that we observe in the inductive evaluation of AAb: in((AAb, CIn) **≈>**PRm/n(AAb →CIn)). The perceptual or observational fact is the result of the quasi-proof of the perceptual or observational judgment in the inductive evaluation: (AAb →CIn), namely CIn is AAb, or the object represented by the cognitive image CIn has the property represented by the concept AAb, i.e., “This is a table.” Here we see that the structure of the fact is the *rule* that provides us with the method for identifying a kind of object such that **(AAb →CIn)**, i.e., “**CIn is AAb**.” Therefore, the controversy about the dichotomy between facts and values or norms of conduct evaporates when we see that facts themselves can be seen as norms or value-based rules. Now, scientific theories are suggested and evaluated on the grounds of such scientific observational facts but this happens at the rational level of self-controlled abstraction and generalization, which is different from the instinctive and practical levels of self-control.

But assuming (AAb →CIn) as the basic scientific fact, what is the difference between facts of the physical (natural) sciences and facts of the psychological (“human” or “social”) sciences? I would suggest that physical facts are quasi-proved or proved in the confrontation between the perceiver or scientist and physical reality (with its real objects), and this can be called the relation between *first person* and *third person*. Analogically, psychological facts are quasi-proved or proved in the relationships between the perceiver or scientist, another person or group of people with their physical bodies (e.g., brains) and/or other physical objects (e.g., land, buildings, machines, experimental or measuring instruments) that are perceived and represented (this can be called the relation between *first person*, *second person*, and *third person*). The reason why the physical *third person* is needed in order to prove psychological facts is because the operation of interpretation cannot take place without an external restriction on the cognitive interpretative operation.

**[10]** **The Basic Scheme for the Cognitive Structure of a Physical Fact:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

| Scientist😐 Cognitive Interpretative Operation →Perceptual Judgment Observation **= Physical Fact |**  | (*first person*) I ||\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ I\_\_\_\_\_\_\_\_\_\_\_|

I Representing

∇

**Physical Reality: ΦR**

(*Third person*)

Hence, the empirical meaning or content of the physical fact is the cognitive structure of the proof operation that proves that the physical fact is a true proposition representing some physical reality (cf. Nesher 2002, chaps. II, V, X; on the proof as the meaning or content of its conclusion, comp. Peirce, 5.448 n1; Wittgenstein [1921] 1975, 192; cf. Nesher 1994, 4.3).

**[11]** **The Cognitive Proof of Facts Representing Reality:** (see also [8])

Interpretation and **Representation**: (AAb→CIn) is True to **Psychological Reality Ab(C, A→C)=>A)**

>================================================🡺

**Ab(C, A →C)=>A)** + Dd((A →C), A) =>C) + In((AAb, CIn) **≈** **>**PRm/n(AAb →CIn))

[Initial Sign] **Icon** [Feeling], Index [Emotional], **Icon**, **Index** [Emotional] **Symbol** [Logical]

Mental Reality *Truth Conditions* = Duality = Comparison *♠*

Incoherency/Coherency *❙*

⇓ ⇓ *❙*

Hesitation ⇒ Doubtfulness Assurance ⇒ Assertion

(Confrontation with Reality)

I **Representation** of Reality

I **ΦR** by (AAb →CIn) = **This is** **ΦR**

∇

**Physical Reality**

Here **CIn** is the cognitive image of physical objects and **AAb** is the concept sign of a physical property of these objects such that what is proved, (AAb →CIn), is an observational fact representing physical reality. The entire cognitive proof operation is the scientist’s interpretative proof of the truth of the observational fact representing physical reality. This operates by instinctive self-control of the duality of the **icon**: the iconic qualities of feelings and **index,** and the indexical reactions of emotions, which are either coherent or incoherent and thus detect the truth of the symbolic assertion of the perceptual or observational judgment (comp. Thagard 2000, chaps. 2-4).

Now the structure of the psychological fact is explained when a *second person* perceives the *first person* and interprets that individual as perceiving a physical object which they also perceive. Hence, the second physical fact that contains the structure of the first physical fact as its subject matter becomes a psychological fact when interpreting and representing the *first person’s* psychological reality, with or without the *first person*’s perceptual judgment. Yet the *second person* who perceives the *first person* along with the physical object also perceives the *first person*’s bodily behavior within the shared environment as a complex physical object and thereby interprets their intentional behavior and intentional judgment about the physical object being perceived.

The epistemological structure of psychological facts is more complicated, since we have to explain the cognitive interpretation of the *second person*, the hearer or the scientist observing cognitive behavior, which is itself an operation of interpretation either implicitly, in the basic perceptual cognitive operation, or explicitly, in the rational interpretation of texts or of axioms in a formal proof.

The type of social science in which we are interested is an *empirical science* of concrete *reality* (*Wirklichkeitswissenschaft*). Our aim is the understanding of the characteristic uniqueness of the reality in which we move. We wish to understand on the one hand the relationships and the cultural significance of individual events in their contemporary manifestations and on the other the causes of their being historically *so* and not *otherwise*. Now, as soon as we attempt to reflect about the way in which life confronts us in immediate concrete situations, it presents an infinite multiplicity of successively and coexistently emerging and disappearing events, both within and outside ourselves. (Weber 1949, 72; cf. 75)

Here it is evident that social (or psychological) science is taken to be an empirical science that deals with reality “both within and outside ourselves,” which begs the question of what constitutes these realities “within” and “outside” ourselves?

After dealing with the “reflective knowledge” of one’s own “experience”, Weber writes:

The distinction that the “inward aspect” of the action which is to be analyzed is directly given to her in her own memory, whereas we must “interpret” the action of a third party from the “outside,” is, despite the naive prejudice to the contrary, only a gradual continuous difference in degree of accessibility and completeness of the “data.” (Weber 1949, 179-180)

I would interpret this as emphasizing that the basic psychological fact contains the *first person*’s reflective self-interpretation and that this is the “inward aspect” which, when their action is available to *second persons*, they must “interpret” the first person’s action as though they were “a third party,” the *third person*, from the “outside,” and firstly as a physical body, to complete the “data” of the psychological fact. Yet we perceive and interpret the other person as a unity of both physical body and mind such that through their bodily behavior we also perceive and interpret the conduct of their mind. Hence I understand “the action of a third party” to mean “a gradual continuous difference in degree of accessibility and completeness of the data” that includes their bodily behavior as a *third person* and their cognitive conduct as a *second person*. Therefore, the three aspects, *first person*, *second person*, and *third person*, combine to explain and complete the structure of human behavior whose basic unit is the psychological fact.

Now there are two kinds of psychological facts: the first (1) is evident when the *first person* represents **physical reality (ΦR**) by using their perceptual judgment and when the *second person* interprets their representation by representing their **psychological reality (ΨR**)but in respect to the represented **physical reality (ΦR**) (e.g., “This is a window”). The second kind (2) is evident when the *first person* represents/expresses their own **psychological reality (ΨR**) through an intentional judgment, e.g., “Please open the window” and the *second person* interprets their intentional judgment and represents their **psychological reality (ΨR**) by a judgment of **psychological fact**, e.g., “She is asking to open the window,” in respect to the same **physical reality (ΦR**), the window, which they represent in their intentional judgment (comp. Habermas 1981, 308-309).

The following is the general scheme of the judgment of a basic **psychological fact**:

**[12]** **The General Scheme of the Cognitive Structure of the Basic Psychological**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Interpreter**: |Interpretation \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Observational|

Hearer or the | of The **Subject** | **Subject’**s cognitive- Perceptual/Intentional | or Intentional | Scientist: | Operation ➔ |interpretative operation ➾Subject’s Action/Judgment|➔ Judgment |

(*first person*) | |**Psychological Reality: ΨR** ==> **Representing**=∥=> |**Psychological** |

Interpreting and | | (*second person*) ∥ | **Fact** |

Representing |\_\_\_\_\_\_\_\_\_\_\_\_\_\_|\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ∥\_\_\_ |\_\_❙\_\_\_\_\_\_\_\_\_| ❙

Describing/ Describing

Representing ❙

▼  **Physical Reality: ΦR**

(*Third person*)

[Body/Brain, Physical Object]

It should be noted that the perceptual judgment of the *first person* (i.e., regarding the window) is implicitly included in their intentional judgment (i.e., about opening the window). The judgment of psychological fact is the *second person*’s judgment that interprets the *first person*’s judgment.

Thus, the interlocutor or psychological scientist, the *second person*, interprets and represents the psychological reality (**ΨR**) of the *first person*, or subject, while at the same time describing the same physical reality (**ΦR**) that the *first person* describes or represents through their respective cognitive behavior. This *first person*, in representing the same physical reality (**ΦR**) through their perceptual judgment about a physical object, simultaneously reflects and self-controls, thus instinctively representing their psychological reality (**ΨR**). The scientist (*second person*) and the subject (*first person*) have a relationship with physical reality (**ΦR**) which is crucial for them to both relate to the same **ΦR** as the common ground for the objectivity of their interpretation and representation of the *first person’s* psychological reality (**ΨR**)**.**  (This is what Max Weber called the *naturalistic* component of the social sciences). This is the case whether the scientist is interpreting and judging the subject’s cognitive behavior or asking about their intentional act or about what they can see on a computer screen, for example. Thus, the scientist’s propositional judgment about a subject’s cognitive behavior is the basic psychological fact, in any psychological science, otherwise known as social or human sciences. In this scheme we can see the relation between the interpretation and representation of psychological reality (**ΨR**) and the representation or description of physical Reality (**ΦR**), without both of which there could not be any quasi-proof of the **judgment of** **psychological fact** as a true representation of psychological reality (**ΨR**) (comp. Habermas [1970] 1988, 13-16). It seems that in the above analysis of psychological facts, they have the same function as the “elementary factors” that Weber cites in his explanation of the pursuit of psychological science.

…in the social sciences we are concerned with psychological and intellectual (*geistig*) phenomena the empathic understanding of which is naturally a problem of a specifically different type from those which the schemes of the exact natural sciences in general can or seek to solve...

Let us assume that we have succeeded by means of psychology or otherwise in analyzing all the observed and imaginable relationships of social phenomena into some ultimate elementary “factors,” that we have made an exhaustive analysis and classification of them and then formulated rigorously exact laws covering their behavior. (Weber 1949, 74-75)

This formulation would provide a serious epistemology of psychological (or social) sciences if the ultimate elementary “factors” were indeed “observed” psychological facts. But since Weber cannot show how perceptual judgments in scientific observation can be ultimate elementary factors, they must only be hypothetical elementary elements. In this case, we must extend our scientific explanation to the vague historical elements that cannot be classified as Weber’s ultimate elementary “factors” either (cf. Weber 1949, 75-76).

**7.** **How Can Pragmaticist Epistemology Explain Communication?**

According to the above analysis, the physical fact of the *first person*’s perceptual judgment and the *second person*’s judgment of psychological fact can be seen as a combined fact representing the two persons’ modes of representing reality. This chunk of reality includes the two persons’ bodies and the objects to which they indexically relate. To make this relation clear let us say that they are both holding the same object, e.g., a ball. Here the meaning of one’s perceptual operation is the content of their experience with the object of their perception and the meaning of the relation of the behavior of the *first person* regarding the ball and the *second person*’s behavior regarding the same ball is ultimately an experience involving three objects that are combined into a complex event.

Thus as the *first person* feels their own true representation of the real object (the ball), through their instinctive self-control of the perceptual operation, the *second person* does the same. But at the same time, the *second person* also interprets the *first person’s* bodily expressions, feeling their respective psychological reality and intentional relation with the object. It should be noted that the perception of the human body is itself the cognitive sign of this body and the perceptual or observational judgment (i.e., the interpretation of this cognitive sign) is itself a meaningful representation of the physical body..

In the above analysis, the empirical meaning or content of the psychological fact is the cognitive structure of the proof operation that proves that the psychological fact is a true representation of some psychical reality (cf. Nesher 2002, II[x], V[x], X [x]). However, in order to truly represent such psychological reality the operation of the proof of psychological facts must be self-controlled by the subject, *first person*, at the instinctive and practical self-conscious level and also self-controlled by the *second person* that proves the proposition of the judgment of the psychological fact. This self-controlled phenomenon occurs at the instinctive and practical self-conscious level if it is a common-sense or folk-psychological quasi-proof of its truth. But in psychological sciences observations should be at the rational level of self-control if the scientific proofs of psychological facts are to control the relevant parameters of the experimental operation. This is so because without such self-control an error can creep into such operations due to nonstandard, abnormal, conditions of perceptual and observational operations, and the proof or the quasi-proof of the fact cannot be performed.

However, in order to explain the proof operation of the judgment of the psychological fact we should analyze its cognitive structure, as in [12] above, and the *truth conditions* for the proof, or quasi-proof, of the truth of such a judgment. The relevant truth conditions are a combination of the truth conditions of the *first person*’s judgments and the *second person*’s judgment about the first person’s. The *first person*’s judgments are both the perceptual judgment and the intentional judgment when the perceptual judgment of the *first person* (i.e., about the window) is implicitly included in their intentional judgment (about opening the window). This is the case since without true perceptual judgment of the object (window) the intentional speech act cannot operate. The truth conditions of the *second person*’s judgment of psychological fact are the truth conditions of their interpretation of the *first person*’s perceptual and intentional judgments. For this interpretation the *second person* has to perceive the *first person* and the object in question and thereby know the truth conditions of his perception. The truth conditions of the *first person*’s intentional judgment that contains the truth conditions of their perceptual judgment are presented in the following scheme:

**[13]** **The Truth Conditions of the *First Person*’s Intentional Judgment**: **Ab(CAb) = The Subject Intention**:

Interpretation/**Expression**: (AAb →CIn) is True tothe Cognitive Reality **Ab(C, A →C) =>A)**

>================================================🡺

Ab(CAb, A →C) =>AAb)+ Dd((A →C), AAb) => CDd) + In((AAb, CIn) **≈>** PRm/n(AAb →CIn))

**Percept** Sign Icon Index Icon Index Perceptual Judgment

**Intentional** Sign **Feeling Emotion**  **Feeling Emotion** **Intentional Judgment**

Cognitive Reality  *Truth Conditions* = Duality = Comparison ♠

Incoherency Coherency I

⇓ ⇓ I

Hesitation ⇒ Doubtfulness Assurance ⇒ Assertion

[Confrontation with Reality]

I **Representation** of Physical Object

Iand the Hearer[**ΦR**] by (AAb →CIn)

♥

**Physical Reality** [**ΦR**]

Physical Object and Hearer’s Body

Assuming that the perceptual operation concerning the physical object (the window) is accomplished, then the *first person* interprets the cognitive *intentional* sign by virtue of their *feeling* about it; consequently their *emotion* (volition) interprets the previous cognition in respect to the initial *intentional* sign. The duality of *feeling* and *emotion* constitutes the *truth conditions* of the interpretation of the initial *intentional* sign such that the coherence or incoherence between them indicates the truth or falsity of this cognitive interpretation. Yet the question remains: How does the *first person* feel and know that their intentional operation is coherent and can be accomplished properly in their speech act or communicative action of intentional judgment? The intentional speech act is intended for the hearer, the *second person* interlocutor who is also perceived as an object by the *first person* subject: “Please [hearer] open the window [physical object].” And yet, the hearer as the *second person* is a subject that can conceive the intentional judgment and react to it but is also a bodily person as a *third person* that can be perceived. Thus, in the communicative situation every human being is all three of these *persons* though their roles change in different communicative situations such that one can play only one *person*’s role at a time. One cannot be the speaker and the hearer simultaneously but one can be the interpreter of oneself since this is the essential structure of the cognitive operation, although one does not ask oneself to open the window but only intends to do so.

However, the *second person* does not have any direct access to the truth conditions of the *first person*’s perceptual and intentional judgments since we cannot reflect on our own cognitive feeling and emotional operation. However, if our cognitive operations are not concealed in an inner realm of subjective consciousness but are manifest in facial expressions and body movements then we can interpret and understand the other person’s thoughts, though we cannot control their cognitive operations from outside as they self-control their own. As Ayer (1956, 209) puts it, “... from the fact that one cannot literally share the experiences of another person it does not follow that one cannot understand what he says about them.” However, what is really meant by this? Can Ayer’s argument hold if I cannot even tell that there is any correlation between my own inner experience and outer behavior (cf. Ayer 1956, 214ff.)? An individual cannot perceptually experience another person’s *perceptual experience*; but we also have *intentional experience* and express our intentions in communicative action, and it is quite possible that we can share this kind of experience. This reminds us of Austin’s “constatives” and “performatives”. However, these are two aspects of the unified operation aimed at either physical or psychological realities. Thus, we are dealing with either essential or functional perceptual judgments (assertive force) or intentional Judgments (expressive force) (comp. Austin’s “expositive force” and “commissive force”, 1962, 33). This goes against Wittgenstein’s argument that we cannot know our inner experience (which he based on the premise that knowledge can only be public). Perhaps this could hold in phenomenological-internal realist epistemology, but then if we cannot show or prove our knowledge of external physical reality, how can we know about our inner experience (Nesher 2002)? The Cartesian assumption that there are two kinds of substances, a mental substance (the innermind) and a physical substance (the outer body), entails that we can observe the latter but not the former, which is known to its owner alone (cf. Ayer 1956, 222). But if mind and body are just two aspects of the same entity, in Spinoza’s terms, then the mind is not hidden from observers after all and we can perceive or conceive another’s mind as well as we can perceive their body (perceptual judgment *perceiving* objects vs. intentional judgment *conceiving* minds [Descartes 1984, II:21-23]). We perceive the body and conceive (grasp) the mind *together* as two aspects of a person’s behavior, and here I use these terms *perceiving* and *conceiving* in order to emphasize the distinction between the way we perceive physical objects and events and conceive psychological expressions of feelings, emotions, and thoughts. Thus, instead of the Cartesian real divide between the two attributes of thought and extension, I accept the Spinozist real distinction between the two aspects of substance, Nature and its finite modes, that determine the union of mind and body in human beings such that our feelings and emotions represent modifications of the body and thus we can explain why we say that we feel and emote through our bodies (cf. Spinoza 1985, II, III). How, though, does a person know their body? Is one’s body in the world and one’s mind outside of it? A Spinozist suggestion would be that one can know one’s body from its union with one’s mind or from outside as an object, or both when the latter offers a constraint that allows for the objective knowledge of the former.

Can I know the truth conditions of the *first person*’s having pain? Their knowledge of these truth conditions come from their reflective self-control, but I cannot make the same reflection because I cannot reflectively control their own cognitive operation. Do I employ different truth conditions pertaining to the other’s condition of having pain? Are their facial expressions and intentional judgments the truth conditions of her feelings and emotional experience (Avramides 2001, 34-35)? The *radical divide* that Descartes sets up refutes skepticism by securing *subjective truths* but this radical divide makes it impossible to know “other minds” without God’s help. Without this help this divide leads us to solipsism (Descartes [1644] 1985, I:213, 16641; 1984, II:21-22.) Avramides suggests that we start from the conceptual presuppositions of the social activities of *second persons* and abolish the starting point of the individual *first person* that precedes epistemology. Yet, without an epistemological explanation of where we obtain these presuppositions, we cannot explain our knowledge of ourselves, objects, and others.

And in our questioning of our capacity to know the way the world is, the existence of other subjects and the world is presupposed. Which questions we then choose to ask is conditioned by these presuppositions. On this picture the radical skeptical possibility is not raised. It is in this way that conceptual considerations derive epistemological projects. (Avramides 2001, 42; cf. Avramides 2001, 36-42-44)

Avramides’ suggestion to begin with social relations essentially confuses the Cartesian radical divide between the two substances of mind and body with the epistemological understanding that knowledge starts as an individual perceptual, actional, and conceptual operation (necessary in order to have knowledge of another person so that communication becomes possible). Therefore, it seems that the *second person* can interpret and understand the first only by comparing the *first person*’s facial expressions and body movements to their intentional judgment, and the coherence or incoherence of this relation forms the truth conditions of their intentional judgment. This is because, through a confrontation with external reality, they indicate the truth of the intentional judgment in relation to the initial intentional operation (cf. Nesher 2002, I, II, III, V, X). However, the basic question is how do we interpret and understand the facial expressions and body movements of another person? Are they physical events or cognitive meanings, and if the latter holds do we directly interpret them as qualities of feeling and as the meanings of emotional reactions? Do we learn this by *analogy* to the relation between our inner cognitive operations and external bodily physical motions, or rather, do we interpret them as we interpret our cognitive operations, though without having any self-conscious self-control over them as they are operated by the *first person* but only in our own interpretation of them (comp. Thagard 2000, 4.4; cf. Avramides 2001, 36-44). We can solve this predicament in terms of a Spinozist epistemology such that the relation between two persons can be explained either physically using the attribute of extension or cognitively using the attribute of thought. Only the latter can explain the communicative relation between the *first* and *second person*; in other words, thought alone enables us to understand the meaning of another’s facial expressions, body movements, and verbal language as meaningful communicative action. Hence, the cognitive meaningful sequence of the qualities of feeling and emotional reactions, and their synthesis in thoughtful reasoning, is not merely an operation that takes place within the individual mind but can be transformed from one mind to another. Yet this explanation only holds if these cognitive operations are also representations of physical objects, which are the external constraints for proving or quasi-proving the truth of our representation of reality (psychological and physical), thus enabling objective communication between persons.

The conditions of their truth and falsity are indicated by “the real assertive force” of perceptual judgment and the real *illocutionary force* of the intentional judgment of the speech act which the speaker reflectively cognizes in their sincerity of feeling, which indicates coherence, or without this feeling, indicating incoherence, between their *intention* and its *performance*; for the hearer, truth conditions stem from the relation between the bodily expressions and movements they perceive and their intentional judgment.

The truth of the intentional judgment, indicated by its “illocutionary force,” is felt in the speech act by the hearer and thus enables the speaker’s communicative action to achieve the communicative goal. The “illocutionary force” of the speaker’s intentional judgment indicates that their speech act truly represents their psychological reality and has the same function as Frege’s “real assertive force” of the indicative proposition in truly representing physical reality. However, there is the question of what constitutes the *truth conditions* of the hearer or the *second person*’s judgment of psychological fact interpreting the intentional judgment of the *first person’s* speech act? According to the above epistemological analysis of the cognitive structure of the judgment of a psychological fact, the *second person* (as presented in [12]) contains the intentional judgment of the *first person*. Therefore, the *truth conditions* of the hearer-interpreter’s judgment of the psychological fact must contain (1) the *truth conditions* of the perceptual judgment/intentional judgment of the speaker as the *first person* with (2) the *truth conditions* of the hearer as the *second person* who interprets the speaker’s perceptual judgment about physical reality and their intentional judgment about their psychological reality. So we are looking first into the *truth conditions* of (1) the speaker’s (*first person’s)* perceptual judgment and intentional judgment as explained above, and the *truth conditions* of (2) the judgment of the psychological fact made by the *second person* interpreting the *first person’s* perceptual judgment and intentional judgment; there is, therefore, a perceptual operation in which both the body-behavioral relation of the *first person* to the physical object in question and the second person’s representation of the same physical object are equally important. However, in this case the hearer-interpreter, the *second person*, perceives the *first person*’s physical body which functions as a *third person*. Indeed, the crucial question is how the *second person* interprets the intentional judgment of the *first person* from the perceived physical bodies and objects so as to represent the *first person’s* psychological reality by their own judgment of the psychological fact. The answer is essentially based on the baby’s cognitive operations in learning their first verbal language on the basis of sensorimotor experience of physical objects and adults ostensively teaching their names (cf. Wittgenstein 1953, ##30, 43; Nesher 1992); Nesher 1987b, V; 1992, 37-38, 1999).

It is important to note that in everyday communicative speech acts the hearer-interpreter does not judge propositionally, “She is asking me to open the window” unless they are requested to explain the illocutionary act. And yet in a psychological observational report the scientist-interpreter, or *second person,* explicitly expresses their judgment of the psychological fact.

**[14]** **The Truth Conditions of the *Second Person*’s Judgment of the Psychological Fact**: **Ab(CAb) = The Human Subject’s Intention:**

Interpretation/ **Representation**: (AAb →CIn) is true tothe Cognitive Reality **Ab(C, A →C) =>A)**

>================================================🡺

Ab(CAb, A →C) =>AAb)+ Dd ((A →C), AAb) => CDd) + In((AAb, CIn) **≈>** PRm/n(AAb →CIn))

**Intentional** Iconic **Feeling** Indexical **Emotion** **Feeling/Emotion** **Intentional Judgment**

Mental Reality *Truth Conditions* = Duality = Comparison ♠

Incoherency Coherency I

⇓ ⇓ I

Hesitation ⇒ Doubtfulness Assurance ⇒ Assertion

[Confrontation with Reality]

I **Representation** of Physical Object

Iand the Hearer[**ΦR**] by (AAb →CIn)

♥

**Physical Reality** [**ΦR**]

Physical Object and Hearer’s Body

We can understand a psychological fact as a basic representational and communicative action whereby the *first person*, the hearer or scientist, who interprets the *second person*, is also a *first person*, truly representing the same real object (of reality) represented by the *first person*, while at the same time acting as the so-called *third person* in their quasi-proof operation, using perceptual judgment. However, the problem lies in explaining how the *first person* knows that the *second person* (as another *first person)* is representing the same real object? For in order to possess this knowledge, they would have to first interpret the second person’s cognitive psychological reality, but to do this they would already have to know that they are both representing the same real object. There are two central questions about “communicative actions” (to use Habermas’ concept). fFirst, how does the *first person* know that the second *person* is representing the same object, and secondly, how do they represent the object as a *third person*, which can also be a real person, with “the same” or similar concepts (comp. Searle 1986; Brandom 1994, chap. 8; Habermas 1981, chap III esp. 307ff.; 1998, chap. V; 2003, chap. 3.V). If the *second person* can prove, or quasi-prove, that they are both using the same concept, then he or she truly represents the psychological reality (**ΨR**) of the *second person*. Therefore, the *first person* quasi-proves in their perceptual judgment the physical contact between the *second person* and the object, or a person-body, he reacting to as the *third person*, is the same or similar to her physical contact reaction to it. This can be achieved since we have already shown that in the perceptual operation the *first person* can quasi-prove the truth of their perceptual reaction to the real physical object that may also include a perceptual judgment. Secondly, the *first person* has to truly interpret, with a quasi-proof, the *second person’s* inner psychological reality (**ΨR**) (comp. Habermas 1981, 308 on one’s *own* world, *our* world, and *the* world). As I developed in my Peircean theory of meaning, the meaning or content of our basic cognitive concepts comes from our empirical experience with the objects of which we form concepts. Thus, according to Peirce, every symbol attached to our iconic qualities of feelings and indexical reactions of emotions is combined with our primordial pre-linguistic sensorimotor cognitive experience with objects (cf. Peirce, 5.119; Nesher 2002, II). Now the question is how can the *first person* know, or quasi-prove, that the *second person* has the same or a similar symbolic-concept of the same object? This can be learnt from their pre-linguistic sensorimotor behavior toward the same object and from their use of language in the context of this behavior, if it is similar to their own use of language. Thus, I reconstruct Wittgenstein’s famous diction in *Philosophical Investigations* (#43) that we explain, or prove, the meaning of a word orconcept according to its use in representing physical objects from within the language (cf. Nesher 1987b, II, IV, VI; 1992). Moreover, the primordial quasi-proof of the truth of perception and the communicative action developed through it can be operated with pre-verbal languages of the mind, similar to the way in which animals communicate or a baby learns their first verbal language. In this way, the verbal language can develop from the natural pre-verbal language. Thus, we can see that the structure of the psychological fact is at the core of psychological sciences that aim to explain how human beings communicate and use social conventions based on true perceptual judgments, without requiring any mysterious transcendental presuppositions involving *rational communicative action* or “plural subjects” (cf. Nesher 2002, chaps, II, X; comp. Habermas 1968, 155-156;1970; 1981, 307ff.; 2003, chap. 3.V; Gilbert 1989, 1-3, V.2, VI.2, 7, 8, VII).

Habermas addressed this crucial problem of human communication. The main shortcoming of the theory of communicative action, he argued, is that it cannot explain how the true representation of objects and persons is possible on the basis of a seemingly already existing *lifeworld* without perceiving them first as physical objects. One should have a true representation of one’s interlocutor in order to interpret their cognitive behavior through their sensorimotor behavior and their use of language. Only through such perceptual representation can the problem of coordinating communicative actions be achieved (cf. Habermas 1981, I: III; 1981 II: VI).

“Dialogue” is seen as the model for an exchange between interlocutors reaching mutual understanding about something in the world. In dialogue, the inter-subjectivity of a shared lifeworld, rooted in the reciprocity and interchangeability of the perspectives of first and second persons, is interconnected with reference to something in the objective world that is being talked about. As Humboldt already realized, there is a dimension of referential relation [*Sachbezung*] inherent in communication. (Habermas 2003, 71-72)

However, this “referential relation [*Sachbezung*] inherent in communication,” namely a reference to something in the objective world that is being talked about, is a prerequisite for the possibility of dialogue between the *first* and *second persons*, and it is the *third person*, a physical body or object in the real world, not in the phenomenological “lifeworld” expressed through language, that ensures the reality of the communicative situation and makes it possible to be certain of what is truly being talked about. Moreover, before the dialogue can even start, the *first person* and *second person* must be aware of one another as separate physical entities, as real objects existing independently of being represented cognitively. Thus, without perceptual *confrontation* with reality and without proving or quasi-proving the *truth* of one’s cognitive representation of it, there cannot be any dialogic communication between two persons (cf. Habermas 1981, 307ff.). This is also the basis of the objectivity of the psychological facts we explained above. And yet there is still a belief that we can explain the epistemology of psychological science from within our social lifeworlds without confrontation with the external physical world.

Above all, it must help change our conception of science from one in which an individual knower faces a neutral world of facts to one in which knowledge is seen as an ongoing social and historical accomplishment. Such changes are already being shaped by recent sociologies of scientific knowledge. (Bohman 1991, viii)

But without there being “an individual knower” who “faces a neutral world of facts” there can be no social agreement or consensus and therefore no “ongoing social and historical accomplishment.” The world is indeed independent of its being represented because otherwise there could not be any cognitive truth or constraining parameters, which are necessary for knowledge of worldly objects that individuals can agree about. Even so, there are no facts in the world since, as I developed in my theory of truth, facts are propositions that we have proved or quasi-proved and are relative to their proof conditions that include the truth condition for such proofs (Nesher 2002, X). The following is a general crude scheme of the cognitive proof of psychological facts.

**[15]** **The Cognitive Proof of Psychological Facts**: **Ab(CAb) = The Human Subject**

Interpretation/**Representation**: (AAb ➞CIn) is true tothe Cognitive Reality **Ab(C, A ➞C) =>A)**

◂—————————————————————————————

**Ab(C, A ➞C) =>A)** + Dd((A ➞C), A) =>C) + In((AAb, CIn) **≈>**PRm/n(AAb ➞CIn))

[Initial Sign] **Icon** [Feeling] **Icon**, **Index** [Emotional] **Symbol** [Logical]

Mental Reality Truth Conditions = Duality = Comparison ▲

Incoherency Coherency ❙

▼ ▼ ❙

Hesitation => Doubtfulness Assurance ➠ Assertion

[Confrontation with Reality]

❙**Representing** of Physical Reality

❙  **OR** by (AAb ➞CIn)

▼

Physical Reality: **OR**

Habermas, despite his claim that Max Weber was not interested in the relationship between natural and cultural sciences from an epistemological point of view, elaborated on Max Weber’s conception of social sciences:

At the same time, and in contradistinction to natural processes, regularities of social action have the property of being understandable. Social action belongs to the class of intentional actions, which we grasp by reconstructing their meaning. Social facts can be understood in terms of motivations. Optimal intelligibility of social behavior under given conditions is not, of course, of itself, proof of the hypothesis that a lawlike connection does in fact exist. Such a hypothesis must also be proved true independently of the plausibility of an interpretation in terms of motivation. Thus the logical relationship of understanding and explanation can be reduced to the general relationship between hypothesis and empirical confirmation. Through understanding, I may interpolate a rationally pursued goal as sufficient motivation for an observed behavior. But only when a resulting assumption of behavioral regularity occurring under given circumstances has been empirically substantiated can we say that our understanding of motivation has led to an explanation of social action. (Habermas [1970] 1988, 11)

Here we can see that first we have to understand basic social behavior as an intentional-interpretive operation and as a behavioral act. Habermas’ only requirement for accepting a fact is to understand it as a specific mode of psychological-social behavior, but I would argue that we should also prove it as a fact that enables us to empirically substantiate our psychological-social theory. For there are different conceptions of facts. According to Habermas, Popper, and other empiricists, we take the “empirical basis” as a given, and for Habermas this means events in the world. But as I explained elsewhere, we cannot merely point to such events and expect a common understanding; rather, we have to represent an event propositionally and moreover prove it as a true representation of reality. Thus, I explain facts as true propositions that we either quasi-prove in perception as observational propositions or rationally prove on the strength of these basic facts (cf. Nesher 2002, X). Then we can introduce the social fact (or generally, psychological facts) into theinterpretative and proof procedures we use to prove whether or not our hypothesis is true to these basic facts. In the above scheme of psychological inquiry, we introduce only the scheme of a set of facts.

Of course, in order to accept the subject’s judgment about their action the scientist should know that the subject self-controls their operative action, as Habermas pointed out in his discussion of Weber’s conception of social theory requirements:

This logical connection [between understanding and factual explanation] also makes clear why Weber accorded methodological primacy to purposive-rational action. As a rule, the interpretively interpolated goal, the assumed intention, will lead to an empirically convincing explanation only if the goal provides a factually convincing motive for the action. This is the case when the action is guided by the intention to achieve a result to be realized through means chosen in a purposive-rational manner, thus in na type of purposive-rational action that is oriented to the choice of adequate means to achieve an end grasped with subjective clarity. (Habermas [1967] 1988, 11)

The subject’s self-controlled operation or failed self-control is crucial for the proof of psychological facts and theories in the explanation of any human cognitive behavior. And yet not every human being’s cognitive self-controlled behavior is *purposive-rational* in terms of the propositional calculation of the intended goal and the means to achieve it, since in many cases we self-control our behavior instinctively and practically without failing to achieve our goals (cf. Nesher 1994 , 2001, 2002a; 2002b, chaps. II, III).

With the illocutionary force of an utterance a speaker can motivate a hearer to accept the offer contained in his speech act and thereby to accede to *a rationally motivated binding* (or bonding: *Bindung*) *force*. This conception presupposes that acting and speaking subjects can relate to more than only one world, and that when they come to an understanding with one another about something in one world, they base their communication on a commonly supposed system of worlds. In this connection I have proposed that we differentiate the external world into an objective and a social world, and that we introduce the internal world as a complementary concept to the external world. The corresponding validity claims of truth, rightness, and sincerity can then serve as guiding threads in the choice of theoretical perspectives for distinguishing the basic modes of language use, or the functions of language, and classifying the speech acts that vary with individual languages. (Habermas 1981, I: 278)

But this raises the question: How is it that, with the illocutionary force of an utterance “a speaker can motivate a hearer to accept the offer contained in his speech act and thereby to accede to *a rationally motivated binding* (or bonding: *Bindung*) *force*”? As I explained above the *illocutionary force* of the speaker’s intentional judgment only indicates that the speaker’s speech act truly represents her psychological reality and that it has the same function as Frege’s *real assertive force* of the indicative proposition in representing physical reality. And yet, the truth of the speech act should be explained epistemically if the relation to the real world is not to remain a myth or a transcendental presupposition. (cf.Miller, 1987)

**8. Conclusion: Facts of Physical and Psychological Sciences and the Proof of Theories**

Our judgments of psychological facts represent psychological reality, which therefore necessarily exists before the interpretational operation even starts. Its existence is thus independent of its being interpreted and represented. However, one can argue that our knowledge of psychological reality depends on the interpretational operation, and it would follow that the meaning of psychological reality depends on how it is interpreted.

This could be a viable argument if the interpretation of psychological reality were arbitrary such that everyone could interpret it differently, without any objective constraints operating on the interpretation. Hence, its meaning would depend on its interpretation, and the subject matters of psychological sciences would not exist independently of our representation of them. In this conception of the relativity of meaning, “anything goes” because there is no objective truth about psychological reality.

However, our perceptual and scientific interpretations are true to psychological reality when their quasi-proofs or rational proofs are based on their proof-conditions and are self-controlled. Under these conditions there are no arbitrary interpretations if they are proved true. In such cognitive operations one’s interpretations depend on an independently existing psychological reality and on the proof conditions that are relative only to our background knowledge that constrains these interpretations, independently of any particular interpretation of psychological reality. The following scheme explains that our cognitive interpretation and representation of psychological reality is objective and true, but only relative to the implicit or explicit proof conditions of these cognitive operations:

**[16]** **The Cognitive Proof of Psychological Facts**: **Ab(CAb) = The Human Subject:**

Interpretation/**Representation**: (AAb ➞ CIn) is true toCognitive Reality **Ab(C, A ➞ C) =>A)**

◂—————————————————————————————

Ab(CAb, A ➞C)=>A)+ Dd((A ➞ C), A) =>CDd) + In((AAb, CIn) **≈>**PRm/n(AAb ➞ CIn))

[Initial Sign] **Icon** [Feeling] **Icon**, **Index** [Emotional] **Symbol** [Logical]

Mental Reality Truth Conditions = Duality = Comparison ▲

Incoherency Coherency ❙

▼ ▼ ❙

Hesitation => Doubtfulness Assurance ➠ Assertion

[Confrontation with Reality]

❙**Representing** Physical Reality

❙ **OR** by (AAb ➞ CIn)

▼

Physical Reality: **OR**

Where the Abductive **CAb** is the psychological fact which contains the cognitive mind of the human subject, and **AAb** is the psychological theory.

It is important to note that psychological facts, as explained above, are independent of the proof of psychological theories since they have been proved and exist before we start to prove our theories on their basis. However, two questions remain: How can proofs of theories and the interpretation of psychological facts be objective, and how should the scientific community control them?

To sum up the main findings of this investigation, it can be concluded that the truth of the intentional judgment of a proposition is its “illocutionary force” in a speech act, namely, in the communicative action that enables the speaker to achieve their communicative goal. The “illocutionary force” of the speaker’s intentional judgment therefore represents their psychological reality, with the same function as Frege’s “real assertive force” (which determines the truth of the indicative proposition in representing physical reality). As we have seen, for Habermas the main shortcoming of the theory of communicative action is that it cannot explain true representations of objects and persons belonging to a seemingly already existing “lifeworld” without the initial perception of them as physical objects. One interprets the cognitive behavior of one’s interlocutor through their sensorimotor behavior and use of language. Therefore, it is only through such perceptual representation that the coordination of communicative action is made possible.

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**Chapter 7**

**The Epistemology of Proving Our “Empirical Basis” or Scientific Hypotheses by the Trio of Abduction, Deduction, and Induction**

**1. Introduction: Can We Prove the Truth of Our Scientific Hypotheses by Basic Propositions?**

In a discussion of the problems of confirmation and induction in science, we can ask: Why do we speak of the *confirmation* of hypotheses rather than *proof*, and what is the function of induction in the development of scientific theories? In analytic philosophy, the required confirmation of empirical theories hinges on the dichotomy between the deductive proof-inference of the truth-result conclusions of formal sciences, and the non-demonstrative unprovable confirmation of hypotheses in empirical sciences. Accordingly, this approach is based on the belief in absolute truth, which holds only in formal sciences but not in empirical sciences, with their non-demonstrative induction. In my pragmaticist theory of truth, the *proof* of the *truth* of any proposition or hypothesis is always relative to its *proof-conditions* which are the *method of proof* and its *truth-conditions*. From this perspective, there is no absolute truth, only local truth, though the truth of our theories can be extended by new theories. Still, if induction is the logic behind our evaluation of hypotheses, what is the logic behind their discovery and prediction? Furthermore, if induction includes all these logical inferences, how are different functions accomplished by the same logical rule? To avoid this difficulty, we can follow a Peircean epistemology approach to which only the trio sequence of abductive logic (of discovery), deductive logic (of Prediction), and inductive logic (of evaluation) can complete the proof of our hypotheses. Hence, we cannot prove any one of these types of reasoning by another (for instance we cannot prove induction deductively), and surprisingly, only the entire sequence can prove itself. I claim that by self-controlling our local proofs as true representations of reality we can establish that this *trio* is conducive to truth relative to our proof-conditions, and thus we can claim that the trio sequence of reasoning constitutes a true method of proof (Nesher 2016, 2018).

Logical positivism and analytic philosophy are both neo-Kantian components of Kant’s transcendental epistemology in which formal logic and pure mathematics are only forms separated from the matter of empirical experience. Moreover, even his epistemology of science cannot explain how the transcendental a priori components of reason and understanding reach our sensual experience in reality.

In the neo-Kantian tradition, Karl Popper tried to explain the epistemology of science from a Kantian perspective, as opposed to Hume and other philosophers who assumed that inductive logic provides the basis for the objectivity of our scientific knowledge. This latter position results in skepticism, which prompted Kant in his Copernican Revolution to suggest that the logic of science is deduction. Additionally, Popper relied on this conception to argue for what he called “the empirical basis” (Popper 1959). However, though Popper considered himself to be a neo-Kantian, he probably overlooked the specific connotation of abduction when he used this to justify his transcendental *a priori* cognition, which stems from neither formal deduction nor transcendental logic of pure reason. Moreover, Kant admitted that he could not bridge the epistemic *gap* between forms of transcendental a priori concepts and rules and cognitive sensual experience, though he attempted to do so in the First Critique by introducing the schematism and in the Second Critique by assuming that unknown *noumena* form the connection between the fact of pure reason and moral laws, and between absolute freedom and the empirical moral subject’s deed. However, the attempt to bridge these gaps was unsuccessful.

This schematism of our understanding, i.e., its schematism regarding appearances and their mere form, is the secret art residing in the depth of the human soul, an art whose true stratagems we shall hardly ever divine from nature and lay bare before ourselves. Only this much can we say: The *image* is [here] a product of the productive imagination’s empirical ability. (Kant[1781-1787] 1929, A141/B180-1; cf. A121, B185-187)

Here is the schema for Kant’s explanation of cognitive knowledge of reality:

**[1]**



Hence, formal logic deduction cannot work in Kant’s transcendental epistemology and the question is whether Popper’s neo-Kantianism and analytic philosophy can solve the problem of explaining scientific knowledge of reality.

I readily admit that only observation can give as ‘knowledge concerning fact’, and that we can (as Hahn says) ‘become aware of fact only by observation’. But this awareness, this knowledge of ours, does not justify or establish the truth of any statement. I do not believe, therefore, that the question which epistemology must ask *is* ‘… on what does our *knowledge* rest? …or more exactly, how can I, having had the *experience* S, justify my description of it, and defend it against doubt? This will not do, even if we change the term ‘experience’ into ‘protocol sentence’. In my view, what epistemology has to ask is, rather: how do we test scientific sentences by their deductive consequences? And *what kind of consequences* can we select for this purpose if they in their turn are to be inter-subjectively testable? (Popper 1959, 98)

However, these *consequences* are the results of the deductive inference from axiomatic scientific theories or experiential propositions.

**2. The Logic of Scientific Discovery: Obtaining Objective Knowledge by Conjectures and Refutations**

Logical proof leads to an infinite regress according to Popper, since he reasons in terms of the *formal logic* of Kant and not the *epistemic logic* of Peirce. Thus, in formal logic one has to go back to the initial base to prove the truth of the conclusion. Popper’s alternative seems to be the empirical basis, which can be used to support scientific hypotheses and theories in their representation of reality.

The doctrine that the empirical sciences are reducible to sense-perception, and thus to our experiences, is one which many accept as obvious beyond all question. However, their doctrine stands or fails with inductive logic, and is here rejected along with it. I do not wish to deny that there is a grain of truth in the view that mathematics and logic are based on thinking, and the factual sciences on sense-perceptions. But what is true in this view has little bearing on the epistemological problem. And indeed, there is hardly a problem in epistemology which has suffered more severely from the confusion of psychology with logic than this problem of the basis of statements of experience. (Popper 1959, 93)

Popper refuses to accept that perceptual judgments and other empirical propositions are basic facts of science, since he considers them to be psychological, not logical, and therefore they cannot be clear and distinct like the claims of formal logic. However, he accepted propositions of the empirical basis, not as being true but only as potential falsifiers of theories due to their being generally accepted as scientific conventions (neither soft like perceptual judgments, nor hard like the conclusions of formal logic). Yet, statements of formal logic cannot have meaning or content, only form, due to the Kantian gap between matter and form, and their meanings must come from cognitive empirical experience, which Popper calls psychological.

If we demand justification by reasoned argument, in the logical sense, then we are committed to the view that *statements can be justified only by statements*. The demand that *all* statements are to be logically justified (described by Fries as a ‘predilection for proofs’) is therefore bound to lead to an *infinite regress*. Now, if we wish to avoid the danger of dogmatism as well as an infinite regress, then it seems as if we could only have recourse to *psychologism*, i.e., the doctrine that statements can be justified not only by statements [logic] but also by perceptual experience (Popper 1959, 93-94).

It is interesting to compare Popper’s epistemology of knowledge and his concept of “empirical basis” with Kant’s general convention “the touchstone whereby we decide …” ([1781-1787] 1929, chap. 2, #3).

**[2] The Popperian Deductive Formal Logic System:**

*Axiomatic Theory*

Intuitive **♠**  **/** I **\** Deductive

**Scientific**  **/** I **\** Logic

**Discovery /**  I **\Formally Inferring**

from **/ ∇ \** the *Consequence*

Experiential **/***Conclusions***\🡺 Evaluation-Refutation-Corroboration** in

Facts  Observational “Empirical Basis” (Popper 1963)

Indeed, according to Popper we cannot deductively prove the truth of a theory or hypothesis since what we deduce from axioms cannot touch reality. Nevertheless, we can falsify it upon the available “*empirical basis*”. Since Popper rejects *inductivism* and *psychologism*, sciences, he argues, must be proved by deduction. And yet, he holds that by deduction we cannot prove the truth of our theories or hypotheses, and therefore we can only refute our false scientific theories; if we cannot do so, we continue to work with them in our scientific enterprises as an approximation to truth. However, the force of Popper’s “*empirical basis*” is not clear enough since if it is not true, how can it provide the basis for refuting our theories, and if it is true, why can it not offer the basis for proving the truth of our theories or scientific hypotheses? Popper’s answer to this difficulty may be that, while he rejects *psychologism*, he nevertheless, as a neo-Kantian, accepts aa radical separation between the *form* of transcendental concepts and rules (as in his conception of logic and mathematics) and the *matter* of our sensual experience.

Experience contains two quite heterogeneous elements: viz., a *matter* for cognition, taken from the senses; and a certain *form* for ordering this matter taken from the inner source of pure intuition and thought. (Kant[1781-1787] 1929, B118/A86)

Popper, however, did not detect the difficulty with Kant’s transcendental philosophy: it is unable to bridge the gap between a priori pure reason and understanding and the sensual intuitions of empirical experience, and thus lacks a theory of truth and cannot explain our knowledge of reality, as Kant himself alluded to:

“The project on which I am now working… must be completed, or else a gap will remain in the critical philosophy.” (Kant to C. Grave, September 21, 1798b).

Peirce understood this and reacted against Kant’s idealist Copernican Revolution by proposing his semiotic empiricist epistemology which can be developed into epistemic logic. Peirce showed how from experience we develop and prove all knowledge of reality based on our available relative proof-conditions, such that scientific knowledge evolves as our proof-conditions are extended. Thus, Einstein did not refute Newton’s theory, as Popper claimed, but extended it to fit wider (yet still relative) proof-conditions. Indeed, one could say that the neo-Kantians who accepted some of the transcendental or phenomenal aspects of Kant’s reasoning did not refute his epistemology but extended it, e.g., Wittgenstein (in *Tractatus*, *Philosophical Investigations*, and *On Certainty)*, the logical positivists, and analytic philosophers.

To reiterate, Kant does not offer any theory of truth, and Popper, as a Neo-Kantian with his formal logic and deductivism, cannot prove the truth or falsity of axiomatic scientific hypotheses since they cannot deductively reach the scientific “empirical basis” in order to do so. Popper’s problem is to explain how science works and what is meant by his empirical basis, which can be used to refute hypotheses and theories but cannot prove them. Popper thinks that truth must be absolute and that *empirical bases* are relative and cannot be the final criterion of scientific objectivity. From this perspective, empirical bases are comparable to truth and subject to change in the progress of science, as demonstrated in the evolution from Newtonian classical objects to Einsteinian sub-classical particles, waves, and energetic fields (Popper 1976, #20; Nesher 2010, #3).

How, then, does Popper view knowledge acquisition in science? Does this happen through intuitive conventions or through rigid formal logic?

Methodological rules are here regarded as *conventions*. They might be described as the rules of the game of empirical science… two simple examples of methodological rules may be given. They will suffice to show that it would be hardly suitable to place an inquiry into method on the same level as a purely logical inquiry.

(1) The game of science is, in principle, without end. He who decides one day that scientific statements do not call for any further test, and that they can be regarded as finally verified, retires from the game.

(2) Once a hypothesis has been proposed and tasted, and has proved its mettle, it may not be allowed to drop out without ‘good reason’. A ‘good reason’ may be, for instance: replacement of the hypothesis by another which is better testable; or the falsification of one of the consequences of the hypothesis. (Popper 1959, 53-54)

Thus, Popper regards the methodological rules of empirical science as *conventions* in the game accepted by scientists, a game that might change either because of an intuitive decision or because the methodology is improved and replaced as our logical scientific reasoning develops. It does not, therefore, seem to be a sterile closed game of formal logic. Rather, it reflects a sort of Kantian transcendentalism that remains grounded in the phenomenal cognition of *sensual intuitions* without reaching noumenal reality. This is different from the Peircean realist semiotics I developed using *epistemic logic* to prove the truth of our cognitive knowledge of reality.

**3. The Shortcomings of Formal Logic and the Sterility of Deductive Inferences**

Since Popper has no theory of truth that can be applied to scientific hypotheses, he proposes an alternative with the concepts of *corroboration* and *convention* which are instrumental in accepting or rejecting hypotheses. Thus, his *scientific game* continues forever, perpetuated by the development of empirical sciences. Popper’s intuitions about empirical science are based on his conception of formal logic, which is a closed formal game dependent upon intuitive axiomatic hypotheses, and he endeavors to accept or reject these by formal deduction. However, by deduction we can only infer their possible conclusion, without being able to prove their representation of reality. Therefore, Popper intends to refute those hypotheses or “theories” that cannot work in empirical science, but as a neo-Kantian he cannot bridge the gap between their formality and the materiality of the empirical experience. To overcome this Kantian difficulty, he elaborates his conception of *the objectivity of the empirical basis* which suffers the same difficulties as the formal axiomatic hypotheses: without proving the truth of the *empirical basis*, as a representation of reality, it remains a convention only, in the *game* of empirical science.

And finally, as to *psychologism*: I admit, again, that the decision to accept a basic statement, and to be satisfied with it, is causally connected with our experiences—especially with our *perceptual experiences*. But we do not attempt to *justify* basic statements by these experiences. Experiences can *motivate a decision*, and hence an acceptance or a rejection of a statement, but a basic statement cannot be *justified* by them—no more than by thumping the table. (Popper 1959, 105)

Indeed, this consideration must also apply to the *empirical basis*, since our basic statements of *perceptual experience* cannot be *justified* by basic experiences. And yet, if we had recourse to a realist theory of truth, which was able to prove the truth, falsity or alternatively doubtfulness of a hypothesis, then we would not need conventions for corroboration. For *epistemic logic* works to prove the truth of our cognitive representations as knowledge of reality, including the logic itself (Nesher 2002, X, 2016, 2018).

The empirical basis of objective science has thus nothing ‘absolute’ about it. Science does not rest upon solid bedrock. The bold structure of its theories rises, as it were, above a swamp. It is like a building erected on piles. The piles are driven down from above into the swamp, but not down to any natural or given ‘base’; and if we stop driving the piles deeper, it is not because we have reached firm ground. We simply stop when we are satisfied that the piles are firm enough to carry the structure, at least for the time being. (Popper 1959, 111)

Indeed, Popper thinks it is impossible to prove absolute true theories on the basis of relative proof-conditions; for him, the *empirical basis* is comparable to a relative true fact upon which we cannot base any proof of the absolute truth of a scientific hypothesis; it is not possible to know nature completely and absolutely, but our knowledge can always be extended by new theories. This differs from Peircean pragmaticist epistemology, since Popper’s methodology lacks a theory of truth and thus promotes a sort of absolute relativism based on feelings and intuition, similar to Kant’s *sensual intuitions*, without being able to represent noumenal reality. On the other hand, pragmaticist epistemology is based on a theory of proof whereby hypotheses can be proved true on the basis of relative proof-conditions, which themselves are proved true by the perceptual judgments of proven true facts (Popper 1959, 111; Nesher 2002, X; 2016, 2018).

The illusion of deductive formal proof lies in the hidden tacit acceptance of axioms from which theorems are deductively inferred, but with the result that their relation to reality is hidden and mysterious. Thus, Popper’s epistemology and his theory of falsification ignore the inability of deductive inference to explain our confrontation with reality; in his view, the actual intuitive acceptance of the “empirical basis,” i.e., the class of propositions that may function as our basic facts, is the basis of accepting the truth or falsity of our hypotheses. However, a deductive-axiomatic system can never explain the discovery of axioms, their truth, and the evaluation of the conclusions they predict. That is why Popper proffered the “crucial experiment” to avoid their inductive evaluation, but the problem here is about the rationality of human cognition (Russell 1907). The internal power of deductive-axiomatic systems is much stronger than any material rules of inference, but this powerful system is sterile since it is isolated from any experience upon reality.

The attempt has often been made to describe theories as being neither *true* nor *false*, but instead more or less *probable*. Inductive logic, more especially, has been developed as a logic which may ascribe not only the two values ‘true’ and ‘false’ to statements, but also degrees of probability… (Popper1959, 251)

Popper explains that *probability logic* uses induction to evaluate the degree of probability of a statement without any proof in respect to the realist’s proven true basic facts representing reality. In Popper’s view, instead of talking about probability, we should assess how far a hypothesis stands up to tests. Popper’s epistemology of science is based on the deductive inference of knowledge, but the notion of absolute truth brings him to skepticism and solipsism, like Wittgenstein, Russel, and others.

I speak of the *‘corroboration*’ of a theory; and corroboration can only be expressed as an appraisal. (In this respect there is no difference between corroboration and probability.) Moreover, I too hold that hypotheses cannot be asserted to be ‘true’ statements, but that they are ‘provisional conjectures’ (or something of the sort); and this view, too, can only be expressed by way of an appraisal of these hypotheses. (Popper 1959, 265)

Popper, then, suggests that the logic of induction and corroboration can be used to support scientific theories or hypotheses. However, his objections to the probability theory could be used against his *empirical basis* because it faces the same problem of proof. Without the proof the truth of basic *perceptual facts*, as distinct from the *empirical basis*, no criterion is able to carry our hypotheses and prove their true representation of reality (Popper 1959, 265; Nesher 2002, X; 2016, 2018)

(1) It is easy to obtain confirmations, or verifications, for nearly every theory ‒ if we look for confirmations.

(2) Confirmations should count only if they are the result of risky predictions; that is to say, if, unenlightened by the theory in question, we should have expected an event which was incompatible with the theory‒ an event which would have refuted the theory. (Popper 1963, 7)

In a departure from Popper, the pragmaticist argues that *epistemic logic* can prove the truth of hypotheses, and this means they are true forever, but only in respect to their accepted proof-conditions, which are always relative to contemporary knowledge, as evidenced in the examples of Newton and Einstein’s scientific hypotheses with their proof-conditions in line with the accepted basic knowledge of their time. And so one can argue against Popper’s epistemology of refutation by suggesting that Einstein does not refute the Newtonian physical paradigm but extends it under new proof-conditions (Nesher 2010, 2018, 2020).

Einstein did not develop his theory of general relativity in the expectation that future observations might reveal his predictions to be false, as Popper’s conception of scientific theories would suggest. His general hypothesis was put forward not only to explain known facts but also to reveal unknown phenomena concerning gravitation and its effect on light rays. In the process of inquiry, the scientific community discovered some unexpected new facts which meant they had to look for new hypotheses. Popper’s vision is that a scientific theory should be expected to uncover an unknown fact; they aim at the truth and attempt to describe reality, but in his view we should accept that it is impossible to ever obtain certain truth. According to pragmaticists a theory can be proven to be true, and its truth is always relative to its proof-conditions. But Popper does not have any theory of truth and thus, in his conception, scientific theories are constructed for refutation and not for explanation.

We feel our axioms as true and we must somehow implicitly operate non-formal inferences of material logics to discover our axioms and to evaluate their conclusions (cf. Gödel 1944).

But formal logic must not be too purely formal; it must represent a fact of psychology, or else it is in danger of generating into a mathematical recreation (Peirce, 2.710).

**4. The Formal Conception of Induction and the Paradoxes of Confirmation**

What is the nature of the logic of induction, and what is its function in cognitive behavior and science? Carnap assumes that he can explain the inductive logic of confirmation syntactically and that the meanings of sentences can be analyzed without relation to reality (Carnap 1966). The shortcomings of formal semantics are related not only to the problems of truth and falsity but also to the meanings of sentences, since without experiential confrontation with reality they cannot have any cognitive meaning. The Carnapian idea that pragmatics can be added to syntax and semantics only after the analysis of language assumes that the meanings of language are already known. This is based on the phenomenalist understanding of language that fails to address the fact that it is only in confronting reality experientially that we acquire the meanings of our language (Carnap 1928). The paradoxes of induction are not merely problems to be solved by syntactic corrections but also symptoms of its fundamental inadequacy (Holland 1986).

The logic of confirmation and the raven paradox are concerned with the conditions of adequacy for any relation of confirmation in which the *equivalence condition* and logically equivalent hypotheses are confirmed by the same evidential propositions. “All ravens are black” is logically equivalent to “All non-black things are non-ravens.” Thus, a non-black non-raven can be used to confirm the hypothesis that “All ravens are black,” and consequently the paradox arises: the two formulations are formally equivalent yet they are about different subjects.

Indeed, these formalisms are *equivalent* formally but not experientially, and if we want to prove the truth of our propositions and hypotheses this cannot be done by formal logic (a sterile closed game) or by intuitions based on formal logic. Rather, it has to be achieved by epistemic logic. Yet, in this context, the discussion is focused on confirmation and not on the proof of the truth of hypotheses in accordance with their relative proof-conditions. Like Kant and the neo-Kantians, those who emphasize a contradiction between intuition and inductive logic lack a theory of proof about our representation of external reality and rely instead on phenomenal intuitions (Peirce 1902; Nesher 2003, X; 2018, 2020).

We can overcome these formal paradoxes only by introducing *epistemic logic* so that with its *form* and *matter* our cognition is anchored in reality, through abduction and induction, unlike the Kantian dichotomy which cannot reach reality. Thus the epistemic role of induction is not only to refute our hypotheses but also (and mainly) to prove the truth of their representations of reality in respect to our known proof-conditions. The meaning of the relation of discovered ideas to their observational evaluation is determined by their being anchored in reality (Levi 1980). Through experiential confrontation with reality iconic and indexical meanings present real objects. Hence, we cannot have any experience of the *propositional logic* of *not-q* and *not-p* as per the paradox of induction, and without meaningful components we cannot represent reality. Carnap’s semantics assumes meanings from nowhere, and without a Peircean understanding of our cognitive confrontation with reality the logic of science remains an empty logical recreation (Popper 1934, ##7, 25-30; Nesher 2002, X, #8; 2007c).

According to Popper, we cannot prove the truth of our propositions and hypotheses by *deduction* and equally we cannot prove the *absolute truth* of our hypotheses by *induction*, but we can refute them using the formal inference of *modus tollens*: “If *P*, then *Q*; Not *Q*;Therefore, not *P*.” Here *Q* is the accepted *empirical basis* which we accept only intuitively without proving its factual truth. Therefore, hypotheses or theories are acceptable, in Popper’s view, in terms of their degree of confirmation, which he calls *corroboration*, whereas it is impossible to prove that they are true. The problem with the neo-Kantian position is that it remains stuck at the level of the *formal logic* of syntax or semantics without the *matter* of empirical experience (Nesher 2007).

Indeed, if we were to prove the *empirical basis* as our *perceptual fact* it would be an inductive refutation of P, but according to Popper we cannot prove the truth of a proposition and therefore we cannot refute our scientific hypotheses inductively. Moreover, we cannot prove them deductively and, thus, we are left without any criteria for establishing truth in the sciences and for guaranteeing their eventual progress. Thus, neo-Kantian epistemology, like Kant himself, is unable to explain how our scientific knowledge represents *noumenal* reality (Popper 1959, chap. 7; Nesher 2002, X, #8, 404).

**5. The Pragmaticist Conception of Complete Proof: Functions of the Material Logic of Abduction, Deduction, and Induction in Epistemic Logic**

An analysis of the literature shows that various cognitive functions with different kinds of inferences come under the umbrella of induction. In Peircean epistemology there are three rules of inference. With different logical rules for performing their specific cognitive functions, it can be expected that different mathematical calculations will achieve abductive *plausibility*, deductive *verisimilitude*, and inductive *probability*.

**[3] Abductively Proving a Hypothesis by Deductive Prediction and Inductive Evaluation:**

Abduction((CAb(A*→*C)=>AAb)+Deduction((A*→*C) A)*→*CDd)+Induction((A, CIn) Pr. m/n ≈> (A*→*C))=X [Subjective antecedent plausibility] [Verifiable prediction] [Testing of a hypothesis]

As opposed to the Kantian and Popperian conceptions of deduction and induction, the Popperian realist conception of scientific knowledge is based on the proof of semantic epistemic logic.

**[4] Epistemic Logic: Proof of the Truth of the Proposed Hypothesis in analogy to deduction:**

*Hypothesis*

**♠** **/** I **\**

**Abductive**  **/** I **\ Deductive**

**Discovery /**  I **\Prediction** the

from **/ ∇ \**Happen Inductive

Experiential **/***Conclusions***\🡺 Inductive Evaluation**

Proved True Facts  in Observational Facts

Carnap, Quine, and others confuse the abductive logic used to discover hypotheses with the inductive logic of evaluating them (Carnap 1966; Levi 1980). In some contexts, Carnap’s induction acts as a type of logic of discovery in which “creative ingenuity is required” and for which “there cannot be an inductive machine,” and in other contexts his inductive logic of evaluation comes into play and works mechanically (Carnap 1966; Neal 2000). The probability calculation of the inductive inferential evaluation is still a controversial question in the philosophy of science (Putnam 1990). There is also a misunderstanding about the operation of cognitive generalization, for it is sometimes viewed as if it were an inductive one. Additionally, confusion persists about the pragmaticist conception of abduction, for some hold the view that it supports both “the discovery and justification of scientific theories”; but “*multiple* abduction” cannot evaluate scientific theories since its sole function is to discover them (Holland 1986). To overcome the misunderstanding of Peircean epistemology we should explain the trio as the complete proof of our hypotheses relative to their truth-conditions.

The epistemology of the proof of our cognitive representations of reality has to show their truth or falsity through their very confrontation with reality, and it must prove that our perceptual judgments are basic facts. This entire trio process operates at different levels of self-consciousness and self-control; it is instinctive, practical, and rational in the generation of proofs. This is “inference to the best explanation” of external reality. The complete proof of the truth of the hypothesis AAb is shown in the following schema:

**[5] Epistemic Logic of Abductive Discovery, Deductive Prediction, and Inductive Evaluation:** **The Structure of the Complete Proof of the True Representation of Reality**:

Ab(CAb, A *→*C)=>AAb+ Dd((A*→*C), A) =>CDd + In((AAb, CIn)≈**>**PRm/n(AAb *→*CIn))

Suggested Hypothesis Predicting Results Hypothesis/Facts Evaluating Hypothesis

**Truth Conditions** = Duality = Comparison ▲ Incoherency/Coherency ❙

▼ ▼ ❙

Hesitation => Doubtfulness Assurance ➠ Assertion

[Confrontation in Logical Reality]

❙**Representing** Physical Reality

❙ **Object**P by Description (AAb ➞CIn): ▼ “This [CIn] is a stone [AAb]”

[Confrontation with Reality]

**Physical Reality**: **Object**P

when AAb is the abductive hypothesis generated by the observational facts CAb; and CDd is the deductive prediction of the likelihood of abstract facts being inferred from the hypothesis AAb. The inductive inference ((AAb*→* CIn) ≈**>**PRm/n(AAb *→*CIn)) is the evaluation of the suggested hypothesis AAb upon the extended observational facts. Finally, CIn≈**>** PRm/n(AAb–>CIn) is the evaluated probability of AAb such that the measured relation of (AAb≈**>** CIn) in the evaluation PRm/n(AAb*→*CIn) PRm/n is the norm for judging the truth of the abductive hypothesis.

**6. Conclusion: Every Proof of the Truth of a Cognitive Representation Is Relative to its Proof-Conditions**

The truth of our propositions and theories is always relative to their *proof-conditions* in the historical situation in which they are proved. Thus it is possible that propositions or theories that were once proved true in specific historical situations can become false in different ones, hence truth is simply “lost” (Quine 1995). Can truth correspond to external reality and also be relative to our ability to prove it? The falsification of true propositions or theories only shows that their truth is limited to the specific proof-conditions. We are mistaken in our intention to extend the truth of our theories beyond their limitations because we cannot know what their limitations are before we have detected them through more comprehensive theories in the progress of our inquiry. To complete this theory of “truth with a human face” we have to show that there are historical continuities in the formation of *methods of proofs* and that the evolution of *truth-conditions* constitute the *proof conditions* of human knowledge.

As I have theorized elsewhere, facts are our proven true propositions and genuine facts are our quasi-proven true perceptual judgments in the basic contexts in which we prove the truth of our interpretations of other propositions and theories (Nesher 2002, X). Therefore, contexts are not given arbitrarily; they are not self-proven or self-defined but proven true in our cognitive confrontation with reality. The *proof* of the *truth* of any proposition or hypothesis is always relative to its *proof-conditions* (Hirsch 1967; Wachterhauser 2002). The relative advantage of one true interpretation over another is in respect to how their different *proof-conditions* comprehend the subject matter of the interpretation and representation (Thom 2000). There is no absolute proved truth. We can only obtain local truths, although, as in our scientific, aesthetic, and other cognitive activities representing reality, they evolve and extend as we develop the proof-conditions to better represent reality (Croce 1901; Nesher 2002, X). Therefore, it is similar to our interpretive activities, when we develop proof-conditions of a text to better understand its meaning by proving the truth of the interpretation; thus, true interpretations with different *proof-conditions* may continue indefinitely (Stout 1982; Margolis 1995; Nesher 2002; Krausz 2002, Habermas 2003).

Peircean epistemology shows that the *trio* of abduction, deduction, and induction provides a basic and complete epistemic method to prove the truth of our interpretations of texts as a representation of reality. Hence, the truth of this method itself cannot be proven by one of these logical inferences, nor can any one of them prove another, and thus, surprisingly, only when the trio comprises the entire sequence of these inferences can we prove its truth. I claim that, by self-controlling our local proofs as true interpretations and representations of reality, in the long run we are able to prove that this *trio* conduces truth relative to our truth-conditions, hence it can be described as a relative true method of proof.

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**Chapter 8**

**On the Concepts of Space and Time: Looking for a New Picture of Physical Reality**

In memory of **Adolf Grünbaum,** leading philosopher of physics, colleague and friend.

The results of the work of Faraday, Maxwell, and Hertz led to the development of modern physics, to the creation of new concepts, forming a new picture of reality. (Einstein and Infeld 1938, 125)

*Is light a wave or a shower of photons? Is it a beam of electrons, a shower of elementary particles or a wave? These fundamental questions are forced upon physics by experiment. In seeking to answer them we have to abandon the description of atomic events as happenings in space and time, we have to retreat still further from the old mechanical view.* (Einstein and Infeld 1938, 297)

The unified field theory has been put into retirement... This state of affairs will last for many more years, mainly because physicists have no understanding of logical and philosophical arguments. (Einstein to Maurice Solovine [1951] 2007, 514).

**1. Introduction**

In the course of my research in epistemology I started to read the general works of contemporary scientists, including Penrose, Greene, Davies, Moffat, Smolin, Barbour, Woit, Randall, Magueijo, Wilczek, and others, as well as earlier scientists such as Mach, Poincaré, Planck, Einstein, Heisenberg, Weyl, Wheeler, Bohm, and others. In Lee Smolin’s *The Trouble with Physics* (2006) I found that the problem of understanding the concepts of space and time lies in the grand picture of physical reality, and in the blowing up of the dimensions of string theories. I would argue that an essential problem of contemporary physics is philosophical and epistemological, that is to say it concerns the basic principles of our picture of physical reality. I therefore believe there is a need to reevaluate the concepts of space and time as understood in classical and modern physics.This chapter suggests an alternative picture of physical reality, namely in terms of a *dynamic plenum*, along with the Spinozist conceptions of space, time, and number as “common notions,” i.e., methods for measuring its components.

**2. The Problem with Conceptions of Space and Time: Leibniz’s Relative “Ideal Things” versus Newton’s Absolute Metaphysical Entities**

Accordingly, I started to examine various conceptions of space and time, and considered the ways in which physicists speak about them, e.g., in works such as Hawking’s *The Beginning of Time* and Barbour’s *The End of Time*. In my philosophical inquiries into the epistemology of the scientific enterprise I found that the connection between a scientist’s general background knowledge and their epistemological attitude is of fundamental importance in their approach to formulating new scientific hypotheses and creating a comprehensive picture of reality. Thirty years ago, I published a paper (in Hebrew) titled “On the ‘Common Notions’ in Spinoza’s Theory of Knowledge and Philosophy of Science” (Nesher 1979). It deals with Spinoza’s notions of size, space, time, and number as indications of our commonly accepted methods of counting and measuring the physical components of Nature (Spinoza [1663] 1985, II P44), which can be compared with Kant’s vision of space and time as transcendental forms of the subject’s pure intuitions (Kant [1781-1787] 1996, Part I). I came to see that Spinozist epistemology and his comprehensive picture of Nature offer a good starting point for discussions on theories of knowledge and the philosophy of science. Now I come back to his basic picture of Nature, which can be construed as an *infinite* and *eternal* dynamic plenum, containing no real void, similar to Bohm’s conception of physical nature (e.g., Bohm 1980, 191; Smolin 2006, chap.10; Wilczek 2008, chap. 8; comp. Mach 1883, 230-231). Leibniz probably took this starting point from Spinoza, but due to their essentially different philosophies, theological and natural respectively, Leibniz did not fully understand Spinoza’s conceptions of space and time, though he uses similar terms to explain them. Leibniz conceives of space and time as “ideal things”; he sees their application to real physical entities as metaphysical, and not as notions of measurements (Leibniz [1715-16] 1989, Fifth Paper ##27, 47). Leibniz corresponded with Spinoza and visited him in 1676, and commented meticulously on Spinoza’s *Opera Posthuma* (Leibniz 1707, 272-281).

The Spinozist concepts of space and time offer a third path between Newton’s substantivalism (and absolutism) and Leibniz’ relationalism and idealism. In this alternative conception, human beings, as finite modes in Nature, measure the finite dynamic modes, or components of the *infinite* and *eternal* dynamic plenum of Nature, by using our relatively stable frames of reference and relatively constant standards of measurements. However, we change them according to the development of our proof-conditions, i.e., the methods of measurement and proof and the discovery of new proof-conditions by proving the truth of new facts, e.g., Eddington’s eclipse of 1919 (Nesher 2002, X.10).

To what extent, then, does Leibnizian theological epistemology affect his scientific conceptions of space and time (Barbour 1982, ##1, 2; Smolin 2008, 7-10)? In contrast to theological discussions with Newtonians, regarding physical reality, it seems that the gap between Leibniz and Newton is narrower, if we compare Leibniz’s *ideal* space and time with Newton’s *absolute* space and time, for both are *abstract* criteria employed to explain the motion of bodies.

The problem lies in deciding how to interpret Leibniz’s conceptions of space and time and their consistency. According to Leibniz’s metaphysics, entities and substances are only accidents, there is no real relation between them, and it is through abstract systems of relations that ideal space and time can accommodate the actual dynamics of physical things (Leibniz [1715-1716] 2000, Fifth Paper #47; [1704] 1996, II: #17).

I will here show, how men come to form for themselves the notion of space. They consider that many things exist at once, and they observe in them a certain order of coexistence, according to which the relation of one thing to another is more or less simple. This order is their situation or distance. When it happens that one of those co-existent things changes its relation to a multitude of others, which do not change their relation among themselves, and that another thing, newly come, acquires the same relation to the others, as the former had, we then say, it is come into the place of the former [Compare, Einstein’s “interval,” Einstein 1934, 278]…

It may also be said, without entering into any further particularity, that place is that which is the same in different moments to different existing things when their relations of coexistence with certain other existents which are supposed to continue fixed from one of those moments to the other agree entirely together. And *fixed existents* are those in which there has been no cause of any change of the order of their co-existence with others, or (which is the same thing) in which there has been no motion. Lastly, *space* is that which results from places taken together. And here it may not be amiss to consider the difference between place and the relation of situation which is in the body that fills up the place. For the place of A and B is the same, whereas the relation of A to fixed bodies is not precisely and individually the same as the relation which B (that comes into its place) will have to the same fixed bodies; but these relations only agree. For two different subjects, as A and B, cannot have precisely the same individual affection, it being impossible that the same individual accident should be in two subjects or pass from one subject to another. But the mind, not contented with an agreement, looks for an identity, for something that should be truly the same, and conceives it as being extrinsic to the subjects: and this is what we call *place* and *space*. But this can only be an ideal thing, containing a certain order, wherein the mind conceives the application of relation. (Leibniz [1715-16] 2000, Fifth Paper #47)

Thus, *space* is an ideal container that the mind conceives as “ extrinsic” ; it remains the same at different moments when existing things maintain the same relations among themselves, while the *relation of situation* refers to the actual physical relations of bodily affection.

The parts of time or place, considered in themselves, are ideal things, and therefore they perfectly resemble one another like two abstract units. But it is not so with two concrete ones, or with two real times, or two spaces filled up, that is, truly actual (Leibniz [1715-16] 2000, Fifth Paper #27)

If space and time are “ideal things” in our minds, how do we apply them to natural objects to sustain our physical theories? I interpret the above sections of Leibniz such that *ideal space* is a configuration of abstract *geometry* by which we describe (or represent) the constant relations between physical bodies “in which there has been no motion.” Thus, in order to measure the actual *relation of situation* between physical bodies, we apply the postulated abstract “parts of time or space” which are considered in themselves as “ideal things,” and with our units of measurement (taken to be unchanging) they function like Newton’s mathematical space and time and Einstein’s absolute-universal speed of light, as standards to explain the actual relative motion of things (Leibniz [1715-16] 2000, Fifth Paper #27). Since, according to Leibniz, there is no empty space in the world, “truly actual” space and time cannot be without physical objects and vice versa.

I don’t say that matter and space are the same thing. I only say, there is no space, where there is no matter and that space in itself is not an absolute reality. Space and matter differ as time and motion. However, these things, though different, are inseparable. (Leibniz [1715-16] 2000, Fifth Paper #62)

Thus, actual space and matter change with the motion of material bodies in time, which we measure by our ideal space and time. We can see the similarity between Leibniz and Einstein regarding the nature of space and time as inseparable from matter or the distribution of mass in the universe. Yet, the question remains: What is the relativity of space and time in Leibniz’s philosophy of science? It seems to be concerned with the *dynamic relations* of physical objects, their motion in any situation whatsoever, as measured by the accepted standards of ideal space and time in *kinematic geometry* (Wheeler 1990; Davis 1995). Thus, for Leibniz actual space and time are systems of relations between physical bodies, while for Spinoza our accepted “common notions” for measuring the components of material Nature and their relations are numbers, rods and clocks (Rescher 1981). In this respect, a central issue in contemporary discussions of relationalist conceptions of space and time is the interpretation of Einstein’s theories of relativity (e.g., Wheeler 1990, 9-15; Earman and Norton 1986).

When spacetime is taken as the background, inertial frames substitute absolute rest as the fundamentally absolute notion. In this sense, Einstein’s theory was not that radical an overthrow of Newtonian physics: it was a reformation rather than a revolution. However, this is true for the particular theory of Einstein, which has come to be known as special relativity. In general relativity, in which gravitational phenomena are also included, spacetime loses its absolute character: it becomes a dynamic quantity, subject to laws of motion and change by its interaction with matter. This *is* a radical overthrow of previous ideas about space and time: the distinction between kinematics and dynamics originating from Newton’s theory is lost, to mention the least of the changes. (Anastopoulos 2008, 121-122; cf. Wheeler 1990, 9-15)

This raises the question: To what extent is the substantive conception of space and time still at work in Einstein’s general relativity?

**3. Special and General Relativity and the Concept of the Absolute Speed of Light**

In my inquiry, I came to think about how Einstein replaces Newtonian absolute space and time with his absolute speed of light and, according to Greene, absolute spacetime, i.e., C = space x time (Greene 2004, 58-61; Cox and Forshaw 2009, chap. 4). We should have criteria for our definitions and measurements of objects and their motion, but why must they be absolute? A sort of paradox can be observed in Einstein’s conception of the constant speed of light. Taking Einstein’s equation E = mc2, the energy of a light wave can be expressed as EL and we can assume its mass to be mL such that EL = mLc2. Now it cannot be that mL = 0 since its energy (EL) would also equal zero, and therefore, if there is a light wave this necessarily entails that mL > 0. But then, according to Einstein, the speed of this mass of light could never reach the speed of light, therefore cM < cL, i.e., *the speed of light* is less than *the speed of light*. Alternatively, light waves have no mass, or rather, the energy of acceleration contributes to the velocity of light and not to its mass, since there is no vacuum in nature and it should thus be continuous,as opposed to Einstein’s basic formula. This shows that the absolute speed of light in special ­­relativity cannot hold in the dynamical explanation of general relativity. Hence, when mL = m0/ 1 – v2 / c2, then m0 is only relative to a specific coordinate system and to our methods of detecting such changes in velocity or in the amount of energy, i.e., a scientific abstraction; therefore, a light wave cannot be massless, thoughwe might not be able to detect it in the experimental proof-conditions (Einstein 1946, 339-341; Grünbaum 1963, chap. 12 (C); Magueijo 2003, #12; Anastopoulos 2008, #3.5). Two different questions need to be addressed. First, how does accelerating energy contribute to the velocity or mass of light in EL = mLv2? Second, how much energy exists in or is released from any kind of mass in Em = mc2? However, with regard to the latter, it is not *mass* that should be considered but another aspect of *matter*; not the size and weight of material bodies but their inner chemical *atomic structure*, e.g., uranium, radium, etc. , namely: EAs = mAs c2. Indeed, this is not connected to Einstein’s special relativity (1905), which deals with the motion of bodies or mass in Newtonian terms, but with the atomic *structure* of matter for releasing energy. But has c2 ever been proved true experimentally or does it remain a myth? (cf. Einstein’s letter to President Roosevelt, August 2, 1939).

If every measurement of natural components and their relations is relative to our accepted coordinate system, which is itself a component of the actual existing proof-conditions, i.e., our proved true observational facts and the available scientific methods used to prove our hypotheses, then we may not accept the speed of light as an absolute, universal law of nature; therefore, we cannot say that a light wave can never be at rest and that it is absolutely massless (Nesher 2002, X; 2007). Moreover, this also holds for our tools of measurement, such as clocks and rods that can be “perfect” only when they are our best available standards for measuring the components of the dynamic plenum of nature and their relational motion (Wilczek 2008, chap. 11). Yet we can imagine that the speed of light reaches zero at the event horizon of a black hole, namely when the light wave evaporates, and thus in a different physical environment it could reach different speeds according to the interaction between the light wave and other material components of nature, e.g., a light wave in the vicinity of the sun (e.g., Magueijo 2003, 242; cf. Einstein and Infeld 1938, 240). The absolute speed of light in a vacuum, à la Maxwell and Einstein, is either a misunderstanding of Michelson and Morley’s experiment of 1887 (i.e., that it took place in limited proof-conditions) or it is an axiom that cannot be evaluated experimentally since no absolute vacuum or void exists that would make such experiments possible (Cox and Forshaw 2009, chaps, 2, 3, p. 91). In any case, if nature is a dynamic plenum of matter in all possible formations, there can be no real vacuum in nature and, accordingly, no universal constant speed of light.

Indeed, it is common to say that m in this equation is at rest, mv0, i.e., with v0, but if a light wave has any mass at all, as all other components or elements of nature have, then according to Einstein’s equation E = mc2, even in such an abstraction the equation should hold. Moreover, it is only by assuming that a light wave has mass that we can explain its being affected by gravitational fields. Yet it could be suggested that light is not directly affected by gravitational fields as other massy objects are, since there is no mutual attraction, but rather it is indirectly affected by the curvature of space near massive objects. But then the spacetime curvature must be a dynamic component of physical reality, and thus a material entity (Wheeler 1990 vs. Anastopoulos 2008). How, then, are we to understand the concepts of “mass” and “energy” in Einstein’s equation? It seems to me that we still lean on classical conceptions, namely that they are two different definitions of discrete structures of *matter*. This is not necessarily problematic if we have different methods to measure *mass* (particles) and *energy* (waves); but if all physical matter is made up of particle-wave structures and if we were to find a common method to measure all such structures, then Einstein’s equation E = mc2 would become a relic of special relativity, inapplicable to general relativity or any unified theory of macro and micro structures of matter. However, since the velocity of light also depends upon the medium it moves in and thus it is not the constant c, we have to convert the mass energy relation to E = mv2, but of course in respect to our accepted proof-conditions (Cox and Forshaw 2009, chap. 5). We can understand the concepts of “mass” and “energy” as being based on two different methods for measuring the structures of *matter*, the *dynamic plenum*,or Einstein’s *cosmological fluid* of nature. Moreover, it seems that the difficulty with the claim that the speed of light is constant lies in the unjustified conceptual transformation from kinematic special relativity to dynamic general relativity, in which every physical entity is subject to interaction with other entities (Magueijo 2003).

**4. Einstein’s Spacetime of General Relativity as a Physical Entity**

It seems that to avoid the paradox of the absolute speed of light, arising from the different conceptual foundations of the special and general theories of relativity, Einstein creates a fictional entity: the geometrical-geodetic (*differential geometry*) structure, which seems to be a form of theoretical formalism representing the assumed substantial space-time, as well as physical reality. As such it affects physical objects, including the light wave line, despite its masslessness. In effect, Einstein identifies gravity with the curvature of spacetime (cf. Wilczek 2008, 8-10; Baker 2005; Earman and Norton 1987, #3). However, geometry is not a material entity but at most a mathematical structure by which scientific hypotheses endeavor to represent physical matter (Anastopoulos 2008).

Once the concept of the solid object is formed in connection with the experiences just mentioned [“visual and tactile impressions”]—which concept by no means presupposes that of space or spatial relation—the desire to get an intellectual grasp of the relations of such solid bodies is bound to give rise to concepts which correspond to their spatial relations... These spatial relations are obviously real in the same sense as the bodies themselves... The interval [between two bodies] is thus shown to be independent of the selection of any special body to fill it; the same is universally true of spatial relations. It is plain that this independence, which is a principle condition of the usefulness of framing purely geometrical concepts, is not necessary *a priori*. In my opinion, this concept of the interval, detached as it is from the selection of any special body to occupy it, is the starting point of the whole concept of space. (Einstein 1934, 278; cf. Smolin 2008, #4)

Thus, from the experiential conception of space Einstein elaborates a substantive conception of physical reality, similar to that of Leibniz (Leibniz [1715-16] 2000, Fifth Paper #62; Smolin 1997, chaps. 16, 18).

According to general relativity, the concept of space detached from any physical content does not exist. The physical reality of space is represented by a field whose components are continuous functions of four independent variables—the coordinates of space and time. It is just this particular kind of dependence that expresses the spatial character of physical reality. (Einstein 1950, 348)

Hence, according to Einstein the spacetime of general relativity is an essential material component of physical reality affecting other such components and being affected by them.

The message of Einstein has two parts: spacetime tells mass how to move, and mass tells spacetime how to curve. If these ideas are correct, all physical phenomena must at bottom be local, and physics only looks simple when it is described locally. But, we protest, the Sun undeniably does hold the Earth in orbit; surely that is not local, that is action at a distance. No, every bit of the physics is local, Einstein will reply. The mass in the Sun curves spacetime where the Sun is. This curvature curves spacetime just outside the Sun. That curvature curves spacetime still farther out and so on. Thus spacetime even as far out as the Earth partakes of a small curvature. Spacetime there, with that small curvature, acts on the Earth, telling her what to do… In brief, distant action arises through local laws. (Wheeler 1990, 12; cf. Mach [1883] 1902, 230-231, 547)

How does it come about that “spacetime tells mass how to move, and mass tells spacetime how to curve” (Wheeler 1990, 12)? Is this language merely metaphorical or does it also offer a causal physical explanation, whereby spacetime and mass are both physical components that are always acting locally on each other?

Space and time, as it turns out, are not simply “there” as an unchanging backdrop of nature; they are *physical* things, mutable and malleable, and, no less than matter, subject to physical law. (Davis 1995, 16)

With his theory of spacetime curvature Einstein intended to overcome Newton’s mysterious gravitational force acting from a distance, for there was no explanation as to how physical forces can act through empty space. Thus, in a way, he appeals to a Cartesian contact mechanism (Einstein and Infeld 1938, 236-238; Moffat 2008, #2).

That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophical matters a competent faculty of thinking could ever fall into it. (Newton, 1692, in his third letter to Bentley)

Hence one can interpret the theory of general relativity in such a way that gravity simply becomes a distortion of Newtonian–Euclidian space and time (Magueijo 2003, 52-53). But then physical spacetime is itself natural matter, and the question remains as to whether we can measure it experimentally, separately from the mass that tells it how to curve? Indeed, how can we explain the dynamic relations between the curvature of spacetime and the mass of objects affected by it; or is it too arcane a metaphor? The conception of the gravitational field and its gravitons with their speed of light suffers the same problem, for it is explained by local dynamical-causal effects on material components (Wheeler 1990, 9-15; Magueijo 2003, 212-219; Moffat 2008):

Einstein’s theory of gravity is thus much more than the story of a master-slave relationship between mass and spacetime. It tells us that spacetime geometry is a new participant on the scene of physics with a dynamic of its own, as real and lively as the dynamic of the electro-magnetic field. What Einstein gave us, then, is more than general relativity, more than a geometric theory of gravity; it is a theory of *geometrodynamics*. Different as these three names are, all refer to the same theory, and so I shall use them as synonyms. (Wheeler 1990, 13-14; cf. Mach [1883] 1902, 230-231, 547)

On the whole, it seems that a conceptual confusion underlies Einstein’s theory of spacetime, and attempts to grasp the conceptual framework of general relativity tend to do so with an amended epistemology and a different comprehensive picture of physical reality. However, it is possible for physicists to measure and calculate the motion of bodies in gravitational fields using Newtonian or Machian conceptions of the distribution of mass, as if the curvatures of spacetime are only illustrations of such distributions, without considering Einstein’s conceptual understanding of spacetime as a real player on the physical stage of nature (Davis 1995, 16). In the long run, this Einsteinian concept of spacetime may block the way to “forming a new picture of reality” because in order to achieve a unified theory of matter “*we have to abandon the description of atomic events as happenings in space and time*” (Einstein and Infeld 1938, 125, 297), and, as I suggest, we can embrace instead the Spinozist concepts of space and time.

Yet our sensual experience is bound to the solidity of physical bodies in our environment and not to the entire material components of the dynamic plenum of nature. Unlike waves and fields and other varieties of matter, only the properties of the solid components of the dynamic plenum form the basis of the concept of the geometric structure of space (cf. Leibniz [1715-16] 2000, Fifth Paper #47; Mach 1906; Einstein 1934). Moreover, since there is no real interval or void in the dynamic plenum of nature, the sensual conception of space corresponds only to classical solid bodies and cannot explain the extreme macro and micro components of physical reality which we cannot experience sensually (comp. Einstein 1934, 278). It seems that Einstein’s conception of physical, substantive spacetime is a relic of classical Newtonian ideas about metaphysical, substantive space and time, and thus remains grounded in material substance (Einstein 1934, 282; Smolin 2008). This basic conception persists among contemporary physicists, substantivalists and relationalists, in contrast to the suggested epistemological alternative, the Spinozist system, in which space and time are seen as the methods human beings use to measure the finite modes and components of the infinite, eternal dynamic plenum of Nature. In nature we can say that there is no space, i.e., no “interval,” vacuum, or void, between rigid bodies but an infinite continuous plenum of varied natural components (e.g. Wilczek 2008, chap. 8). Thus, Heraclitus’ picture of nature as hydrodynamic flow without any space between its components can be considered as the “substance” of nature’s dynamic plenum (Barbour 2001, 51, 411-413, 443). However, it would be difficult for modern substantivalists to reconstruct fields as classical objects moving inside the “space container,” as Einstein suggests, and for relationalists to reconstruct fields as substantive “relations” among classical objects (cf. Einstein and Infeld 1938, 242-243; Leibniz and Clarke [1715-16] 2000, Fifth Paper, #47).

What, then, is the status of geodetic spacetime? Is it a physical component of nature or just a geometrical representation of mass distributions and their gravitational fields? If spacetime is a substantive object of nature which is curved by near massive objects, then the light wave path is affected by this and bent according to the curvature; but if it is just a representation of the distribution of mass in nature, then its curved path is due to the effects of mass, and, à la Newton, caused by gravitational forces from a distance. In any case, without mass the light wave cannot change its direction near objects with mass in gravitational fields. Therefore, light photons must have mass to be affected in gravitational fields (Magueijo 2003, 37; Smolin 2006, chap. 3; Wilczek 2008, 73; cf. Baker 2005). Perhaps the difficulty here is that in the shift from special relativity to general relativity, the effect of gravity on the mass of light photons is neglected and so the above paradox regarding the absolute constant speed of light can be developed, based on the discrepancy between these two theories (Magueijo 2003, chap. 3).

Of course, one can argue that photons are particles with zero rest mass. That means that if they are not moving (which abstractly means they are at rest) they have zero mass. However, it also means that if they are moving they do have mass; and if they move at velocity c they should have infinite mass (Ohanian 2008, 103, n27). Yet, where does this mass come from? This is like asking where does nature or the universe come from, since physical nature is material, and mass and energy can be defined as two different methods to measure the units of matter of any component of nature. Therefore, light is material, has mass-energy, and is affected by gravity. Hence, photons and other light components, including gravitons, all have mass-energy (“mass has energy and energy has mass” – Einstein and Infeld 1938, 244). But what is the structure of the massless photon at rest before it receives its momentum? It cannot be nothing, like the universe before the Big Bang, though it could be an incident in the infinite plenum of nature. The emission and absorption of photons change the mass of the system and show that its energy-mass is expressed in Einstein’s equivalence, EL = mLc2. Hence it can be suggested that what we call the *mass* and *energy* of the components of nature are rather a *wave-particle* structure, which is actually *matter*, although measured by different methods in terms of mass-particles or energy-waves. The unity of particle and wave combines the two aspects of physical reality that affect and are affected differently in different force fields (Magueijo 2003, 214-219, 223-232; Planck 1902; Frank 1957, 8.4). With this understanding of matter as a comprehensive conception of physical reality in its entirety, we can even question Heisenberg’s uncertainty principle, since this is based on the view that subatomic matter is composed of particles that have a classical position and momentum that we want to measure. But if we see such matter as having a wave-particle dual nature in quantum fields, we should find different methods to measure their energies with different parameters. However, Einstein’s idea that we can obtain knowledge about physical reality without “disturbing” it experimentally is very ambiguous since we do indeed use our instruments by affecting the objects of experimentation. However, we can see these instruments themselves as components of physical reality and measure their effects on the observed objects; yet since all knowledge is relative to our proof-conditions, we have to find a mode of measurement whose effect on the components is negligible (Nesher 2002, IV).

**5. Mass, Energy, Particles, and Waves: What Constitutes Matter?**

Classifications such as mass, energy, particles, waves, and other physical components, seem based on current methods of measuring them. But eventually, under certain future proof-conditions, we might reach a situation where we will be able to measure all of them by the same method; then we would understand them as different structures of universal *matter* and identify them according to their other properties. The question is whether we can formalize such a unified theory of physical reality for particle-wave force fields, differently from the classical picture of matter (Einstein and Infeld 1938, 297). Under such a *new picture of physical reality* it would be difficult to measure the components of matter by assuming the accepted classical conceptions of *mass* and *energy* in *space* and *time* according to the epistemologies of the substantivalists or relationalists (Newton 1687; Leibniz and Clarke [1715-1716] 2000, esp. Fifth Paper #47; Mach 1906; Einstein 1934). Indeed, it is not clear whether we can reduce all representations of natural matter to the classical conceptions of space and time.

The problem lies in explaining how we can truly measure and represent *matter* and whether or not the measurement itself is a component of matter. Moreover, when it comes to minimal scales we cannot take measurements on the basis of the classical concepts of energy and mass; we have to overcome a “measurement problem” to penetrate the minuscule structures of the components of matter (the minuscule structures of reality), and learn how to understand the status of the observer through the experimental device in reality:

The conformity of the formalism of quantum electrodynamics with the interpretation of the idealized field and charged measurements has of course no immediate relation to the question and scope of the theory and of the actual possibility of measuring the physical quantities with which it deals… yet, an ultimate limitation of the consistent application of the formalism is indicated by the necessity of introducing forces of short range in nuclear theory, with no analog in classical electrodynamics, and by the circumstance that the ratio between the electron mass and the rest mass of the quanta of the nuclear field has the same order of magnitude as the fundamental parameter e2 /hc of quantum electromagnetics. The further exploration of such problems may, however, demand a radical revision of the foundation for the application of the classic dual concepts of field and particles. (Bohr and Rosenfeld [1950] 1983, #4)

Accordingly, the limited proof-conditions of Newton and Leibniz’s metaphysical and physical theories (bound to their classical conceptions of space and time) affect their comprehensive intuition of picturing physical reality; this hindered later physicists and philosophers in their efforts to discover a new general theory able to unify the macro and micro realms of nature (Bohm and Hiley 1993, chap. 1.1; Damour 2006, 160-163). Assuming the existing scientific situation of physics, with the limitation of its proof-conditions and our willingness to break the so-called existing picture of physical reality, is it possible to accept the idea of massless particle-waves even if we cannot measure their mass in such existing conditions?

Light consists of particles called photons. According to relativity’s second postulate, this speed is the same for all observers… Likewise, there is no way that you can decelerate a photon until it is at rest. A box packed with photons does not make any sense. Photons exist only because they move. In some sense they are pure motion, unable to be at rest. For this reason, we say that photons have zero rest energy or mass: They are *massless.* (Magueijo 2003, 215)

However, behind the suggestion of a dynamic plenum picture of nature there is the idea that everything is in motion, and it is thus absolute, whereas being at rest is only relative, and nothing is absolutely at rest. Due to human limitations, rest and motion are always relative, measured in respect to our frame of reference. Thus, the idea of absolute rest is only an *abstraction*, and so it is with photons too. There is no such thing as pure motion (or pure energy) just as there is no such thing as pure mass (at rest) since there is only the *unity of energy and mass*, but we tend to conceive of them as being separate due to our different methods of measuring them (and our sensual experience with them) (cf. Einstein and Infeld 1938, 244).

The situation with our existing proof-conditions and methods of measurement is that we cannot yet detect the mass of a particle. Assuming the unity of mass and energy, to speak about massless particles is to suggest that they have no energy either, which might lead us to say that they are not material components of nature, only spiritual ones. But this is reminiscent of the erroneous conclusion Einstein drew from Michelson and Morley’s experiment: since changes in the speed of light cannot (yet) be detected, then it must be constant, which entails that neither “dark energy” nor “dark matter” exist as fields in the dynamic plenum that affect light waves (Einstein 1934, 280). So far, this has not been proved wrong by experimental results based on existing proof-conditions; but since we cannot assume that there are absolute elementary components, or basic “building blocks,” in the infinite and eternal dynamic plenum of nature, we cannot attain a definitive theory of infinite nature (Weinberg 1992; Nesher 2002, X). This is so since our conception of basic elements is based on our everyday sensual experience of classical physics, which cannot belong to the above picture of physical reality, and such principles therefore hamper our philosophical-epistemological understanding of nature (Heisenberg [1968] 1989, 50-52).

*Matter, mass, energy, field*: how are the relationships between them to be understood? It can be suggested, Spinozistically, that the *mass-energy* unity is *matter*, as distinct from *mind*; thus, all kinds of physical components are *matter*, that is to say that physical reality is wholly independent of its cognitive representation. It seems that philosophically and conceptually Einstein and Infeld are not clear about the concept of matter, mechanistically understood, and somehow they assume that fields are not matter; so are they spiritual or scientific representations of matter? (Einstein and Infeld 1938, 242-243; Wilczek 2008, chap. 4)

We have two realities: *matter and field*. There is no doubt that we cannot at present imagine the whole of physics built upon the concept of matter as the physics of the early nineteenth century did. For the moment we accept both concepts. Can we think of matter and field as two distinct and different realities? (Einstein and Infeld 1938, 240)

We cannot build physics on the basis of the matter-concept alone. But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? (Einstein and Infeld 1938, 241)

It seems that Einstein and Infeld hesitate between classical conceptions of bodies and their movement (perceived sensually as matter) and the theoretical inclination to explain physical reality by leaning on the conception of energy fields without recourse to any general picture of physical reality.

What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created. Its final aim would be the explanation of all events in nature by structure laws valid always and everywhere. A thrown stone is, from this point of view, a changing field, where the states of greatest field intensity travel through space with the velocity of the stone. (Einstein and Infeld 1938, 242-243)

*But we have not yet succeeded in formulating a pure field physics. For the present we must still assume the existence of both: field and matter*. (Einstein and Infeld 1938, 245; comp. Einstein 1954, 366-377)

This intuitive picture of physical reality seems to be based on our sensual experience and the classical conceptual distinction between *mass* and *energy* where *matter* is taken to be a synonym for *mass* or *bulks* *of* *energy* i.e., a “concentration of energy.” The alternative picture of physical reality is a *pure field physics*.

As long as the separation between the massive and the massless persisted, a unified description of the physical could not be achieved. (Wilczek 2008, 9; cf. 8-10)

In my understanding, if *E = mc2*, there are no massless components of nature though there are components that we cannot measure in terms of classical mass. Due to the classical conception of mass, what we cannot measure classically as massive has no mass, e.g., photons, gravitons, etc., but since they have energy they also have mass (Ohanian 2008, 103 n27). The problem here is that we have two meanings for the term *mass*: matter measured as classical inertial *invariant mass*, or as gravitational *relativistic mass*. We can measure the weight of an object but how can we measure its mass? Moreover, can we measure the mass of energy as a photon? It seems that though the classical conceptions of mass and energy are separate, Einstein treats them as being quantitatively equivalent in his famous formula (Wilczek 2008, 8-10; Alexander 1956, liv-lv).

For the time being, we have to admit that we do not possess any general theoretical basis for physics, which can be regarded as its logical foundation. The field theory, so far, has failed in the molecular sphere. (Einstein 1940, 334)

But why do we expect field theory to be based only on the picture of a medium with ripples, which is itself based on the model of sound waves in air or water waves in the sea, or mass in spacetime (Cox and Forshaw 2009, #2; Einstein and Infeld 1938, 242-243)? The question is whether this conception of field, or even Einstein’s *pure field physics*, presents an accurate picture for explaining nature’s dynamic plenum.

But the relation *E = mc2* tells us more than that [nuclear fission]. It not only implies that mass is a congealed form of energy, but also that all kinds of energy have mass. Thus, energy and mass are equivalent: wherever there is a mass, there is energy; and wherever there is energy, there is mass. Mass and energy are two facets of the same thing–in modern terminology, they are two facets of a generalized concept of energy. (Ohanian, 2008:152-153)

However, I suggest that the generalized concept of *energy* and *mass* is *matter*, while other structures are identifiable as constituents of matter thanks to the different methods we have at our disposal for measuring structures of the plenum. Moreover, we identify the components of *matter* according to our methods of detecting and measuring them, and hence we tend to see matter as aggregates, particles, waves, fields and more, but there are other components of matter, like *grey* and *black* *matter*, which can be detected indirectly even though we do not yet know their inner structure (for experimental intervention has not been possible). We can assume that *matter* is not amorphous but that it has internal powers that determine the changing structures of the *physical plenum of reality.* Moreover, experimentalists can be considered segments of such powers because they create physical-material structures and observe the resulting components of *matter* (Wheeler and Zurek 1983, chap. IV; Lockwood 1989, #11).

If every physical component in the dynamic plenum of nature is in motion, it has *relativistic mass* and every component or “body” has *velocity-dependent mass: m= E/c2*. Its *invariant mass* is independent of its velocity. But if every component is in motion, how can we measure its *invariant mass*? Furthermore, measuring *relativistic mass* depends on measuring *invariant mass*. If every component of nature is constantly in motion, its *invariant mass* must be related only to its *relative rest*. Yet if it is only relatively at rest it must have *velocity energy* in its relatively *invariant mass*. It seems that the amount of matter of the component depends on the method used to measure its mass-energy and so on. But if we unify the concepts of the known aspects of *matter* (mass-energy-field), then the two kinds of mass, Newtonian *inertial mass* and Einsteinian *relativistic mass*, belong to their respective “closed” proof-conditions. However, can we attain a unified conception of matter whereby dynamic quasi-wave-particle components are seen to be in continuous motion and measurable on macro and micro scales in units of energy-cum-mass?

**6. Looking for a Comprehensive Conceptual Picture of Physical Reality**

It is claimed that a photon never stands still and can move only at the speed of light; it exists only because it moves and therefore has zero rest energy or mass. However, it can be argued that this is based on the picture of the classical stable objects of sensual experience. Yet in a more comprehensive picture of nature as a dynamic plenum, all its components have duration, are in continuous motion and undergo change, which could eventually be formulated as “metric fields,” the “topology of fluid flows,” or fields of forces, instead of Einsteinian materially substantive space-time geometry (Einstein and Infeld 1938, III; Wilczek 2008, 6, chap. 8; Nesher 1979). Thus, there is no absolute rest and the relative rest (mo) of natural components is only an abstraction. They are “absolutely” in motion, which we can measure relatively in different frames of reference. Some would claim that the speed of a light wave is constant and absolute and that photons cannot be in a state of relative rest, so that the mass-energy equation cannot apply to them (Frank 1957, 8.4). However, the question remains: Why can we not consider a light wave to be at relative rest such that EL = moLc2 (considering other components of nature, which are also at relative rest). Of course, one can argue that the speed of light is constant and an absolute criterion for all measurements of motion with regard to all other components of nature (Einstein and Infeld 1938, 176-192). Hence, the speed of light is the absolute criterion for evaluating all other motion in nature, and Einstein’s equation is not applicable, since there is no other reference frame in which light can be at relative rest. However, if all motion is relative to other motion or to the accepted standard criterion of frame of reference, then the physical motion of light is also relative to other motion and frames of reference. But then if this reference frame is the speed of light itself, it must be independent of all other frames of reference and therefore its motion is absolute, like Newtonian space and time. Moreover, it seems that “empty space,” i.e., the absolute void, has a similar function to Newtonian “absolute space” in which we can define absolute motion or the absolute-constant velocity of light (Einstein 1954, 370). The question is whether the constant speed of light is an absolute value in nature, according to an absolute law of nature, or whether it is only a conventional constant in the context of our temporal proof-conditions due to the limitations of our methods of measurement? Moreover, is it possible for certain waves, fields, or other components of nature to have a higher speed than light, which cannot be detected by our existing proof-conditions?

Can we preserve the two basic principles of Einstein’s theory of relativity: the postulate of the relative nature of motion and the constancy of the speed of light?

His [John Moffat’s] leading principle was the preservation of the pillars of Einstein’s relativity theory: the postulate of the relative nature of motion and the constancy of the speed of light. But how could one have a varying speed of light and not conflict with the second of these principles? It appeared to be a hopeless contradiction in terms. John’s clever approach went right to the heart of the matter and asked what the constancy of c really means. As I said before, it means that the speed of light is the same regardless of its color, the speed of its source or the observer, and when and where it was emitted or observed. But what does “light” mean in this statement? In Einstein’s initial formulation it means nothing but the usual stuff you call light, not just visible light but also any other form of electromagnetic radiation, such as radio waves, microwaves, or infrared radiation. (Magueijo 2003, 214)

Accordingly, Einstein’s postulate of the constancy of the speed of light is about the physical reality of such radiation. Yet we can overcome the seemingly “hopeless contradiction in terms” surrounding the hypothesis of the varying speed of light if we distinguish between the variation in speed of the physical reality of electromagnetic radiation and the constancy of c: this distinction could offer a conventional criterion, or a frame of reference with which to measure the motion of other components of nature. From this perspective, under Einsteinian existing proof-conditions, variations in the speed of light are negligible (Grünbaum 1963, chap. 2 (B); Ohanian 2008, 102, 103 n27).

The velocity of light in empty space always has its standard value, independent of the source or receiver of light. (Einstein and Infeld 1938, 176)

However, in the suggested picture of physical nature, the dynamic plenum, which might be explained by a unified theory of fields, absolute empty space cannot exist, neither can absolute motion nor an absolute speed of light in a vacuum. We can compare Einstein’s conception of the speed of light in a vacuum with Newton’s conception of absolute space in that they are vacuous axioms for both epistemological and experimental reasons.

Yet could we not say that it was an arbitrary decision on Einstein’s part to use an objective and absolute criterion for all observers of physical nature (e.g., Davies 2005, 52)? But then it could be suggested that all criteria for observers are only relative to the available *proof-conditions*. Therefore, this might also apply to the motion of light waves: a relatively constant speed could be taken to be the criterion for formal calculations in a specific referential framework. Light that travels through transparent matter does so at a lower speed than *c*, the speed of light in a colloquial vacuum. However, since nature is a dynamic plenum, and there is no absolute vacuum or void, the speed of light cannot be constant; in other words, it is relative to the medium, the different fields it travels in, metaphorically called *ether*, e.g., “dark matter” and “dark energy” (Magueijo 2003, chaps. 4, 5; Wilczek 2008, chap. 4). Moreover, another relatively absolute criterion could be suggested for measuring the motion of natural components, for example the early universe could be taken as a constant in respect of which we could measure the varying speed of light (e.g., Magueijo 2003, 246-252). Perhaps we have to separate our decision on the stipulated criterion of measurement, or the rules for the reference frame, from our theory of the laws of nature, and reduce Einstein’s famous formulation by saying that the limit of acceleration is just a standard human convention. Thus, as per Newton, “All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any Change”(Newton [1687] 1953, #2). This convention of the ideal and absolute standard of measurement is supposed to be, like Einstein’s speed of light, the same for all observers in objectivizing (or intersubjectivizing) the measurements of the theory in question. The reason why we lean on such conventions is that our accepted proof-cognitions do not enable us to detect such changes, although we do not rule out the possibility of developing new methods capable of detecting them (Frank 1957, 134-135; Grünbaum 1963, chap. 12 (D); Magueijo 2003, 223-232).

If the speed of light is only a standard convention for measuring other motions in nature, then it changes physically in different physical environments, as the hypothesis of the varying speed of light (VSL) suggests. Hence we are dealing with the speed of a natural component of nature, not an absolute and universal speed of nature, which entails that it could be even faster than our convention stipulates, and the photon itself might not be able to reach an infinite mass. And if the barrier of the speed of light collapses, we have to overcome the Einsteinian picture of physical reality in accord with his own philosophical epistemology (e.g., Magueijo 2003).

In general relativity all motion is, or should be, explained within the context of the gravitational field, and there is no difference between inertial and gravitational mass that moves through this field. Yet inertial mass is no longer Newtonian mass in absolute empty space, since all mass comes under the forces of the gravitational field. The problem is with quantum matter, with its mass-energy/particle-wave dualities and their dimensions, because, according to Einstein, the “spaces of sub-atomic extension cannot be measured” (Einstein 1954, 366). We need to answer the question of whether there is continuity or a break between special and general relativity. If the latter is the case, then why does the constancy of the speed of light still hold, given that the gravitational field always acts on all mass including that of the light-wave-particle? (cf. Einstein 1954, 372-373). Perhaps Einstein needs an absolute coordinate system, a frame of reference, for his theories, but in infinite nature and relative scientific investigations coordinate systems are always relative to our known and accepted proof-conditions (Nesher 2002, X; 2017, 2019).

It could be concluded that there cannot be an absolute speed or an absolute criterion for observers’ measurements of motion, and even if there is absolute motion we will never be able to identify it as we are only finite modes or components in our infinite and eternal universe (Spinoza [1677] 1985; Smolin 2006, haps. 13, 14). Indeed, those conceptual-epistemological attitudes, which prevent us from intuiting nature absolutely, do not affect the calculations and predictions of a theory thanks to its specific proof-conditions, including the degree of precision of the existing methods, as per Newton’s conception of absolute space and time and Einstein’s absolute speed of light in the material spacetime (Magueijo 2003, 34-35; Greene 2004, 58-61). However, these restricted conceptualizations affect our intuition for picturing nature, and thus hamper and block us in our efforts to discover a new comprehensive picture and a general theory to unify the macro and micro realms of nature (Bohm and Hiley 1993, chap. 1.1; Damour 2006, 160-163; cf. Einstein and Infeld 1938, 297).

It seems to me that Einstein’s intuition about the constancy of the speed of light being the absolute limit of motion is based on the rigid dichotomy between massless waves and massive particles which, in my understanding, we can no longer hold onto, since matter has a particle-wave structure (cf. Wilczek 2008, 76-90; Bohm and Hiley 1993; Greene 1999, chaps. 4, 5; 2004, 90; Nesher 1999, 2002;; comp. Einstein and Infeld 1938, 51, chap. III, 197-199).

In the final analysis discrepancies seem to exist between the basic principles of special and general relativity. Indeed, we can only know what we can measure; if we are not able to measure the mass of certain physical components we might conclude that they are massless, likewise with energy and other forces. Nevertheless, we can create our comprehensive picture of physical reality because of our background knowledge and intellectual intuitions, and then endeavor to discover and explain new physical structures by new methods of measurement. Since there is no real void in physical nature, i.e., no real space-vacuum between components, the relations between them are what we measure, and their physical relations are dynamical and not dimensional.

**7. The Alternative Picture of Physical Reality as a Dynamic Plenum, and the Spinozist Conceptions of Space, Time, and Number**

How can we develop and claim knowledge of a general picture of reality? We might put forward a hypothesis based on previous physical and philosophical knowledge but which contains difficulties and paradoxes; we might discover new proof-conditions, observational and methodical, and prove them to be true, as in, for example, the shift from the Newtonian to the Einsteinianpicture of physical reality. Yet, how can we prove the truth of the pictorial hypothesis in order to continue with our scientific enterprise? It seems that as long as we continue to prove our scientific hypotheses by assuming our new picture of reality, as distinct from theories based on a previously accepted picture of reality, this new picture can be supported or even proven to be true according to our accepted general epistemological proof-conditions at the time, and for the time being.

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683–684)

Indeed, in every scientific revolution scientists adopt certain philosophical perspectives and epistemological attitudes, in accordance with the Kuhnian concept of a paradigm, which affect their general picture of nature. Hence, it is interesting to investigate how far such paradigms affect the discoveries of new hypotheses; in particular, to what extent do our conceptions of space and time affect mathematical formalism and experimentation in the testing of hypotheses (Planck 1933, III; Heisenberg 1968). Hence, we can take Newton’s conceptions of absolute space and time and ask what function they have in his theory, likewise with respect to Einstein’s conceptions of spacetime, the absolute vacuum, and the absolute-constant speed of light, as well as our contemporary grand hypotheses in physics (Ohanian 2008, chap. 4)? Moreover, a misguided comprehensive picture of physical reality, e.g., encompassing past conceptions of space and time, may disturb the progress of discoveries of potential new comprehensive revolutionary hypotheses (Barbour 1999, 2001, 2008; Magueijo 2003; Greene 2004; Randall 2005; Davies 2005; Smolin 2006; Woit 2006; Majid 2008).

If this is so, those concepts can be questioned and perhaps replaced by the Spinozist epistemological conceptions of space, time, and number in their capacity as “common notions,” i.e., our methods of measuring the components of the infinite and eternal dynamic plenum of nature. Hence, we cannot speculate on indefinite dimensions of natural components and their relations, on the basis of mathematical formalism, as in string theories, i.e., hypotheses that cannot be proved experimentally. We can only work with magnitudes that we can measure in the process of attempting to prove the truth of our new hypotheses in order to explain the components of nature’s dynamic plenum (Leibniz and Clarke [1715-1716] 2000, Fifth Paper, #29; Mach 1893, IV #iv-9). Moreover, we must follow Spinozist scientific naturalism without recourse to God’s help in epistemological explanations of nature, which is something that Descartes, Leibniz, Newton, and others resorted to. In this case we can epistemologically prove that nature is infinite and eternal and has no beginning or end of any sort, contrary to what is proposed by the story of creation in *Genesis* or even the Big Bang theory, and other theological fantasies or mathematical formalist speculations (Broad 1946).

The distinction between Spinoza and Leibniz’s conceptions of *space* and *time* is as follows: For Leibniz, they are in our minds but are themselves *physical realities*, while for Spinoza the real relations between natural finite components are *dynamic causal relations*, which we explain with our notions of spatial distance and counted time, which constitute our experimental methods of measurement. The long-standing argumentation in the history of physics and philosophy over whether space and time are relational or absolute is a false dilemma if a third option, as suggested by Spinoza, can show that both sides of the dilemma are wrong. The basic question is whether space and time are metaphysical or natural physical entities, or simply notions we have created for measuring the components of nature’s dynamic plenum (Wheeler and Zurek 1983, chaps. IV-VI)?.

What, then, are the essential relations in nature’s dynamic plenum? They are the *dynamic causal relations* between components of the infinite dynamic plenum that we, as its finite components, explain by measuring them with the devices available to us, such as rods, clocks, and numbers. We can measure only the finite *sizes* of such finite components using finite rods and their finite *durations* using finite clocks since we, as finite components in the infinite dynamic plenum of nature, cannot measure infinite sizes or infinite duration. Thus, we indicate these measures numerically and use them to elaborate our theories about this dynamic physical nature. Put differently, this is an epistemology of devices, space-meters, time-clocks, and numbers-numerals: Spinoza’s “common notions” of our measurement operations (Nesher 1979).

The conception of “background-independent theory” seems to be specifically formulated against the assumption of Newtonian absolute space and time and not as a basic epistemological understanding of the evolution of scientific comprehensive pictures of reality for the formulation of hypotheses and their experimental evaluation (Smolin 2006, 80-83; cf. Nesher 2002, X; 2008). However, it seems that the intention of the “background-independent theory” is to avoid the unsatisfactory traditional background theory in order to eventually open the actual research program to a new general picture of physical reality. How, then, may scientists discover and formulate their comprehensive picture of reality and the principal background tenets of their hypotheses, and how do they know whether or not they can test them experimentally? Above all, before considering the geometry of space and time, the central question remains: What indeed are space and time?

Dynamic relations exist between the natural components of matter and their duration in the dynamic plenum, but space, time, and numbers are “common notions” created by human beings and not components of physical reality; in other words, they are signs and devices we employ, based on our cognitive concepts of measuring these dynamic relations. Thus, measures of *distances*, the so-called *space*s and *times* of dynamic *durations*, are the results of our measuring the components and relations of Nature’s Dynamic Plenum. Moreover, the results of these measurements provide the dimensions of our coordinate systems that we need to develop our theories of physical reality, and there cannot be any mathematical speculations about it.

In practice, we measure nature’s components and their relative motion with our coordinates in a local frame of reference, but it seems that in our elaborate comprehensive picture of physical reality we replace these coordinates with our conceived dimensions of space and time, thus making our measuring devices substantival entities of nature. This raises the question as to why physicists transform coordinates of measurement into such material substances as space and time? This could be due to our sensual experience with discrete and separate rigid bodies that eventually lead to speculative principles that result in mathematical formalisms, concluding with abundant extra dimensions that cannot even be measured (e.g., Leibniz [1715-1716] 2000, Fifth Paper #47; Mach 1906; Einstein 1934; cf. Green 1988, 123-139).

My view is that philosophically and epistemologically intuitive pictures of physical reality in contemporary physics have inherited their difficulties from Einsteinian and Bohrian conceptual traditions. Though revolutionary at the time, they hamper physicists today in understanding nature correctly and in envisaging a unified theory of the dynamic plenum of physical nature (comp. Moffat 2008, introduction; Nesher 2002, 1999). The basis of any scientific revolution is the replacement of an existing limited picture of reality by a new one, and the theory-paradigm with its mathematical formalism and its experimental results for proving its truth (comp. Nesher 2012, 2018). Yet physicists usually seem to go from one unsatisfactory formalism to another more satisfactory one, without attempting to leap into a new comprehensive picture to overcome the difficulties of their main picture, which still basically belongs to the prior paradigm (e.g., Heisenberg [1968] 1989, 36-39, 44; Feynman 1985, 149-152; Green 1988, 123-139; Randall 2005; Moffat 2008).

Confusion about what we call “space” and “time” puts physical theories of nature in difficulty, giving rise to paradoxes, which means that the basic philosophical pictures are problematic and should be revised (comp. Tarski 1944; Nesher 2002, V). Einstein wrote about the failure to understand and formulate a unified field theory for gravity and electromagnetics:

This state of affairs will last for many more years, mainly because physicists have no understanding of logical and philosophical arguments. (Einstein, cited in Ohanian 2008, 326)

In his later years Einstein seems to accept the conception of *field* as the *matter* of the general theory of relativity, and eventually of all physical reality, in the hope that it will provide a solution to the unified theory of macro and micro components of physical reality. However, *matter* seems better understood as physical reality and *field* as its structure as represented in specific physical theories (cf. Einstein 1954, 375, 376-377). Yet the question is whether Einstein, at this stage, identifies *field* with *matter*, or rather with *spacetime* as a physical reality, but still different from energy or mass?

There is no such thing as an empty space, i.e., a space without field. Spacetime does not claim existence on its own, but only as a structural quality of the field (Einstein 1954, 375).

It requires the idea of the field as the representative of reality, in combination with the general principle of relativity, to show the true kernel of Descartes’ idea; there exists no space “empty of field.” (Einstein 1954, 376)

Yet, the question remains as to whether the “pure gravitational field” of general relativity, which coexists with spacetime, can be without matter or mass. Therefore, we have to go one step further and say that in his later writings Einstein’s conception of the field as the basic element of physical reality is very close to the Spinozist dynamic plenum.

It is common to all these attempts, to conceive physical reality as a field, and moreover, one which is a generalization of the gravitational field, and in which the field law is a generalization of the law for the pure gravitational field… By this I mean a theory which describes exhaustively physical reality, including four-dimensional space, by a field. (Einstein 1954, 376)

But it seems that Einstein’s identification of “pure field” with spacetime is an obstacle to a new grand picture of physical reality where space and time are no longer components of such reality but only “common notions” or concepts of methods used to measure this physical reality. Indeed, Einstein’s conception of general relativity basically remains bound up with the natural substance of spacetime when the accepted conceptions of space and time are based on our sensorimotor experience in a classical environment, so to speak. However, a more comprehensive picture of nature might suggest different conceptions . Thus, after centuries of confusion about the nature of space and time, with wrong epistemologies applied to scientific research, scientists today use space and time as metaphysical or basic physical entities to explain the laws of nature. However, we can better picture nature as a dynamic plenum, like Einstein’s *cosmological fluid*, which may be explained conceptually and described formally according to our measurements of its specific components and their relative motion (Magueijo 2003, 65). Thus, we have to overcome the myths of space and time by adopting a better epistemology of scientific inquiry in order to reach an accurate conceptual picture of physical reality (Spinoza 1663; Nesher 1979).

If both gravity and quantum mechanics are based on misguided conceptions of space and time, and on confusion about the structures of the material components of nature, our picture of reality is unsatisfactory. How, then, can we merge two incorrect pictures of reality to form one true theory (cf. Magueijo 2003, chap. 12)? I would suggest that in the picture of the dynamic plenum of infinite and eternal nature, *space* and *time* are not physical substances but methods we use to measure the fluidity of this physical reality (versus Moffat 2008, chap. 16). However, it is essential to make a clear distinction between our comprehensive picture of physical reality and the theory and formalism that represents it.

There remains an epistemological puzzle: How can it be that Newton and Einstein’s scientific theories and mathematical formalisms make it possible for us to predict and explain facts of physical phenomena, despite their limited and problematic comprehensive pictures of physical reality? Their relatively true paradigmatic theories and conceptions of space and time enabled them to formulate theories that did not contradict their representations of physical reality under their limited proof-conditions coupled with their methods of measurement. Nevertheless, we can see that their *closed theories* have their limitations, based as they are upon specific *proof-conditions*, including problematic conceptions of space and time, which the next revolution in physics will need to overcome if progress is to be made (cf. Heisenberg [1968] 1989, 36-39, 44; Nesher 2002, X; 2008). This can be achieved through the discovery of a new comprehensive picture of reality.

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**Chapter 9**

**The Role of Productive Imagination in Creating Artworks and Discovering Scientific Hypotheses**

**1. Introduction: Probing Kant’s Ideas on the Role of Productive Imagination in Artistic and Scientific Creation and in the Discovery of New Modes of Representing Reality**

In this chapter I elaborate on Kant’s conception of *productive imagination* in creating artworks and I generalize it to explain the scientist’s *intellectual intuition* in the discovery of new hypotheses. Kant proposes that *intuition* is a faculty of the imagination, and *productive imagination* is necessary for the creation of fine art.

For the imagination (as a productive cognitive power) is very mighty when it creates, as it were, another nature out of the material that actual nature gives it (Kant [1790] 1987, 314).

Kant develops the conception of *intellectual intuition* and connects this to supersensible objects of reason, as distinct from empirical ones. I apply this transcendental concept to cognitive operations to explain them experientially. Hence the role of *productive imagination* lies in the artistic creation of new exemplary artworks, and the role of *intellectual intuition*, as productive imagination, lies in scientific discoveries. From a pragmaticist’s point of view, I explain that artists and scientists use their productive imaginations differently in their respective enterprises to construct their different modes of representing reality. These two kinds of imaginary productive operations are based directly and indirectly on the perceptual images of empirical objects. To understand the artistic creation of exemplary artworks, and the scientific discovery of new hypotheses, we have to elucidate the roles of productive imagination in these enterprises by analyzing the structures of the artistic *aesthetic reflective judgment of taste* and scientific *logical reflective judgment of coherence*. I criticize Kant’s narrow conception of *judgment* and suggest that epistemic logic offers a complete proof of truth.

**2. Kant on *Theoretical* and *Aesthetic* Judgment: Problems with the Conception of Judgment**

**2.1. Kant’s division between *theoretical logical judgment* and *aesthetic reflective judgment***

Kant’s dichotomy between art and science is based on the epistemological division between *theoretical (logical) judgment* and *aesthetic (reflective) judgment*, where the former is an objective and true representation of reality while the latter is subjective yet universal in the aesthetic experience of human beings, without representing reality. This is based on the metaphysical division between the *determinism* of scientific mechanical rules followed in the development of theories, and the *freedom* of the artistic genius’s productive imagination in creating exemplary fine art. Kant expresses this division as *lawfulness* versus *free play* (Kant [1790] 1987, ##35 36). This dichotomy between art and science,between *free* *productive imagination* in creating artworks and scientific *determinate mechanical rules* of formulating theories, is elaborated in our traditions of phenomenological “artism” and analytical “scientism.”

**2.2. Difficulties with Kant’s Conception of Judgment in His Three Critiques**

Kant’s epistemology developed around his general conception of judgment:

I then find that judgment is nothing but a way of bringing given cognitions to the objective unity of apperception (Kant [1781-1787] 1987, B141-142).

Kant has three conceptions of judgment: the *theoretical logical judgment* of science, the *aesthetic reflective judgment* of fine art, and the *practical judgment of the commands of moral law*. In all three, judgment has a reflecting role, for we process representations coming from our faculties of imagination, understanding, and reason. Thus, we detect harmony or disharmony as the subjective conditions for adequate or inadequate judgments. However, not every cognitive operation determines objective judgment, since aesthetic reflective judgments are not concerned with objective knowledge of reality but only subjective reflection on the ideas of the faculties of the imagination and understanding to compare their harmony or disharmony ensuing from the feelings of aesthetic pleasure and displeasure (Kant [1790] 1987, 237'-238'). The difficulty with Kant’s three types of judgment is that, because of his phenomenalist epistemology, there cannot be any external restriction to ensure objectivity and, therefore, he must assume transcendental principles, concepts, or rules, based only on faith. I claim that Kant’s judgment of taste in the *Third Critique* is the same as Peirce’s *abductive inference*  in the suggestion of new concepts or hypotheses; the *moral* judgment of the *Second Critique* is *deductive apodictic inference*; and the *inductive determinative inference* of *theoretical* judgment in the *First Critique* is equivalent to Peirce’s basic *inferences*.

**2.3. Overcoming Kant’s Narrow Conception of Judgment Using Epistemic Logic and the Trio of Inferences**

Not one of Kant’s different judgments is complete proof of its truth, validity, or universality. To overcome *a priori* epistemology I shall show that only the sequence of inferences, the *trio* of the abductive logic of discovery, the deductive logic of necessity, and the inductive logic of evaluating hypotheses, can confront reality and constitute complete proof (Nesher 2007). This epistemic logic of cognition constitutes complete proof of any judgment without recourse to any transcendental a priori assumptions. Our basic cognition happens through the perceptual operation of the *trio*:

**[1] The Operation of Perception is the Quasi-Proof of Perceptual Judgment, namely the Trio of Inferences:**

**Abduction**((CAb(A→C)=>AAb)+**Deduction**((A→C) A)→CDd)+**Induction**((A, CIn)≈>(AAb→CIn))

Thus, => is the abductive plausibility *connective* that suggests the concept AAb, which in turn is the deductive *necessity connective* inferring the abstract object CDd, and ≈> is the inductive *probability connective* evaluating the relation of the concept AAb to the new experiential object CIn. Since Kant does not combine the three inferences into complete proofs of the truths of theoretical, ethical, and aesthetic judgments, he has to justify their a priori assumptions separately (Kant [1781-1787] 1987, A84ff; [1788] 2002, 42; [1790] 1987, ##30, 31). Thus, a complete cognitive proof enables us to confront reality through the abductive material logic of discovering cognitions and the inductive material logic of their evaluation, which can justify them empirically without any a priori justification. Kant’s frustrated attempt to unify human reason “to derive everything from one principle” is solved by the Peircean epistemic logic of the *trio of inferences*. With the pragmaticist’s epistemic logic we can better understand how the scientist arrives at hypotheses and how the artist creates artworks.

**3. Genuine Artistic Productive Imagination in Creating Fine Arts and Aesthetic Experience**

**3.1. Can the Artist Exercise Free Play of Productive Imagination** **in the Creation of Exemplary Artwork?**

Kant’s aesthetic theory of fine arts is divided into two parts: the artist’s creation of the artwork and its evaluation in the reflective judgment of taste. How can a genuine creation of artwork be the result of the free play of the faculties without following rules? After all, the artist is purposely and academically trained to control his work. It can be shown that free creation is self-controlled by habitual rules, and generally, according to Spinoza, personal freedom is inner determination (Nesher 1999). Kant cannot accept such a conception of freedom since his critical philosophy is based on the dichotomy between the determinism of nature and the freedom of the transcendental subject. Yet we cannot explain the role of the artist’s *productive imagination* without the free play of self-control in creating artworks.

**3.2. Aesthetic Experience and Creativity**

The artist’s aspiration in creating an artwork is to make his abstract ideas of reality sensible by exhibiting them aesthetically through the individual characters and situations of the artwork. The artist has the motivation and theme to turn his intellectual *ideas* into the imaginatively created *aesthetic ideas* embodied in the artwork. Yet these intellectual ideas with their intuitive meaning and content come from the artist’s experiential confrontation with reality. The artist wants to create the epitome of a lover or a cruel person, as Dostoevsky does in *The Idiot* and *The Devils* respectively, but not by representing a particular personality but a type of human character, a “sensible expression” in which anyone can find something of themself. In this way a piece of art may aesthetically represent reality by exhibiting the workings of the human mind and behavior.

My fantasy can in the highest degree differ from the reality that took place, and my Pyotr Verkhovensky may in no way resemble Nechayev, but it seems to me that in my astonished mind imagination has created that character, that type, which corresponds to this crime (Dostoevsky, on *The Devils*, October 8, 1870).

We have to explain how the artist playing freely with *productive imagination*, reflecting continually on his experience and evaluating the beauty of the work in its creation, can achieve harmony between the *rationality* of *intellectual ideas* and the *sensuality* of *aesthetic ideas*.

**3.3. Reflective Self-Control of the Productive Imagination in Creating the Aesthetic Product**

However, if spiritual motivation is *aesthetic ideas* emulating *intellectual ideas* to create beautiful artwork, it must have reflective self-control to achieve *harmony* between them:

**[2] Genuine Creation of Artwork with Reflective Free Play of Productive Imagination**:

-----Reflective Act of Comparison----

DEDUCTION: A QUASI-INFERENCE

Free Play of *Productive Imagination* to Reach Harmony of **Intellectual** and **Aesthetic** Ideas

HARMONY or DISHARMONY

~⎯⎯⎯⎯⎯⎯⎯⎯∧⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

*Understanding Productive Imagination* *Artwork*

**Creation Rule (Intellectual Ideas** ==============> **Aesthetic Ideas)**

(Conception of Artwork) (Exhibited Artwork)

When an artwork is created through *productive imagination*, the artist’s *intellectual ideas* harmonize with their created *aesthetic ideas* which can be achieved by playing freely with them. Intellectual ideas include rich experiential and general meanings, along with the theme of the intended artwork. The artist uses the latter in the meaningful components of pre-conceptual imagery to quasi-deduce and exhibit aesthetic epitomes, and this involves subsuming the exemplified particulars under general ideas. This is done with the elements that will attune to the initial intellectual ideas. To evaluate these elements in the creative operation the artist continuously has recourse to their *general knowledge of reality* and to their *sensual intuition*. Since this productive imagination is an unstated operation, there are no formal rules to control the exhibition of aesthetic ideas, but habitual quasi-rules are instinctively and practically self-controlled and *aesthetic ideas* are adequately inferred from *intellectual ideas*. Yet the criterion for achieving beauty is only a true aesthetic representation of reality.

**4. Discovery of New Modes of Representing Reality: Intellectual Intuitive Productive Imagination and Genuine Creative Discovery as a Metaphor**

**4.1.** **Sensual and Intellectual Intuition in the Discovery of New Concepts and Hypotheses**

Epistemically the role of intellectual intuition in the abductive logic of new scientific hypotheses is analogous to our sensual intuition in the perceptual discovery of new concepts (Nesher 2001). The scientist’s *intellectual intuition* operates with *productive imagination* and scientific background knowledge to solve difficulties in explaining reality. This is achieved because *productive imagination* operates through instinctive and practical self-control to recombine the iconic and indexical imagery, meaning, and content of background knowledge, thus arriving at a new picture of reality. The scientist formulates this into a new abstract hypothesis, so we do not need to view *a priori* intuition as a miracle, à la Einstein and Popper. We can understand the intuitive discovery of new aesthetic ideas and scientific hypotheses as *metaphors*. In creating and discovering new ideas, artists and scientists still use old expressions, such as the terminology of *space and time*, but they change the meaningful components of the imagery to elaborate new pictures, so as to replace the classical picture of physical reality with the relativist picture. The new accepted theory loses its metaphorical character as something newly discovered, and becomes merely an analogy to the old theory, e.g., the analogical pictorial imagery of Newtonian gravitational forces and Einsteinian relativity with spacetime curvatures: we use both of them, but in accordance with different proof-conditions.

In his first paper on atomic theory in 1913, Bohr emphasized that although Newtonian mechanics is violated, its symbols permit visualization of an atom as a minuscular solar system. Bohr based all of his reasoning on the following visual metaphor: The atom behaves *as if* it were a minuscule solar system (Miller 1996, 225).

But the source of the meaning and content derived from intellectual intuition lies in sensual intuition, otherwise it would remain empty abstract formalism.

**4.2.** **The Role of Intellectual Intuition and Productive Imagination in the Recombination of Scientists' Background Knowledge to Discover New Hypotheses**

The role of intellectual intuition in genuine scientific breakthroughs lies in overcoming the difficulties with interpreting scientific background knowledge to reach a new comprehensive picture of reality in order to formulate new hypotheses. The *productive imagination* of *intellectual intuition* operates on the *imagery* components of the symbols to abductively recombine these components, detecting new iconic similarities and indexical analogies in order to obtain new combinations with components of background knowledge. For example, in looking for a new intellectual image of the components of quantum theory, instead of separating images of waves and particles, the scientist can imagine a dynamic continuum of particle-wave components (Bohm and Hiley 1993). Similarly, Cervantes combines two different characters in *Don Quixote*: a brave fighter for justice and a ridiculous fantasist, a combination we can find, to differing degrees, in every one of us. Thus, *intuitive productive imagination* can play freely with different components of our experiential knowledge to create new aesthetic characters.

**[3] Intuitive Recombination of Background** **Knowledge Using Productive Imagination to Abductively Suggest a New Picture of Reality and New Intellectual Scientific Hypotheses**:

**Abductive** Intuitive Recombining Imageries  **Symbol**\*

of Background Knowledge ⎛ Icon2 ⎞

into **New Picture** | Index**1** |

**Difficulties** in Intellectual **⎛***Recombined* ⎞ | Icon**F** **|** **The New**

**Background** **Knowledge (? =** **⎨** Icons & **⎬**→**BK) Inverting to Symbols =>** | Icon**j**  **| = Intellectual**

**⎝** Indices ⎠ (From Intellectual Imagery to | Index3 | **Symbol**\*

To Scientific Formalism**)**  ⎝ Index**n** ⎠

This abductive path toward a new scientific hypothesis is the first stage of the entire scientific discovery; it continues with the deductive inference of a theoretical prediction and inductive evaluation to prove its truth. Einstein describes how he plays with productive imagination:

In the following, I am trying to answer in brief your questions as well as I am able…

(A) The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychological entities which seem to serve as elements in thought are certain signs and more or less clear images which can be “voluntarily” reproduced and combined. There is, of course, a certain connection between those elements and relevant logical concepts...

(B) The above mentioned elements are, in any case, of visual and some of muscular [kinesthetic] type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be produced at will.

(C) According to what has been said, the play with the mentioned elements is aimed to be analogous to certain logical connections one is searching for.

(D) Visual and motor. In a stage when words intervene at all, they are, in my case, purely auditive, but they interfere only in a secondary stage as already mentioned.

(E) It seems to me that what you call full consciousness is a limited case which can never be fully accomplished.

*I am enough of an artist to draw freely on my imagination. Imagination is more important than knowledge. Knowledge is limited. Imagination circles the world.* (Einstein to Hadamard, 1945)

*“Intuitive thinkers have made many of the breakthroughs in science.”* (Louis de Broglie)

**4.3. The Conscious Self-Control of Intellectual Intuition in the Discovery of a New Hypothesis**

Einstein’s expression of *thought* without words can be understood as a distinction between *imagination* and *reasoning* (Einstein 1949, 7-9). The idea is that one’s cognitive operation is meaningful when the meanings of its elements are felt in such a way that the entire operation can be communicated to others. Yet without verbalizing such an operation we hardly remember or articulate it, though we can elaborate upon it *habitually*, albeit with recourse to some explanation for it as an unconscious process, e.g., as the work of a god, muse, or some supernatural element (e.g., Plato, Kant). Thus, we say that there is no mystery in such an ingenious scientific operation. How, then, can we understand Einstein’s *unconscious thought* in the scientist’s creative imagination (Einstein 1949, 7)?

Certain obvious features of the phenomena of self‑control… can be expressed compactly… by saying that we have an occult nature of which and of its contents we can only judge by the conduct that it determines, … and since we are conscious of what we do deliberately, we are conscious *habitualiter* of whatever hides in the depths of our nature; and… that a sufficiently energetic effort of attention would bring it out. Consequently, to say that an operation of the mind is controlled is to say that it is, in a special sense, a conscious operation. (Peirce *CP*, 5.440‑441)

Yet all self-control of mental operations must maintain some level of self-consciousness to connect the phases of intuitive creativity in order to discover, elaborate, and rationally prove a hypothesis.

**5. Different Roles of “Productive Imaginations” in Artistic Creation and Scientific Discovery**

**5.1. The Roles of “Productive Imagination” in Artistic Exemplary New Representations of Reality**

The role of artistic *productive imagination* in the creation of aesthetic representations of reality lies in the artist’s *deductive* interpretation of his *intellectual ideas* into *aesthetic ideas* as epitomized artwork. This happens through the quasi-proof of this operation to ensure that the artwork is a true aesthetic representation of reality.

**[4] The Artist’s Creation of an Artwork and Reflective Free Play to Harmonize Intellectual Ideas with Aesthetic Ideas**: **The Role of Productive Imagination**

~⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯Reflective Art Creation ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯~

ABDUCTIVE, DEDUCTIVE, AND INDUCTIVE TRIO OF INFERENCES AS PROOF

Harmony or Disharmony

~⎯Reflective Suggestion⎯~ ~⎯⎯Free Play Comparison⎯⎯~ ~⎯⎯Quasi-proof⎯⎯~

**Abduction**  **Deduction Induction**

*Productive Imagination*

*Discovery* *of* *Intellectual Ideas* *Creativity Aesthetic Representation*

**((CAb (AAb**→**CAb)=>AAb)=Spirit(Intellectual Ideas=Aesthetic Ideas)=Artwork: Truth & Beauty**

(Artist Presents Reality) (Animating Principle) (Exhibited Epitomes) (Quasi-Proof Artwork)

**V 5.2. The Roles of “Productive Imagination” in the Scientific Discovery of a New Picture of Reality**

However, the role of the scientist’s *productive imagination* lies in the exercise of *intellectual intuition*, which abductively recombines the imagery-related components of background scientific knowledge to overcome its difficulties. This means discovering a new picture of reality and formulating a new hypothesis to prove its truth.

**[5] The Genuine Discovery of a Scientific Theory Using the Intellectual Intuition of Productive Imagination (Solving the Difficulties with Previous Theories)**:

~⎯⎯⎯⎯⎯⎯⎯Reflective Act of Discovery⎯⎯⎯⎯⎯⎯⎯⎯⎯~

ABDUCTIVE, DEDUCTIVE, AND INDUCTIVE TRIO OF PROOF

**Abduction**

*Productive Imagination*

*Difficulties* in  *Discovery*   *Logical Representation*

**Rule (Background Knowledge****Recombination** **Imagery Ideas)****Symbolic New Hypothesis** **Proof**

(Analysis of Theoretical Difficulties) (The New Visual Model) (Proving New Theory)

But taken from the psychological view-point, this combinatory play seems to be the essential feature in productive thought–before there is any connection with logical construction in words or other kinds of signs which can be communicated to others. (Einstein 1945)

Scientists reach the coherency of the new scientific picture only by evaluating the hypothesis experimentally, but in the imaginative phase they only feel it in regard to background knowledge.

**5.3. Artists** **and Scientists Represent Reality Through Their Cognitive Confrontation with Reality**

We can explain that our aesthetic judgments of beauty are due to the artwork’s true aesthetic representation of reality, and can be indicated through the harmony of intellectual ideas and aesthetic ideas in the creation and evaluation of artworks. But without confrontation with reality there is no ground for the objective and true creation and evaluation of artworks in our judgments of taste and in the scientific feeling of the coherence and beauty of hypotheses (Nesher 2002).

**6. Conclusion: Genuine Artistic and Scientific Works are Different Modes of Representation**

**6.1. Fine Art and Science Are Different Cognitive Procedures of Representing Reality**

There is a similarity between scientific theories and fine arts and even myths in their representations of reality. In the creation of artworks, and in their reception, comparisons are continuously made with concrete experience. The difference between artists and scientists in representing reality is that the former only instinctively quasi-prove the truth of artworks, while scientific hypotheses are proved rationally. This explains why artworks are regarded as fiction; we feel their truth only implicitly, while in science we prove it explicitly.

Every natural science will be worthless if its claims could not be tasted by observation of nature; every art would be worthless if it was no longer able to move men, no longer able to illuminate for them the meaning of existence (Heisenberg 1948, 88).

Hence, from our sensual experience and inquiries into the nature of reality we develop scientific theories and aesthetic artworks to represent reality truly and to elevate our lives within it.

**6.2. Art and Science Are Different Modes of Representing Reality: “Aesthetically” and “Logically”**

*Aesthetic* and *scientific* modes of representation differ in that the artist *aesthetically epitomizes* characters and situations, while the scientist *formulates* general theories through *logical abstraction*. Artworks evoke feelings and emotional reactions of pleasure by virtue of which we aesthetically judge them to be beautiful; they indicate their beauty and truth in an aesthetic representation of reality. The proof and truth of scientific formulations from logical abstraction are proved true at the rational level of self-control of the discovery, elaboration, and evaluation of hypotheses, yet are always relative to the accepted proof-conditions and truth-conditions, as well as the methods of proving theories.

**6.3. Art and Science Both Prove the Truth of Their Representation of Reality and Thus Have Truth in Beauty and Beauty in Truth**

Wherein lies the beauty of scientific formulas and their proofs? The icons of aesthetic representations in art and science show some similarities, and so do the indexical analogies between them in the process of representing reality. Therefore, we can hypothesize that in both cases the feeling of aesthetic pleasure that arises can be explained as a reaction to a true aesthetic representation of reality, though the modes of representation of art and science differ: the former is grounded in an individual’s attempt to epitomize something specific and the latter aims at general formalization.

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**Chapter 10**

**Gödel On Truth and Proof: Epistemological Proof of Gödel’s Conception of the Realistic Nature of Mathematical Theories and the Impossibility of Proving Their Incompleteness Formally**

No calculus can decide a philosophical problem. A calculus cannot give us information about the foundations of mathematics. (Wittgenstein [1933-34] 1969, 296)

**1. Introduction: Pragmaticist Epistemological Proof of Gödel’s Conception of the Realist Nature of Mathematical Theories and the Impossibility of Proving Their Incompleteness Formally**

In this chapter, I attempt a pragmaticist epistemological proof of Gödel’s realist conception of mathematical theories as representing facts of external reality. Gödel brought about a realist revolution in the foundations of mathematics by attempting to formally prove the distinction between complete formal systems and incomplete mathematical theories. According to Gödel’s Platonism, mathematical reality consists of eternally true facts that we can grasp with our mathematical intuition, which is an analogue of our sensual perception of physical facts. Moreover, mathematical facts force us to intuitively accept true mathematical axioms, which are analogues of physical laws of nature, and through such intuition we evaluate the inferred theorems on the basis of newly grasped mathematical facts. However, grasping abstractions by means of such mysterious pure intuition is beyond the capacity of human cognition. Employing pragmaticist epistemology, I will show that formal systems are only *radical abstractions* of our cognitive operations and therefore cannot explain how we represent external reality. Moreover, in formal systems we cannot prove the truth of axioms but only assume their truth dogmatically, and their inferred theorems are logically isolated from external reality. Therefore, if Gödel’s incompleteness theories hold, then we cannot know the truth of the basic facts of reality by means of formal proofs. Hence Gödel’s formal proof of the incompleteness of mathematics cannot hold, since the truth of basic facts of mathematical reality cannot be proved formally and thus his unprovable theorem cannot be true. However, Gödel separates the *truth* of mathematical facts from mathematical *proof* by assuming that mathematical facts are eternally true such that the unprovable theorem seems to be true. Pragmatistically, realist theories represent external reality, not abstract reality and not by formal logic but by the *epistemic logic* of the complete proof of our perceptual propositions and realist theories. Accordingly, it can be shown that all knowledge starts from our perceptual confrontation with reality, without assuming any *a priori* or “given” knowledge. Hence, mathematics is also an empirical science; however, the reality it represents is neither that of *ideal objects* nor that of *physical objects*, but rather that of our operations of counting, grouping, and measuring physical objects which we perceptually quasi-prove to be true as basic mathematical facts (Nesher 2002, V, X).

**2. Gödel’s** **Platonism and the True Conceptual Facts of Mathematical Reality**

Gödel’s insights into the fundamental nature of mathematical reasoning come from his position as a mathematical realist. For him mathematics is a science that represents an objective mathematical reality and not just a conventional formal system. Yet, Gödel's Platonist mathematics constitute an abstract science that represents true ideal mathematical entities, although it is analogous to the empirical sciences (Gödel 1944). As a *metaphysical realist*, Gödel separates the mathematical reality of true abstract facts from formal proofs, and it is only by pure intuition that we can grasp these facts. Figure 1 presents Gödel’s different conceptions of logic and mathematics:

**[1]** **Gödelian Epistemology Encompassing Three Conceptions of Logic and Mathematics:**

A B C

Russell: *Principia*

Wittgenstein-Logic: *Tractatus* Frege, Hilbert, Gödel-Platonism: Conceptual Realism

Carnap-Syntax Wittgenstein-Language: *Tractatus* of Scientific Mathematical Theories

[(Tautological) (Analytical) (Realist)]: Gödel: 1951

**Axiomatic**  **Axiomatic**  **Theories:**

**Syntactical**  **Semantical**  **Platonic** **Realism,**

**Formal System**  **Formal System**  **or** **Empirical Realism**

and its language

/|\ Representing Reality

Miraculously Assumed/|\ Miraculously Assumed /|\ Axioms ///\\\

Set of Axioms /||||||\ Set of Axioms /||||||\ / ////\\\\\

/|||||||||||\ /|||||||||||\ / | \

Rules of Inferences / | \ Rules of Inferences / | \ Rules of Proofs /Deductive\

(As Proofs) /Deduction\ (As Proofs) /Deduction\ / Consistent \

/ | \ / | \ / Inference \

/\_\_\_\_\_\_\_\_\_\_\ /\_\_\_\_\_\_\_\_\_ \_\ / ⎪ \ **Intuitive** Evaluation

/ \ Inference / \ of Conclusions on the Basis

**No External Reality** / Conclusions \ of Axioms / Conclusions \ Mathematical Facts

Simile-Reality | Assigned |

**Isolated from** | Interpretation of | **Intuitively** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**External Reality** | Intended Model | Grasped / P r o s i t i o n a l F a c t \

(Weyl 1925) ⎛ / \

(Zach 2003) Representation ⎨ / Mathematical \ (Hintikka 2005) ⎝ /  **EXTERNAL REALITY** \

Gödel’s tri-partitions are between (A) *complete* analytic formal systems with their formal syntactic *tautologies*, (B) *complete* formal semantic *analyses*, and (C) the *incomplete* realist theories of *conceptual* mathematics (Gödel [1951] 1995, 319-323; Poincaré 1902, chap. I).

The two significations of the term *analytic* might perhaps be distinguished as tautological and analytic (Gödel [1944] 1995, 139, n. 46).

Epistemologically, the *tautological* and *analytic* aspects of *complete formal systems* are, respectively, ***syntactically* closed** by their fixed axioms and formal rules of inference and ***semantically closed*** byaxioms, formal rules, and the assigned model. The *realist* *incomplete theory* is only ***relatively closed***by its relative proof-conditions, formal proofs, operations of pure intuition, and conceptual facts of external reality (Nesher 2002, X). Since Gödel’s mathematical theories are regarded as axiomatic formal systems with formal inferences, their external reality can only be grasped by pure intuition (Gödel [1931a] 1986, 203; [1964] 1990, 268).

For Gödel, pure mathematical intuition has three functions: (1) to grasp the true ideal mathematical facts of mathematical reality, (2) to use these to establish the true axioms of mathematical theories in order to infer theorems formally, and (3) to assess whether these theorems truly represent facts of mathematical reality (Gödel 1953-54, fn. 34; Nesher 2001a, 2010).Gödel’s conception of mathematical intuition is based on his mathematical experience, which he calls the “psychological fact of the existence of an intuition,” but as a “given” without any explanation.

However, the question of the objects of mathematical intuition (which incidentally is a replica of the question of the objective existence of the outer world) is not decisive for the problem under discussion here. The mere psychological fact of the existence of an intuition which is sufficiently clear to produce the axioms of set theory and an open series of extensions of them suffices to give meaning to the question of the truth or falsity of propositions like Cantor’s continuum hypothesis (Gödel 1964, 268).

How, then, can we grasp the pure meanings of such propositions with mathematical intuition? This is the essential problem confronting Gödel's conceptual realism (Gödel 164, 268).

**3. Gödel’s Incomplete Distinction Between Formal Systems and Realist Theories**

Gödel revolutionized our understanding of the nature of mathematics through his distinction between complete logical formal systems and incomplete mathematical theories (Gödel [1931] 1986, 195, 1964). However, he did not conclude this revolution, because he accepted formalist methods of mathematical proof and the subjective conception of pure intuition owing to his Platonist realism (Gödel [1931] 1986, #1).

[2] **Epistemological Gap between Logical Formal Systems and Mathematical Theories**

**Realist Theory**

**Formal System** representing reality

and its language /|\

Miraculously Assumed /|\ Axioms ///\\\

Set of Axioms /|||||||\ /////\\\\\

/|||||||||||||\ / ⎟ \

Rules of Inference / ⎟ \ /Deductive \

(As Proofs) /Deducting / Formal \

/ ⎟ \ / Inference \

/\_\_\_\_\_\_\_\_\_\_ \ / ⎟ \

/ \ Axioms *force* / \

/ Conclusions \ themselves / Conclusions \ Conclusions

Quasi-Reality | Assigned |  **Intuitively** ­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Evaluated

**Isolated from** | Interpretation in | / **Intuitively** Grasped \  **Intuitively**

**External Reality** | Intended Model | / P r o s i t i o n a l F a c t \

/ **External Conceptual Reality** \

The difference between a formal system and a realist theory lies in their proof-conditions. The formal system is by definition **hermetically closed** by its fixed formal proof-conditions without relation to external reality, whereas the mathematical realist theory is **relatively closed** by its proof-conditions. The mathematical facts of external reality, formal inferences, and pure intuition complete the representation of reality, while the axioms change as we continually grasp new mathematical facts. However, formal systems are *artificially* abstracted from human mathematical operations and cannot explain them. Thus, they can never be “ideal machines” for they lack human cognitive self-consciousness and have no self-controlled operations for representing reality (Gödel [1931) 1986, 195 and n. 70; [1951] 1995, 310; Feferman 2006; Putnam 2011; Penrose 2011). Apparently, Gödel did not completely account for his epistemological revolution as he considered the threeclasses of logico-mathematics**,** A, B, and C, to be formal systems, while neglecting the essential distinction between formal systems and mathematical theories.

The development of mathematics toward greater precision has led, as is well known, to the formalization of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system of *Principia mathematica* (*PM*) on the one hand and the Zermelo-Frankel axiom system of set theory… on the other. These two systems are so comprehensive that in them all methods of proof today used in mathematics are formalized, that is, reduced to a few axioms and rules of inference. (Gödel [1931] 1986, #1; cf. [1931a] 1986)

Gödel’s incompleteness theorem essentially shows that *PM* and *ZF* are mathematical theories, not formal systems. However, since they use formal inferences, without the help of conceptual intuition, those systems are isolated from mathematical reality. According to pragmaticist epistemology, formal inference is only one component of epistemic logic, which also includes abductive and inductive material inferences to form a complete proof of the basic mathematical facts of external reality. Yet, even after proving the incompleteness of mathematics, Gödel continued to oscillate between mathematics as axiomatic formal systems and as scientific theories, and thus he could not complete his realist revolution of mathematics (Gödel [1953-54] 1995, II; Feferman 1984, 9-11).

**4. Gödel’s Paradoxical Formal Proof of Incompleteness, Based on Separating Truth from its Proof**

If Gödel’s incompleteness holds, then mathematics is a theory and not a formal system. How, then, can Gödel formally prove incompleteness in mathematical theory when this cannot formally prove true theorems (Hintikka 2000, V)? Gödel’s formal proof of incompleteness actually uses “arithmetization of syntax,” in an attempt to prove his epistemological conception of the nature of mathematics. But Gödel’s incompleteness is a general claim that can only be proved epistemologically, and not by a specific theory about itself. Perhaps with respect to a special mathematical theorem it can be proved that a specific theory (e.g., PM or ZF) is incomplete in respect to a specific proposition and given true mathematical facts; but incompleteness theorems cannot provide a general proof of the nature of mathematics (Gödel 1944, 121).

Gödel demonstrated that the proof of the undecidable proposition G1: “I am unprovable,” can be arithmeticized by means of a *metamathematical description* in order to prove this unprovable mathematical proposition. “We therefore have before us a proposition that says about itself that it is not provable [in *PM*]” (Gödel 1931, 151). The question is whether this formal proof can be considered proof of G1: “I am unprovable”? There are two problems here: (1) Can there be metalanguages at all, since meta-descriptions of mathematical languages can, at most, describe physical-syntactical signs, following Tarski, and not their meaning or contents, which we can only interpret in respect to experience and not through abstract models (Wittgenstein [1921] 1961, [1933-34] 1969, II.12; Gödel 1953-54, fn.34, p.203; Nesher 1987; 2002, V)? (2) Can G1 be meaningful and “contentually true” such that it may eventually represent a true mathematical fact (Gödel 1931a, 203)?

If G1: “I am unprovable” is proved formally *true* in *PM*, then its claim of being unprovable is *false* because it has been proved true[in *PM*] and therefore cannot be unprovable, but when G1 is *false* then being unprovable in *PM* is *true* as claimed, thus presenting something like the classical liar’s paradox, and Gödel’s trick of using a paradoxical argument fails.

The analogy of this argument with the Richard antinomy leaps to the eye. It is closely related to the “Liar” too. (Any epistemological antinomy could be used for a similar proof of the existence of undecidable propositions.) (Gödel 1931, 149)

Since any epistemological antinomy is void of truth, this means that its proof is also void of truth. It seems that Gödel grappled with this difficulty, and his way out of this paradoxical situation was to locate the *proof* in metamathematical arithmetical language, thus separating this formal proof from the language of G1 with the assumed truth of its bizarre meaning.

From the remark that [R(q);q] says about itself that it is not provable, it follows at once that [R(q);q] is true, for [R(q);q] is indeed unprovable (being undecidable). Thus, the proposition that is undecidable *in the system PM* still was decided by metamathematical considerations. (Gödel [1931] 1986, 151)

Why did Gödel take recourse to this “epistemological antinomy” and use it in the manner of a trick without proving the incompleteness of *PM* by showing that propositions “of the type of Goldbach or Fermat” are unprovable in it (Gödel [1931a] 1986, 203)? It seems that Gödel intended to establish a general proof of the nature of all mathematical theories in respect of their infinite mathematical reality (Agazzi 1974, 24). Gödel’s Platonist realism leads him to formulate his proof with the suffix *able* as his “provable” and “unprovable” terms. This means that since there are eternal and infinite true mathematical facts that may eventually be grasped by pure intuition, they are either provable or unprovable in any mathematical theory (Hintikka 2000, 29). In such Platonic epistemology, *truth* in reality and *proof* in theories are *separate*, which enables Gödel to separate the *proof* of G1 from the *truth* of the mathematical fact it is to represent in order to prevent the classical liar’s paradox.

Finally, it should be noted that the heuristic principle of my construction of undecidable number theoretical propositions in the formal systems of mathematics is the highly transfinite concept of ‘objective mathematical truth’ as *opposed* to that of ‘demonstrability’…, with which it was generally confused before my own and Tarski’s work (Gödel in a letter to Wang, December 7, 1967, in Wang 1974, 9; Feferman 1984, 106-107; Franzén 2005, 2.4).

Hence, Gödel leans on the distinction between the liar’s proposition PL: “I am lying” and the unprovable proposition PU: “I am unprovable” since in the former we reach the liar’s paradox (if it is true then it is false and vice versa), whereas there is no such paradox of truth and falsity in the latter, since proof and truth are distinct (Gödel 1934, #7; 1951, 322-323; Hintikka 2000, 35-36; Devlin 2002).

So we can see that the class α of numbers of true formulas cannot be expressed by the propositional function of our system, whereas the class β of provable formulas can. Hence α ≠ β and if we assume β ⊆ α (i.e., every provable formula is true) we have β ⊂ α, i.e., there is a proposition A which is true but not provable. ∼A then is not true and therefore not provable either, i.e., A is undecidable. (Gödel 1934, 363)

Generally, Gödel separates the truth of mathematical facts, which can be grasped intuitively, from the formal *proof* of propositions in mathematical theories. Thus, he can also separate the attempted formal *proof* of G1 from its apparent representation of the *truth* of a fact in the mathematical reality of *PM*. Leaning on his Platonic realism, he avoids the problem of G1 being both true and false, as in the Tarskian liar’s proposition.

Thus if truth for number theory *were* definable within itself, one could find a precise version of the liar statement, giving a contradiction. It follows that truth is not so definable. But provability in the system *is* definable, so the notions of provability and truth must be distinct. In particular, if all provable sentences are true, there must be true non-provable sentences. The self-referential construction applied to provability (which is definable) instead of truth, then leads to a specific example of an undecidable sentence. (Feferman 1984, 106)

However, if the notions of truth and proof are not separated, there are no “true non-provable sentences” and “the self-referential construction” of G1 leads to an “epistemological antinomy,” a kind of liar’s paradox. Metaphysical realists, such as Platonists and formal semanticists (e.g., Tarski), assume that truth is independent of proof and, by virtue of the bivalence of truth values, or the principle of excluded middle, identify truth with reality, yet not for complete formal systems (Gödel 1929, 63; Penrose 2011, 342-343). Pragmaticists, however, argue that for human beings the truth and falsity of propositions are determined by what has already been proved as such, for we cannot know truth from a divine perspective (Nesher 2002, V). Since there is no separation between truth and being proved, we have to drop the expressions “provable” and “unprovable” from our epistemology. This terminology belongs to metaphysical realism, such as Gödel’s conceptual realism and Popper’s absolute truth, among others, as distinct from the pragmaticist’s representational realism (Nesher 2002, III, V, VIII).

Therefore, without being proved true or false, propositions remain doubtful, and since no one has proved the truth or falsity of the *liar’s* *proposition*, it is doubtful and there cannot be any paradox (Nesher 2002, V). Hence the separation of truth from proof is epistemologically untenable and so is the separation between the liar’s paradox and the unprovable-provable antinomy. Thus, with the doubtful *unprovable* proposition we cannot prove anything (Hintikka 2000:31-35). Although Wittgenstein sensed the paradoxical difficulty in Gödel’s alleged proof of incompleteness, he could not explain it without having an epistemology of truth (Wittgenstein 1937; Nesher 1992; Floyd and Putnam 2000; Floyd 2001; Berto 2009, # 9).

How can Gödel prove that his crucial proposition is not logically provable by using the very same logic? And how we can know that the proposition in question is true if we cannot prove it? (Hintikka 2000, 29)

What, then, would be the meaning of G1 if we were able to prove that it represented a true conceptual fact of mathematical reality? And can we specify the true fact that the alleged meaning or content of G1 is supposed to represent? In fact, G1 cannot represent any mathematical fact, since it is not a proposition with real subject matter or clear content and it has a shadowy meaning at best (Gödel [1931a] 1986, 203; Weyl 1949, 51; Feferman 1984, 106). However, if G1 (“G1 is unprovable”) is void of real meaning and not “contentually true” then it cannot represent any intended “mathematical objects or facts,” according to Gödel’s criticism of the syntactic conception of mathematics (Gödel [1931a] 1986, 203; [1953-54] 1995, #30; Agazzi 1974, 24; Feferman 1984, 103). Hence the arithmeticized proof of G1 is only mechanically connected to the object language and has nothing to do with its meaning (Tarski 1944; Nesher 1987, 2002, V; Floyd 2001, III). Then if G1 can be proved formally, any sentence can be proved emptily and the system or theory in which it is proved is inconsistent (Gödel [1931a] 1986, 203).

This formulation of the non-feasibility of the syntactic program (which also applies to finitary mathematics) is particularly well suited for elucidating the question as to whether mathematics is void of content [in the sense that no mathematical objects or facts exist]. For, if *prima facie* content of mathematics were only a wrong appearance, it would have to be possible to build up mathematics satisfactorily without making use of this “pseudo” content. (Gödel [1953-54] 1995, #30; Hintikka 2000, 29)

However, the meaning and contents of scientific theories are based on our experiential confrontation with external reality and mathematical reality, as well. Thus, the basic facts of mathematical reality cannot be proved formally in a theory using axioms and the question is: How can we prove their truth? Can we indeed grasp their truth by pure mathematical intuition (Gödel [1944] 1990, 21)?

It has turned out that (under the assumption that modern mathematics is consistent) the solution of certain arithmetical problems requires the use of assumptions essentially transcending arithmetic, i.e., the domain of the kind of elementary indisputable evidence that may be most fittingly compared with sense perception. (Gödel [1944] 1990, 121; cf. Gödel [1953] 1995, #34)

Gödel’s insight fits the pragmaticist understanding of the role of proofs in epistemic logic in all empirical sciences, mathematics included (Gödel [1947] 1990, 182-183; 1964, 268-269; Nesher 2002, 2007; Chihara 1982). The central problem in the epistemology of mathematical theories concerns the quest for an explanation of mathematical reality: What is it and how do we prove the propositional facts of mathematics (Kitcher 1984; Nesher 2002, X)? Since this reality cannot be described by any known axiomatic mathematical theory, there may be other methods of grasping it, such as Gödel’s mathematical intuition, which leads us to accept true mathematical facts, or rather the epistemic logic we operate to quasi-prove the truth of our perceptual judgments representing mathematical reality (Agazzi 1974, 24).

(Assuming the consistency of classical mathematics) one can even give examples of propositions (and in fact of those of the type of Goldbach or Fermat) that, while contentually true, are unprovable in the formal system of classical mathematics. Therefore, if one adjoins the negation of such a proposition to the axioms of classical mathematics, one obtains a consistent system in which a contentually false proposition is provable. (Gödel [1931a] 1986, 203)

The discrepancy between Gödel’s intuition about the realist nature of mathematics and his attempt to formally prove propositional facts can be resolved by the Peircean epistemic logic of complete proofs. Through this, we can prove the truth of the basic propositional facts of mathematics, discover hypothetical axioms, and evaluate their truth on the basis of the true facts of mathematical reality.

Nevertheless, why was Gödel’s formal proof of the incompleteness of mathematical theories accepted almost without questioning the problematic “epistemological antinomy?” It may be that Frege and Hilbert’s generation, and the next one, were captivated by the deductivist-formalist agenda and by analytic formal semantic epistemology with its hierarchies of metalanguages, which could not seriously reevaluate this proof (Dawson 1984). Since the realist’s conception of mathematics expresses the intuition of mathematicians about their work, Gödel’s incompleteness theorems are accepted naturally, i.e., there are “contentually true” propositions in the language of theory that cannot be proved except by extended axiomatic theories (Hintikka 2000, V).

**5. The Pragmaticist Epistemology of Cognitive Empirical Representations of External Reality**

It could be suggested that *formal systems are merely realist theories in disguise* or that they are *utopian* in the sense that they can be described as *impossibly “ideal machines*” to different degrees (Dawson 1984, 79; Nesher 2001b).

By the turn of this century mathematics, ‘the paradigm of certainty and truth’, seemed to be the real stronghold of orthodox Euclideans. But there are certainly some flaws in the Euclidean organization even of mathematics, and these flaws caused considerable unrest. Thus the central problem of all foundational schools was: ‘to establish once and for all the certitude of mathematical methods’ (Hilbert 1925, 35). However, foundational studies unexpectedly led to the conclusion that a Euclidean reorganization of mathematics as a whole may be impossible; that at least the richest mathematical theories were, like scientific theories, quasi-empirical. Euclideanism suffered a defeat in its very stronghold. (Lakatos 1978, 30)

Although formal systems aim to increase the power of formal computations, their efficiency is inferior to the cognitive operations of human beings in their representation of reality. The advantage of our cognitive operations lies in our having self-consciousness and self-control in confronting mathematical, physical, and other realities, which enable us to correct errors and expand human knowledge (Gödel [1972] 1990, 305-6; Nesher 1990, 1999; Hintikka 1997, 5.7; 2000, X; Putnam 2011, 15.4). From this perspective, we can understand the epistemology of the “Exact Sciences,” the subject of the Königsberg Conference of September 1930, at which Gödel announced his discovery of incompleteness; namely, that even mathematics is not pure science and is only relatively exact (Nesher 2002, X).

…as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. (Einstein 1921)

Gödel’s incompleteness theorem is about the relativity of any mathematical theory in respect of its proof-conditions in representing mathematical reality.

There is in fact in the light of hindsight a major puzzle about Gödel’s insights and about the way he put them to use. One of his greatest achievements, arguably the greatest one, was to show the deductive incompleteness of elementary arithmetic. (Hintikka 2005, 536)

Hintikka obscures the issue that the incompleteness of any scientific theory, including elementary arithmetic, is not merely due to the incompleteness of formal deductive inferences; scientific theories with their complete epistemic logical proofs are also incomplete and are true only in relation to their specific proof-conditions. Therefore, they are incomplete in respect to the reality we endeavor to represent. All knowledge of reality is based on perceptual experience, and our mathematical experience confronts mathematical reality, which cannot be comprised of Platonic abstract objects. The distinction between the completeness of axiomatic formal systems and the incompleteness of mathematical and other scientific theories is not logical but, rather, epistemological and can be proved with pragmaticist epistemic logic (Nesher 2002, 2007; Wittgenstein [1933-34] 1969, 296).

The nontrivial-ity of the proof of completeness for limpid logic must have forcefully presented the possibility to Platonist Gödel that there were propositions that were *arithmetically* true but not provable within a formal system of arithmetic. (Goldstein 2005, 154)

According to Gödel, true propositions in specific formal systems are “evident without proof,” illustrating that our cognitive confrontation with external reality cannot be formalized. Thus, basic true mathematical facts can be grasped intuitively, and on the strength of these, axioms are intuitively accepted as true without proofs.

Of course, the task of axiomatizing mathematics proper differs from the usual conception of axiomatics insofar as the axioms are not arbitrary, but must be correct mathematical propositions, and moreover, evident without proof. There is no escaping the necessity of assuming some axioms or rules of inference as evident without proof, because the proofs must have some start point. (Gödel [1951] 1995, 305)

However, since there is no truth without proof, this can only be undertaken by using quasi-proofs of basic perceptual judgments representing reality in complete epistemic logic, and this brings into play the trio sequence of logical inferences: abductive discovery, necessary deductive inference and the material inference of inductive evaluation (Nesher 2002, V, X). Hence, the impossibility of formally proving in metamathematics the theorem of unprovability is also due to the impossibility of formally proving the truth of propositional facts of external mathematical reality, “because the proofs must have some start point” and their proved truth is the “start point.” Russell also hints at the empirical assumptions of mathematics, and points out that Gödel cannot formally prove G1 in the context of an incomplete mathematical theory (Russell 1914; Nesher 2002, V). With cognitive epistemic logic, we start from the quasi-proof of the basic perceptual facts of our knowledge of reality without any miraculous “given.” Thus, we can discard transcendental *a priorism* for all knowledge is empirical (Nesher 2007).

**[3] The Entire Perceptual Operation: Complete *Trio* of Abduction, Deduction, and Induction**

**Abduction**((CAb(A→C)=>AAb)+**Deduction**((A→C) A)→CDd)+**Induction**((A, CIn)≈>(AAb→CIn))

Where: => is the abductive *plausibility connective* suggesting the concept AAb, →is the deductive *necessity connective* from which the abstract object CDd is inferred, and ≈> is the inductive *probability connective* evaluating the relationship between the concept AAb and the new experiential object CIn.

From this epistemological position, it is amazing that Gödel, using pure intuition and thus admitting the limitation of formal proofs, nevertheless attempted to prove the incompleteness of mathematical theories by incomplete formal inference ([1931] 1986, #1; [1951] 1995, 304-306; Dawson 1984, #2; Hintikka 2005, 536).

Indeed, Lakatos and Putnam's conceptions of *quasi-empirical* proofs in mathematics seem analogical to Gödel’s mathematical proofs with his intuitive grasp of true facts and other intuitive inferences. Yet, the question remains: What is proof (Putnam 1975, 61-64)? However, the Peircean epistemic logic of the trio of inferences offers a solution to the limitations of formal logic, not as a quasi-empirical method based on convention, but as an empirical quasi-proof of the truth of the basic propositions of external reality in order to reach convention, which is the only way for realism to function in knowledge, including mathematical scientific knowledge (Lakatos [1967] 1978, 36; Putnam 1975, 63-77). The pragmaticist overcomes Gödel’s Platonism by arguing that all knowledge develops from our sense-perception’s confrontation with external reality, and therefore conceptual realism with its pure intuition is only *disguised* empirical knowledge of reality. Since for Gödel mathematical reality consists of abstract entities, the analogy with the empirical sciences is incomplete. The following schema shows the perceptual quasi-proof of perceptual judgment representing external reality (cf. [3]):

**[4]** **Perceptual Experience of Interpreting Cognitive Signs in Representing Physical Objects: Quasi-Proof of the Truth of Perceptual Judgment**

I n t e r p r e t a t i o n relations evolve hierarchically

From Pre-Verbal Sensori-Motor Signs to Propositional Judgment

Interpretational relations

**Percept**-**Sign** **→Iconic Presenting →Indexical Operating →Symbol: Perceptual Judgment**

**Object Shapes  *Immediate Object* *Concept of Object***

*Feeling* Reaction Thought

\ ⎧ **Iconic**  **Indexical** ⎞ The

\ Replicas ⎨ *Feeling* Reaction ⎬ **Meaning-Content** of

\ ⎪ \  **Iconic** ⎪ **Symbol-Concept**

\ ⎝ \  *Feeling* ⎠

\ \ ⎮

 

Represented **Real**  **Physical Object**

The signs representing a *real object* constitute the *iconic* feeling of *object shapes*, the *indexical* reaction to it being the pre-symbolic representation of the *immediate object*,and their synthesis in the*symbolic concept representing* the ***real object*** through the true*perceptual judgment*. Recognizing that our knowledge springs from our perceptual confrontation with reality, we can understand Gödel’s problem with *grasping* ideal entities through pure intuition. Just as Kantian *intellectual intuition* grasps *supersensible objects*, which is only possible for a *supernatural being* (cf. Gödel [1951] 1995; Dummett 1981, 251-252). It is upon such basic knowledge that all our theories develop through the creation of hypotheses (Nesher 2008).

But despite their remoteness from sense-experience, we do have something like a perception of the objects of set theory, as it seems from the fact that the axioms *force* themselves upon us as being true. I don’t see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense-perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them, and, moreover, to believe that a question not decidable now has meaning and may be decided in the future. (Gödel 1964, 268; emphasis added; cf. Weyl 1949, 235)

We can compare this feeling of *force* to Frege’s feeling of the *force* of truth in indicative sentences:

We declare the recognition of truth in the form of an indicative sentence. We do not have to use the word “true” for this. And even when we do use it, *the real assertive force* lies not in it but in the form of the indicative sentence, and where this loses its assertive force the word “truth” cannot put it back again. (Frege [1918] 1999, 89-90, emphasis added; cf. Nesher 2002, VI.5.)

Such a feeling of the force of truth is the feeling of the self-controlled perceptual quasi-proofs of our perceptual judgments, and “the fact that the axioms *force* themselves upon us” is the feeling of the abductive discovery and inductive evaluation of the axioms as hypotheses, through the instinctive, practical, and rational operation of epistemic logic. Thus, mathematical theories are also based on perceptual experience confronting external reality. The question is, how does mathematical reality differ from physical reality (Putnam 1975, #4; 1994, # 12)?

**6. What Is the *Mathematical Reality* That Mathematical Theories Represent?**

Since all our knowledge of reality is based on perception and introspection, basic mathematical knowledge is also based on such experiences (Wang 1974, VII.3; Nesher 2002, III). *Our operations of counting, grouping, and measuring physical objects* constitute the basic mathematical reality that we initially represent when confronting our environment (Nesher 1990; 2002, V; 2007).

…the primitive man could count only by pointing to the objects counted, one by one. Here the object is all-important, as was the case with early measures of all peoples. The habit is seen in the use of such units as the foot, ell (elbow), thumb (the basis for our inch), hand, span, barleycorn, and furlong (furrow long). In due time such terms lost their primitive meaning and we think of them as abstract measures. In the same way the primitive words used in counting were at first tied to concrete groups, but after thousands of years they entered the abstract stage in which the group almost ceases to be a factor. (Smith 1923, 7)

Hence, arithmetic and geometry stem from historically basic human modes of quantitative operations on physical objects. With our sensual perception, we represent these operations, yet not the physical objects with which they are engaged or the relevant conceptual number signs, but their combination in these operations themselves. Hence, the perceptual representation of these operations, being our basic representation of mathematical reality, is “a kind of visual justification which the Egyptian employed” (Gittleman 1975, 8, 27-31; Parsons 1995, 61). Arithmetical numbers are neither *physical objects* nor *abstract concepts*, but the *conceptual components of our quantitative operations with physical objects*. We assign numbers to these intentional cognitive operations-*cum-*physical maneuvers as signs of these operations. The *discovery* of the first concepts of these operations of enumeration consists of natural numbers; and further *discovering* their expansion through abstractions and generalizations generates new mathematical hypotheses, which will be evaluated upon the extended mathematical reality (Gödel [1944] 1990, 128; 1964, 268; Martin 2005, 207; Spinoza [1663] 1985).

But consider a physical law, e.g., Newton’s Law of Universal Gravitation. To say that this law is true… one has to quantify over such non-nominalistic entities as forces, masses, distances. Moreover, as I tried to show in my book, to account for what is usually called ‘measurement’ – that is, for the numericalization of forces, masses, and distances – one has to quantify not just over forces, masses, and distances construed as physical properties…, but also over *functions from* masses, distances etc. *to real* numbers, or at any rate to rational numbers. In short – and this is the insight that, in essence, Frege and Russell already had – a reasonable interpretation of the *application* of mathematics to the physical world *requires* a realistic interpretation of mathematics. (Putnam 1975, 74)

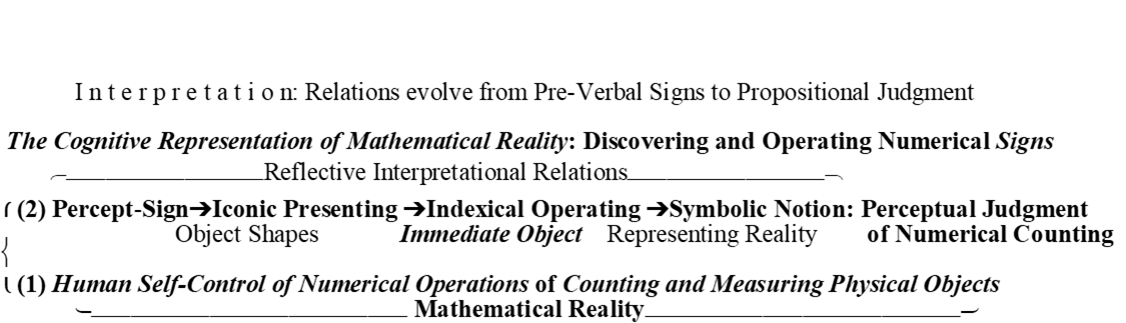
I wish to suggest here a realist view of mathematics in which mathematical reality is not an interpretation belonging to the physical reality of the physical sciences, but is instead comprised of the human operations of counting, grouping, and measuring physical objects and their relations; this is the basic mathematical reality upon whose true representation mathematical abstract and generalized theories are developed (Putnam 1975, 77-78; Weyl 1949, 235).

These basic operations are known by their perceptual representations; however, when we abstract, generalize, and further recombine the arithmetical components of these operations with our intellectual intuition, we continue to self-control them perceptually. Although the new mathematical structures are based on our perceptual confrontation in the reality of operations, when we elaborate them into more complicated kinds of mathematical structures they seem detached from their reality as abstract conceptual entities grasped by pure intuition. In fact, they evolve in hierarchical relations between *sense-perception* and *intellectual intuitions* in our knowledge of mathematical reality without this reality being divided into “two separate worlds (the world of things and the world of concepts)” (Gödel [1951] 1995, 321).

On the other hand, we have a debate between Realism—mathematical things exist objectively, independently of our mathematical activity—and Constructivism—mathematical things are created by our mathematical activity. We want to know how much of this can be regarded as continuous with the practice itself. (Maddy 1997, 191)

The question is about the relationship between our mathematical activity and mathematical structures. If they are part of an external mathematical reality, how can we know them, and if they are our constructions, how can we apply them to our empirical theories (Heyting [1931] 1964, 52-53; Dedekind [1901] 1963, 15-16)? The solution to this predicament between metaphysical realism and phenomenological constructivism is that mathematical reality *exists objectively,* yet *not independently* of our mathematical activity. Mathematical reality is made up of our intentional self-controlled mathematical operations on physical objects (such as 1 apple plus 1 apple equals 2 apples), which are connected with our perceptual representation of these operations as a certain behavioral reality. Hence, we perceptually quasi-prove the truth of our perceptual judgment that “1 + 1 = 2,” representing a mathematical operation, and thereby discover the structures of arithmetical numerical signs. Then, by discovering and proving the true representation of new mathematical operations, we hypothesize general theories, such as Peano Arithmetic; finally, by evaluating them, we extend our knowledge of mathematical reality (Smith 2007, #28.3). In this way we discover the construct of mathematical theories, although constructivists consider theories themselves to be mathematical reality and not representations of mathematical operations (Resnik 1997). Hence, it is only by quasi-proving the truth of perceptual facts representing mathematical operations that we represent mathematical reality.

**[5]** **The Double Layer of Mathematical Operations: (1) Counting Physical Objects; (2) Perceptual Quasi-Proving the Truth of Numerical Signs (Discovering and Operating Them)**

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Gödel considers abstract mathematical theories as analogous to physical theories in that their representation of abstract mathematical reality precedes their application to the empirical world, but this reality is not based on the mathematical operations themselves on physical objects:

…the applications of mathematics to the empirical world, which formerly were based on the intuitive truth of the mathematical axioms … (Gödel [1953] 1995, #12)

In contrast to Gödel’s insistence on the role of intuition in grasping the truth of abstract mathematical facts, we can perceptually prove the truth of propositional facts that represent the reality of mathematical operations (Wittgenstein [1956] 1967, III, 44). When we understand that mathematical reality consists of perceptually self-controlled operations, we can see how Gödel confuses the meaning and contents of mathematical symbols, which are the immediate modes of representing numerical operations, with his Platonist abstract mathematical objects. These immediate modes of representation are the Peircean indexical representations of real objects, which in mathematics are the factual operations of mathematical reality. Here we can discern Gödel’s clear insight into Peirce’s conception of the perceptual “immediate object” component of symbols representing mathematical reality.

It should be noted that mathematical intuition need not be conceived of as a faculty giving an *immediate* knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we *form* our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is *not* or not primarily, the sensations. That something beside the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself, whereas, on the other hand, by our thinking we cannot create any qualitatively new elements, but only reproduce and combine those that are given. Evidently the “given” underlying mathematics is closely related to the abstract elements contained in our empirical ideas. It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality. (Gödel 1964, 268)

Here Gödel’s distinction between sensual perceptions and mathematical intuitions about the reality of abstract mathematical objects is equivalent to the pragmaticist’s distinction between the immediate iconic-sensual sign and the indexical-reaction that together form the “immediate object” and the “abstract element,” which is only the sign *representing* the *real object*. Thus, Gödel's distinction is based on a confused epistemology that replaces the *meaning and contents* of such mathematical propositions with the *external reality they represent* (Gödel [1953/54] 1995 #35). It is Peirce’s conception of the cognitive “immediate object,” representing the real object that Descartes calls “objective reality” as distinct from “formal reality,” the real object, without being able to explain it as a perceptual cognitive representation of external reality (Nesher 2002, II, III, V; Feferman 1998; Parsons 2008, Chap. 6). The following is a schema of mathematical reality represented by the perceptual *immediate object* as the meaning or content of the symbolic sign of mathematics:

**[6]** **Perceptual Representation of the Cognitive Operation of Counting Physical Objects by Quasi-Proving the Truth of the Perceptual Judgment of the Mathematical Operation**

I n t e r p r e t a t i o n relations evolve from Pre-Verbal Signs to Propositional Judgment

***The Cognitive Representation of Mathematical Reality*: Discovering and Operating Numerical *Signs***

Reflective Interpretational Relations

**Percept**-**Sign→Iconic Presenting→Indexical Operating→Symbolic Sign: Perceptual Judgment of**

*Feeling* Reaction Thought **Counting: “2 plus 2 equals 4”**

**Objects Shapes  *Immediate Object* Represent Objects**

\ ⎧  **Iconic**  **Indexical** ⎞ The

\ Replicas ⎨  *Feeling* Reaction ⎬  **Meaning-Content** of

\ ⎟ \  **Iconic** ⎟  **Symbol-Concept**

\ ⎩ \  *Feeling* ⎠ I

\ \ \ I Relation of

⎨Representation

***Human Self-Controlling of Numerical Operations*** **of** ***Counting and Measuring Physical Objects***

**Mathematical Reality**

An echo of this explanation can be observed in Gödel’s insight into the realist nature of mathematics:

. . . [Mathematics] is encountered in its simplest form, when the axiomatic method is applied, not to some hypothetico-deductive system as geometry (where the mathematician can assert only the conditional truth of the theorems), but mathematical proper, that is, to the body of those mathematical propositions, which hold in an absolute sense, without any further hypothesis. There must exist propositions of this kind, because otherwise there could not exist any hypothetical theorems either. For example, *some* implications of the form:

If such and such axioms are assumed, then such and such theorems hold, must necessarily be true in the absolute sense. Similarly, any theorem of finitistic number theory, such as 2 + 2 = 4 is, no doubt, of this kind. (Gödel [1951] 1995, 305; cf. 322)

The perceptual representation of a basic mathematical operation is the quasi-proved true empirical fact of mathematical reality, but not in the ideal absolute sense. Yet this seems to be an unbridgeable gap for Penrose.

…real numbers are called ‘real’ because they seem to provide the magnitudes needed for the measurement of distance, angle, time, energy, temperature, or of numerous other geometrical and physical quantities. However, the relationship between the abstractly defined ‘real’ numbers and the physical quantities is not as clear-cut as one might imagine. Real numbers refer to *mathematical idealization* rather than to any actual physically objective quantity. (Penrose 1989, 112-113; Penrose 2011, 16:1)

Hence, Popper’s amazement as to how mathematics can be applicable to reality is resolved by explaining that mathematics indeed originated in true human perceptual representations of mathematical reality, the “empirical basis” of mathematical theory being more akin to an abstract component of this empirical science (Popper 1963, #9; Dedekind 1901, 17; Poincaré 1902, Author’s Preface, Chap. II).

**7. Mathematics is an Empirical Science Based on True Propositional Facts of Mathematical Reality**

Hence the problem lies in explaining the nature of mathematical science. For example, how exactly should we define “data,” the basic facts upon which mathematical theories develop and are evaluated?

…mathematics has always presented itself, throughout the history, as an abstract discipline, but has nevertheless always dealt with specific subject matter of its own. Considering mathematics in this light one might ask: what kind of knowledge can be attained through it? How can it be said to deal with contents and objects which are offered as ‘data,’ and yet are not data at all from the point of view of sensible experience? We are here confronted with the problem of mathematical intuition, considered as a real source of knowledge, to be clearly distinguished from that further form of mathematical activity which consists in the systematic construction of various theories. Indeed, the most delicate point of this problem is precisely the comparison between the intuitive moment and the moment of theoretical construction, since it is impossible to deny that, in many cases at least, mathematical theories are in fact an exact and systematic codification of what is known intuitively, and that, on the other hand, intuition is not sufficiently reliable unless it is supported by logical proof. (Agazzi 1974, 9-10)

A formal logical proof cannot support or replace the intuitive grasp of the true basic mathematical fact in Gödelian Platonism, and only the epistemic logic of the Peircean trio can quasi-prove the truth of perceptual judgments as basic mathematical propositional facts (Nesher 2002, X). Only this logic can replace the mysterious unexplainable intuition of mathematical facts and prove mathematical truths by the epistemic logic of a complete proof. Thus, it also replaces the assumed roles of such intuition for discovery and evaluation of the axioms of mathematical theories (Agazzi 1974, 12).

From the quasi-proof of the truth of the basic mathematical propositional facts of mathematical reality, mathematical hypotheses are abductively *discovered* to deductively infer their *predicted* theorems, and are *evaluated* inductively on the basis of empirically newly discovered and proved mathematical facts. The following outlines a pragmaticist’s epistemological explanation of the general structure and operation of the theories of mathematical empirical science:

**[7]** **Pragmaticist’s Epistemological Presentation of Mathematical Empirical Theory:**

**Empirical Theory**

//|\\

The Hypothesis ////\\\\

//////\\\\\

/ | \

/ Deductive\

/ Formal \

/ Inference \

Abductive / \

Discovery / The Conclusions \ Inductive

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Evaluation

/ Q u a s i–P r o v e d \

/ T r u e Propositional Facts \

/ of Mathematical Reality \

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

/ O p e r a t i o n s of counting, \

/ grouping, and measuring structures \

 /and motions of components of Nature: \

/ **Mathematical External Reality** \

The *proof-conditions* of *mathematical empirical theory* are grounded in *epistemic logic*, the *trio* comprising the *inferential rules of the complete proof* of the truth of basic *propositional facts* representing *external reality.* With this epistemic logic we also prove the truth of scientific hypotheses (Gödel’s axioms), through their abductive discovery, deductive formally inferred theorems, and their inductive evaluation based upon the basic *propositional facts*. Yet, Gödel’s conception of mathematical intuition covers components of the pragmaticist’s epistemic logic, which he felt through their operations but could not explain, whereas a complete proof of epistemic logic is able to show the truth of these basic propositional facts of mathematical reality and the truth of the axioms (Feferman 1998, #1; Parsons 2008, #5). Hence, empirical theories are only relatively true by being “closed” by their proof-conditions, which can change with newly discovered facts of reality (Heisenberg 1971, 43-44; Nesher 2002, V.5, X.10; Nesher 2020).

Yet if mathematical facts are facts, they must be facts about something; if mathematical truths are true, something must make them true. Thus arises the first important question: what is mathematics about? If 2 plus 2 is so definitely 4, what is it that makes it so? (Maddy 1990, 1)

Although mathematical theory is about mathematical operations of counting, grouping, measuring, and so on, the question is, how do we prove the mathematical facts that represent such operations, i.e., “what is it that makes it so” that 2 plus 2 is definitely 4? We operate in such a manner that we count with our indexical ostensions while representing this operation in our perceptual judgment as a true fact of arithmetical counting. Since all our basic knowledge comprises such quasi-proofs of our perceptual judgments, so too do the truths of our basic mathematical facts (in other words, they represent such operations of mathematical reality).

Indeed, we do not create at will the patterns of mathematical reality, but we discover the mathematical concepts of our counting, grouping, and measuring operations with physical objects in the operations of mathematical reality, and this is “[mathematics] in its simplest form, … mathematical proper, that is, to the body of those mathematical propositions, which hold in an absolute sense, without any further hypothesis” (Gödel [1951] 1995, 305; Dedekind [1901] 1963, 15-16). Epistemologically, we can understand that when we intuit the force of the truth of our basic mathematical propositions we feel that they “hold in an absolute sense,” but without conceiving the epistemic logic behind them we cannot explain them as our own empirically quasi-proved true mathematical propositions (Steiner 2000, 337-339).

Namely, it is correct that a mathematical proposition says nothing about the physical or psychical reality existing in space and time, because it is true already owing to the meaning of the terms occurring in it, irrespectively of the world of real things. (Gödel [1951] 1995, 320)

Gödel is right when he says that mathematical reality consists of neither physical nor psychical realities, but there is a specific connection between them; namely, the mathematical “world of real things” consists of our cognitive operations of quantifying components of physical reality, and the meaning and contents of mathematical signs evolve in this perceptual experience (Wittgenstein [1956] 1967, III, 44; Benacerraf 1973; Tait 1986; Resnik 1992, #1; Martine 2005, 210).

To mention another example, the Pitta-Pitta, a tribe [of aborigines] in Queensland, are able to count the fingers and toes without a system of numerals, but only by the aid of marks in the sand… (Smith [1923] 1951, 7; Gullberg 1997, Chap. 4)

This is evidence that arithmetical facts can be iconic-cum-indexical sensorimotor operations of counting and grouping using pre-conceptual signs of properties and relations. Such signs eventually develop into conceptual components, i.e., the numerical symbols involved in mathematical facts (Gödel [1951] 1995, 320).

From its earliest beginnings science has used mathematics. Counting, measuring, ordering, and estimating are basic mental operations necessary for science as well as for many other human activities, and their nature is mathematical. (Bos 1993, 165)

Hence, mathematics, from “the ubiquitous use of elementary mathematics” to “the great variety of high level applications of mathematics” (Bos 1993, 165-166), is an empirical science of the operational quantification of physical components of nature. Advanced level mathematics has evolved from the elaboration of abstract mathematical theories put forward by scientists working toward the advancement of scientific theories.

**8. Conclusion: Mathematics Is an Empirical Science Representing Its Own Reality, Neither Queen nor Servant to Other Empirical Sciences but Their Quantitative Backbone**

The challenge is to explain the difference between mathematics and other sciences and to describe the ways in which they collaborate, given that they are empirical sciences representing different realities with different roles in developing our knowledge of nature (Wang 1974, VII). Thus, in mathematics we cannot have true theories without proving them on the basis of mathematical reality. Mathematicians develop theories by generating general hypotheses as mathematical formulations of theoretical models, typically in the realm of physics, e.g., pertaining to force fields or fluid flow topology, but also with regard to all other sciences. These theories are evaluated in accordance with the mathematical reality of quantitative operations on predicted physical observations.

The rich interplay between mathematics and physics predates even their recognition as separate subjects. The mathematical work that in some sense straddles the boundaries between the two is commonly referred to as *mathematical physics*, though a precise definition is probably impossible. (Jaffe and Quinn 1993, 4)

Mathematical theories formularize models for theoretical physical hypotheses, but there is a distinction between proving the truth of mathematical theories and proving the truth of the relevant physical theories themselves (Feferman 1998).

For as far as verifiable consequences of theories are concerned the mathematical axioms are exactly as necessary for obtaining them as the laws of nature (cnf. footn. 41). If, e.g., the impredicative axioms of analysis are necessary for the solution of some problem of mathematical physics, these axioms will imply predictions about observable facts not obtainable without them. Moreover, it is perfectly conceivable that an inconsistency with observation may be due not to some wrong physical assumptions but to an inconsistency of these axioms. (Gödel [1953-54] 1995, II: #44)

That it is arbitrary to call mathematics void of content because, without laws of nature, it has no verifiable consequences also appears from the fact that the same is true for the laws of nature without mathematics or logic. Cnf. also #44. (Gödel [1953-54] 1995, II: fn. 41)

Thus, physicists and mathematicians represent different realities with their theories, and a proved true mathematical theory in the measurement of observed physical phenomena is only the condition for the evaluation of physical theories. Thus, as opposed to Gödel's conceptual epistemology of mathematics, according to the above explanation, mathematical reality is also empirical. The truth of mathematical theory enables us to experimentally prove both the truth and falsity of physical theories. We can understand Gödelian epistemic intuition about the nature of mathematical theories from this perspective, but not Quinean “mathematical naturalism,” which confuses mathematics with other sciences and identifies mathematical reality with physical reality.

When there are difficulties with a physical picture of reality and its mathematical model, such that it becomes impossible to make measurable predictions, it is necessary to investigate why we are unable to experimentally evaluate the physical hypothesis (Woit 2007, x-xiii, Chap. 14; Feferman 1998, #2, #4).

I can’t say whether string theory will ever get past its most serious hurdle–coming up with a testable prediction and then showing that the theory actually gives us the right answer. (The math part of things, as I have said, is already on a much firmer ground.) Nevertheless, I do believe the best chance for arriving at a successful theory lies in pooling the resources of mathematicians and physicists, combining the strengths of the two disciplines and their different ways of approaching the world (Yau and Nadis 2010, 304).

Hence, mathematics without the operational measurements of the predicted and eventually observed true facts of reality cannot be true and cannot be “on a much firmer ground” than physics without “a testable prediction.” Both have to prove their own truths according to “their different ways of approaching the world.”

However mathematical intuition in addition creates the conviction that, if these formulas express observable facts and were obtained by applying mathematics to verified physical laws (or if they express ascertainable mathematical facts), then these facts will be brought out by observation (or computation). (Gödel [1953] 1995, 9-III: #16; cf. ##13-15 & n. 34)

How may one understand this depiction of the relationship between intuitive mathematical truth representing its own reality and its application to physical theories to enable observable predictions (Gödel [1953] 1995, II: #15)? In the end, mathematics is neither the *queen* of science nor its *servant* but its *quantitative* *backbone*—that is, the quantified formulations of scientific theoretical models and their operations in scientific observations—without which physical and other theories cannot be evaluated experimentally (Bos 1993, #10). The question of why mathematics is considered an *exact* or *pure* science when it is empirical like other experimental sciences, can be answered by the relative simplicity of its represented reality in respect to physical and psychological realities.

Mathematics may be the queen of the science and therefore entitled to royal prerogatives, but the queen who loses touch with her subjects may lose support and even be deprived of her realm. Mathematicians may like to rise into the clouds of abstract thought, but they should, and indeed they must, return to earth for nourishing food or else die of mental starvation. They are on safer and saner ground when they stay close to nature. (Kline 1959, 475)

This is a poetic metaphor that illustrates the above explanation of the empirical nature of mathematical reality, upon which mathematical theories can be evaluated and proved true. This empirical explanation can be seen in Gödel’s late philosophical writings on the foundations of mathematics:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics... This whole consideration incidentally shows that the philosophical implications of the mathematical facts explained do not lie entirely on the side of rationalistic or idealistic philosophy, but that in one respect they favor the empiricist viewpoint. It is true that only the second alternative points in this direction. (Gödel [1951] 1995, 313)

Hence, we can experientially acquire knowledge of the mathematical facts of mathematical empirical reality.

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**Chapter 11**

**On the Nature of Mathematics and the Limitations of Peano Arithmetic:**

**The Empirical Epistemology of Mathematics and How Confused Epistemologies Affect the Practices of Mathematicians**

It is not the purpose of this work to cover the “firm rock” on which the house of analysis is founded with a fake wooden structure of formalism–a structure which can fool the reader and, ultimately, the author into believing that it is the true foundation. Rather, I shall show that this house is to a large degree built on sand. I believe that I can replace this shifting foundation with pillars of enduring strength. (Weyl 1917; Preface to his 1918)

**1. Peircean Pragmaticist Epistemology and the Empirical Nature of the Sciences**

An essential question (and one of the most difficult) in the epistemology and methodology of mathematical science is whether mathematics is an axiomatic formal game with more or less rigid rules or an empirical science representing mathematical reality. Furthermore, what can be considered proof and truth in the two alternatives? If we choose the former option, we need to ask whether mathematics should be seen as a global axiomatic formal system in which mathematicians purposively change axioms in respect to their intuitive discoveries of new mathematical propositions or, rather, as a federation of enterprises whereby mathematical axioms are intuitively discovered, forming separate systems of games, like overlapping fibers in a thread, à la Wittgenstein? As for the latter option, if mathematics is an empirical science, analogous to other sciences, as Gödel suggested, we may ask: What exactly is this empirical reality that mathematical theories are supposed to represent?

While I can understand certain aspects of Platonic realism and the intuitionistic logic of proofs, I cannot accept their ideal or mental realities, since only empirical realism based on epistemic logic can explain our confrontation with and representation of external mathematical reality (Nesher 2002, 2007a, 2011). The question is this: to what extent do epistemology and methodology affect mathematical science?

The disagreements concerning what correct mathematics is and the variety of different foundations affect seriously not only mathematics proper but most vitally physical science. As we will see, the most well-developed physical theories are entirely mathematical... Hence scientists, who do not personally work on foundational problems, must nevertheless be concerned about what mathematics can be confidently employed if they are not to waste years on unsound mathematics. (Kline 1980, 7)

The above shows how controversy and confusion over the epistemological foundations of mathematics can affect the progress of its scientific theories. With pragmaticist epistemology, we can see mathematics as an empirical science representing basic mathematical reality, this being our counting, grouping, and measuring of physical objects. Through abstractions and generalizations from these basic theories, mathematicians develop formularized structures, called models, to enable other sciences to make predictions and evaluate their theories by employing mathematical operations conducted upon physical reality (Nesher 2011).

Peano axioms, or Peano Arithmetic (PA), can be considered as relatively true with regard to mathematical reality. These axioms do not constitute a complete game with rigid rules of formal inference, which would be considered as proofs in formal systems, as Peano probably believed. Indeed, a theoretical representation of reality cannot be constructed by formal inferences only, and this is why mathematicians practically compensate for their limitations by relying on indefinable intuitions. However, complete mathematical proofs can work by means of what I call epistemic logic, which also makes it possible to prove true propositional facts of mathematical reality; on the basis of these, mathematicians discover hypotheses in order to evaluate whether these basic facts are true theoretical representations of mathematical reality (Nesher, 2002: X, 2007a, 2011).

Thus, PA can only be true relative to its proof-conditions, as expressed in Peano’s standard model of natural numbers (NN), but only as far as this model presents mathematical reality. This is so, since axiomatic formal systems only disguise mathematicians’ real confrontations with mathematical reality; as they do not know it explicitly, they might miss its true representation. Thus, they use their intuition to represent, at least implicitly, the mathematical empirical reality of counting, grouping, and measuring with natural number-symbols, operations that are basic facts of arithmetical theory (Kronecker 1884, 1891; Weyl 1918; Feferman 1998, 1999; Davies 2005; Nesher 2011).

However, PA deviates from these proof-conditions and from the meanings of these basic mathematical facts by introducing formal structures that are incompatible and conflict with them. PA is therefore doubtful and at most can be proved false, as in the Fregean set of sets and the Cantorian continuum hypothesis (CH) (Zach 2006) as I will try to explain below.

**2. Can Sciences be Constructed as Axiomatic Formal Systems? How Can We Understand the Nature of Mathematical Science?**

**2.1. Gödel’s Basic Insight into the Real Nature of Mathematical Truth**

Gödel’s basic insight into the real nature of mathematical truth comes from his position as a metaphysical realist. For him mathematics is a science that represents mathematical reality, not just a conventional formal system. It is an abstract science, based on his Platonic conception of mathematics, and although it represents true ideal mathematical reality, it is analogous to the realistic nature of the empirical sciences (Gödel 1944). Gödel assumes that the mathematical reality of abstract true facts exists separately from mathematical theories and formal proofs, and that it is only by pure intuition that we can grasp the relevant facts and axioms of the theories in question, as well their evaluation in mathematical reality. Hence, Gödel distinguishes between axiomatic formal logical systems and realistic mathematical science. Gödel’s tri-partitions are between (A) Complete syntactic formal systems with their formal tautologies, (B) Complete analytic formal semantics, and (C) Incomplete realist theories of conceptual mathematics (Gödel [1951] 1995, 319-323; Poincaré 1902, Chap. I; Nesher 2011, # 2).

Epistemologically the *tautological* and *analytic* aspects of complete formal systems are, respectively, **syntactically closed** by their fixed axioms and formal rules of inference and **semantically closed** by axioms, formal rules, and their assigned models. A realist incomplete theory is only **relatively closed** by its relative proof-conditions: the formal proofs and pure intuition, which is also operative in grasping the conceptual facts of external reality (Nesher 2002, X, 2011). Gödel's epistemological revolution about the nature of mathematics and its incomplete theories representing mathematical reality shows the limitations of the two types of complete formal systems, syntactic and semantic, as closed games (see, e.g., Hilbert [1925] 1967). Yet Gödel did not complete his revolution and continued to consider mathematics axiomatically instead of hypothetically.

**2.2. The Nature of Mathematical Science: Proofs of Hypotheses, and What** **Theories Represent**

The basic question is about the nature of mathematical science, its proofs, and what mathematical theories represent. However, mathematical practitioners disagree about definitions of proof, truth, and adequate inference in mathematics; furthermore, they differ from philosophical epistemologists in their views of the nature of mathematics, its foundations, and how the differences in approach among mathematical practitioners concerning these issues may be explained. Various epistemological and methodological schools of thought about the nature of mathematics (e.g., the controversies between Kronecker and Cantor and between Hilbert and Brouwer-Weyl) might also affect mathematical practices.

However, it seems that the intuitions of mathematicians are based mainly on their experiences and on the commonly accepted conventions of mathematical operations and proofs, even though there are disagreements about some basic definitions, axioms, hypotheses, and theorems, and their proofs and disproofs. On the basis of different experiential intuitions, there are also epistemological controversies, such as the conception of the continuum, with the CH, and whether a given contention can be proved or disproved if it is epistemologically vague or incorrect (Feferman 2011, 26). Overall, these different approaches might affect intuitions andresult in different views about the global nature of mathematics. As such, it would be helpful to look at the history of mathematics and the development of its philosophical perspectives.

Hence, I suggest that mathematicians and philosophers of mathematics should consider that number-symbols are not freely created from nothing, nor are they ideal entities grasped by mysterious intuitions. Rather, they have been discovered by human beings in their operations of counting physical objects to enable them to exert self-control over their environment; they are not to be considered as objects in themselves, whether physical (Tarski) or mental (Brouwer, Resnik, Chihara), nor as ideal abstract entities (Gödel). Natural numbers have become a sort of fetish, and mathematicians have enslaved themselves to illusory objects instead of being their masters, while at the same time applying them to reality (similar to Marx’s analysis of commodities in *Capital*). Thus, Dedekind, in his conception of the free creation of numbers, confuses their a priori creation with their discovery in the experience of counting physical objects (Dedekind 1888, 15). The difference between these two basic conceptions of natural numbers and other mathematical symbols entails a totally different picture of the nature of mathematics. Hence, it makes a difference whether we operate with number-symbols to count, group, and measure physical objects, or operate with them as objects themselves detached from their basic meanings in mathematical reality and from their roles in other sciences (e.g., Weyl 1949, II.6; Popper 1963; Jaffe and Quinn 1993; Hilton 1997; Brown 1999, Chap. 4; comp. Nesher 2011). The confusion between these two different operations makes for paradoxes such as those of Zeno with regard to the continuum and Russell’s set-theoretic paradox.

**2.3. The Epistemological Problem with Ontological and Domain Models in Formal Semantics**

With the pragmaticist’s epistemology, we can highlight the problem with explaining our cognitive representation of reality and show the reasons why semantic models cannot help. However, we have to distinguish between the roles of *formal models* in logical and mathematical axiomatic formal systems and those of mathematical *theoretical models* in empirical sciences (Giere 2004). The epistemological problem with ontological conceptions and the domain or model in formal semantic axiomatization relates to knowing the meaning of the model. If we do not empirically experience mathematical reality, we cannot explain the meanings of the axioms themselves, since meanings originate from our experiential confrontation with mathematical reality. But even if we instinctively and implicitly confront mathematical reality experientially, we cannot explicitly explain the experiential meanings of the axioms or describe the ways in which they are connected with mathematical reality. Thus, it is not possible to show how the meanings of the intended models themselves relate to this reality.

Moreover, with the concept of mathematical intuitions alone, we cannot epistemologically explain our understanding of the meanings of the accepted axioms and their semantic model; intuition, then, remains a vague concept of a mysterious operation. Dealing with formal semantic axiomatizations, conceptions of ontology and models are merely artificial substitutions for reality, since such a semantic approach cannot epistemologically explain how we experience and know external reality. Metaphysical realists simply assume it and internal realists just avoid it; for the intuitionists and constructivists, mathematical reality consists of mental operations. However, as I suggest, we can explain our knowledge of external reality through Peircean epistemology and its theory of meaning and truth (e.g., Nesher 2002, III, X).

Syntactic and semantic axiomatic systems are only radical abstractions from the real cognitive operations of sciences that isolate them from the relevant reality they intend to represent. Thus, they are artificially abstracted from basic human mathematical operations and cannot explain mathematical scientific knowledge of reality. Mathematicians intuitively assume that axioms are true without proving them, showing a sort of faith, à la Kant. Their formally inferred conclusions cannot be evaluated formally against the basic empirical facts. According to my Peircean epistemology, these basic facts are our quasi-proved, true perceptual judgments; they are themselves propositions that cannot be proved formally.

**3. Peano Endeavors to Formalize Arithmetic in the Euclidean Axiomatic System Following Boole, Frege, and Dedekind**

**3.1. What is the Nature of Peano Arithmetic: A Formal Closed Game or a Disguised Theory?**

Axiomatic formal systems, I claim, are epistemologically sterile in respect to their relevant external reality, and formal “proofs” are only inferences from our unexplainable acceptance of the axioms. Hence, Peano Arithmetic contains the standard model of the sequence of natural numbers without even explaining how we have certain knowledge of this or what it means. How, then, does Peano know the meaning and truth of this arithmetical model and how can he be certain that the theorems inferred from these axioms are true to this model, i.e., if the model represents arithmetical external reality? This model is itself only intuitively assumed from the implicit feeling of reality or from the accepted arithmetic axioms. Therefore, if the axioms represent the seeming facts of the model through the inferred theorems, can this be considered proof of the formal axioms in accordance with the arithmetical model or, rather, is it only the formal circular completion of the expository game (Peano 1889, #1)?

On the basis of his axiomatization, Peano constructed the entire theory of natural numbers. In particular, he showed how the elementary theorems of arithmetic can be obtained from his axioms (Peano 1895-1905). It should be noted that Peano’s axiomatization was to a significant degree inspired by the ideas expressed by R. Dedekind in his treatise (Dedekind 1888).

Peano’s axiomatization is characterized by the fact that it is categorical, in that it possesses completeness of content. In other words, in substantive arithmetic Peano’s axioms determine the system of natural numbers, up to isomorphism [i.e., completely, such that any possible axiomatic system of arithmetic is substantially identical with Peano’s]. (Styazkin [1964] 1969, 279-280)

Yet, this “completeness of content” of PA is based on the complete model, which enables the formulation-cum-interpretation of this axiomatic system. How, then, is this model constructed and how does it obtain its meaning? If one assumes arithmetical external reality as Gödel understands it, PA must be incomplete (Gödel 1931; Paris and Harrington 1977; Kline 1980, IX; Nesher 2011). Thus, the question is about the nature of PA, whether it is a closed game with rigid normative rules as it seems to be, or whether it is an empirical theory disguised as a closed game? However, since a closed game is only an illusion, because it is one of empty signs, then it must be a disguised empirical theory, one that only partially and inconclusively represents mathematical reality.

**3.2. PA as a Formal System is Only a Disguised Empirical Theory that Cannot Eliminate Confrontation with Mathematical Reality**

From the pragmaticist’s point of view, all human knowledge starts from our confrontation with reality. Our basic knowledge develops in our perceptual operations, which are practically self-controlled and thereby quasi-prove our perceptual judgments of facts representing external reality. However, descriptive representations, when established, also contain our rules of habit; we self-control our operations in reality, and thus description and norms are not dichotomously separated as is traditionally accepted in philosophy (e.g., Hume [Hadson 1969]; Pigden 2010). As we acquire knowledge of reality more comprehensively, our norms of behavior evolve such that our self-control in reality is no longer a closed game with rigid normative rules; so, too, with the development of science and mathematics (Nesher 1983, 1994, 1999; Habermas 1998).

In light of all this, how should we understand Peano Arithmetic in respect to arithmetical reality? The problem is at once epistemological and arithmetical, since if we represent arithmetical reality partially and consider it only as sequences of different kinds of numbers, then we continue to be led astray in our interpretations and face difficulties, antinomies, and unsolved hypotheses (e.g., Weyl 1918; Feferman 1998, 1999, 2005, 2011). On the face of it, we consider mathematics as a pure and exact science, but this is only due to its relatively simple reality of counting, grouping, and measuring physical objects. We carry out these processes using our discovered arithmetical numerical symbols and other mathematical symbols in respect to our models of physical and psychological realities. Without understanding the epistemology of mathematics as an empirical science, we run into scholastic mathematics, with which we cannot represent mathematical reality. This also holds for PA and Cantorian scholasticism. Both are misleading because of their view of numbers and their sets as objects constituting mathematical reality, which can be interpreted as either Platonic ideal objects, intuitionist mental objects, or constructivist nominal objects (cf. Weyl 1918, 48; Feferman 1998, 1999; Nesher 2011). In some epistemological perspectives, PA is considered a complete axiomatic formal system, a game we play with normative rigid rules and that can lose contact with arithmetical reality. The epistemological confusion about PA is the undecided question of whether we accept *formalism*, such that PA = > (2 + 2 = 4), or *intuitionism*, such that (2 + 2 = 4) = > PA. Alternatively, we can understand mathematics in terms of realist epistemology, either *Platonic* realism or, better still, *empirical* realism (Maddy 2011, 114; cf. Weyl 1918, 48; Nesher 2011).

**3.3. The Illusion that Mathematics is a Formal Axiomatic Science Curtails the True Representation of Mathematical Reality** **and** **Represents Reality Only Partially and Inconclusively**

From a pragmaticist’s epistemological point of view, our basic knowledge of mathematical reality originates in our perceptual quasi-proofs of the truth of the perceptual judgments representing our operations of counting physical objects and discovering arithmetical numerical symbols. Hence, with a true representation of these operations using the discovered sequence of natural numbers, we do not need the mysterious intuition of mathematicians like Dedekind, Peano, and Gödel or of intuitionists like Brouwer and others, since we can empirically prove the basic facts of mathematics. Nor must we prove them logically with set theory in the manner of Frege, Russell, and others, by assuming *a priori* formal logic and the implicit intuition of the sequence of natural numbers (Nesher 2011; Edwards 1995, 49-52; comp. Parsons 2008, #32).

The structures and inferences of axiomatic formal systems are isolated from external reality, since deductive formal logic cannot prove the truth of the axioms nor can it evaluate the truth of the inferred theorems. These steps can be undertaken only by the abductive logic of discovery and the inductive logic of evaluation; these are aspects of material logic, and the meanings of their components are essential to their work. Thus, the deficiencies of formal systems are connected to our understanding of the conception of number and the nature of mathematics. It can be shown from pragmaticist epistemology that antinomies and paradoxes in mathematics arise from the formal epistemology of science, mathematics, and logic (Nesher 2001, 2002, 2007, 2008, 2011; comp. Feferman 1977, 1998, 2011; also Landry 2012, on definitions from nowhere).

However, along with PA, insofar as we avoid interpretations that contradict empirical arithmetical reality, we can work consistently with it. This is due to the fact that when mathematicians base their conceptions only on intuitions of natural numbers, this separation from our basic empirical experience denatures the experiential meanings of numbers and other mathematical symbols. Without our rational self-control of these operations, we cannot guarantee that this will hold generally and that we would not develop false or at least doubtful new axioms and inferred theorems with PA (Weyl 1918, 45-50; Bouwer 1949, 90; Feferman 1998; 1999;, 2011, 26). Still, given the correct realistic interpretation, PA can provide a basis for mathematically developed theories without the need for set theoretical foundations; from the realistic point of view, PA is also an incomplete theory and cannot be a closed game (Feferman 1992; 1999,15; cf. Paris and Harrington, 1977). As Kronecker suggested, all our mathematical knowledge is based on our operations with natural numbers; by misunderstanding their nature, we can go astray with our mathematical theories (Edwards 1995).

My investigations began with an examination of Zermelo’s axioms of set theory, which constitute an exact and complete formulation of the foundations of Dedekind-Cantor theory... My attempt to formulate these principles as axioms of set formation and to express the requirement that sets be formed only by finitely many applications of the principles of construction embodied in the axioms–and, indeed, to do this *without presupposing the concept of the natural numbers*–drove me to a vast and ever more complicated formulation but, unfortunately, not to any satisfactory result. Only when I had achieved certain general philosophical insights (which, incidentally, required that I renounce conventionalism), did I realize that I was wrestling with a scholastic pseudo-problem. And I became firmly convinced (in agreement with Poincaré, whose philosophical position I share in so few other respects) that *the idea of iteration, i.e., of the sequence of natural numbers, is an ultimate foundation of mathematical thought*–in spite of Dedekind’s “theory of chain” which seeks to give a logical foundation for the definition and inference by complete induction without employing our intuition of the natural numbers. (Weyl 1918, 48)

When we read “*the idea of iteration, i.e., of the sequence of natural numbers, is an ultimate foundation of mathematical thought*” this raises the question: Does this mean that in the *basic operation of* *counting physical objects* we begin *with the natural numbers* we have discovered or that since we already have these numerical-symbols, we just make **calculations** *using the sequence of natural numbers with our operators or functors,* to wit: *+ (a, b), – (c, b), x(a, b), :(c, a) =b, and more.* This distinction is essential in order to understand the nature of mathematical science and its reality and the elaboration of mathematical theories.

**4. Pragmaticist Epistemology of Meaning and Proof in Mathematics Representing Reality**

**4.1. The Nature of Mathematical Empirical Science and the Reality it Represents**

The meanings of natural number-symbols are unexplainable without knowing their experiential origin and their roles in mathematical reality, and without their meanings they appear only as physical objects with specific shapes. Yet we come to know aspects of their meanings implicitly, since we experientially discover them in learning to count and, even more so, in participating actively in mathematical reality. This also holds for Kronecker, Dedekind, Cantor, Frege, Poincaré, Peano, Hilbert, and Weyl.. Yet, Kronecker and Weyl base the meaning of NN on our experience of counting, which is what gives meaning to mathematical symbols. However, without considering what is being counted or the distinction between *counting,* *calculating,* and *iterating*, it retains only partial meaning. Others, though, base their conception of numbers on intuition, but this entails that the experiential meanings of numbers are overlooked (Kronecker 1887, 1891; Weyl 1918; cf. Edwards 1995; Boniface 2005; Fine 1998; Feferman 1998, I, V: ## 12, 13.2; Hinzen 2003; Nesher 1990)*.* In Weyl’s explanation:

A set-theoretic treatment of the natural numbers such as that offered in Dedekind (1888) may indeed contribute to the systematization of mathematics; but it must not be allowed to obscure the fact that our grasp of the basic concepts of set theory depends on a prior intuition of iteration and of the sequence of natural numbers. (Weyl 1918, 24)

Set theory developed and provided a foundation for arithmetic and mathematics in general. If we can establish an experiential empirical basis for arithmetic and thus for more abstract mathematics, then, à la Kronecker, we do not need Cantorian or Zermelonean set theory, and we can escape from antinomies and paradoxes. However, without an epistemological understanding of our involvement in mathematical reality, our basic experience of counting physical objects using discovered arithmetical number-symbols precedes the construction of any set theory. Therefore, we do not require formal logical set theory for the foundations of number theory, since we already have a basic experiential explanation of the roles of natural numbers in the operations of mathematical reality. Formalists do not have an epistemological empirical explanation of the quasi-proved truth of such experiential operations; they prefer to assume such logical structures. Indeed, the meanings of these structures are accepted without any explanation and without understanding the real basis of arithmetical operations. Hence, they neglect the explicit meanings of arithmetic symbols, which they accept as Platonic forms or as mental or syntactic objects of mathematics (Frege, Dedekind, Cantor, Peano, Hilbert, Brouwer, Gödel, and all such traditions [Nesher, 2011, 2013; e.g., McCarty, 1995 on Dedekind]).

If mathematical science represents mathematical reality, then we do not need indispensability arguments for the existence of mathematical entities. Mathematical theories are proved true on the basis of mathematical reality, which is a prerequisite if they are to work in other empirical sciences, and it is not thanks to the latter that they are true. The indispensability of mathematical theories for other sciences is reflected in the essential support they provide for scientific development (e.g., Galileo, Newton, Lagrange, Laplace, Gauss, Lobatchevsky, Euler, Maxwell, and Einstein; cf. Kline 1985; Putnam 1971, VI-VIII; Feferman 1992; Hellman 1992).

The question is whether in the future we might discover and prove new facts of arithmetical reality that would extend or elaborate new arithmetical theories, with the potential to replace or extend Peano Arithmetic. The problem is that basic arithmetical reality is as simple as the basic operations of counting, grouping, and measuring, and therefore arithmetic seems to be an *exact and pure science* or a closed game. However, in order to answer the above question as to whether PA is incomplete in respect to mathematical reality, we need to conduct more research. Without this further elaboration, we may fail to understand its essential meaning and structure and develop remote abstract structures, such as the Cantorian paradise of scholasticism, as expressed by Weyl, and therefore be led astray from the experiential meanings and proofs of mathematical reality (Weyl 1918, e.g., 45-50; Feferman 1998, #13.6, K(α); Nesher 2011, #6).

In the mathematical foundations of the 19th and 20th centuries, there was evident confusion between logical and mathematical theories and their realities, such that symbolic conceptions and their structures were accepted as logical and mathematical reality. This produced difficulties, antinomies, and paradoxes. From this epistemological confusion, the question arises: What are the objects of set-theories? After all, numbers and other logical and mathematical symbols are components of theories and not their objects.

Thus, do we even need the formal logical or intuitional foundations of mathematics if we can show and prove with epistemic logic that mathematics and logic are different sciences representing different realities and, consequently, are based on different empirical foundations (Hintikka 1995; Feferman 1998, 1999, 2011; Nesher 2013)?

**4.2. The Conception of Numbers in Mathematics as Symbols Involved in Our Operations in Mathematical Reality**

We explain the conception of numbers in mathematics as symbols involved in our operations in mathematical reality. How, then, can we understand the notions of definitions, axioms, theorems, and proofs in mathematics? If we can show that the origins of numbers lie in our experience in reality, we can forego a priorism and vicious circles and understand that formal models are only artificial realities. We can then replace the implicit intuitive feeling of validity with material logic inferences, and ostensive definitions can be perceptually quasi-proved experientially.

It was often said that mathematics has to start with definitions and that mathematical propositions have to be deduced from those definitions and from ground postulates. However, definitions per se are already impossibility, as Kirchhoff underlined, because each definition uses its own concepts, which in their turn have to be defined, etc. (Kronecker 1891, in Boniface 2005, 145)

Although Kronecker may suggest that our mathematical knowledge develops from experience, the question remains: Which experience? We do not have to confuse *counting* objects and *iterating* or *calculating* numbers as Boniface seems to do in the following quotation:

For Kronecker, on the other hand, positive integers were the only numbers to be accepted as basic arithmetical objects, because they were the only numbers to be consistent with the experience of counting. It was then not necessary to create other entities which, moreover, would denature the concept of number. Thus, such unnecessary creations were to be avoided. Such was the nominalist aspect of Kronecker’s conception. (Boniface 2005, 149)

It would be preferable to interpret Kronecker’s epistemology as implicit experiential realism, since it is not only the names of numerical symbols that are considered but also the concept of numbers that we actually discover in our experiential operations of counting (Kronecker 1891). Kronecker has difficulty in explaining how our experience of counting NN can supply a solid base for arithmetic theory. He does not offer an epistemological explanation of how ostensive definitions can be quasi-proved true in our arithmetical experience of counting and thus become the true empirical basis of the concept of number-symbols of arithmetic and mathematics in general (Feferman 1998, I, V: ## 12, 13; Nesher 2005, 2011).

Thus, the portion of classical mathematical analysis that can be formalized in (∏ ̊∞ -CA) [the class of all arithmetical formulas], following Weyl’s plan, rests on first-order Peano Arithmetic as a foundation. Since the general notion of real numbers is defined in a wider system, the conservation result shows that such uses of the uncountable in classical analysis can be eliminated. (Feferman 1998, 243 & #12)

Real numbers as mathematical symbols cannot be used for counting physical objects, since they are “uncountable”; however, they can be used for measuring continuous objects. Since they are not objects to be counted, we should not become entangled in the problem of how to count them unless we consider them as *given* physical objects that can somehow be counted. Moreover, we cannot experientially consider the sequence of natural numbers and the sequence of real numbers under the same category of infinite sets, since the meaning of the operational role of N lies in counting, and the meaning of the operational role of R in measuring. Yet we count physical objects, not numbers, and we do not measure numbers but rather physical continuous objects. We cannot have sets of sets, since sets are mathematical symbols, not objects. Along the same lines, we cannot count the physical continuum either, nor build it from R symbols or points, since they are sizeless; we can only measure it with them. Hence, from this epistemological perspective, we can avoid Cantorian set theory paradoxes and the CH, in line with Kronecker, Poincaré, Weyl, Brouwer, and others (Feferman 1998, 1999, 2005, 2011). It seems that these different epistemological approaches can be explained by Hilbert’s distinction between Cantorian axiomatic abstraction and Kroneckerean experiential perspectives, namely as a distinction between axiomatic and genetic methods (Hilbert 1900; cf. Hinzen 2003; Sieg and Schlimm 2004, 4; Feferman 1977, 2005; Giaquinto 2002, II, VI; Landry 2012).

If, as I advocated, we give a precise meaning to the concept of “set,” then the following assertion gains a substantial content: “To every point in a line (given an origin and a unit of length) there corresponds a (distance-measuring) real number… This assertion establishes a noteworthy connection between something given in the intuition of space and something constructed in a logical conceptual way. But, clearly, this assertion far exceeds everything which intuition teaches, or can teach, us about the continuum. For it does not offer a morphological description of what presents itself to intuition (that being, first and foremost, a fluid whole rather than a set of discrete elements). (Weyl 1918, 49)

The difficulties with the Cantorian perspective on mathematics are seen in the CH and, as Cohen proved, the CH cannot be proved from a system of axioms for set theory; therefore, the ZF set theory is incomplete in respect to the CH, which is independent of it. Yet the question remains about the nature of mathematics as a science and whether CH and ZF are both incompatible with the realist-*empiricist* conception of mathematics? Both ZF and CH are built on artificial models, and not on proofs of empirical mathematical reality; thus, the method of “forcing” is also based on constructing an extended artificial model that provides an interpretation of CH proving that it is meaningful and independent of ZF. But if we understand them both within the empirical epistemology of mathematics, can we show that they are not false or true but doubtful and, moreover, mathematically senseless? Hence, I am also looking for “a more profound understanding of the concepts underlying logic and mathematics,” to quote Gödel, since difficulties with the CH and conceptions of set theory seem to be due to epistemological confusions (Weyl 1918, 1921; Brouwer 1930, 1949; Webb 1995; Feferman 1998, 1999, 2011; Giaquinto 2002, II, VI; Nesher 2011, 2013).

As for the continuum problem, there is little hope of solving it by means of those axioms of infinity which can be set up on the basis of principles known today (the above-mentioned proof for the undisprovability of the continuum hypothesis, e.g., goes through for all of them without any change). But probably there exist others based on hitherto unknown principles; […] which a more profound understanding of the concepts underlying logic and mathematics would enable us to recognize as implied by these concepts. (Gödel 1947, 520-521/182)

However, in accordance with my suggestion, it is not a problem of better formalism or formal axiomatic systems, as Gödel and Feferman hinted, but of an epistemological perspective that would help us to understand mathematics as an empirical science and numbers as symbols of its theories. This can be seen in Feferman’s consideration about the nature of the continuum, though he does not deal here with the epistemology of mathematics:

Is CH a definite problem as Gödel and many current set-theorists believe? Is the continuum itself a definite mathematical entity? If it has only Platonistic existence, how can we access its properties? Alternatively, one might argue that the continuum has physical existence in space and/or time. But then one must ask whether the mathematical structure of the real number system can be identified with the physical structure, or whether it is instead simply an idealized mathematical model of the latter, much as the laws of physics formulated in mathematical terms are highly idealized models of aspects of physical reality. (Herman Weyl raised just such questions in his 1918 monograph *Das Continuum* [28].) But even if we grant some kind of independent existence, abstract or physical, to the continuum, in order to formulate CH we need to refer to arbitrary subsets of the continuum and possible mappings between them, and then we are dealing with objects of a higher level of abstraction, the nature of whose existence is even more problematic than that of the continuum. Here we are skirting deep philosophical waters; let us retreat from them for the moment. (Feferman 1999, 110)

A geometrical line is the sign of continuous physical objects, and numbers are mathematical symbols, with which we genuinely operate in counting, grouping, and measuring, such that with cardinal numbers we count physical objects, with ordinal numbers we order objects into groups, and with real numbers we measure physical objects. Arithmetical symbols are not physical objects with which we can construct continuous physical objects just as, for example, we cannot build or fill our bodies with our ideas. With the continuum problem, we intend to confuse the *token* of the geometrical line sign with the physical continuous object, thereby confusing the sign itself with its object. Evidence of this is that we do not measure the sizes of geometrical lines but only assume them to be ideal entities and prove their relationships, e.g., in Pythagorean theorems. Therefore, the Cantorian conception, according to which continuous objects consist of real number cognitive symbols, stems from epistemological confusion because of the assumption that numbers are objects. Symbols or sign-points have no physical size to make up any physical object, and the tokens of symbols are only our physical way of fixing and generalizing our ideas to remember them better and communicate them to others. Thus, CH is epistemologically meaningless as presented by Cantor and other set-theorists, who confuse symbols and objects. In the same manner, in their conception of set theory symbols are considered physical objects such that the symbols of one set become the objects of another, thus speculating a mathematically impossible Cantorian paradise.

The view of the flow consisting of points and, therefore, also dissolving into points turns out to be false. Precisely what eludes us is the nature of the continuity, the flowing from point to point; in other words, the secret of how the continually enduring present can continually slip away into the receding past... So we can gather the following concerning objectively presented time… an individual point in it is non-independent, i.e., is pure nothingness when taken by itself, and existing only as a “point of transition” (which, of course, can in no way be understood mathematically)… When our experience has turned into a real process in a real world and our phenomenal time has spread itself out over this world and assumed a cosmic dimension, we are not satisfied with replacing the continuum by the exact concept of the real number, in spite of the essential and undeniable inexactness arising from what is given. (Weyl 1918, 91-93)

With the discrepancy between the intuitive concept of the continuum and the exact concept of the real number of the continuum, we need epistemological clarification of the conception. This can be achieved by appealing to the distinction between mathematical symbols and the mathematical reality in which they are involved (Weyl 1918, 1921; Brouwer 1930; Feferman 1998; Mancosu 1998; Longo 2001). Feferman’s discussion of these issues runs along the same lines:

My own view, voiced elsewhere, is that CH is what I have called an essentially vague statement, which says something like: there is no way to sharpen it to a definite statement without essentially changing the meaning of the concepts involved in it. But to formulate that idea more precisely within the semi-constructive framework, some stronger notion of formal definiteness may be required. (Feferman 2011, 26, cf. 1999)

Yet for philosophical analysis, we do not need “some stronger notion of formal definiteness” but “a more profound understanding of the concepts underlying logic and mathematics”; namely, empirical realist epistemology from the pragmaticist’s perspective (e.g., Nesher 2002, 2007a, 2011, 2013). Thus, we can see that the continuum is the property of physical objects or processes, and not of mathematics, since mathematical symbols are cognitions without any physical size, except for the phenomenal appearances we give them. They can, though, represent continua in different modes, but such representations by themselves cannot be continua (Weyl 1921, 99). There is a tendency to consider the iconic-physical appearance of symbols as mathematical objects, which leads to nominalistic epistemology and thus to a misunderstanding of the nature of signs and symbols as physical objects. Points and numerals are then confusedly considered as objects that constitute the physical continuum (e.g., Longo 1999).

What, therefore, is the structure of mathematical signs-symbols and what leads mathematicians to accept them as mathematical objects? According to Peircean semiotic epistemology, signs are the components of our cognitive operations. By interpreting them and proving their truth, we represent external reality (Nesher 1982, 2002, 2011).

**[1] The Structure of a Cognitive Symbolic Sign is the Hierarchy of its Components**:

***Cognitive Sign******Physical Appearance***

***Structure of Cognitive Sign***

**Intuitionist** ⎧ Icon = Feeling ⎞ ⎛ Tone = Property ⎞ **Nominalist**

Mind ⎨ Index = Emotional ⎮⎮ Token = Actuality ⎬ Phenomenal

Ideas ⎩ **Symbol = Conceptual Type = Generality** ⎠ Object

**Pragmaticist Structure of Cognitive Symbolic Signs Operating in Mathematical Reality**

Historically, Plato and Pythagoras conceived of numbers as *ideas* and *objects* respectively, but this arises from epistemological confusion: the two aspects ofsigns and numbers must go together, otherwise they are not signs, and we cannot grasp the sign’s meaning without its appearance and cannot understand its appearance without its meaning. The sign in Peircean semiotics is the conjunction of “form” and “matter,” or better, the *sign* has two components, which cannot exist separately. Indeed, without their unity there are no signs. However, mathematicians and philosophers in modern history have not clarified whether numbers are ideas or objects or both; hence, they consider these two aspects as separate entities, such that numbers are ideas and also objects. This confusion about the nature of numbers, i.e., viewing the phenomenal-objective component of the sign-number as the object of its cognitive-idea component, led to the difficulties, ambiguities, and paradoxes of the group-set theory. Thus, if the phenomenon of a number can be the object of that number’s idea, then the number can be the object of itself. This confusion is the basis of Russell’s paradox in set theory, as it assumes that a number can be a member of its own set (Russell 1901). By contrast, if a number is a sign, then it cannot be an object and, of course, it cannot be its own object (Russell 1919).

As distinct from the structure of a cognitive symbolic-type sign, thePeircean semiotic conception of Platonic and nominalist aspects of mathematical signs or numbers is that numbers are obstruct ideas-forms, which we cannot experience, although we somehow grasp them in the Platonic haven and present them to ourselves as nominalist phenomenal objects.

**[2] Peircean Semiotic Conception of Platonic and Nominalist Aspects of Mathematical Signs and Numbers**

“Form” “Matter”

***Cognitive Sign******Physical Appearance***

***Structure of Cognitive Sign***

Erigena’s ⎧ **Realist** ⎧ Icon = Feeling ⎞ ⎧Tone = Property ⎞ **Nominalist** ⎞ Ockham’s

Archetypical ⎨ Platonist ⎨ Index = Emotional ⎜ ⎜ Token = Actuality ⎬ Phenomenal ⎬ Word

**Ideas** ⎝ **Ideas** ⎝ **Symbol = Conceptual ≈ Type = Generality**⎠ **Object** ⎠ **Object**

(e.g., “Understanding the Symbolism of Mathematics,” in *Visible Language*, Vol. XVI, No. 3, summer 1982; Kronecker on nominalism – see Boniface 2005, #3; Hart 2010, Chap. 10, “The Zoology of Reality.”)

Symbolic concepts and propositions consist of a hierarchical structure of meaning or content, i.e., the *iconic* sign and the *indexical* sign, both evolve hierarchically in perceptual experience toward their synthesis in the *symbolic* sign. Hence, through interpretational synthesis, the *iconic feeling* and the *indexical emotional reaction* lead to the *conceptual symbol* presented in a category of *general type*.In this wayour perceptual cognitions become rational judgments representing external reality. Thus, it is only through the *union* of the *cognitive* perceptual components and *physical* components classified into *symbolic types*that wecan have *rational self-control* of our cognitive thoughts in *the proof of the truth of the interpretation of their meanings, and their representation of reality* (e.g., Nesher 2007b).

If we now return to our basic arithmetical operations on objects, using the vocal sign as *tone* and pointing indexically with our fingers as a *token* of *symbolic type* in counting objects enable us to self-control this operation. Yet we would never relate to such *vocalizing* and *pointing* as nominal arithmetical objects but as our primitive arithmetical signs in counting. Thus, we can understand the epistemological confusion of nominalismin mathematics in identifying the *tone-cum-token* appearanceof the *symbolic type* with external *mathematical reality*; it is the confusion that takes mathematical language to be mathematical reality (Mancosu 2010, IV). Epistemological *nominalism* may, then, be considered mistakenly as *realism* or quasi-empiricism. Interestingly, Peirce himself, in discussing the philosophy of mathematics from the mathematical practitioner’s perspective, intuitively adopted *nominalist* epistemology in understanding mathematical reality, which of course is incompatible with his own, mature, *realistic* epistemology centered on the cognitive representation of external reality (cf. Peirce 1992 in Houser's Introduction, xxiv; Weyl 1918, #1 “material content” vs. “formal logic” structure; Giaquinto 2001; Nesher 2001, 2002, 2005, 2007a, 2007b, 2011, 2012). A parallel form of epistemological confusion is intuitionism, which is based on the ***mind*** with the *intellectual intuition* of the Kantian *transcendental subject,*and this is separated from sensual and other cognitive representations of external reality, making for a kind of pure constructivism (Detlefsen 1990; van Stigt 1998).

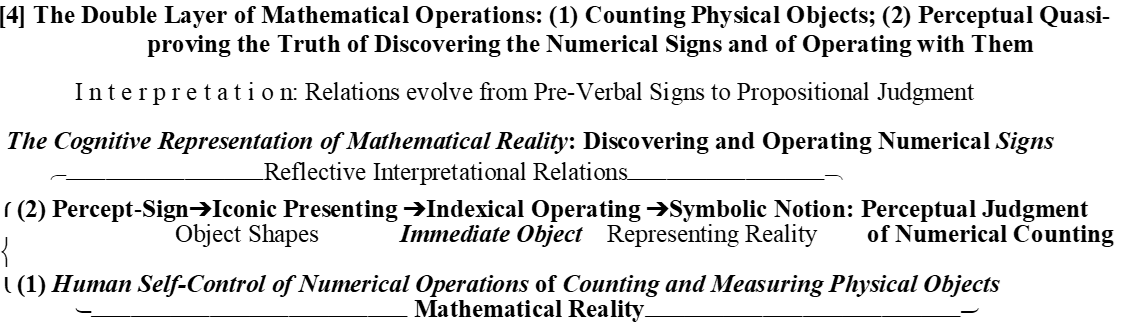
However, formalism is based on nominalism in considering the seemingly meaningless phenomenal structures of signs as *clear and valid* expressions of logical and mathematical calculations. Platonism is the realization of intuitions, which identify our *sensual intuition* of the perceptual “*immediate objects*” (à la Peirce), representing our operations of counting *abstract mathematical objects*. We continue to make calculations with them by employing *intellectual intuition* of *ideal entities*, which correspond to a Platonist *external reality* (Nesher 2011).

“Objects” do not do the job of numbers singly; the whole system performs the job or nothing does. I therefore argue, extending the argument that led to the conclusion that numbers could not be set, that numbers could not be objects at all; for there is no more reason to identify any individual number with any one particular object than with any other (not already known to be a number). (Benacerraf [1964] 1983, 290-291; cf. 294)

If I understand this explanation, numbers cannot be individual objects, nor can they represent objects; therefore, they cannot be element-objects of sets. Thus, the question arises: If we know what numbers cannot be, how can we understand their nature? It seems that if numbers can neither be nor represent objects, they must be components of systems of eternal ideal entities; that is, forms of Platonic reality. How, then, can we grasp and identify them in their systems? According to the pragmaticist’s epistemology, numbers are cognitive symbols. We discover them as systems in our operations of counting, grouping, and measuring physical objects. However, mathematicians and other scientists develop abstract mathematical theories for formularizing other scientific theories in order to enable scientists to predict and operate their experimental evaluations. The epistemological problem is to explain how the mathematical symbols with which we operate in mathematical realty are at the same time components of reality and symbolic components of the theories supposed to represent mathematical reality. How can we use symbols of theories to represent operations with symbols in mathematical reality? Can symbolic theories represent symbolic operations of mathematical reality? If we employ symbols of mathematical theories to represent symbols of mathematical reality, then can we represent representations? Indeed, with mathematical theories we represent our mathematical operations in reality leaning on our interpretation of their symbols. Hence, only by quasi-proving the truth of our perceptual judgments representing mathematical operations (see 2 in figure [3] below) do we represent mathematical reality (see 1 in figure [4] below):

**[3]** **The Double Layer of Mathematical Operations: (1) Counting Physical Objects; (2) Perceptual Quasi-proving the Truth of Discovering the Numerical Signs of the Operation**

I n t e r p r e t a t i o n Relations evolve from Pre-Verbal Signs to Propositional Judgment



The epistemological difficulty is how, if at all, we can represent the behavior of human cognitive operations? We cannot represent cognition itself because meanings can only be interpreted in cognitive operations, and this seems to echo Wittgenstein’s criticism of the Fregean and Russellian conceptions of types of formal hierarchies of languages as well as the Tarskian (and other) epistemologies of formal semantics in regard to the conception of orders of languages and meta-language representation (Wittgenstein [1921] 1961; Nesher 1986; 1990; 2002, V). Since we can represent physical objects but only interpret meanings of behavior and their symbols, the problem lies in determining how mathematical theories represent their reality, which constitutes our cognitive operations with mathematical symbols on physical objects. Thus, through our quasi-proved, true perceptual judgments and theories developed upon them, we *represent* the physical components of mathematical operations on objects while *interpreting* their cognitive behavior with symbols as another aspect of mathematical reality (Nesher 1986; 1990; 2002, I, II, IV, V I, X; 2004; 2007b).

**4.3. Misunderstandings of Mathematical Reality and its Number-Symbols**

According to pragmaticist epistemology, we can understand the meanings of mathematical symbols only through our basic confrontation with mathematical reality and our true representation of it (e.g., Nesher 2007b, 2011, 2013). The essential question is this: If we do not explicitly understand the meanings of mathematical symbols as components of mathematical reality, are we not facing the risk of deviating from their authentic meanings? The seeming difficulty is that in our basic perceptual experience with arithmetical operations on physical objects, we discover and use arithmetical numbers as symbols to represent such operations while at the same time using abstractions and generalizations to consider number-symbols as if they were objects themselves for the purpose of our calculations, and not meaningful signs representing operations on objects. The distinction is that in mathematical abstractions, we operate *with* number-symbols and not *on* them as objects, ideal forms, or mental intuitions. Abstract mathematical theories aim to formularize and build mathematical structures to formally present the scientific, abstract, and hypothetical picture of physical and psychological realities. How, then, can we distinguish between correct abstractions and generalizations, on the one hand, and the original basic meanings of mathematical symbols, on the other. When do we begin to stray into mathematical scholasticism or into “a scholastic pseudo-problem,” or “Cantorian paradise,” so to speak, which throws up difficulties, antinomies, and paradoxes, such as the et-theoretic paradox and the opposition between the CH vs. Kronecherean “mathematics as a natural science” (Weyl 1918, 48; Kronecker 1891; Cantor 1895, 1897; Brouwer 1949; cf. Dauben 1979, ##10, 11; Edwards 1988; Feferman 1998, I, V; Boniface 2005, #1).

Abstraction in mathematics occurs when we operate with mathematical symbols in relative separation from our original operations of counting, ordering, grouping, and measuring physical objects. It is necessary to work with mathematical abstractions in order to develop mathematical theories for formularizing scientific theories. In the end, the former enable scientists to predict and evaluate the latter in experimentation through their representation of mathematical reality, thereby allowing an evaluation of the scientific theories themselves by proving their truth or falsity. However, it is impossible to understand the meaning of number-symbols in complete separation from their original operations, as we can see, among others, in Cantorian, Fregean, Dedekindian, Hilbertian, and Russellian formal axiomatizations, which unavoidably bring about difficulties and paradoxes. In Cantor’s conception of the nature of mathematics, whole numbers are ideal entities of the human mind and the basis of free creations by abstraction of his new kinds of numbers, both finite and infinite; thus, in separating them from any empirical reality, Cantor creates his “mathematical paradise.” How, though, does the meaning of these whole numbers come to be well defined in the mind? What is this mathematical reality if the conception of these whole numbers is separated from our empirical experience in mathematical reality, as opposed to other sciences that relate to their specific realities (Nesher 2011; Feferman 1998, 248)? Dauben explains Cantor’s conception of the nature of mathematics and mathematical reality in these terms:

This reality, which the whole numbers consequently assumed, he described as their *intersubjective* or *immanent* reality (Cantor, 1883c). In contradistinction to this immanent reality was the reality numbers could assume concretely, manifest in objects of the physical world. He explained further that this second sort of reality proceeded from whole numbers as expressions or images of processes in the world of physical phenomena. This aspect of the whole numbers, be they finite or infinite, he termed *transsubjective* or *transient*. (Dauben 1979, 132, cf. Cantor, 1883c)

Cantor’s conception of the nature of mathematics, as I understand it, is a separation between the *immanent reality* of numbers, inborn in human minds, and their *transient reality*, as they proceed from the first reality and become embodied in numerical physical phenomena, in line with Tarski’s conception of numbers as syntactic physical objects (Tarski 1944).

Because of this extraordinary position which distinguishes mathematics from other science, and which produces an explanation for the relatively free and easy way of pursuing it, it especially deserves the name of *free mathematics*, a designation which I, if I had the choice, would prefer to the now customary “pure” mathematics. (Cantor 1883, in Dauben 1979, 132)

This Cantorian epistemology separates mathematics from human experience in reality and allow us to develop mathematical abstractions that distort our mathematical reality and detach its number-symbols from their original meanings. It is interesting to compare this explanation of Cantor’s conception of the nature of numbers with the Peircean theory of symbols and of mathematical symbols, but with a crucial distinction: the basic reality of Cantorian numbers is identified with the ideal entities of the human mind, or Kantian innate concepts, whereas symbols according to Peircean epistemology are discovered in perceptual experience in our confrontation with them in empirical reality, as I explained above, and are not due to a free creation of our so-called transcendental minds.

Cantor thus asserted the freedom of mathematics to accept the creation and the application of new ideas solely on the ground of intellectual consistency. Though there were counterparts to the immanent reality of number in the phenomenological world that did not matter. Instead, the formal consistency of mathematical ideas in the mind provided the ultimate criterion for Cantor in determining the advance of mathematics. Its application to physical phenomena of the external world was of considerable but subsidiary importance. Mathematics was therefore absolutely free in its development, and bound only to the requirement that its concepts permit no internal contradiction, but that they follow in definite relation to previously given definitions, axioms, and theorems. On these grounds, what were the criteria for introducing new numbers? The matter rested entirely in terms of definition. So long as new numbers were distinct and could be distinguished from other kinds of numbers, as well as from each other, then a new number was defined and must be taken as existing. (Dauben 1979, 132-133)

However, Cantor starts from his previously given definitions, axioms, and theorems. How, therefore, can he know their *meanings* and *truths* without proving that they are true? Without quasi-proving the truth of the basic ostensive definitions and without proving the truth of hypotheses rooted in mathematical reality, there is no criterion for their intellectual consistency. Indeed, such an empirical epistemology of mathematics cannot differ categorically from other empirical sciences as Cantor envisages (Nesher 2002, 2007a, 2007b, 2011). In his conception of mathematical freedom, without a realistic epistemology for mathematics, Cantor was able to develop his *scholastic paradise* with its *idle speculations*,but this led to contradictions and paradoxes and revealed the inconsistency of his system (cf. Dauben 1979, 137). Difficulties with Cantor’s view relate to: *symbolic sets as objects of sets*; the *physical continuum* presented by a geometric line composed of infinite *sizeless point-symbols or number-symbols*; and the different *rates of counting sequences*of cardinal numbers and even equal numbers because of the artificial criterion of *one-to-one correspondence* that treats numbers as though they were physical objects (Weyl 1918, e.g., 49; Feferman 1998, 30-35). There are further difficulties with the methodological-logical assumption about complete infinity or the transfinite nature of the sizes and relations of equinumerosity and non-equinumerosity based on one-to-one correspondence that might be eliminated if we measure their relative progressions of enumeration. It seems to me that the criterion of one-to-one correspondence can hold only for finite sequences in order to compare their cardinality (Feferman 1998, #2 Intr., #12, 30-35).

I once asked myself the question: How were the famous axiom systems, such as Euclid’s for geometry, Zermelo’s for set theory, Peano’s for arithmetic, originally obtained? This was to me more than merely historical question, as I wished to know how the basic concepts and axioms were to be singled out, and, once they were singled out, how one could establish their adequacy. One possible approach which suggests itself is to take typical theorems, proofs, definitions, and examine case by case what assumptions and concepts are involved. The obstacle in such an empirical study is… the lack of conclusiveness in both result and justification. The attempt to find an answer to this question led me to some interesting fragments of history. (Wang 1957, 145)

Yet, the historical description of the development of axiomatic systems is itself based on intuitions about their definitions, concepts, axioms, and theorems, without any investigation of their epistemological foundations. Thus, Wang accepts the basic axiomatizations of Dedekind and Frege without questioning the epistemology of their assumptions and without asking whether they can explain our knowledge of mathematics as a science as we explain our knowledge of other sciences. Moreover, these assumptions have to explain the science of logic and how there can be certainty about “the laws of thought” as its basis, whether these are cognitive representations or Platonic forms or something else (Cantor 1884; Dedekind 1888; Frege 1918; Wang 1957, #6; Feferman 1998, Pref. ##2, 12; Nesher 2007a, 2011). However, the basic problem is the tradition of axiomatizing, rather than hypothesizing, scientific theories, since axiomatization cannot emulate and explain scientific knowledge and might endanger its progress.

Thus, if we complete the Gödelian revolution about the incomplete nature of human knowledge and accept mathematics and even logic as sciences grounded in empirical realism, then we have to avoid accepting formal axiomatic systems as an epistemological basis, along with the methodological structure of sciences (e.g., Carnap 1937). All our knowledge and general theories originate from our confrontation with reality and from our quasi-proof of the truth of perceptual judgments as our basic facts; and of discovered hypotheses, their elaboration, and their inferred conclusions as our predictions, which are to be evaluated and proved in order to become propositional and theoretical knowledge. In this vein, the issue is how to understand arithmetical theories and specifically Peano Arithmetic if we actually work with it as a specific mathematical theory representing mathematical reality.

We no longer need set theories as a foundation of mathematics, as we prove mathematical theories and their number-symbols empirically. Instead we can consider *sequences* and *series* of numbers with their original empirical meanings, but not necessarily their sets; the reason is that we only group objects, not symbols, and we can probably do without the conception of sets in most of our mathematical work. We can, though, investigate the structural relations between natural numbers and sets of them according to their ostensively defined meanings and properties (Poincaré 1913; Weyl 1918- [1849]; Brouwer 1923, 1927; Feferman 1998, V). Therefore, our problem is to decide whether it is possible to adhere to a mistaken arithmetical epistemology and, nevertheless, develop correct mathematical theories, as the case of Peano Arithmetic suggests.

In fact, mathematics had developed illogically. Its illogical development contained not only false proofs, slips in reasonings, and inadvertent mistakes which with more care could have been avoided. Such blunders there were aplenty. The illogical development also involved inadequate understanding of concepts, a failure to recognize all the principles of logic required, and an inadequate rigor of proof; that is, intuition, physical arguments, and appeal to geometrical diagrams had taken the place of logical arguments. (Kline 1980, 5)

Indeed, can sciences develop properly without a clear understanding of their epistemologies? To understand the epistemology of mathematics, it behooves not just philosophers but also mathematicians to develop their science more smoothly (cf. Einstein 1949).

In view of the disagreements about what sound mathematics is, why is it effective at all? Are we performing miracles with imperfect tools? …How can we, then, speak of the artificiality and varieties of mathematics? Can the body live on when the mind and spirit are bewildered? Certainly this is true of human beings and it is true of mathematics. It behooves us therefore to learn why, despite its uncertain foundations and despite the conflicting theories of mathematicians, mathematics has proved to be incredibly effective. (Kline 1980, 7-8)

This confusion about the nature of numbers, i.e., viewing the phenomenal-objective component of the sign-number as the object of its cognitive-idea component, led to the difficulties, ambiguities and paradoxes of the group-set theory. Thus, if the phenomenon of a number can be the object of that number’s idea, then are we to say that the number can be the object of itself? This confusion is the basis of Russell’s paradox in set theory, as it assumes that a number can be a member of its own set (Russell 1901). By contrast, as discussed earlier, if a number is a sign, then it cannot be an object and cannot be its own object (Russell 1919).

Peano’s program suffered from the above difficulties. Indeed, how can we prove the truth of definitions, axioms, theorems, and formal proofs themselves? Can our normative rules, models, and implicit intuitive feeling replace empirical quasi-proofs of arithmetical reality (comp. Russell 1919, I; 1907, 282; 1914, III; 1940; Maddy 2011)? We can understand mathematical models as generalizations and schematization of the facts of reality, but how do mathematicians know factual reality itself in order to formulate their models? Indeed, they might not know the facts, not having proved them beforehand, but still have an implicit intuition of them. But do they then have enough control of their intuitive representation of reality in order not to misunderstand its meaning and thus go astray?

Our representation of empirical mathematical reality does not start from primitive definitions of meanings and should not lead us to circular or indefinite regress as in formal axiomatic systems. Formal semantic epistemology cannot explain how *ostensive definitions*are quasi-proved true in our arithmetical experience of counting and, therefore, it cannot provide the true empirical basis of the concept of number-symbols of arithmetic and mathematics in general (Feferman 1998, I, V: ## 12, 13; cf. Nesher 2005, 2011). However, if mathematics starts from ostensive definitions that are quasi-proved true in perceptual representations of mathematical reality, then we start from a true “empirical basis,” which should not bring us to any infinite regress as shown by Spinoza in his refutation of Cartesian deductive formalism (Spinoza 1662, #38; Peirce 1902; Nesher 2002, Intr. Xvii; 2012).

In this understanding of the epistemological situation, the essential point is the extent to which philosophical epistemology is important to the effective development of mathematical theories and scientific theories in general. This understanding can affect our evaluation of the truth of scientific theories, and of Peano’s Arithmetic in particular. With knowledge of basic arithmetical reality, we can then evaluate how much Peano deviated from it and how it can be further developed; of course, this also holds for other creative mathematicians, such as Kronecker, Dedekind, Cantor, and Hilbert.

**5. Conclusion: Does a Misconceived Epistemology Prevent Mathematicians from Working With Mathematics as a True Representation of its Reality? The Reciprocal Relationship Between Epistemology and Science**

It is interesting to note that in spite of many difficulties, paradoxes, and contemporaneous criticisms (Kronecker, for example), the separation of the formalist conception of the axiomatization of mathematics from mathematical empirical reality became the main path of mathematical development, although not without corrections by the intuitionists, structuralists, and constructivists (Weyl 1921; Dauben 1979, Chaps. 6, 10, 11; cf. Carnap 1939, #20; cf. Feferman 1998, I, V). The question is whether sets are “abstract entities” or cognitive symbols with which we operate on physical objects. And this question is connected with the basic epistemology of logic; namely, the ways in which logic can be viewed as an empirical science, a universal science of human knowledge, based on our confrontation with reality and our basic experience of proving the truth of our cognitive representations (cf. Putnam 1971, VI-IX; Nesher 2012).

It seems that because of the epistemological confusion about the nature of mathematics and the different epistemological intuitions about it, we can view mathematical science as a federation of approaches, a family of different understandings and working methods. The question is if and how, with Peircean empirical epistemology, we can understand its nature as an empirical science and elaborate a comprehensive picture of it.

It was often said that mathematics has to start with definitions and that mathematical propositions have to be deduced from those definitions and from ground postulates. However, definitions per se, are already an impossibility, as Kirchhoff underlined, because each definition uses its own concepts, which in their turn have to be defined, etc. (Kronecker 1891, in Boniface 2005, 145)

Epistemologically, the only meanings that we can have in our intended models of arithmetic, and mathematics in general, are those that originate in our perceptual experience, in which we quasi-prove the truth of our perceptual judgments representing our basic arithmetical operations on physical objects and our *ostensive definitions* of the role of number-symbols in these operations. Moreover, abstract mathematical structures for other sciences are based on our experiential confrontation with mathematical reality, these operations of counting, ordering, grouping, and measuring components of physical reality, as distinct from the alleged epistemology of axiomatic formalisms such as those of Dedekind, Frege, and Peano, among others.

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683–684)

With a pragmaticist’s empirical epistemological approach to mathematical science, we can also investigate the correct interpretation of Peano Arithmetic, its incompleteness, its relative truth in regard to its explicit proof-conditions and, therefore, what is false or doubtful in it (Nesher 2002, X; 2012; cf. Brouwer 1949, 90).

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**Chapter 12**

**‘What Makes Reasoning Sound’ is the Proof of its Truth: A Reconstruction of Peirce’s Semiotics as Epistemic Logic, and Why He Did Not Complete His Realistic Revolution**

“Do not block the way of inquiry.” (Peirce 1898)

**1. Introduction: Peirce’s Semiotic Theory of Cognition**

**1.1. Peirce’s Phaneroscopy: A Starting Point in the Development of his Semiotics**

In his philosophical inquiries, Charles S. Peirce endeavors to discover and develop a theory of signs that interpret one another to form a true representation of reality that originates in our basic perceptual operations of interpretation; a process he explains as the quasi-proof of the truth of perceptual judgments in their representation of reality. The essential problem is to explain how, through our cognitive interpretation of sequences of perceptual signs, we may represent external physical reality and reflectively represent the mind’s interpretive operations of signs. In an attempt to develop his pragmaticist epistemology, Peirce starts from basic perceptual experience and, through phenomenological introspection, or Phaneroscopy, cognizes and explains the sequence involved in the interpretation of signs, in which the iconic feeling is itself a sign that is interpreted by the indexical reaction to this feeling, and these are synthesized to form the symbolic thought, that is perceptual judgment. In this account, the incoherence or coherence of the iconic feeling sign, and the eventual image of an object indicate the expectation of the *ego*, while Peirce classifies the indexical emotional reaction to the first sign, which may contrast or agree with it, as *non-ego*. Hence, the latter either disappoints the expectation, and thus may be understood to represent reality negatively, or fulfills it and represents external reality (Nesher 2002b, III). Through phaneroscopical introspection, Peirce shows how, without going outside our cognition, we can represent external reality. With this explanation, Peirce can avoid the pitfalls of various Berkeleyian, Humean and Kantian phenomenologies, as well as those of modern analytic philosophy and hermeneutic phenomenology (Marty 1982; Nesher 2002b, VI; 2004a/b).

**1.2. Does Peirce Overcome Kant’s Transcendentalism with the Epistemic Logic of the Complete Proof of Truth?**

Peirce elaborates logical inferences from different syllogisms to formulize a complete proof that explains the perceptual operation of quasi-proving perceptual judgments using three types of inference. With the abductive logic of discovering the hypothetical concept of the object, we experience the object; with deductive logic, we infer the expectation of what this object could be; and with inductive logic, we evaluate the concept expressed in the proposition of the perceptual judgment to prove the true representation of the real object. This trio of semiotic interpretation is the complete proof of the truth of perceptual judgments, which are our basic empirical facts; and with this method of proof, we do not have to assume any transcendental a priori principles or axioms that we cannot prove (Fisch 1966). Peirce can show that the unifying principle Kant was looking for in his three Critiques is this trio sequence of inferences. Complete empirical proof stands on external reality with its two pillars of material logic, abduction and induction; thus, all scientific knowledge consists of empirically proved true representations of reality, relative to their proof-conditions (Peirce 1931-1958, 6.95, 1903; Nesher, 2001, 2002b, 2007a, 2011, 2016).

**1.3. Peirce’s Realist Epistemology of Reasoning is Shown to be Sound by Proving the Truth of its Representation of External Reality**

Thus, the semiotic logic of the complete empirical proof of the truth of our representation of reality is *epistemic logic*, representing our confrontation with reality, and thus we can achieve knowledge (of reality) and conduct our behavior (in this reality) (Houser 1997). The question is: Why did Peirce not complete his realist revolution to eliminate previously accepted nominalist and idealist epistemologies of formal logic and pure mathematics (Fisch 1967)? Since he elaborates the pragmaticist’s conception of truth as the conclusion of complete proof, we have to reconsider his axiomatic approach to formal logic and pure mathematics, which are based on subjective feelings of correctness and proof as distinct from the realist epistemology of empirical sciences. As Peirce was a genuine philosopher and knowledgeable scientist with a rich imagination and sound reasoning, we have to ask why he did not complete his historical realist epistemological revolution and, following that inquiry, we must then consider how we can reconstruct it.

**2. Peirce’s Semiotics as a Theory of Sign Interpretation and Representation of Reality**

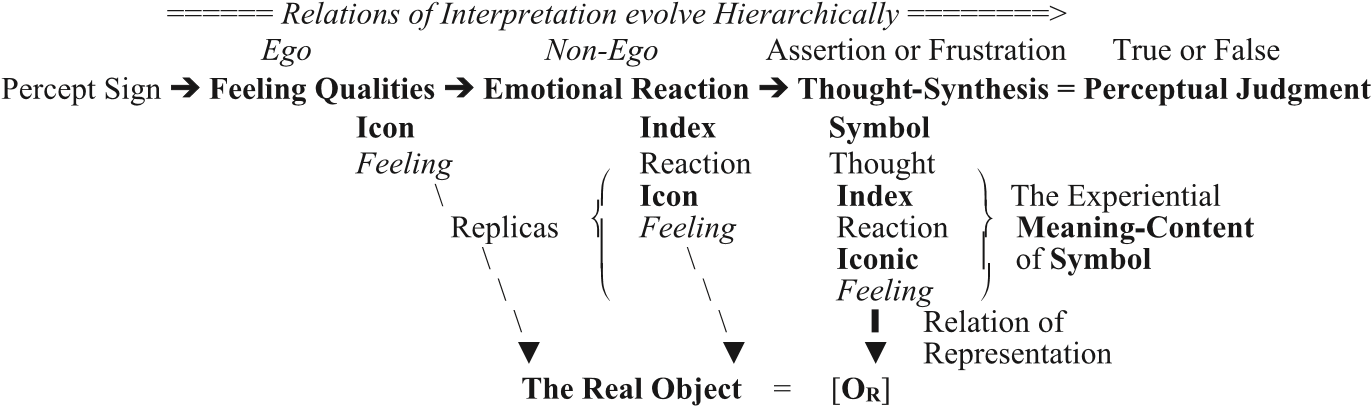
**2.1. Perceptual Subjective Confrontation with Reality Offers Objective Proof of True Representation**

Through our perceptual operations, or basic experiences, we can inquire into signs (and their interpretations) and reflect on their relationships, all of which amounts to phenomenological introspection. This introspection enables us to detect a clash between the perceptual *iconic* sign, presenting *ego* expectation, and the *indexical* sign, which, by clashing with the former, presents the *non-ego*, or external reality. The vividness of the shock of dual reactions is manifest in our *reflective feelings*, from which *reflective consciousness* develops.

The question is what the phenomenon is. We make no vain pretense of going beneath phenomena. We merely ask, what is the content of the Percept? Everybody should be competent to answer that of himself. Examine the Percept in the particularly marked case in which it comes as a surprise. Your mind was filled [with] an imaginary object that was expected. At the moment when it was expected the vividness of the representation is exalted, and suddenly, when it should come, something quite different comes instead. I ask you whether at that instant of surprise there is not a double consciousness, on the one hand of an Ego, which is simply the expected idea suddenly broken off, on the other hand of the Non-Ego, which is the strange intruder, in his abrupt entrance. (Peirce 1931-1958, 5.53, 1903, #11, 1903)

This experience and its semiotic explanation show our perceptual confrontation with reality; and with our instinctive and practical self-control of this operation, we can quasi-prove the truth of our perceptual judgments.

**[1] Perceptual Hierarchical Signs and Their Interpretation and Confrontation with Reality Form the Synthetic Content of Symbolic Representation:**



Peirce shows that the experience of the duality of action and reaction is direct but independent of our deliberation, and that it is not critically inferred from previous cognition. The starting point of his inquiry about reality is the direct experience of the duality of action and reaction, which is beyond our control and, therefore, depends on something different from us. We can cognize the indirect relation between mind and external reality in the perceptual experience of the duality between our expectations and the direct objects that are presented (Nesher 2002a).

**2.2. Semiotics As Epistemic Logic: From Introspection and Confrontation with Reality to the Complete Trio of Inferences that Quasi-prove Our Perceptual Judgments**

Peirce’s phaneroscopical inquiry is an essential break from traditional and contemporary approaches to tackling the difficulty of how to understand our representation of external reality without going outside our cognitive skins. However, only the entire *trio* sequence of abduction, deduction and induction, with its two material logical components, can provide the complete proof of the truth of human cognition, which must originate in our pre-rational operations in order to quasi-prove their perceptual judgments (Peirce 1931-1958, 5.121–145, 1903; Nesher 2002a).

**[2] The Complete Cognitive Operation: *Trio* Sequence of Abduction, Deduction and Induction**:

**Abduction**((CAb(A→C)=>AAb) + **Deduction**((A→C) A)→CDd) + **Induction**((A, CIn) =❥ (A→C))

Thus, => is the *plausibility connective* suggesting the concept or theory A; → is the *necessity connective* deducing the abstract object or fact C; and =❥ is the *probability connective* evaluating the relation of the concept or theory A to the new experience of objects or proved facts C. Yet, to explain scientific, moral, and aesthetic human knowledge as determinative, the complete *trio* of cognitive operations must work on different levels of self-consciousness and to varying degrees of self-control, from instinctive and practical quasi-proofs to rational proofs of the truth of judgments in these different domains (Peirce 1931-1958, 5.121ff., 1903; Nesher 1983b).

**2.3. The Transition from Phaneroscopy to the Complete Quasi-proof of the Truth of Perceptual Judgment Representing Reality**

Peirce develops his semiotics into the epistemic logic of our perceptual confrontation with reality, manifest in the duality of the *ego* and *non-ego*, by interpreting our genuine signs as complete proof of the true representation of external reality, conditioning the *validity* of the interpretation and the *soundness* of the proofs.

**[3] Confrontation with Physical Reality Through the Coherent Interpretation of Meanings Derived from the Three Inferences Used in the Quasi-proof of the Truth of Perceptual Judgment**:

**Validity** of MeaningInterpretation and **Soundness** in Proving theTruthof **Perceptual Judgment**

*Percept sign**Perceptual judgment***,** CIn is AAb

**Ab(C, A ➞C) ➾A)** + **Dd**((A ➞C), A) ➞C) + **In**((AAb, CIn) =❥Pr. m/n (AAb ➞Cin)) = **Falsity or Truth**

[Initial Sign] **Icon Index Icon, Index Symbol:** **Perceptual Judgment**  ⇘ ⇙ ▲

**Truth Conditions** = Duality= Comparison ❙

*Incoherency Coherency* ❙

▼ ▼ ❙

Hesitation Assurance ➠ Assertion

**Logical Reality: *Confrontation in Reality***

❙  **Representing** Physical Reality

❙ **Object**P by **Description** **(AAb**

▼ “This [CIn] is a stone [AAb]”

**Physical Reality**: **Object**P

We find that through our cognitive clash with reality, we first become conscious of the reality external to us: this is our negative knowledge of reality, whereby we cognize the existence of something that contradicts our expectation, yet we still lack a positive true representation of it.

And what do we mean by real? It is a conception which we must have had when we discovered that there was an unreal, an illusion; that is, when we first correct ourselves. (Peirce 1931-1958, 5.311, 1868)

Thus, the perceptual cognitive operation in which we discover our error, which cannot come only from ourselves, provides proof of negative knowledge of external reality. This explanation can be considered to be a philosophical proof of the existence of something external that is independent of the way we present it; and when we interpret the *coherency* of themeanings of iconic and indexical signs, we can prove our positive knowledge of this external reality. Hence, *semiotics* is a form of *epistemic logic that represents our confrontation with reality*;it is the *methodeutic* of all our *true representations of external reality* (Peirce 1869, 136–137; 1878).

**3. Pragmaticist Realism: Can Mathematical Reasoning Be Sound Without Being A True Representation of Reality?**

**3.1. The Gap Between the Nominalist/Platonic Conception of Mathematics and Realist Empirical Sciences**

Peirce revolutionized philosophy by developing a realist epistemology of the true representation of reality in contrast to Cartesian metaphysical realism and Kantian transcendental phenomenalism. He developed his semiotics as a form of epistemic logic capable of representing our cognitive confrontation with external *reality*, thus enabling us to prove the truth of our cognitive representation of *positive reality* (Nesher 1981; 2002b, II, X; 2005). Hence, unlike nominalism, we can realistically quasi-prove the truth of our perceptual judgments and, on the strength of this, prove that true scientific theories represent reality, with *general natural kinds* and *general laws of nature*. This realist epistemology is the basis of all our knowledge of reality. However, since pure mathematics and formal logic do not confront reality experientially, according to Peirce, he cannot explain how such subjective reasoning can *determine* the *identity* and the *truth* of their propositions (Nesher 2016).

Every reasoning takes place in some mind. It would not be that mind’s reasoning unless it satisfied that mind’s feeling of logicality… But as long as it does that, nothing can be gained by criticizing the reasoning any further, since there is no other possible sign by which we could know that it was good than the feeling of logicality in the reasoner’s mind… Consequently, since every reasoning satisfies the reasoner’s feeling of logicality, every reasoning is as good as any reasoning can be. That is, there is no distinction of good and bad reasoning. (Peirce 1893-1913, #17, 243–244; 1903)

In his mature realism, Peirce concedes that our reasoning cannot be sound without truly representing external reality, but then this is incompatible with his conceptions of pure mathematics and formal logic. Accepting Peirce’s understanding that validity cannot be controlled only by the feeling of the reasoner, one is surprised that his conception of pure mathematics is itself based on subjective feeling without objective criteria for truth (Peirce 1931-1958, 4.227–245; 1902; Murphey 1961, XII). Later in life, Peirce considered aesthetics, ethics and mathematics as pure forms of cognition separated from experienced reality.

Yet the maxim of Pragmatism does not bestow a single smile upon beauty, upon moral virtue, or upon abstract truth; the three things that alone raise Humanity above Animality. (Peirce 1893-1913, 465; 1913)

Historically there have been prominent examples of an alliance between nominalism and Platonism… The reason of this odd conjunction of doctrines may perhaps be guessed at. The nominalist, by isolating his reality so entirely from mental influence as he has done, has made it something which the mind cannot conceive; he has created the so often talked of “improportion between the mind and the thing in itself.” And it is to overcome the various difficulties to which this gives rise that he supposes this *noumenon*, which,being totally unknown, the imagination can play about as it pleases, to be the emanation of archetypal ideas. The reality thus receives an intelligible nature again, and the peculiar inconveniences of nominalism are to some degree avoided. (Peirce 1867-1893, 100; 1878; 1893-1913, 260; 1903)

I suggest that Peirce accepted “this odd conjunction of doctrines,” the alliance between *ideal realism* and *phenomenal nominalism*, to explain the practitioner’s intuition of pure mathematics and formal logic (Fisch 1967, #1).

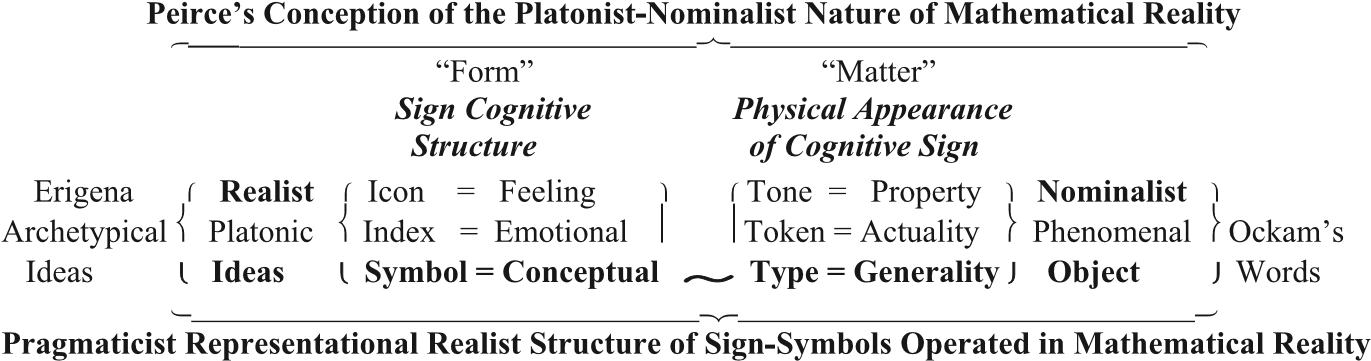
**3.2. Peirce’s Odd Conjunction of *Ideal Realism* and *Computational Nominalism* to Explain Pure Mathematics**

There is a tendency among mathematicians to consider the phenomenal appearances of the symbols they use as mathematical objects; their feelings of implicit meanings lead to a nominalist/Platonic epistemology. They misunderstand the nature of symbols by perceiving them as reality itself rather than representations of reality (Peirce 1867-1893, 85; Fisch 1967; Haack 1992; Engel-Tiercelin 1992; Forster 2011; Beth 1965, #XIII; Nesher 2012).

Ockam’s nominalism may be said to be the next stage in English opinion. As Scotus[Erigena]’s mind is always running on forms, so Ockam’s is on logical terms; and all the subtle distinctions which Scotus effects by his *formalitates*, Ockam explains by implied syncategorematics (or adverbial expressions, such as *per se*, etc.) in terms. Ockam always thinks of a mental conception as a logical term, which, instead of existing on paper, or in the voice, is in the mind, but is of the same general nature, namely, a *sign*. (Peirce 1867-1893, 93; 1878)

Here we can detect the misleading analogy between the pure mathematical perception of the phenomenal appearance of symbols and the observational experimentation of empirical sciences (Gödel 1944, 121).

**[4] The Structure of a Cognitive Symbolic-type Sign is a Hierarchy of its Components: The Distinction between Platonic Realism, Scholastic Nominalism, and Empirical Realism:**



The distinction shown above can be considered as being between the particular and the universal, Aristotelian matter and form, words and their meanings; but then, what are the nominalist meanings of “words” or “logical terms”? If pure mathematics is isolated from experience in reality, mathematical expressions must be meaningless epistemologically, and mathematicians have to treat them as *phenomenal objects*, in keeping with the scholastic conception of *syncategoremata*, as words without individual meanings. Since pure mathematics is detached from experienced reality, mathematicians work without explicit objective criteria for accepting the constructions of their science, like the intuitionists’ conception of mathematics, without any real proof of truth (Russell 1901; Putnam 1975, #4; 2004, 65-67; Burgess and Rosen 1997; Mancosu 2010, ##9, 14; Nesher 2016; Parsons 2014, Chaps. 8, 11; Hacking 2014, Chap. 7. B.).

First, the mathematician’s experiments being conducted in the imagination upon objects of his own creation… Secondly, the assurance of the mathematician is due to his reasoning only concerning hypothetical conditions, so that his results have the generality of his conditions. (Peirce 1931-1958, 5.8; 1902)

Peirce considers mathematics as *necessary reasoning*, but this consists only of formal inferences, which cannot constitute the complete proof of truth, and therefore pure mathematics based on subjective feelings cannot be a science.

In reasoning, a man may feel sure he is right; but to "rest" that confidence on nothing but itself is to rest it on nothing at all. If the fact that we must use our reasoning instinct in criticizing reasoning proves that we must appeal to nothing else in such criticism, it actually proves that we ought to follow the lead of that instinct without any logical control at all, which would be as much as to say that we ought not to reason at all. A man cannot criticize every part of reasoning, since he cannot criticize the act of reasoning he is performing in the criticism, it is true. But he can criticize steps whose validity he doubts; and in doing so, ought to consider in what characters the validity of reasoning consists, and whether the reasoning in question possesses those characters. (Peirce 1931-1958, 2.209; 1902)

As I have discussed elsewhere, the character in which the validity of reasoning consists is the coherence of the interpretation of meanings in the operation of proving their truth or falsity by proving their truth in representing external reality (Nesher 2007b, 2016, above [3], and #3).

Nothing can be more completely false than that we can experience only our own ideas. That is indeed without exaggeration the very epitome of *all* falsity. Our knowledge of the things in themselves is entirely *relative*, it is true; but *all experience and all knowledge* is knowledge of that which is, independently of being represented. (Peirce 1931-1958, 6.95; 1903; my italics)

Peirce probably detects the incompatibility of his representational realism with his Platonic-nominalist conception of pure mathematics when he endeavors to eliminate nominalism from pragmaticism (Fisch 1967):

My plan for defeating nominalism is not simple nor direct; but it seems to me sure to be decisive, and to afford no difficulties *except the mathematical toil that it requires*. (Peirce 1931-1958, 4.1; 1898; my italics)

But without any criterion of objectivity for the mathematician’s subjectively created potential Platonic world, and without proving any true representation of mathematical reality, mathematical reasoning cannot be sound. We have to understand the essential argument, the gist, of Peirce’s mature philosophy and the solution he aimed for in his intellectual inquiry. However, we also have to understand that he could be mistaken in stepping away from epistemological realism with respect to mathematics and formal logic. The main question arises from Peirce’s essential philosophical epistemology: How can we avoid his mistaken ideas in order to reconstruct his philosophy?

**3.3. What Is the *Mathematical Reality* that Mathematical Theories Are Supposed to Represent?**

How can we realistically overcome the Platonic-nominalist epistemology of pure mathematics?

Everybody ought to be a nominalist at first, and to continue in that opinion until he is driven out of it by the *force majeure* of irreconcilable facts. (Peirce 1931-1958, 4.1; 1898)

Two modes of representation correspond to the two components of reality: perception of the physical objects of external reality and reflective introspection of the cognitive signs of mental reality (Nesher 2004a). Thus, we can distinguish between physical objects, which philosophers understand as existing “in space and time” (the real existents), and mental signs, which philosophers consider as real abstract entities and regard either Platonically or cognitively. However, since they are not perceived in causal relation to other physical objects, nominalists consider them as non-existing (e.g., Burgess and Rosen 1997, §I.A.I.a.). Hence, we can understand the controversy between phenomenal nominalism and Platonic realism as tension between two opposing camps: on the one hand, there is the view that the particular phenomenal objects of perception are the only existing entities, and on the other hand there is the miraculous grasping, so to speak, of general-universal *abstract* or *ideal* entities. Hence, as observed in the empirical realism of pragmaticist epistemology, neither Platonism nor nominalism can explain our knowledge of and conduct in reality, since neither transcendentalism nor phenomenology can explain the meaning of our cognitive signs and the ways in which they represent external reality. However, it seems that the early Peirce and his interpreters tend to make a general distinction between realist Platonism and phenomenal nominalism by somehow rejecting Platonism and confusing nominalism with empirical realism (Michael 1988). Indeed, such quasi-realists cannot prove our representation of external reality; they are either Platonists, à la Gödel, or phenomenal realists, like all neo-Kantians. It is quite possible that Peirce’s difficulty in doing away with nominalism comes from his Kantian erudition and his conception of pure mathematics in which mathematical phenomenal signs are operated on and treated as objects, in an “odd conjunction” with Platonic ideal meanings (Peirce 1867-1893, 100; 1878). However, by employing the pragmaticism of realist epistemology, we can show through perceptual operations how we quasi-prove our knowledge of physical objects and, through introspection, how we identify our cognitive signs. But their truth can only be proved together, like Siamese twins. As a result, we prove our knowledge of external physical reality and cognitive reality as though they exist “independently of being represented” (Peirce 1931-1958, 6.95; 1903; Nesher 2004b, 2007b). Therefore, we may conclude that nominalist phenomenology and Platonic idealism, construed respectively as internal realism and metaphysical realism, are sterile epistemologies and must be replaced by the empirical realist epistemology of pragmaticism, which is able to prove our true representation of external reality (Burgess and Rosen 1997, C.2.; Nesher 2002b, III; 2004a). Hence, our true perceptual judgments represent real *particular objects*, and scientific theories represent reality with *general laws of nature*, though only relative to our accepted proof-conditions, and both conduct our self-controlled behavior in reality (Peirce 1931-1958, 5.525; 1905; Boler 1963, 32-36; Haack 1992, 41-43; Burgess and Rosen 1997; Nesher 1982b, 1994, 2002b, 2007a, 2016).

The real is the object of an absolutely true proposition. Thus we obtain a theory which, while it is Nominalistic, inasmuch as it bases universals on signs, is yet quite opposed to that individualism which is often supposed to be coextensive with nominalism. For there is nothing to prevent universal propositions from being absolutely true, and therefore universals may be as real as singular. (Peirce 1868, “Questions of Reality” in Michael 1988, 319-320)

Since all our knowledge of reality is based on *perception* and *introspection*, mathematical knowledge is also based on such experiences. Hence, it can be shown that the basic *mathematical reality* we initially represent consists of *our operations of counting, grouping, and measuring physical objects* when confronting our environment. As Smith observed,

…the primitive man could count only by pointing to the objects counted, one by one. Here the object is all-important, as was the case with early measures of all peoples. The habit is seen in the use of such units as the foot, ell (elbow), thumb (the basis for our inch), hand, span, barleycorn, and furlong (furrow long). In due time such terms lost their primitive meaning and we think of them as abstract measures. In the same way the primitive words used in counting were at first tied to concrete groups, but after thousands of years they entered the abstract stage in which the group almost ceases to be a factor. (Smith 1923, 7)

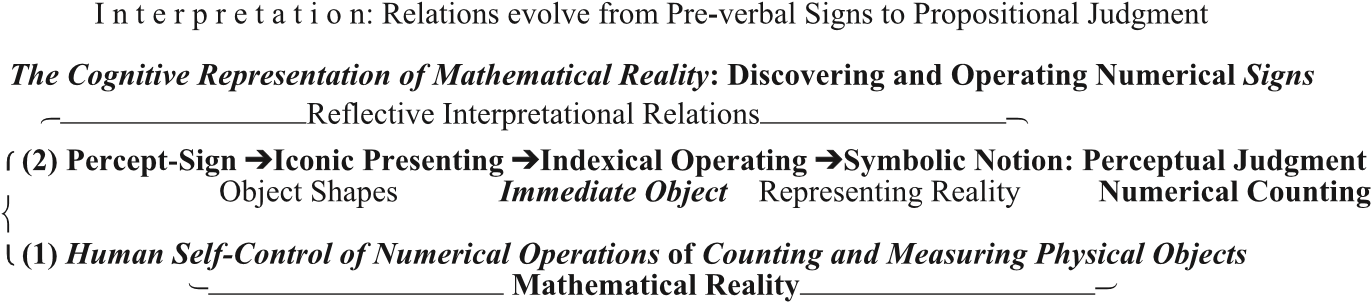
Arithmetic and geometry stem from historically basic human modes of quantitative operations on physical objects. With our sensual perception, we represent these operations, yet not the engaged physical objects and not the involved conceptual number signs alone, but their alignment in these operations themselves.

But the sole uses of the cardinal numbers, are, first, to count with them, and second to state the results of such counts. Of course it is impossible to count anything but clusters of acts, i.e. events and things (including persons); for nothing but reaction-acts are individual and discrete…

But the system of numerals having been developed during the formative period of language, are taken up by the mathematician, who generalizing upon them creates for himself an ideal system after the following precepts. (Peirce 1897, 116, 2010)

The *discovery* of the first concepts of these operations of enumeration consisted of natural numbers, and the further *discovery* oftheir expansion through abstractions and generalizations constitutes our new mathematical hypotheses, which will be evaluated on the basis of the extended mathematical reality. Hence, only by quasi-proving the truth of perceptual facts representing mathematical operations can we represent mathematical reality. The perceptual representation of these operations is our basic representation of mathematical reality.

**[5]** **The Double Layer of Mathematical Operations: (1) Counting Physical Objects; (2) Perceptual Quasi-proving the Truth of Discovering the Numerical Signs and of Operating with Them**



If we accept that mathematical reality consists of perceptually self-controlled numerical operations on physical objects, we can see how Peirce and Gödel confuse the meaning or content of mathematical symbols with Platonic mathematical abstract forms as objects. Here we can discern a Platonic echo in Peirce’s conception of “immediate object” as the component of a symbol representing numerical operations in mathematical reality. Arithmetical numbers are neither *physical objects* nor *abstract concepts* but *conceptual components of our quantitative operations on physical objects;* this is the mathematical reality on which we lean to establish the truth or falsity of our abstract mathematical hypotheses. From these are derived the abstract mathematical models of scientific hypotheses whose predictions are to be evaluated experimentally. However, it seems that string theories, with their twelve or more dimensions, cannot be measured and evaluated experimentally (Byl 2003).

Let us now approach the subject of logic, and consider a conception which particularly concerns it, that of *reality*…That whose characters are independent of how you or I think is an external reality. There are, however, phenomena within our own minds, dependent upon our thought, which are at the same time real in the sense that we really think them. But though their characters depend on how we think, they do not depend on what we think those characters to be. (Peirce 1867-1893, 136–137; 1878)

Therefore, mathematical reality comprises a combination of cognitive and physical realities, our cognitive operations with symbols on physical objects, in *counting, grouping, and measuring physical objects*. Hence, the proposition and theories that we prove to be *true* represent reality, physical or cognitive, particular or universal; and this is Peircean-pragmaticist realist epistemology, or *representational realism* (Nesher 2002b, V.5.; Boler 1963, Chaps. I, esp. pp. 19-20, n. 4-22, and VI).

**4. Epistemic Logic: Representing Our Confrontation with Reality (the Methodology of All Our Knowledge)**

**4.1. Epistemic Logic as the Methodology of Perceptual and Scientific Operations in Proving True Representations of Reality to Guide Human Conduct**

In a pragmaticist sense, every cognitive operation consists of descriptive and normative components that compose both the rules of habit of our cognitive operations and the rational norms embedded in every rational judgment, including scientific theories, which promote our rational conduct in self-controlling ourselves in reality (Peirce c. 1902, 1.281; 1893-1913, #14, 198-199; 1903; Nesher 1982a, 80-82; 1983b; 1990, 24-26).

That which any true proposition asserts is *real*, in the sense of being as it is regardless of what you and I may think about it. Let this proposition be a general conditional proposition as to the future, and it is a real general such as is calculated really to influence human conduct; and such the pragmaticist holds to be the rational purport of every concept. (Peirce 1931-1958, 5.432; 1905)

However, in considering this pragmaticist conception of semiotics, it is essential to understand the epistemological deficiency of syntactic and semantic axiomatic formal systems. Formal systems cannot explain how our cognitive operations prove our true representation of reality to guide human conduct (Nesher 2004b, 2011).

In order to gain a clear understanding of the origin of the various signs used in logical algebra and the reasons of the fundamental formulae, we ought to begin by considering how logic itself arises. (Peirce 1867-1893, 200; 1880)

The epistemic difference between formal logic and epistemic logic lies in their different proof-conditions, the formal system being *hermetically closed* by its fixed formal proof-conditions, which are detached from external reality; epistemic logic is only *relatively closed* by its proof-conditions, since it is the method of complete proof and thus also quasi-proves the truth of our perceptual judgments as basic facts. Thus, *formal systems* are *complete* and *sterile*, and human perception and science based on *epistemic logic* are *incomplete* but *true* in representing reality relative to accepted proof-conditions (e.g., Peirce 1931-1958, 4.582; 1906).

In the first place, all our knowledge rests upon perceptual judgments… Now consider any other judgment I may make. That is a conclusion of inferences ultimately based on perceptual judgments, and since these are indisputable all the truth which my judgment can have must consist in the logical correctness of those inferences… To say that a proposition is certainly true means simply that it never can be found out to be false, or in other words that it is derived by logically correct arguments from veracious perceptual judgments. Consequently, the only difference between material truth and the logical correctness of argumentation is that the *latter* refers to a single line of argument and the *former* to all the arguments which could have a given proposition or its denial as their conclusion…

These three kinds of reasoning are Abduction, Induction, and Deduction. (Peirce 1893-1913, 204-205; 1903; cf. Nesher 2002b, II, X)

This is the distinction between the inferences of *formal logic*, which is isolated from reality and unable to hold truths about it, and the *epistemic logic* of complete proof, be it true or false, which consists of the *trio* of abduction, deduction, and induction. Complete proof, then, stands on reality with the support of abductive and inductive material logical inferences (Nesher 2001, 2002b, II, X; 2007a, 2011, 2016).

It does not seem to me that mathematics depends in any way upon logic. It reasons, of course. But if the mathematician ever hesitates or errs in his reasoning, logic cannot come to his aid. (Peirce 1931-1958, 4.228; 1902)

However, the semiotics of our cognition are bound by epistemic logic, which is the science of reasoning. Therefore, mathematicians cannot make their reasoning sound, for they control the logic of their operations in confronting mathematical reality.

And to say that mental phenomena are governed by law does not mean merely that they are describable by a general formula; but that there is a living idea, a conscious continuum of feeling, which pervades them, and to which they are docile. (Peirce 1931-1958, 6.163; 1892)

From the perspective of pragmaticist epistemology, all human behavior and conduct, perceptual and scientific, is based initially on *logica utens*, as our habitual reasoning is instinctively and practically self-controlled, which evolves into *logical ducens,* whose rules are formulized, and reasoning is rationally self-controlled (Peirce 1867-1893, 141; 1878).

**4.2. Our Propositional Meanings Proved Clear and Distinct by Proving their True Representation of Reality**

Philosophical and logical sciences develop together in our experience and enable us to understand their basic contributions to knowledge and to the conduct of our life within nature. Thus, we prove that *epistemic logic* is our basic science, for it represents our confrontation with reality, from perceptual operations to all other sciences when they prove the truth of their representations (Peirce 1893-1913, 256–257; 1903). In this conception of epistemic logic, all knowledge is proved to be a true representation of reality, and this applies to logical knowledge as well. However, we can prove our cognitive representations to be either *true* or *false*; and if we do not prove them, they remain *doubtful.* Thus, *truth* cannot be separated from being proved, which is in contrast to classical formal logic, whose propositions are either true or false *independently* of being proved (Nesher 2002b, V; 2011). Therefore, we can no longer accept the *principle of the excluded middle* and cannot prove the *provability* of any proposition but only its real values, and thus every proposition remains doubtful (Peirce 1893-1913, 168-1903, 351-1905; Gödel 1931; Heyting 1956, 18f.; Brouwer 1981, 5, 92; Kleene 1952, #13; Weyl 2012, 188-189; Nesher 2011). Hence, the meanings of *validity*, *proof*, and *truth* in epistemic logic differ from their meanings in classic logic (Nesher 2016).

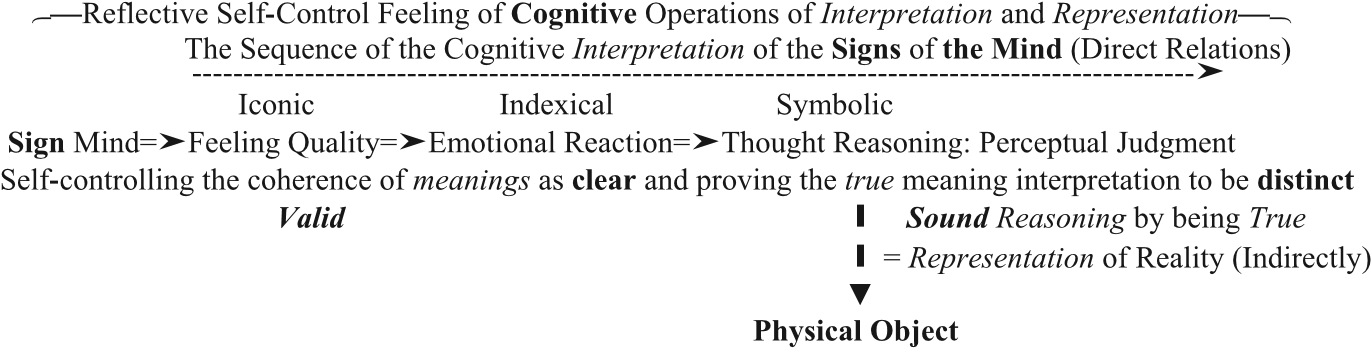
A sign (under which designation I place every kind of thought, and not alone external signs) that is in any respect objectively indeterminate (i.e., whose object is undetermined by the sign itself) is objectively *general* in so far as it extends to the interpreter the privilege of carrying its determination further. (Peirce 1931-1958, 5.447; 1905)

The interpreter’s *determination* of a sign lies in proving the true interpretation and representation of its object, and this holds for propositions and their sign components as well. The *identity* of a sign is encapsulated in the way its meaning will be comprehended (made *clear*) through further interpretation, while the meanings of signs are made *clear* and *distinct* by proving the truth of their interpretation in their representation of reality, and this is the *soundness* of the reasoning. However, the *validity* of these operationsis manifest in the *coherence* of interpretations of *meaning,* and the *soundness* of this reasoning is the *proof* of their *truth* in representing external reality. This contrasts with the Cartesian *subjective feeling* of *clearly* and *distinctly* *intuiting* the *truth* of propositions, which lack *objective criteria* for their meanings and truth (Descartes 1628, Rule Three, 13-15; 1644, Part One, ##43-50; Peirce 1878, #1;1931-1958, 5.448, 1905).

The very first lesson that we have the right to demand that logic shall teach us is, how to make our ideas clear; …To know what we think, to become master of our own meaning, will make a solid foundation for great and weighty thought. (Peirce 1867-1893, 126; 1878)

Hence, we can make the meanings of our ideas *clear* by *valid* interpretation, and **distinct** by proving their *truth* in representing external reality in *sound* reasoning (Nesher 2002b, III.3.3; Gaukroger 1989, 60-71).

**[6] The Interpretation of Signs to Determine Their Clear and Distinct Meanings Using Sound Reasoning**:



We have, hitherto, not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear, but they may be ever so clear without being true. (Peirce 1867-1893, 141, 1878)

Meaning is made *clear* by its *coherent* interpretation andis*distinct*by being proved a *true interpretation* because it *truly represents reality*, such that the true proposition enables us to exercise self-controlled conduct in reality.

Logic is the theory of self-controlled, or deliberate, thought; and as such, must appeal to ethics for its principles. It also depends upon phenomenology and upon mathematics. All thought being performed by means of signs, Logic may be regarded as the science of the general laws of signs. It has three branches: (1) *Speculative Grammar*, or the general theory of the nature and meanings of signs, whether they be icons, indices, or symbols; (2) *Critic*, which classifies arguments and determines the validity and degree of force of each kind; (3) *Methodeutic*, which studies the methods that ought to be pursued in the investigation, in the exposition, and in the application of truth. Each division depends on that which precedes it. (Peirce 1893-1913, 260; 1903)

In the realist interpretation of cognitive signs, there cannot be complete and absolute determination of their meanings, since all proofs of true meaning are relative to the accepted proof-conditions, the real context in which we operate, and the method of proof. However, logical reality cannot be the same as the physical reality that physical sciences represent, nor the cognitive reality that psychological sciences represent, nor any ideal metaphysical reality (Hintikka and Sandu 2006).

Logic does rest on certain facts of experience among which are facts about men, but not upon any theory about the human mind or any theory to explain facts. (Peirce 1903, 5.110)

Indeed, this essentially sums up pragmaticist epistemic logic, the implicit *logica* *utense* and explicit *logica docens* being the basis of all human knowledge, the perceptual and the scientific, including mathematical science. Epistemic logic is, we could say, bound by the Boolean “laws of thought,” representing our *cognitive confrontation* with reality to allow for knowledge and to sustain our conduct in reality (Nesher 1983a, 244-250, 2002b, 2016).

**4.3. The Role of *Meaning* in the Operations Concerning *Validity*, *Proof* and *Truth: the Soundness of Epistemic Logic***

We actually learn the components of epistemic logic in our basic experience, and we naturally start by reflecting on our basic inference of the implication of the perceptual operation of signs. In formal semantics, if the antecedent is accepted as true, then its implied consequent is also true, although if the antecedent is false, then the entire implication is true. According to the pragmaticist’s explanation of *implication*, the conditional relation is such that we interpret the meaning of the *antecedent* using the meaning of the *consequent*; we self-control their *coherency*. This is the *validity* of the interpretation, yet it is not a tautology, which can only be a repetition and not an interpretation of content. The connection between the validity of such arguments and the *forms* of their expression is the *meaning* involved in the laws of the mind, without which the formalizations remain meaningless.

The last objective criterion of the validity of cognitive meanings is the proof of the truth of their interpretation in representing reality. However, different proof-conditions can result in different meanings and different relative truths (Peirce 1869, # 15; 1903; Nesher 2007b). Hence, by being separated from reality, formal syntax has no theory of *meaning* based on experience, and formal semantics has no theory of *truth* based on confrontation with reality; although we intuitively understand their meaning and their truth, we cannot prove their *validity* and *soundness*. Hence, we have to look for a logic that can conduct and explain our cognitive confrontation with reality, and we find this in Peircean semiotics: our epistemic logic, as I understand it.

In formal systems, we start by assuming that the primitive definitions, the axioms, and the rules of inferences are true, but in the sciences, by virtue of epistemic logic, we do not have to assume these truths, since we can obtain them by proving their truth. However, in *epistemic logic*, our premises are hypothetical and can be proved true only at the end of our reasoning process, through the *material logic* of inductive evaluation, based upon the available proved true facts and the *perceptual facts* themselves; finally, it is on the strength of all this that we prove knowledge of all kinds (Peirce 1867-1893, Chap. 8 #I; 1878, 350-354; 1905; Nesher 2002b, II, III, X).

The ultimate purpose of the logician is to make out the theory of how knowledge advanced… so *Methodeutic* which is the last goal of logical study, is the theory of the advancement of knowledge of all kinds. But his theory is not possible until the logician has first examined all the different elementary modes of getting at truth and especially all the different classes of arguments, and has studied their properties so far as those properties concern [the] power of the arguments as leading to the truth. (Peirce 1893-1913, #17, 256; 1903)

These different classes of arguments are the trio sequence of the abductive logic of discovery, the deductive logic of consistency, and the inductive logic of evaluation, which compose the complete proof of truth. Without the methodology of epistemic logic,mathematical hypothesescannot be proved true or false according to the proved facts of reality. In this way, mathematics depends on the habitual rules of epistemic logic and its rational formulations for proving the truth of mathematical theories in order to make their reasoning sound. However, epistemic logic itself, in confronting its reality, is the *Methodeutic* of all our knowledge (Kerr-Lawson 1997; Nesher 2002b, X; 2007c).

It is interesting to consider Russell’s conceptions of formal logic and pure mathematics: from his perspective, without realist epistemology we cannot have logical and mathematical sciences, although we can work with them implicitly by using intuition; but then we might slip into sterile scholasticism, as Russell detected in the paradox of Cantorian set theory (Nesher 2012).

Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is, of which it is supposed to be true. Both these points would belong to applied mathematics. We start, in pure mathematics, from certain rules of inference, by which we can infer that *if* one proposition is true, then so is some other proposition. These rules of inference constitute the major part of the principles of formal logic. We then take any hypothesis that seems amusing, and deduce its consequences. *If* your hypothesis is about *anything*, and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. (Bertrand Russell [1901] 1919, 75)

From the above context, we can analyze Russell’s epistemology of pure mathematics, as distinct from applied mathematics. The first proposition suggests the rule of formal inference: *if* proposition P is true of *any* x, it is true of the particular a: (x) (Px 🡪 Pa). Thus, pure mathematics is built on formal logic, and it applies vacuously to *anything* without relation to any mathematical reality, since it is pure and not applied mathematics, as it is regarded in the so-called positive sciences. And since, accordingly, there is no reality that pure mathematics endeavors to represent, we have no objective criterion for the truth of its deduced propositions. Hence “*If* your hypothesis is about *anything*, and not about some one or more particular things, then our deductions constitute mathematics” and thus pure mathematics holds vacuously for everything and yet is actually about nothing. The problem is with the deduction of formal logic and its role in pure mathematics, since we have no objective criterion for validity in pure formal logical inferences. The reason for this is that without perceiving their meanings, we cannot apply them and cannot have any theories of meaning and truth for formal logic and for pure mathematics (Russell 1901, 75-76; Nesher 2002a, 2007a, 2011, 2012, 2016).

Indeed, Russell comprehends that pure mathematics is based on formal logic, whereby “the primitive ideas of logic and its propositions are deduced from the general axioms of logic, such as the syllogism and the other rules of inference.” But then the question is, what are the meanings of the primitive ideas and meanings and truths of the axioms upon which pure mathematics is built? Moreover, how do we know that all the rules of inference of pure logic and pure mathematics are valid (Kline 1980, XV)?

But today one cannot derive much comfort from the current confusion about what valid mathematics is. This is why Hilbert sought so desperately to restore truth in the sense of objective, unassailable reasoning. As he put it in his paper of 1925 “On the Infinite”: “And where else would reliability and truth be found if even mathematical thinking fails?” He repeated this concern in a talk he gave at the International Congress in Bologna (1928):

For how would it be above all with the truth of our knowledge and with the existence and progress of science if there were no truth in mathematics? Indeed there often appears today in professional writings and public lectures skepticism and despondency about knowledge; this is a certain kind of occultism which I regard as damaging … The future of mathematics has never been of greater promise; the nature of it has never been less clear. The subtle analysis of the obvious has produced a spiral of never-ending complications. But mathematicians will continue to struggle with foundational problems. (Kline 1980, 326)

Indeed, if we cannot prove the truth of the interpretations, validity, and soundness of reasoning of all those logical and mathematical operations, then how can we work with them?

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683–684)

Hence, without epistemological foundations in the realms of formal logic and pure mathematics “we never know what we are talking about, nor whether what we are saying is true.” Moreover, according to the Peircean realist revolution, we cannot count them as gateways to knowledge, nor can we work with them explicitly as sciences. To overcome “skepticism and despondency” in this regard, let us continue Peirce’s realist revolution in epistemology with his *Methodeutic*, i.e., the *epistemic logic* of our knowledge.

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**Chapter 13**

**Epistemic Logic and How It Can Explain** **Mathematical Knowledge**

…mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. (Russell [1901] 1919, 75).

The reciprocal relationship of epistemology and science is of noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is—insofar as it is thinkable at all—primitive and muddled. (Einstein 1949, 683–684)

**1. Introduction to Logic and Its Role in Mathematics**

The basic epistemological questions are: What is logic and what is its role in human affairs? Epistemic Logic is the basic science that represents our confrontation with reality by proving that we truly represent it. Formal systems are merely *closed games of argumentation* in which the truth or falsity of the initial *propositions* of the *syllogisms* or *axioms* are assumed, and by assuming the *validity* of the inferences made, we might reach their conclusions. The difference between *formal systems* and *realist theories* lies in their different proof-conditions. *Formal systems* are bound by their fixed axioms which cannot be proved true, and their formal rules of inference cannot evaluate the truth of their theorematic conclusions. Hence, *axiomatic formal systems are* *complete* and isolated from reality. *Realistic theories*, on the other hand, *are* *incomplete* in a Gödelian sense but can be proved true relative to their proof-conditions: the proved true facts of reality together with the methods of proving their hypotheses. However, if *mathematics is to be a theoretical science* it cannot be a *pure* *axiomatic closed system* isolated from reality; only if mathematics is taken to be an empirical science can mathematicians avoid the ambiguity, contradictions, and paradoxes in creating mathematics from unbasted axioms (Byers 2007).

**2. What Is Logic and What Is Its Role in Human Affairs?**

In his book *Logic*, Kant summarizes his conception of logic as an a priori pure discipline of rules of thinking. This work influenced the following generations of philosophers, logicians, and mathematicians who accepted certain aspects of his philosophical system and became known as neo-Kantians, contributing in various ways to a tradition which still dominates philosophy, logic, and mathematics today.

If, however, we set aside all knowledge that we can only borrow from *objects*, and reflect simply on the exercise of the understanding in general, then we discover those rules which are absolutely necessary, independently of any particular objects of thought, because without them we cannot think at all. These rules, accordingly, can be discerned *a priori*, that is, *independently of all experience*, because they contain merely the conditions of the use of the understanding in general, whether pure or empirical, without distinction of its objects. Hence, also, it follows that the universal and necessary laws of thought can only be concerned with its *form*, not in anywise with its *matter*. The science, therefore, which contains these universal and necessary laws is simply a science of the form of thought. And we can form a conception of the possibility of such science, just as of a *universal grammar* which contains nothing beyond the mere form of language, without words, which belong to the matter of language. This science of the necessary laws of the understanding and the reason generally, or, which is the same thing, of the mere form of thought generally, as we call *Logic*.(Kant, Logic, 1800: 171-172)

According to Kant, the science of logic discovers the *a priori* *necessary* rules of our faculties of understanding and reason, but the rules of other sciences are *contingent* on and connected to our experience with *particular objects* and can change respectively. However, according to Kant’s transcendental epistemology, for the logical rules of our pure cognition to be *necessary* and valid they must be separated from our sensual experience; they are *formal* without the *matter* of sensual experience. Thus, those *pure* rules remain meaningless to us. This epistemology of logic is, in a nutshell, Kant’s essential influence on the philosophy of logic and on logic itself that followed him historically, as we can see in Frege, Hilbert, Russell, Carnap, Tarski, and more (Hintikka 1973, #VIII). Hence, it makes formal logic sterile, leaving it Platonic, syntactical, and intuitionist, faced with difficulties, due to a lack of any objective control of its inferences and so-called proofs (Krantz 2011). Indeed, such logics are closed systems isolated from our experience in reality, that is, argumentation which starts from axiomatic assumptions and reaches conclusions without any objective criteria for the *validity* of the inference and the *truth* of the conclusion (Hintikka 1996; Nesher 2002, 2011, 2016, 2017). Indeed, Kant does not have any comprehensive theory of truth to prove the validity of the rules of formal logic, and he must accept them as *absolute* and *necessary,* *independently of all experience*, but without knowing their meanings we cannot think rationally (Kant 1800, 171).

**3. Axiomatic Formal Systems Are *Artificial* by Abstraction from Cognitive Operations and Cannot Provide a True Representation of Reality to Guide Us in Directing Our Conduct**

Axiomatic formal systems cannot explain and direct the mental operations involved in proving our cognitive representations of reality; such systems, therefore, cannot guide human conduct.As explained previously, *formal systems* are by definition *closed games* with rigid rules and axioms that cannot be formally proved, since deductive rules of inference cannot evaluate the truth of theorems according to reality. The epistemological basis of axiomatic formal systems lies in the conception of truth and its acceptance, in the assumption that truth and falsity are ideal and determine whether our sentences are true or false. Hence, every sentence is bivalent and can be either asserted or unasserted, and, in accordance with the *principle of the excluded middle*, can either be true or false. In practice, however, formal logicians do not live in any Platonic haven, and to discover axioms and rules of inference, they use their experiential intuitions (which remain vague) to compensate for their formal rigid rules. Due to the abstraction and sterility of logical formal systems, logicians are divorced from reality, and thus might go astray and face antinomies and paradoxes. Axiomatic formal systems are *artificially* abstracted from our cognitive operations, but logicians try to accommodate their formal systems by intuiting new axiomatic modes of logic, without ever being able to reach reality (Hintikka 1996, #2).

Difficulties with the concepts of validity and truth in formal logic can only be overcome in *epistemic logic*, where the meanings of the logical components that are essential for the proof of truth originate in our basic perceptual experience of our confrontation with external reality. However, there is an epistemological distinction between the conception of interpretation in Peircean semiotics (the interpretation of signs as *meanings* and proved *true* representations of reality) and the formal Tarskian semantic interpretation based on the representation of artificial models. The intuitionist conception of interpretation is also a question of proof derived from inner mental activity, a hermeneutic interpretation isolated from reality (Tarski 1969; Nesher 2002, II, V). Accordingly, formal systems are complete only in respect of their assumed true axioms and valid inferences, and not with regard to any representation of external reality, unless we pretend that the axioms cover the facts of reality by being identical with the model itself. Hence, we cannot hold onto the picture of model theory, which floats above the world without any known support and without the realistic approach that already belongs to the Gödelian revolution in mathematics, and eventually in logic, as well; but then logical and mathematical realities cannot be Platonic entities that come from nowhere, à la Gödel (Gödel 1951, 313; Nesher 2002, X, 2011).

**4. Peirce’s Semiotics: Epistemic Logic Developed from Introspection of Our Perceptual Operations Using the Complete Trio of Inferences to Quasi-Prove Our Perceptual Judgments**

Peirce’s Phaneroscopy is an essential break from traditional and contemporary approaches to tackling the difficulty of how we can logically understand our representation of external reality. Indeed, only epistemic logic in its entire *trio* sequence (the abduction of discovery, the deduction of prediction, and the induction of evaluation) can provide the complete proof of the truth of our cognitive representations, originating in our pre-rational operations, to quasi-prove our perceptual judgments (Peirce 1931-1958, 5.121–145; 1903).

**[1] Complete Cognitive Operation: the *Trio* Sequence of Abduction, Deduction, and Induction**:



Thus, => is the *plausibility connective* suggesting the hypothesis A, when → is the *necessity connective* deducing the abstract object or fact C, and =❥ is the *probability connective* evaluating the relation of the concept or theory A to the new experience of objects or proved facts C. Peirce developed his semiotics into an epistemology of our perceptual confrontation with reality, manifest in the duality of the expectation of the iconic sign of the feeling (indicating the presence of *ego*) and the indexical sign of the emotional reaction (*non-ego*), which together interpret our genuine signs in their *coherent* synthesis. Thus emerges a complete proof of the true representation of reality, conditioning the *validity* of the interpretation of *meaning* and the *soundness* of the proofs.

**[2] The Confrontation with Physical Reality Through a Coherent Interpretation of the Meanings of the Three Inferences in the Quasi-proof of the Truth of Perceptual Judgment**:



We find that through our cognitive clashbetween the iconic sign of*ego and* the indexical sign of *non-ego*, we first become conscious of the reality that is independent of and external to us:

And what do we mean by real? It is a conception which we must have had when we discovered that there was an unreal, an illusion; that is, when we first correct ourselves. (Peirce 1931-1958, 5.311; 1868)

This explanation can be considered philosophical proof of the existence of something external that is independent of the way we initially present it; and when we interpret the *coherency* of themeanings of iconic and indexical signs, we can prove our positive knowledge of this external reality (Peirce 1867-1893, 136–137; 1878).

**5. Epistemic Logic Empirically Explains Our Confrontation with Reality and Forms the Basis of the Realist Conception of Truth, Eliminating the *Principle of the Excluded Middle***

Axiomatic *formal systems* are *complete* and isolated from reality, whereas *realistic theories* are *incomplete* and true relative to their proof-conditions. Epistemic logic is a basic and universal science whose rules represent our self-controlled method of truly representing reality, hence refuting Berkeley’s *solipsism* and Kant’s *a priorism*. The basic conceptions of *epistemic logic* hold that every instance of knowledge is a proven true representation of reality, and thus we prove our cognitive representations to be either *true* or *false* such that if we do not prove them, they remain *doubtful*. Therefore, we can no longer accept the *principle of the excluded middle*, and *truth* cannot be separated from being proved, as opposed to the logic of formal systems and all kinds of metaphysical and internal realism (Nesher 2002, III, 2011). Since the validity of logical inferences depends on the coherency of the meanings of their signs in respect of the *proof-conditions* in which their true interpretations are decided, then all inferences are made valid by the coherency of their meanings in true interpretation. However, different proof-conditions can have different meanings and truths; thus, if P does not include the meaning of C, then we cannot infer C, since the implication P ➞ C is not valid. With the rules of inference, Pi ➞ Ci, Pi ∧ Ci, and Pi ∨ Ci, the epistemological and logical question is: How can the elimination of the *law of excluded middle* in the realist theory of *truth* affect *deductive inference* when it operates in pragmaticist epistemic logic. If the propositions Pi and Ci are proved true or false or doubtful, what conditions are required for the inferences Pi ➞ Ci, Pi ∧ Ci, and Pi ∨ Ci to be *valid*? Thus, Pi ➞ Ci is valid when the meaning of the consequent Ci is contained in the meaning of its antecedent Pi and when their *truths* are proved only by the trio of complete proof in the common proof-conditions of Pi and Ci. For if they were proved true on the grounds of different proof conditions, the truth of Pi could not entail the truth of Ci, since the complete true interpretation of meaning depends on the entire proof of truth. In epistemic logic the deductive rule of inference {[(Pi ➞ Ci), Pi] ➞Ci}, Pi and Ci are evaluated by induction [(Pi Ab, Ci In) =❥Pr. m/n (Pi Ab ➞ Ci in)], when empirically proved true. But this is not entailed by formal semantic conventional *Truth Tables*, since in epistemic logic the *truth* and *falsity* of propositions are proved by confrontation with reality. Therefore, a formal semantic language including such terms as “if,” “suppose,” “provable,” “unprovable,” etc. is meaningless and not allowed (Gödel 1931; Hintikka 1996, 46-87; Nesher 2011, 2016).

**6. The Epistemology of Mathematics and the Conception of Pure Mathematics as Being Isolated from Reality: How Can it Be Theoretical Science?**

The problem with Euclidean geometry and formal mathematics is that they were created to investigate some structures and properties of reality but remained pure sciences with their a priori assumptions, without any confrontation with reality (Russell 1919, Chap. XVIII-204; Nesher 2017).

Now, the intuition which pure mathematics lays at the foundation of all its cognitions and judgments which appear at once apodictic and necessary are space and time. For mathematics must first present all its concepts in intuition, and pure mathematics in pure intuition; that is, it must construct them. If it proceeded in any other way, it would be impossible to take a single step; for mathematics proceeds, not analytically by dissection of concepts, but synthetically, and if pure intuition be wanting there is nothing in which the matter for synthetical judgments *a priori* can be given. Geometry is based upon the pure intuition of space. Arithmetic achieves its concept of number by the successive addition of units in time… (Kant [1783] 1950, 282-283; Hintikka 1973; schema [4])

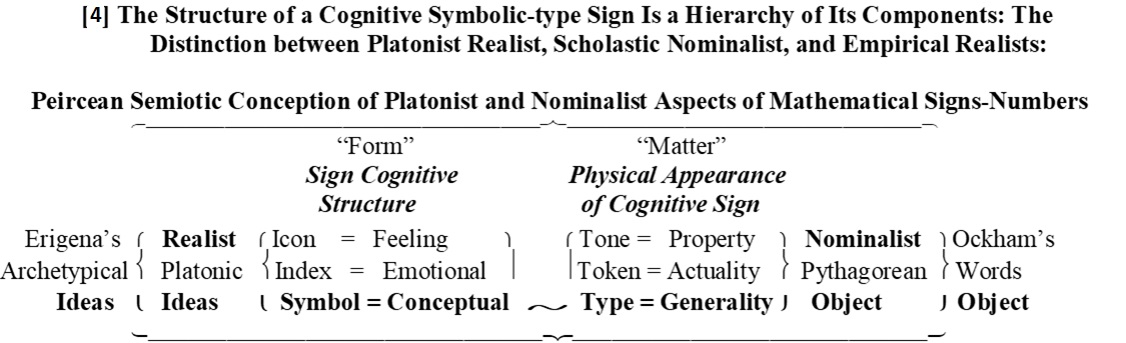
Indeed, Kant bases his epistemological conception of pure mathematics on his analysis of its syllogistic structure and operations; put differently, his conception is based on the *axiomatic systems of transcendental logic and mathematics*. In this way, *pure mathematics* is viewed as a closed game, isolated from any reality, which cannot prove any truth (Kant [1787] 1996,B316-7; Nesher 2011, 2012, 2016).

**[3] The Kantian Conception of Knowledge Based on Pure Concepts and Empirical Sensations: The Evolvement of Empirical Concepts from Blind Sensual Intuitions and Empty Pure Concepts, and their Synthesis in Perceptual Judgment:**

This schema can help to explain how the synthesis of the indeterminate meaning of the *blind object* and the *empty pure concept* makes the concept meaningful and the object determinate, and thus the *empirical object* can be determined by being subsumed under the *empirical concept*. The evolvement of the empirical concepts in perception from sensual intuitions to pure concepts, and with imagination to their Synthesis in perceptual judgment relieves Kant’s difficulty with the epistemology of empirical concepts (Kant [1787] 1996, #24-B150-151). However, Kant bases his transcendental epistemology on the *mystical* conception of *Schematism*, to bridge the gap between *form* and *matter*, for without it his philosophical system cannot hold. Pure *a priori* knowledge includes his conception of *pure mathematics*, but formalism cannot work without empirical matter, that is, without the meaning of the form. Since Kant assumes that mathematics is a pure science based on transcendental pure intuition, he has difficulties in explaining this intuition. In his *Critique of Pure Reason* (B-1787), he explains basic mathematical intuition empirically, by counting fingers or dots.

In thinking merely that union of seven and five, I have by no means already thought the concept of twelve; and no matter how long I dissect my concept of such a possible sum, still I shall never find in it that twelve. We must go beyond these concepts and avail ourselves of the intuition corresponding to one of the two: e.g., our five fingers or (as *Segner* does in his *Arithmetic*) five dots. In this way we must gradually add, to the concept of seven, the units of the five given in intuition… For then it is very evident that, no matter how much we twist and turn our concepts, we can never find the [number of the] sum by merely dissecting our concepts, i.e., without availing ourselves of intuition. (Kant [1787] 1996, B14-15)

The first epistemological difficulty is that we have to decide whether numbers are ideas or objects, and reference can be made here to schema [4] which shows the semantic structure of the signs and symbols: therealist Platonic ideas on the left and the nominalist phenomenal object on the right. The epistemological problem of mathematics lies in the challenge of answering the questions: *What are numbers?* Are they objects of signs, or signs of objects? Furthermore, what is *mathematics* and what is *proof* in mathematical systems (Russell 1901)?



**The Pragmaticist Structure of Cognitive Symbolic-Signs Operating in Mathematical Reality**

Historically, Plato conceived of numbers as *ideas* and Pythagoras regarded them as *objects*, but this indicates epistemological confusion. The two aspects of *signs and numbers must go together* because otherwise *they are not signs*; we cannot grasp the *meaning* of the sign without its *appearance*, and cannot understand the appearance of the sign without its meaning. The sign in Peircean semiotics is the conjunction of “form” and “matter,” or better, the **sign** has two componentsthat cannot exist separately. Moreover, for mathematicians and philosophers in modern history it is not clear whether *numbers* are signs or objects. Indeed, the two aspects are taken to be two separate entities, such that numbers are signs and also objects. This confusion about the nature of numbers evokes the difficulties, ambiguities, and paradoxes of set theory, namely by considering the phenomenal, objective, component of the sign of the number as the object of its cognitive component, or the idea of it (Nesher 2012). Thus, the number’s phenomena are assumed to be the object of the number’s idea, making the number an object of itself. This confusion is the basis of Russell’s paradox in set theory, which is based on the assumption that a number can be a member of its own set, but if a number is a sign, then it cannot be an object and certainly not an object of itself (Russell 1901, 1919). Moreover, the formalist epistemology of logical positivism and analytic philosophy, which assume that cognitive signs and language, with their syntactical and semantical aspects, can be represented by meta-signs and meta-language, brings about further difficulties and paradoxes (Byers 2007). Hence, cognitive signs and languages are not physical objects that can be cognitively represented; we can only interpret their meanings and prove their truth or falsity (Wittgenstein [1921] 1961, 3.33-3.34; Nesher 1986).

**7. On the Nature of Mathematics: Mathematical Proofs at the Crossroads Between the Pure Formal Game and Empirical Theory**

Indeed, the signs of numbers cannot be the signs of objects of empirical experience; they are the discovered components of our empirical operations of counting, grouping, and measuring physical objects (Nesher 2011). The *discovery* of the concepts of these operations of enumeration contains natural numbers, and the further *discovering* oftheir expansion through abstractions and generalizations constitutes our mathematical hypotheses, which will be evaluated and proved on the basis of the extended mathematical reality (Krantz 2011). Hence, by proving the truth of perceptual facts in their representation of mathematical operations, we represent mathematical reality.

**[5]** **The Double Layer of Mathematical Operations: (1) Counting Physical Objects; (2) Perceptual Quasi-proving the Truth of Discovering the Numerical Signs and of Operating with Them**

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If we accept that mathematical reality consists of perceptually self-controlled numerical operations on physical objects, we can see how Peirce and Gödel confuse the meaning and content of mathematical signs with Platonic mathematical abstract forms as objects. Arithmetical numbers are neither *physical objects* nor *abstract concepts* but *conceptual components of our quantitative operations on physical objects*; this is the mathematical reality on which we lean to establish the truth or falsity of our abstract mathematical hypotheses (Nesher 2012).

**8. Conclusion: Mathematics Is an Empirical Science, Neither Queen nor Servant to Other Empirical Sciences but Their Quantitative Backbone**

The challenge is to explain the difference between mathematics and other sciences, and to describe the ways in which they work together, given that they are all empirical sciences representing different aspects of reality but with basic epistemic logic in developing our knowledge of reality. Thus, in mathematics we cannot have true theories without proving them on the basis of mathematical reality. Mathematicians essentially develop theories by discovering hypotheses as formulations of theoretical patterns, typically in the realm of physics, but with regard to all other sciences. These theories are evaluated in accordance with the mathematical reality of quantitative operations on predicted physical observations. Thus, physicists and mathematicians represent different realities with their theories, and a mathematical theory that has been proved true in the measurement of observed physical facts is the condition required for the evaluation of physical theories. The truth of mathematical theory enables us to experimentally prove the truth and falsity of theories. We can understand Gödelian epistemic intuition about the nature of mathematical theories from this perspective, but not by confusing mathematics with other sciences and identifying mathematical reality with physical reality.

When there are difficulties with a physical picture of reality and its mathematical model, such that it becomes impossible to make measurable predictions, then it is necessary to investigate why we are unable to experimentally evaluate the physical hypothesis. Hence, mathematics without the operational measurements of the predicted and eventually observed true facts of reality cannot be true and cannot stand on a much firmer ground than physics without a testable prediction. Both have to prove their own truths according to their respective realities.

However mathematical intuition in addition creates the conviction that, if these formulas express observable facts and were obtained by applying mathematics to verified physical laws (or if they express ascertainable mathematical facts), then these facts will be brought out by observation (or computation). (Gödel [1953] 1995, 9-III: #16)

How are we to understand this depiction of the relationship between intuitive mathematical truth representing its own reality and its application to physical theories to allow for observable predictions? In the end, mathematics is neither the *queen* of science nor its *servant* but rather its *quantitative* *backbone*—that is, the quantified formulations of scientific theoretical patterns or models and their operations on scientific observations, without which physical and other sciences cannot be evaluated experimentally. This empirical explanation can be seen in Gödel’s late philosophical writings on the foundations of mathematics:

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics… This whole consideration incidentally shows that the philosophical implications of the mathematical facts explained do not lie entirely on the side of rationalistic or idealistic philosophy, but that in one respect they favor the empiricist viewpoint. (Gödel 1951, 313)

Hence, we can experientially acquire knowledge of the mathematical facts of mathematical empirical reality.

However, if mathematics is to be considered a theoretical science, it cannot be composed of pure axiomatic closed systems isolated from reality, but rather an empirical science based on our experience of counting, measuring and timing, these being our Spinozian “common notions.” In this way it can be considered to be the backbone of all science, and mathematicians can thus avoid the ambiguities, contradictions, and paradoxes of mathematical systems.

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**Chapter 14**

**Epistemic Logic: The Cognitive Representation of Our Experiential Confrontation with Reality**

Logic, I should maintain, must no more admit a unicorn than zoology can; for logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. (Russell 1919, 169)

**1. Introduction: Epistemological Explanation of How Logic Guides Human Conduct in Reality**

* 1. **The Science of Logic and its Role in Human Affairs – A Phaneroscopical Analysis of Experience**

Epistemic logic is the basic science of knowledge, the method behind our genuine perceptual experience of *confronting* *reality*, through which we prove the truth of our original cognitive representations and scientific hypotheses, and by proving their true representation of reality, we can self-control ourselves in this reality. We thus refute Berkeleyian *solipsism* and Kantian *a priorism*. In this manner, epistemic logic provides a way of understanding *rationality* in human affairs, which is supposed to guide our conduct in reality (Nesher 2007b). Peirce’s Phaneroscopical inquiry constitutes an essential break from traditional and contemporary approaches to tackling the difficulty of how to understand our representation of external reality, without going outside our cognitive skins (Davidson 1996, 312; Peirce 1906, #26, 260-370; Nesher 2002, VI). In contrast to *formal deductive logic,* only the entire *trio* sequence of inferences – abduction, deduction, and induction, along with its two components of *material logic* – can provide the complete proof of the truth of our cognitive representations, which must originate in our pre-rational operations in order to quasi-prove our perceptual judgments (Peirce 1903c, 5.121–145; Nesher 2002). Thus, using epistemic logic, we quasi-prove our *perceptual judgments* as basic *facts* about reality. These *facts* are based on the signs of sensual iconic feeling that resemble objects and the process of interpretation whereby indexical signs emotionally react to iconic signs; once interpreted, these signs evolve hierarchically and become content or meanings, synthesized into symbolic conceptual judgments representing reality (Nesher 1999; 2002, X).

**1.2. Basic Conception of Epistemic Logic as Distinct From Formal Logic**

According to epistemic logic, it is only by proving the truth of our representation of reality that we can achieve knowledge of it, thereby enabling us to classify our cognitive representations as either true or false. Without such proof, our cognitive representations would be doubtful. Therefore, *truth* cannot be separated from the fact of being proved, and we can no longer abide by the *principle of excluded middle*, as is done in formal semantics and in all versions of metaphysical realism. In contrast, intuitionistic logic is based on subjective intellectual intuition and a feeling of correctness in suggesting constructions of proofs – a logic that is epistemologically encapsulated in the transcendental metaphysical subject (Wittgenstein [1921] 1961).

## **1.3. The Meanings of *Validity*, *Proof*, and *Truth* in Epistemic Logic**

*Epistemic logic* is a basic science that enables us to prove the truth of our cognitive representations of reality, allowing us to prove the truth of epistemic logic itself, albeit experientially and not formally, as is the practice in axiomatic systems. In this manner, we avoid the vicious circularity and indefinite regress that characterizes the practice of logical intuitionists, whose working logic is developed without any objective criteria in which they might anchor their decisions. Hence, the general issue is to explain *validity*, *proof*, and *truth* by examining the meanings attributed to these concepts in epistemic logic in contrast to those attributed to them in classical logic, and then to consider the ramifications of these differences. To this end, we happily consider the price we have to pay for our rejection of metaphysical realism, axiomatic syntax, and semantic formal systems, as well as the intuitionist internal realism of the “brain in a vat” scenario (Putnam 1981). As there is no truth without proof, we have to be able to epistemically prove the truth of epistemic logic itself.

# 2. The Epistemological Deficiency of Syntactic and Semantic Axiomatic Formal Systems

## **2.1. Formal Systems Can Neither Explain Nor Perform Human Cognitive Operations**

Formal systems cannot explain the cognitive operations involved in proving our true representation of reality, which is intended to guide human conduct. *Formal systems* are – by definition – *closed games* with rigid rules and axioms that cannot be formally proved true, while at the same time, deductive rules of inference cannot evaluate the truth of theorems in accordance with reality. The epistemological basis of axiomatic formal systems relies on the assumption that truth and falsity are ideal and determine whether our sentences are true or false. For this reason, every sentence is *bivalent*, it can be either asserted or unasserted, and, in accordance with the *principle of excluded middle*, must be either true or false (Haack 1996, I.3).

In practice, however, formal logicians do not live in some insulated Platonic haven, since in order to discover axioms and rules of inference, they use experiential intuitions, which remain epistemologically vague, to compensate for their rigid formal rules that cannot explain or direct human conduct. Thus, abstract logical formal systems, as closed games with rigid rules and baseless axiomatic systems, are inevitably sterile and divorced from reality, with the consequence that logicians are liable to go astray and face ambiguities, antinomies, and paradoxes.

## **2.2. Difficulties with the Conceptions of Meaning and Truth in Formal Systems**

Formal logicians, of course, do not live in an ideal world and yet they still adhere to the abstraction and sterility of their logical formal systems. Their rules and axioms are applicable to reality only ambiguously. Without rational self-control, such logicians can go astray. Since in reality they lack the ability to prove the truth of their hypothetical axioms, they have to keep changing their intuitive assumptions without ever having a clear idea of where they are going (Russell 1902; Tarski 1969; Nesher 2011).

In order to gain a clear understanding of the origins of the various signs used in logical algebra and the reasons for fundamental formulae, we should begin by considering how logic itself arises. (Peirce 1880, #13: 200)

The epistemological explanation of the fundamental rules of logic faces the problem of showing how the truth of these rules can be established on the strength of their inferential operations and conclusions. To achieve this, we are required to use our genuine, instinctive, and practical operations in order to elaborate these rules for further scientific work. Logic is not just the art of reasoning, as Boole and even Peirce sometimes view it. According to Peirce’s mature epistemology, logic should be considered as the basic act of cognition, since it is operated on the basis of pre-verbal signs, instinctively and practically controlled, up to the level of rational self-control of symbolic reasoning. Thus, there must be some implicit operation that enables formalist logicians to work with their systems despite their lack of an epistemology with which to explain them. We can see with Gödel, Tarski, and others, that their basic experiential intuitions enabled them to touch reality, albeit vaguely and without being able to explain how logical rules arise or how to follow them explicitly (Nesher 2002, 2011).

**2.3. The Difference Between the Ways in which Axiomatic Formal Systems and Realist Theories Represent Reality**

The difficult problem is to explain the experiential origin of formal logic: if all knowledge is grounded in the proof of our experiential confrontation with reality, what do formal propositions of logic and their rules of inference mean, given our conclusion that they are declaratively divorced from reality due to their ideal truth and falsity? Without knowledge of the original meanings of logical propositions, we are liable to interpret them incorrectly and misuse them in practice.

The difference between a logical formal system and the logic of realist theories lies in their proof-conditions. Formal systems, by definition, are *hermetically closed* by their fixed formal proof-conditions without any explicit relation to external reality (Nesher 2002, III, X). Hence, formal systems are *artificially* abstracted from human experiential operations and cannot explain them. Indeed, such systems are devoid of cognitive self-consciousness or self-controlled operations in regard to the epistemology of their logic. By contrast, a realistic theory is only *relatively closed* by its available proof-conditions, namely, the proven true facts of reality, the methods of discovery, and the proof of the hypotheses through the epistemic confrontation with reality. Hence, *formal systems* are *complete* and isolated from external reality, whereas *realist theories* are *incomplete*, à la Gödel,but true relative to their empirically proved proof-conditions (Gödel 1929, 1931; Tarski 1941; Nesher 2002, 2011, 2017).

# 3. Can Intuitions Compensate for the Deficiency of Formal Inferences in Representing Reality?

# 3.1. Axiomatic Formal Systems Are Artificially Abstracted from Human Cognitive Operations

Axiomatic, semantic formal systems are *artificially* abstracted from human cognitive operations in reality and cannot explain them explicitly. However, because logicians live in reality and confront it, they rely only on their intuitions and accommodate their formal systems to reality by continually assuming new axioms and new modes of logical operation. The question is whether intuition can really compensate for this deficient or incomplete representation of reality and whether logicians can rationally self-control these intuitive operations to ensure the truth of their disguised virtual proofs.

If logicians cannot have knowledge of the reality that their logics represent, their proposed *models* are unfounded, as they derive these from the meanings of their intuitive axioms of formal systems, which are themselves unfounded. Hence, the intuitive conceptions of *ontology* and the related *models* are but artificial substitutions for *reality* in the formalist conception, and they can neither replace nor explain our cognitive representation of reality (Russell 1919, 169; Quine 1953; Tarski 1956). If we say that propositions and formulas are true if they are true in a model, we can then ask whether models themselves are true representations of reality, and what are the criteria for their truth? For without such proofs we need another level of models to satisfy the first level, and so on *ad infinitum*. In formal semantics, the validity of inferences is established when their consequences are true in all possible models of the system; thus they “[demonstrate] the extensional adequacy of the deductive system in question” (Etchemendy 1999, 3-4). The essential question, though, concerns the nature of the logic of interpretation or satisfaction on which a model is based, since any formal inferences from the conclusions deduced from the systems cannot prove the truth of the assumed models, which are external to them (Tarski 1944; Etchemendy 1999, Introduction).

**3.2. Can Intuitions Compensate for The Deficiency of Formal Inferences to Represent Reality?**

The question requires us to explore conceptions of the *validity* of formal inferences and its criterion in formal semantics: Is it the *truth* of the consequence inferred from the axioms or rather its interpretation in the models of the *formal semantic* paradigm?

The intuitive concept of consequence, the notion of one sentence following logically from others, is without doubt the most central concept in logic. It is what has driven the study of logic for more than two thousand years… The fact that neither the model-theoretic nor the proof-theoretic account of consequence alone captures the genuine notion does not mean they are useless for studying this very same concept. (Etchemendy 1999, 6-7)

But what is the way out of the antinomy? In order to prove the *validity* of inferential consequences, it must be shown that their *true* consequences themselves depend on their validity. The difficulty with *formal semantic* epistemology is that logical inferences are derived from formal deduction alone, and thus they are without meanings. In Tarski's theory of truth, the proposition “P” is true in language L iff “P”, but how can we be certain of “P”? Yet, this proposition can be true if it has a satisfactory interpretation in, say, model M. These two stages in defining truth in formal languages complete Tarski’s theory of truth to satisfy proposition “P” in the semantic model. How, then, do semanticists construct their models, and how can the meaning of proposition “P” be interpreted such that it is satisfied in model M? Moreover, in formal language, Tarski cannot show how we prove the truth of “P”, e.g., "Snow is white"; he can only assume its truth in model M, which is isolated from reality, and thus he cannot have a theory of truth. Any theory of truth must be based on proof of the truth of our perceptual judgments of “P”, and this cannot be done in axiomatic formal theory (Tarski 1936, VIII). Therefore, in formal semantics, we cannot explain and consistently use conceptions of validity, interpretation, consequence, truth, and so forth (Etchemendy 1999, ##3, 4; Nesher 2002, V; 2011, 2017).

Epistemic logic resolves the antinomy and deficiency discussed above by showing that inferences can be proved valid by being interpreted meaningfully and coherently. But then the question is: What is the criterion for their coherence? According to Peircean semiotic realism, we can prove the truth of the inferential interpretation of meaning by proving that the conclusion is a true representation of external reality, and thus we can rely on an objective criterion for the validity of the inferential interpretation (Etchemendy 1999, ##3, 4; Nesher 2002, 2007a, 2007b, 2017).

**3.3 The Abstraction and Sterility of Logical Formal Systems as Closed Games with Rigid Rules**

The difficulties surrounding *validity* and *truth* in formal logic can only be overcome in *epistemic logic*, in which the meanings of the logical components that are essential for proof originate in our basic perceptual experience of confrontation with external reality. However, there is an epistemological distinction between Peircean conceptions of interpretation in semiotics, i.e., the interpretation of signs as *meanings* and *true* (proved) representations of reality, and the formal Tarskian semantic interpretation representing elements of artificial models. In addition, as previously noted, the latter relies on an intuitionist conception of interpretation in which inner mind activity provides the proof of validity, a sort of hermeneutic interpretation isolated from reality (Tarski 1941, #37; Etchemendy 1999, #12-*Conclusion*; Nesher 2002, II, V; 2007b).

Identifying logical consequence with model-theoretic consequence is as mistaken as identifying it with derivability… Though the model-theoretic account may sometimes get the extension exactly right, as may deductive characterizations, this is not because either of them captures, or comes close to capturing, the genuine concept.

Tarski’s conflation spawns as many confusions, as many distracting issues, as Carnap’s. Take, for example, the so-called problem of the logical constants. We saw how this alleged problem immediately evaporated once we recognize exactly why the model-theoretic account is sometimes right and sometimes wrong. The reason has nothing to do with any shared characteristic of the expressions held fixed, but rather with facts about the world. (Etchemendy 1999, 157-158)

The difficulties with formal semantic epistemology and model theoretic truth are not only related to understanding facts about the world, but, more prosaically, the *logical reality* upon which we lean to prove logical truths. However, formal systems are complete only with respect to their assumed true axioms and valid inferences, and not with respect to any representation of external reality, unless we assume that the axioms cover the facts of reality by being identical with, or represented by, the model itself. But then we already know the basic facts of reality before we intuitively discover our axioms, which are to be proved by the known true facts of reality in contradistinction to the traditional conception of formal logic and its consequent conclusions (Nesher 2002, X; 2017). Hence, we cannot hold a picture of the true facts of the world (reality) together with a model-theoretic picture , which floats above the world without any known support. The realistic approach already belongs to the Gödelian revolution in mathematics and will eventually exist in logic as well. However, this revolution can be completed only by understanding that formal systems are solely artificial, disguised scientific theories, and that we cannot rationally control their representation of external reality. And yet, logical and mathematical realities cannot be Platonic entities that come from nowhere, à la Gödel, (Nesher 2002, X; 2007a, 2011; cf. Etchemendy 1999, 4-6*-*7).

# 4. Pragmaticist Epistemic Logic Is the Comprehensive Logicof Complete Proofs Under Specific Proof-Conditions

# 4.1. Cognitive Epistemic Logic Proves Truth in Representing Logical Reality by Using the Trio of Inferences

Pragmaticist cognitive *epistemic logic* consists of the *triadic* sequence of inferences: the abductive logic of discovering hypotheses, the deductive logic of consistency inferring the predicted eventual percepts or facts, and the inductive logic of evaluation of the prediction through perceptual confrontation with reality. However, the inferences of abduction and induction are forms of ampliative *material logic*, based essentially on the meanings of the interpreted signs in our perceptual confrontation with reality. Thus, withepistemic logic, which stands on reality because it is held up by these two pillars of material logic, we prove the truth or the falsity of our hypothetical cognitive representations in confronting external reality. Therefore, we do not have to assume the undefined meanings of primitive terms; rather, we can perceptually quasi-prove the truth of our *ostensive definitions* by means of epistemic logic*.* Accordingly, we do not have to assume the *truth* of axioms, since we can consider them as hypotheses and can prove that their truth is grounded in our perceptual judgments, our basic facts of reality, be they physical, cognitive, mathematical, or logical realities (Tarski 1936, VI; Nesher 2002, 2005b, 2007b, 2011, 2017).

The Peircean empirical explanation is that with complete proofs in epistemic logic, we quasi-prove our *perceptual judgments* as basic *facts*, inferred from our sensual, pre-verbal, iconic feeling signs with regard to an object and its interpretation by the indexical, emotional reaction sign. These cognitive operations form the content and meaning that then becomes synthesized and generalized into a symbolic proposition to be proved in the perceptual judgment. This propositional structure evolves *hierarchically* in the experiential perceptual operation, from pre-verbal meanings into verbal perceptual judgments, i.e., *propositional thoughts* representing reality.

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Thus, the signs that sequentially represent the real object, the iconic feeling of the object, along with the indexical reaction, are our genuine experiential meanings of thesymbolicsigns, our perceptual judgments, and their meanings are carried into all of our intellectual representations of reality. With this analysis of the hierarchical evolvement of propositions, it can be shown that only by abstraction can we separate the symbolic general structure as the verbal *form* of the proposition from its experienced *meaning or content*. *Form* and *content* cannot be separated and remain human cognitive signs representing reality. Hence, our pre-verbal, initially vague sensual feelings and our interpretations mediated by emotional reaction are not blind; rather, they constitute empirical content, which when synthesized in the symbolic proposition becomes clear and distinct as a quasi-proved true representation of external objects. With this understanding of our perceptual judgments, we can avoid the Kantian dichotomy between *empty concepts* and *blind intuitions* as two severed components of the transcendental epistemology of cognition, which cannot explain human knowledge (Peirce1903c, 5.142; Nesher 1999, 2002, III, V, X; 2005a; Detlefsen 1990, 502).

**4.2. Through *Epistemic Logic*, Supported by the Two Forms of *Material Logic* Grounded in Reality, We Can Prove the Meaning, *Truth*, Or *Falsity* of Our Cognitive Representations**

We can prove the *validity* of the three different types of inference belonging to the complete proof of *truth* in representing reality, when: abduction is the logic of discovery, inference to (➾) the best *possible* hypothetical explanation A; deduction is the necessary inference to (➞) a predicted, meaningful conclusion C; and induction is the logic of evaluation, or inference to( =❥) the most probable true result A➞ C:



In contrast to the *axiomatic method*, which starts from unproved *assumptions* to prove questionable validity and so-called true theorematic conclusions, the method of *epistemic logic* starts by attempting to prove hypotheses and employs the complete sequence of the trio of inferences to inductively evaluate the hypotheses, which once proven, can substantiate their truth. Thus, only at the end of the complete trio can we prove the truth of the initial hypotheses, with which we started the entire proof. However, by proving the *truth* of the inductive conclusion (A ➞ C)In with the inferential rule of *probability* =❥ in representing reality, we also prove the *validity* of the abductive rule of *possibility* **➾,** inferring the hypothesis AAb, and hence we also demonstrate the *validity* of the *implication* rule of inference (A ➞ C)Ab, which is the deductive rule of *necessity* ➞. Thus, in employing the *probability*connective =❥,we can prove the *validity* of all three rules of inference. Contrary to the axiomatic formal system, in which the *validity* of the formal rule of the necessary implication is assumed, with epistemic logic, we can prove every inferential component.

Since we see that the conditional *necessary* ➞ connective operates through the entire sequence of the trio operation of proof, the epistemological question now becomes: How do we come to know this necessary inference? We cannot understand its validity from its form and the assumed truth of its consequence, given that truth in epistemic logic cannot be assumed but only proved at the end of the sequence of the trio of inferences in the inductive evaluation. Therefore, the criterion for the validity of the necessary inference can only be the coherence of meanings, the congruity of the components of meaning that the inference entails. *It is through the interpretative proof of the truth of the entire perceptual operation that we control the coherency of its components according to their meanings, thus avoiding the contradictions and paradoxes that plague formal systems.*

**4.3. In *Epistemic Logic*, Validity and Truth Must Be Proved True to Represent Reality, and thus We Can Change Conceptions in Classical Logic Which Are Based On Axiomatization**

In Pragmaticist epistemic logic, which is based on the realist theory of truth, *validity* is a meaningful inferential interpretation *relative to the truth of interpretation in representing reality*, whereas *soundness* is a *true representation of reality* but only *relative to its proof-conditions* (Peirce 1903b; Nesher 2002, 2007b, 2017). The reality that epistemic logic represents by proving the truth of its hypothetical structure cannot be physical reality, which the physical sciences represent, nor cognitive reality, which the psychological sciences represent, and – needless to say – it is not any ideal metaphysical reality (Putnam 1968; Kripke 1975; Stairs 2012). Epistemic logic relies, let us say, on Boolean “laws of thought,” representing our *cognitive confrontation with reality through our perceptual judgments representing objects and, at the same time, the cognitive operations of such representations, which, taken together like Siamese twins*, enable us to sustain our self-controlled conduct in this reality, and it is precisely this *confrontation* that epistemic logic represents.



**Cognitive Proof-Condition**:

In our perceptual experience, we *reflect* intuitivelyupon its operation to *self-control* the *meaning-coherency* of the two perceptual components, the sight and the touch of the eventual object, be it the Peircean *immediate object* or the Kantian *blind object*, and thus we can cognize the *validity* or *invalidity* of the interpretation. Hence, when we feel its *validity,* we continue to interpret signs in the symbolic conception of our perceptual judgment, to quasi-prove its *soundness* in the true representation of reality. In the realist interpretation of cognitive signs, there cannot be complete and absolute determination of the signs’ meanings, given that all proofs of meaning, by proving the truth of our perceptual judgments as basic perceptual facts, are always relative to the accepted proof-conditions, the real context in which we operate.

Logic does rest on certain facts of experience among which are facts about men, but not upon any theory about the human mind or any theory to explain facts. (Peirce 1903, 5.110)

Indeed, Peirce’s pragmaticist *epistemic logic* is rooted in the conceptual analysis of this basic perceptual operation, which he calls Phaneroscopy, which forms the basis of all human knowledge, perceptual and scientific, including mathematical science (Nesher 1983, 244-250, 2002, 2011).

# 5. The Role of *Meaning* in the Operation of *Validity*, *Proof*, and *Truth* in Representing Reality

# 5.1. Peirce’s Phaneroscopy and the Basic Rules of Epistemic Logic

As demonstrated above, only the entire *triadic* sequence of abduction, deduction and induction, with its two *material logic* components, can provide the complete proof of the truth of our cognitive representations, which must originate in our pre-rational yet self-reflective operations, in order to quasi-prove our rational perceptual judgments (Peirce 1903, 5.121–145; Nesher 1990, 2002, 2017). In his philosophical inquiries, Peirce endeavors to discover and develop a theory of cognitive signs that interpret one another, situating the first part of his theory in our basic perceptual operations of interpretations, which quasi-prove the truth of perceptual judgments in their representation of reality. This is Peirce’s Phaneroscopic inquiry, the rational analysis of perceptual phenomena, which provides a crucial means to steer clear of the traditional and contemporary dilemma of how to understand our representation of external reality without going outside our cognitive “skins” (Davidson 1996, 312; Nesher 2002, VI).

The essential problem is that we need to explain how, through a cognitive interpretation of the sequence of perceptual signs, we can represent external physical reality and reflectively represent the mind’s operations of signs. In his endeavor to develop a pragmaticist epistemology, Peirce starts from his basic perceptual experience and, through phenomenal introspection, or Phaneroscopy, cognizes and explains the sequence of sign interpretations: the iconic feeling interpreted by the indexical reaction to this feeling, and then synthesized into the symbolic thought of perceptual judgment. In this interpretation, the incoherency and coherency between the *iconic* sign (quality of feeling) and the image of an eventual object presents *ego* expectation, along with the indexical emotional reaction to this first sign, which when contrasted with the first sign, is what Peirce calls *non-ego*. Hence, the latter either disappoints the expectation, and thus may be understood as representing reality negatively, or fulfills it and represents external reality (Nesher 2002, III; 2017). With this Phaneroscopic introspection, Peirce shows how, without going outside our cognition, we can prove the truth of our cognitive representations of external reality. Thus, with this realist epistemology we can avoid the difficulties of Berkeleyian and Humean phenomenology and Kantian transcendentalism, as well as those of modern hermeneutic phenomenology and analytic philosophy (Marty 1982; Nesher 2002, VI; 2004).

**5.2. Learning Epistemic Logic and Interpreting its Components in Basic Experience**

With this understanding of our true perceptual judgments (as being the complete proof of our basic knowledge), we can avoid both the formal semanticist’s ideal truth and the intuitionist neo-Kantian phenomenalist’s *subjective intuitions* in constructing proof, which cannot explain how we reach human knowledge of reality. Hence, we have to investigate how we actually learn these components of epistemic logic in our basic perceptual and scientific experience. The natural starting point is to reflect on the perceptual operation of our perceptual cognitive signs and the basic inference of implication. Doing so, in turn, requires us to analyze and understand this specific operation (Boole 1860).

Sextus: [4] And those who judge by implication (έμφασιϛ) say that a true conditional is one whose consequent is contained potentially in its antecedent. According to them the statement “If it is day, it is day” and similarly every conditional which is repetitive (διφορομενον) will apparently be false; it is impossible for a thing to be contained in itself. (Sextus, *Pyrrhoneiae Hypotyposes*, ii 110-12, in Kneale and Kneale 1962, 129)

In formal semantics, if the antecedent is accepted as true, then its inferred consequent is also true, as with *modus ponens*, but we should distinguish between logical implication (A ➞ C) and deductive inference {[(A ➞ C), A] ➞ **C**}, in which the latter is a component of the former, yet not identical to it. However, if the antecedent in implication were not true (**~**A ➞ C), could the consequent potentially be contained within it? According to the pragmaticist, the conditional relation is such that we interpret the meaning or content of the antecedent through the meaning of the consequent by self-controlling the coherency of their relation. This is the concept of the *validity* of the interpretive operation, yet it is not a tautology, which would be repetition, for it is distinct from the interpretation of the meaning and content (Wittgenstein [1921] 1961; Nesher 1990, 2007b).

The task of realist epistemology is to show what we mean by the *validity* of the three types of inference in *epistemic logic* (abduction, deduction, and induction). Indeed, their *validity* differs from the concept of *validity* *of inferences* in formal logic systems. As in epistemic logic, the *necessary* deductive inference component of expectation, in the triadic complete proof, cannot preserve the truth in its operations because the truth is not axiomatic and can only be proved at the end of the complete proof in the inductive *probable* inference of evaluation. However, in *formal systems*, the deductive inference is not necessarily valid, because the components of the deductive formal inference are based on their *logical form*, which without explicit experiential meaning lack any coherency of meaning. Without such coherency, the components of the deductive formal inference cannot be meaningfully valid. Moreover, the deductive inference alone cannot prove true representations of reality since we cannot formally prove how our deduced conclusions relate to it. Indeed, formal logic lacks the epistemic logic foundations for establishing what is meant by *validity,* *necessity*, and *proof* (Marion 2012, 96). Hence, we can make the meanings of our ideas clear by *valid* interpretation, and **distinct** by proving their *truth* in representing external reality using *sound* reasoning (Nesher 2002b, III.3.3; Gaukroger 1989, 60-71).



We have, hitherto, not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear, but they may be ever so clear without being true. (Peirce 1878, 141)

Meaning is made *clear* by its *coherent* interpretation andmade *distinct*by being proved a *true representation of reality*, such that true propositions enable our self-controlling our conduct in reality.

In contrast, the rules and structure of formal logic did not come into being *ex nihilo* and thus our knowledge of their use has to originate in some generalization from our behavior in natural language. But then, by divorcing the rules and structure of formal logic from their origins, that is their experiential meaning, we can err in using the *logical* form, which is empty of meanings, for argumentation in life. Even if we had some vague intuitions about the meanings of the operational rules of formal logic, what role could it play in our lives? Indeed, formal logic is just a *closed game of argumentation* that assumes the *truth* and the *falsity* of the initial propositions of the syllogisms, and we reach the conclusions of such arguments merely by assuming the validity of the inferences.

It has been said in the foregoing section that in virtue of the laws of thought as expressed in signs, the forms under which our conceptions, judgments, and conclusions are expressible become in a more or less absolute sense determinate. As a particular illustration of the truth, it is shewn that there is a connection between the validity of that species of argument called the syllogism [and] the form of its expression. (Boole [1860] 1997, 132)

In the same way, we understand the structure of *formal logical systems* by assuming the *truth* of their axioms and the *validity* of their rules of inferences, so that we can deduce the relevant theorems and conclusions. By being formal, they are meaningless and so too is their validity, but without being able to prove their truth as objective knowledge of reality, *formal logical systems* remain only a game of argumentation.

The merits and the defects of the Aristotelian theory are both due to the same cause. We must regard it less as a science than as an Art… (Boole [1860] 1997, 136)

Accordingly, we have to distinguish between the *formal logic of argumentation* thatcannot help us in proving the truth of our cognitive representations in order to grasp the reality in which we live and the *theory of* *epistemic logic* that enables us to prove our true knowledge of reality. The connection between the *forms* of such arguments and their *meanings* allow for the *validity* of their expression within those laws of thought. Without this connection, the formalizations remain meaningless and we are left with indeterminate validity. With the above understanding of the experiential evolvement of the formulation of logical laws of thought, it also becomes clear that the meanings of the notions of *validity* and *truth* originate in our perceptual experience. This occurs through the introspection of our instinctive, practical, and rational quasi-proof of the true interpretations of meaning regarding these notions, operated in the symbolic thought of the perceptual judgment. Thus, we self-reflect on our native behavior in order to operate it rationally as our basic method of securing knowledge, and we compare and evaluate this method with the development of scientific theories and their experimentally proved truth (Spinoza 1662, ##29-31; Peirce 1903b; Nesher 2002, xv-xx, Chap. I).

... the knowledge of the laws of mind does not require as its basis any extensive collection of observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances... [A] general truth in Logic… is made manifest in all its generality by reflection upon a single instance of its application. (Boole [1854] 1997, 4)

The central question is this: How do we discover, rationally control, and prove the truth of the inferential rules of our habitual conduct? Indeed, it can be shown that the rules of our cognitive habits are felt in terms of the relationships between their cognitive components, i.e., the feeling of the coherency between the meanings of signs and the *validity* of logical inferences of propositions whose *truth* we first quasi-prove in our habitual signs of perceptual operations. Indeed, only when we introspect and formulate inferential rules rationally can we prove the *validity* of these rules and the *soundness* of their relationships. However, the proof of the *truth* of *epistemic logic* rules of inference can only be achieved in the complete proof of the *truth* in the triadic sequence of logical rules of inference, that is: the abductive discovery of hypotheses, the deductive prediction of their results, and the inductive evaluation of their truth via a confrontation with reality. With the above understanding of the empirical evolvement of the formulation of logical laws of cognition, as well as the meanings of the notions of *validity*, *soundness*, and *truth*, which also evolve from our perceptual experience, by reflecting on our natural behavior, we can quasi-prove their truth by introspecting about our instinctive, practical, and rational operations (Peirce 1869, 78-81; 1906, 260-370; Nesher 2002, 2007b, 2017; Kneale 1962, XII.4).

**5.3. Epistemic Logic as the Basis of Knowledge and Self-Controlled Conduct in Reality**

The pragmaticist reflects phenomenally on the cognitive operations of our confrontation with reality in order to develop an epistemic logic that proves our knowledge of reality to direct humans in self-controlling their conduct in life. From this perspective, we have to reevaluate the formalist and intuitionist logics; to what extent can they contribute to human life in reality, if at all? According to the pragmaticist- realist theory of truth, what we prove true represents reality, whereas what we prove false does not represent reality. Additionally, if hypotheses are proved neither false nor true, they are considered at best doubtful or even erroneous. Hence in the realist theory of truth, which is the core of *epistemic logic*, we have to eliminate the formal logical principles of *bivalence* and the law of *excluded* *middle* because we cannot axiomatically assume *truth* or *falsity* without proving them in accordance with reality, and this relegates us to a position in which we have to accept the third option of *doubtfulness* (Nesher 2002, II, III, V, X; 2011, 2017; Béziau 2003).

Intuitionists do not have a theory of truth, and their theory of proof amounts to a theory of construction based on inner subjective feeling, a sort of intellectual intuition. Therefore, they use '*de jure*' constructive proof and not '*de facto*' proof of the truth of facts, propositions, and theories. As they do not have any objective criterion for truth, they use the *principle of excluded middle,* situated between constructive proof and assertion. Nonetheless, given the absence of any external criterion, it is based only upon subjective intuition (Heyting 1956, 17-18; Brouwer 1949, 92; Kleene 1950, #13).

*But with regard to the principle of the excluded third, except in special cases, the answer is in the negative, so that this principle cannot in general serve as an instrument for discovering new mathematical truths*.

Indeed, if each application of the *principium tertii exclusi* in mathematics accompanied some actual mathematical procedure, this would mean that each mathematical assertion (i.e., an assignment of a property to a mathematical entity)could be *judged*, that is to say could either be proved or be reduced to absurdity. (Brouwer 1949, 5)

The intuitionists’ constructions by *rules* are just *de jure* proofs, and the *principle of excluded middle* merely offers an absurd alternative. Hence, it is only by construction that we can prove assertions, and we cannot prove absurdity. The intuitionist cannot err in construction, but only in assertion, because the subject creates mathematical and logical constructions separately from any model or reality. Without any objective criterion to distinguish between valid and invalid operations, there remain only inner intuitive feelings (Kleene 1950, #13). This is different from classical logic in which logicians prove the truth of a proposition *de facto*, but they do so only by representing their formal rules or constructed models, while proving the falsity of a proposition through the principle of excluded middle (Heyting 1956, 18-19).

However, according to realist epistemology, because we cannot know the *truth or falsity* of propositions that we have not proved as such, then the *bivalence* *of truth and falsity* evaporates, as with the *principle of excluded middle*. This is also what the intuitionists suggest, yet somewhat ambivalently. Without an explicit theory of truth, the intuitionists prefer to talk about “constructing proofs” rather than “proving truths” (Brouwer 1949, 5; Heyting 1956,18-19, 102; Kleene 1950, #13; Etchemendy 1999, Introduction; Nesher 2002, III, V, X; 2011, 2017; Burgess 2009, Chap. 6).

Therefore, with the elimination of ideal *truth* and *falsity* we also have to eliminate the principle of excluded middle, and the semantics of *truth tables*. Accordingly, we must change the meanings of the connectives, the rules of inference, and the conception of proof as accepted in classical logic. The challenge is to process these changes to see which kind of science *epistemic logic* belongs to, though we know that it is based on the realist theory of truth (Nesher 2002). How, then, can we explain the new meanings of the classical connectives and the logical inferential components, ∨**, ¬, ∧, ⇒,** ⇔, |–, |=, as well as the relevant conceptions of *meaning, proof, truth,* and *validity*? For example, the negation connective **¬** is not bivalent as in “*if* P is true, then **¬** P is false,” since “*if*” can no longer hold, because P is doubtful without its truth being proved. Furthermore, its negation, **¬** P, cannot be false without being proved as such, as I showed with Gödel’s incompleteness theorem: we cannot use the term “provable,” only “proved” or “not proved” are acceptable (Hintikka 1996, Chaps. 4, 7, 9; Nesher 2011, 2017).

STRANGER: So, when it is asserted that a negative signifies a contrary, we shall not agree, but admit no more than this—that the prefix ‘not’ indicates something different from the words that follow, or rather from the things designated by the words pronounced after the negative (Plato, Sophist: 257: b-c in Haack 1996, 45-46; cf. Hintikka and Sandu 2006, #15; Nesher 2007b).

The natural explanation is that the negation of the proved true P, namely **¬** P, has a different meaning in different contexts, that is, under different proof-conditions (Nesher 2002, V; 2017).

# 6. *Epistemic Logic* and the Conceptual Changes of Formal Logic for a True Representation of Reality

# 6.1. The Role of *Meaning* in *Validity* ofInterpretationand *Soundness* inProofin *Epistemic Logic*

Under the pragmaticist realist theory of truth, all the connectives of formal semantics have to be reevaluated or eliminated with respect to the new epistemic logic theory of truth. As perceptual facts are basic truths, all propositions and theories that have been proved true are facts. Hence, we cannot negate or contradict them formally or falsify them experimentally, à la Popper, but only explain their limitations relative to their accepted proof-conditions (Peirce 1902, 2.141; 1903, #14-204-206; Nesher 2002, xiv, VIII.7; 2017). At the same time, if we prove the falsity of our hypotheses, then negating them does not lead to any truth, but actually drives us to discover new hypotheses.

Therefore, we cannot use negation **¬** in *epistemic logic* in the same way as it is used, as an integral element in formal semantics built on the conception of truth in metaphysical realism. Indeed, in realist epistemology, all connectives are developed and proved valid in perceptual judgments and in natural language behavior, and the role of *negation* in *epistemic logic* cannot be based on any ideal truth and falsity. I suggest that in P➞ **¬**q, the negation of q is only in respect to the validity of coherency of meaning, when the interpretation of the meaning of q contradicts the meaning of P. The criterion for the coherency or incoherency of meanings is not only grounded in subjective feelings; the criterion is also based on the initial meanings developed and proved true in perceptual experience, and on the ostensive teaching of Wittgenstein, not rooted in language-games but rather in the fact of confronting reality (Wittgenstein 1953, ## 27-34; Nesher 2005b). Therefore, the objective criterion for the valid comprehension of cognitive meanings is their coherency, enhanced by proof of the truth of their interpretation in representing reality (Nesher 2007b, 2017).

**6.2. The Validity of Logical Inferences Depends on the Coherency of the Meanings of Their Signs**

We have established that the validity of logical inferences depends on the coherency of the meanings of the relevant signs with respect to the *proof-conditions* in which their true interpretations are proved. However, under different proof-conditions, there may be different meanings and truths (Putnam 1968, VI). Thus, when P does not include the meaning of C, then we cannot infer C from P, since in this instance the implication P ➞ C is meaningless and not valid (Nesher 2007b). Hence, when the propositions Pi and Ci are either true or false or doubtful, we have to reconsider the conditions of validity for the inferences Pi ➞ Ci, Pi ∧ Ci, and Pi ∨ Ci (Kleene 1950, #64; Putnam 1957). The problem is that we need to evaluate whether different kinds of logic have developed in natural behavior and from classical logic, and if so, can they be considered aspects or branches of epistemic logic?

It nevertheless appears that the right project is to develop richer logical languages rather than devise different sets of special-interest modes of reasoning. If and when this is done, the result is likely to be a richer idea of what logic is – or of what it can be. (Hintikka and Sandu, 2006: 32)

To reiterate: we have established that the validity of logical inferences depends on the coherency of the meanings of the applicable signs in respect to the *proof-conditions* in which their true interpretations are proved; we have observed that, under different proof-conditions, we can detect different meanings and truths; we have likewise noted that when **C** is not included in the meaning of P, we cannot infer C from P, since the implication P ➞ C becomes meaningless and invalid. Given the above, with regard to the rules of inferences Pi ➞ Ci, Pi ∧ Ci, and Pi ∨ Ci, the epistemological and logical question is this: Within the framework of the realist theory of truth, how does the elimination of the *law of excluded middle* affect *formal connectives* as they operate in pragmaticist *epistemic logic* (Nesher 2002, 2007a, 2011, 2012)? In other words, if the propositions Pi and Ci are *true*, *false*, or *doubtful*, what conditions must be met to prove that inferences Pi ➞ Ci, Pi ∧ Ci, and Pi ∨ Ci are *valid* (Kleene 1950, #64; Putnam 1957)?

As to establishing the *validity* of the *deductive* logical rule of inference Pi ➞ Ci according to *epistemic logic*, we have shown that it is valid when a consequent’s meaning is contained in the meaning of its antecedent, and its *truth* can be proved only by the trio of the complete proof under the relevant proof-conditions, but not by using conventional *truth tables*. Given that truth in *epistemic logic* is proved through confrontation with reality, reliance on formal semantic language that includes “if,” “suppose,” “provable,” “unprovable,” etc., is meaningless and disallowed (Kleene 1950, #13; Nesher 2011). In this manner, *epistemic logic* acts to overcome the unrealistic epistemologies of formal semantics and intuitionistic logic. Finally, the last stage in proving the validity of the *deductive* logical rule of inference Pi ➞ Ci is to consider the conditional interpretation of propositional signs representing reality under specific proof-conditions, in order to determine whether *validity of interpretation* and *truth of representation* coincide (Nesher 2007b).

1. When Pi in Pi➞ Ci is false or doubtful it cannot entail the truth of Ci, and thus the inference is invalid, because under different proof-conditions their *interpretations do not coincide with the truth of their representations*.

2. When Pi is true in Pi ➞ Ci, it cannot entail Ci if the latter’s proof-conditions differ from those of Pi .

3. If Pi has proof-conditions which include those of Ci and both are true, then Pi ➞ Ci is a valid inference through the interpretation of meaning, yet not true if not proved so.

4. When both Pi and Ci are true under different proof-conditions, then the *logical rule of inference*, Pi ∧ Ci, is invalid, because their meanings are not coherent in the two contexts.

5. When the meanings of both Pi and Ci are true interpretations under different proof-conditions, then the *logical rule of inference* Pi ∨ Ci is valid, because they can be unconnected meaningfully.

6. When one of the propositions Pi or Ci is true and the other is doubtful, i.e., neither true not false, then the *logical rule of inference* Pi ∨ Ci is devoid of validity, i.e., it does not work.

7. When one of the propositions Pi or Ci is true and the other is false, then the dilemma ∨ (“or”) is meaningless and the *logical rule of inference* Pi ∨ Ci is neither valid nor invalid, but rather empty of validity.

8. When both Pi and Ci are false or doubtful then the *logical rule of inference* Pi ∨ Ci is invalid.

We can compare these interpretations of the connectives with the intuitionist explanations of *constructions* and *transformations,* but the latter lacks proofs of truths. However, with pragmaticist *epistemic logic*, such proofs can be achieved under specific proof-conditions, namely, the truth of *interpretation* is dependent on the *truth of the representation of reality* (Makinson 1973, 4.2., 4.3.; Nesher 2002, X).

**6.3. The Role of Meanings in the Basic Inferences of *Epistemic Logic***

In formal systems, we start by assuming that the primitive definitions, the axioms, and the rules of inference are true, but in sciences and even implicitly in *epistemic logic*, we do not have to assume these truths since we can obtain them by proving the truth of their hypotheses. However, in *epistemic logic*, in contradistinction to *formal logic*, our premises are hypotheses, which are discovered by the *material logic* of abduction. Additionally, in the *formal* deductive inference, we deduce from such hypotheses our expectation or predictions, and these can be proved true only at the end of our reasoning through the *material logic* of inductive evaluation (on the strength of the relevant proved coherence of meaningful percepts as true facts). It is essential to explain how, in perceptual operations, we inductively evaluate and prove the *perceptual facts* themselves through assessment of the percepts through which we implicitly prove their truth (Schemas [3] [4], Nesher 1999; 2002, X). In the Inductive evaluation, it is essential to accept the new independent experienced percept CIn as factual, if we wish to prove our basic perceptual facts. In other words, this is the genuine criterion for obtaining knowledge of reality. Therefore, when the independently experienced percept CIn has a *similar meaning* to that of the abductive percept CAb, namely, when their similar meanings (CIn CAb)are ensured, the *validity* of the interpretation of the probability of the inference from AAb to CIn\* is high and we can prove the truth of m/n=❥(AAb ➞ CIn\*). In this case, the proof of the true representation of reality is sound. Thus, by proving the truth of the perceptual hypotheses, we provide a foundation for proving all of our knowledge (Peirce 1878, Chap. 8 #I; 1905, 350-354; Nesher 2002, III, X; 2017).

In *epistemic logic*, in keeping with the realist theory of experiential meaning and truth, all logical operations (for example, the logical rule of inference Pi ➞ Ci) are valid when their consequent meanings are contained within the meaning of their antecedents, and their *truth* is proved only under their common proof-conditions. However, due to the conception of proof as well as the conception of truth and falsity in epistemic logic, the inference Pi ➞ Ci cannot work using the formal semantic convention of ideal conceptions in the form of *truth tables*. As previously stated, truth in epistemic logic is based on proof that depends upon a confrontation with reality, rendering formal semantic language meaningless and therefore disallowed (Kleene 1950, #13; Nesher 2011). In this manner, epistemic logic acts to overcome the unrealistic epistemologies of formal and intuitionistic logics.

# 7. Conclusion: We Can Prove the Truth of *Epistemic Logic* as the Basis of all our Knowledge

## **7.1. Can *Epistemic Logic* be Proved Empirically?**

As all our knowledge is empirically proved true, this condition also holds for our knowledge of *epistemic logic*, which is the basis of all knowledge. Spinoza recognized this and endeavored to refute Cartesian formalism by proving the true logical method of experiential knowledge (Spinoza 1662, ##29-34; Nesher 2002, I). From the perspective of this realist approach, Peirce argues that we exercise self-reflection on perceptual operations before analyzing those operations. This happens through our interpretation of the relationship between the iconic sign representing *ego* and the indexical sign as *non-ego*, and the coherency between these is the criterion for the validity of interpretation, synthesized in quasi-proving the truth of perceptual judgments representing external reality (Peirce 1906, #26). In generalizing this analysis in *epistemic logic*, in contrast to axiomatic formal logic, our premises are hypothetical and can be proved true only at the end of our reasoning in the *material logic* inference of inductive evaluation. Thus, perceptual judgments that have been proved true are themselves original *perceptual facts*, and we prove all evolved knowledge on their basis.

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Throughout the entire series of the trio of inferences, it is essential that the interpreted meanings of the components cohere as a whole to validate the proofs. Thus, the newly experienced percept CIn of the inductive evaluation, when observed independently, should share *similarity of meaning* with that of the abductive percept CAb, meaning (CIn CAb),to ensure the *validity* of the interpretation and the *soundness*of the evaluation, thereby proving the truth of the hypotheses.On the basis of this*similarity of meaning* between the percepts of abduction and induction, (CIn CAb), we prove by inductive *evaluation* the truth of the perceptual judgment and, with it, the truth of the abductive hypothesis (Nesher 2017).

These different modes of inference comprise the triadic sequence of the abductive logic of the *possible* discovery of the hypothesis, the deductive logic of *necessary* consistency, and the inductive logic of evaluation of *probability*, and together they form the complete proof of truth or falsity in representing reality. Without the methodology of epistemic logic,the experiential hypothesescannot be proved true or false, since they depend only on the habitual rules of experience. However, with the rational formulation of our basic rules for proving the truth of perceptual hypotheses, we can make our reasoning sound through confrontation with reality. Indeed, epistemic logic itself can be proved philosophically by reflectively controlling the operation of the perceptual experience, directly confronting reality. Hence, after proving *epistemic logic* as the basis of all our knowledge, it becomes the *methodology* for empirically proving all three of our basic sciences: *theoretic, ethics*, and *aesthetics* (Peirce 1903a, ##11, 14;Nesher 2002, X; 2007a, 2007b, 2017).

## **7.2. Realists Can Refute *A Priorism* and *Solipsism* by Proving Our Confrontation with Reality**

Thus, we can be realists, refuting and overcoming *a priorism* and *solipsism* by proving that our basic epistemic logic enables us via our confrontation with reality to represent it truly (Spinoza 1662; Boole 1854; Peirce 1902; Nesher 2002, Introduction, X, 2017). But how are we to understand the place of *epistemic logic* among the sciences and its relation to philosophy? Using philosophical epistemology, along with Peircean Phaneroscopy, we discover that with epistemic logic we cognitively represent both the basic inborn and acquired rules of operation that we apply in our confrontation with reality. Thus, through self-reflection, we observe these habitual operations and are able to formalize the rules that constitute the complete proof of the truth of our representation of reality. In this sense, epistemic logic allows us to conduct basic rational self-control of our operations of knowledge, and thus of all other sciences. This also includes philosophy, which, according to Peirce, is an observational science in which we work with such logic implicitly to discover its rationality (Boole 1860, 133; Peirce 1905, 5.440-441; 1906, #26; Nesher 1994, 2001, 2002; Hart 2010, #10).

Under the last head of difference would fall such questions as the following viz. whether the rules in question apply to all the processes of thought or only to that of reasoning, whether in their application to reasoning they take account of all the forms of inference or only of the syllogistic form etc. (Boole [1860] 1997, 133).

The basic structure of epistemic logic is originally acquired through our experience, and with it, we implicitly and habitually quasi-prove the truth of our perceptual knowledge and conduct our behavior, and eventually philosophize with it. Hence, we can say that by joining the philosophical and logical sciences, we gain insight into their influence on the way we conduct our lives in nature.

## **7.3. The Place of Epistemic Logic Among the Sciences and Its Relation to Philosophy**

How, then, do we understand the place of epistemic logic among the sciences and its relation to philosophy? As explained above, using philosophical epistemology, along with Peircean Phaneroscopy, we discover via self-reflection the basic inborn and acquired rules of operation that we apply to our confrontation with reality. Hence, by reflecting on our habitual operations, we can formalize these rules into a complete proof of our true representation of reality (Hintikka 1997; Nesher 2002, xv-xx; 2017). Epistemic logic, therefore, is universal logic that can explain and control our behavior in reality and, thus, it underpins the rational self-control that we apply to all our operations of knowledge, including philosophy, an observational science in which we work implicitly with such logic, and we formulate this rationally.

Epistemic logic, then, forms the basis of all our knowledge, habitual and scientific: *Logica est Scientia scientiarum et ars atrium* (Ockham 1323)

In this its highest conception therefore Logic might be said to be the Philosophy of *all* thought which is expressible by signs whatever the objects of that thought, whatever the nature of those signs may be. (Boole [1860] 1997, 126)

With epistemic logic we implicitly and habitually conduct our basic behavior, and eventually philosophize with it. Hence, we can say that philosophical and logical sciences come together in epistemic logic to enable our understanding of reality and of the ways in which we self-control our conduct in this reality.

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1. See also the historical description offered by Joong Fang in *Hilbert,* *Towards a Philosophy of Modern Mathematics II*, Chapter XI “Foundational Problems” (New York: Paideia Press, 1970), pp 168 ff. [↑](#footnote-ref-1)
2. See Y. Bar-Hillel’s discussions in *Problems in the Philosophy of Mathematics*, edited by Imre Lakatos (Amsterdam: North Holland Publishing Company,1967), e.g., “...the most adequate philosophy of mathematics, viz. the strictly non-ontological conception of mathematics by Carnap, Kerneny and others...” (p. 136). I must admit that in Popper’s later writings there is a change towards a realistic approach to formal science. In his *Objective Knowledge* under “Appreciation and Criticism of Brouwer’s Epistemology” (Oxford: Oxford University Press, 1972), he writes: “Brouwer’s other great achievement, from a philosophical point of view, was his antiformalism: his recognition that mathematical objects must exist before we can talk about them.”(p. 134, cf. pp. 137-138). [↑](#footnote-ref-2)
3. See Immanuel Kant, *Critique of Pure Reason*,translated by Werner S. Pluhar (Indianapolis: Hackett Publishing Company, [1787] 1996). “As regards the formal element, we can determine our concepts in a priori intuition, inasmuch as we create for ourselves, in /pure/ space and time, through a homogeneous synthesis, the objects themselves— these objects being viewed simply as quanta.” B751. “We are not here concerned with analytic propositions... but with synthetic propositions, and indeed with just those synthetic propositions that can be known a priori. For I must not restrict my attention to what I am actually thinking in my concept of a triangle (this is nothing more than the mere definition); I must pass beyond it to properties which are not contained in this concept, but yet belong to it. Now this is impossible unless I determine my object in accordance with the conditions... of pure intuition.” B746. [↑](#footnote-ref-3)
4. Louis O. Kattsoff expresses a similar idea in the following: “It appears that what cannot be in reality cannot be expressed in an adequate language; and if we have such a language then what cannot be expressed in it could not be.” See *Logic and the Nature of Reality*, Second edition (The Hague: Martinus Nijhoff, 1967), pp. 51-52. [↑](#footnote-ref-4)
5. We may also have a kind of categorical gap between Kant and Popper in regard to determinism and necessity. Kant applies his determinism to the theories of nature while Popper’s indeterminism is applied to an extra-theoretical nature, the world-in-itself. See *The Philosophy Of Karl Popper*, Book II, edited by Paul A. Schilpp (The Hague: Martinus Nijhoff, 1974), p.104. [↑](#footnote-ref-5)
6. Compare my position with Bunge’s words on metaphysics: “Although a metaphysical theory can be neither confirmed nor refuted by empirical data, it can be either relevant to science or pointless in regard to it. The metaphysical presuppositions of scientific theory do not hang in mid-air: they are not free speculations but come and go with the theory.” See Mario Bunge, *Method, Model and Matter* (Dordrecht: D. Reidel publishing Company, 1973), p.146. [↑](#footnote-ref-6)
7. This research was partially supported by the Research Funds of the Faculty of Humanities, University of Haifa, Grant 1290-971. [↑](#footnote-ref-7)
8. Baruch Spinoza, *Ma’amar `al Tiqun ha-Sekhel* (translated from the Latin by N. Spiegel; edited by and with introduction and commentary from Y. Ben-Shlomo), Jerusalem 5733 (1972-73). [↑](#footnote-ref-8)
9. See H.A. Wolfson*, The Philosophy of Spinoza*, Harvard 1934, Introd. and Chap. I; in a personal conversation with the author on this subject, which took place at Harvard in 1972, he repeatedly emphasized his own regressive historical approach. [↑](#footnote-ref-9)
10. See D. Bidney, *The Psychology and Ethics of Spinoza*, New York 1962, Introd. [↑](#footnote-ref-10)
11. K. R. Popper, *Conjectures and Refutations*, New York 1968, Chap. II [↑](#footnote-ref-11)
12. The term “conceptualism” is narrower than the term “rationalism,” and befits normative systems built through a method of deductive logic, which can be further designated as “formal intuitionism,” which can be seen as a combination of features drawn from the formalism of D. Hilbert and the intuitionism of L. E. J. Brower. This will be further explored over the course of this discussion. [↑](#footnote-ref-12)
13. If one accepts this analysis’ separation of Spinoza’s philosophy into three linguistic systems, distinct from an ontological methodological point of view, the question of the connection between them becomes a problem – but one which we shall not discuss in the present context. It is interesting, and perhaps surprising, to note the similarity between the interpretation of Spinoza’s system offered here and the theories of Juan Huarte, the sixteenth-century Sephardi doctor discussed by Noam Chomsky in his book *Language and Mind*, New York 1968, pp. 8-9. It would be fascinating indeed if there proved to be any historical connection between these two theories. [↑](#footnote-ref-13)
14. That is, a language’s general nouns are not divorced from their communicative context and are not used metaphorically or analogistically. [↑](#footnote-ref-14)
15. A comment on this issue can be found in E. E. Kramer’s book, *The Nature and the Growth of Modern Mathematics*, Vol. II, Chap. 29, Greenwich, Conn. 1970, p. 457): “The Aristotelian logic born of natural classification would be adequate to the theory of finite collection and would not go beyond it.” Cf. W. V. Quine, *Ontological Relativity*, New York – London 1969, pp. 62-63. [↑](#footnote-ref-15)
16. See N. Goodman, “A World of Individuals,” in *Philosophy of Mathematics, Selected Readings*, Englewood Cliffs, New Jersey 1964. [↑](#footnote-ref-16)
17. Compare, for example: R. Carnap, “Notes on Semantics,” *Philosophia*, Vol. II 1972, Nos. 1-2, a XXIX. [↑](#footnote-ref-17)
18. W. Sellars discusses this sense of proto-language in his writings. [↑](#footnote-ref-18)
19. This view of Plato’s is characterized in section 249d of *Sophist*: “he [the philosopher] must quote the children's prayer, ‘all things immovable [‘Forms’ and ideas] and in motion [physical objects],’ and must say that being and the universe consist of both.” From *Plato in Twelve Volumes*, Vol. 12 translated by Harold N. Fowler. Cambridge, MA, Harvard University Press; London, William Heinemann Ltd. 1921. [↑](#footnote-ref-19)
20. See Quine (note 8, above), p. 49. Quine calls language which determines the objects of another language or theory “background language”: “We need a background language, I said, to regress into…”; *Ethics*, Part II, Prop. XXXIX; Prop. XL, Note 2; Part V, Prop. XII. [↑](#footnote-ref-20)
21. In my view, Spinoza’s use of the term “common notions” is crucial to understanding his later philosophy, and the key concept in understanding his philosophy of science. It is hard to know when precisely the change in the meaning of the term “common notions” occurred, and when it acquired the meaning it carries in *Ethics*. The same concept appeared in the works of philosophers who preceded Spinoza, albeit with a different meaning, one closer to that of accepted axioms in a theoretical system. Descartes used the term with a meaning identical to that of “primary notions” – metaphysical or logical first premises. To the best of my knowledge, this concept first appears in Spinoza’s works in letter 4, his response to H. Oldenburg in late 1661. Unfortunately the letter’s Hebrew translation fails to emphasize the technical nature of this concept, which is better served by A. Wolf’s English translation, *The Correspondence of Spinoza*, Frank Cass & Co. Ltd 1966 (pp. 82, 377-378). In this context, for unclear reasons, Spinoza brings up the difference between “common notions” and axioms. It may be that in this context that the concept is still being used in its Cartesian sense, but it is not identical to axioms; rather, it constitutes a subgroup of this system’s first premises, one foreign to the subgroup of axioms. [↑](#footnote-ref-21)
22. See L. Wittgenstein, *Remarks on the Foundation of Mathematics*, Cambridge, Mass. – London, England 1967, pt. IV, §49, p. 156e: “It is quite true: the numerical sign belongs with a concept-sign and only together with this is it, so to speak, a measure”; see also Letter 37; for the English translation above, note 14; likewise, see *Ma’amar `al Tiqun ha-Sekhel* §108, III; in the same context, see *Cogitata Metaphysica I*, an appendix to B. Spinoza, *The Principles of Descartes Philosophy*, The Open Court Publishing Company, La Salle, Ill. 1961. [↑](#footnote-ref-22)
23. See Letter 6, pp. 74-75, 77; *Ma’amar `al Tiqun ha-Sekhel* §27. [↑](#footnote-ref-23)
24. *Ethics*, Part II, Prop. XL, Note 1. [↑](#footnote-ref-24)
25. Terms are “transcendental” or “universal” if one derives them from one linguistic nexus and uses them in another. In our context, the use of metaphysical terms in the realm of natural, quotidian language renders them “transcendental,” whereas the use of *scientific terms* in natural language renders them “universal”: “universal images of things” (*Ethics,* Part II, Prop. 40, sch. 1). [↑](#footnote-ref-25)
26. For more on this, see *Ethics*, Part V, note on Prop. 37. [↑](#footnote-ref-26)
27. *Ethics*, Part II, Schol. [III.B(iii)]] on Prop. 49. For the importance of distinguishing between concepts and objects, note especially: “So the thing to note here, above all, is how easily we are deceived when we confuse universal with singulars, and beings of reason and abstractions with real beings.” [↑](#footnote-ref-27)
28. *Ethics*, Part II, Prop. 40, Note 2; Part V, Prop. 36, Schol. I. [↑](#footnote-ref-28)
29. See also: *Theologico-Political Treatise*, Chapter 5, p. 60. [↑](#footnote-ref-29)
30. I shall not expand on this idea here. For more on the matter, see especially: *Treatise* § 22 99-104, 108, and the remarks by the Hebrew editor (Y. Ben-Shlomo) in notes 273-275, 414, 736. [↑](#footnote-ref-30)
31. Due to the ontological changes that took place between Plato’s earlier and later dialogues, usage of the adjectives “Platonic” and “realistic” is likely misleading. “Realistic” entered use, so it would seem, because of a supposition of the objective existence of the Forms (notions); but in this there is nothing that would differentiate between what I call a “normative method” and a realistic method (like those of Frege, A. Church, Popper, or even Russell and Carnap at certain points). The difference as regards the real existence of notions, whether they are a human invention or possess an eternal non-dependent existence in the Platonic heavens, is trivial from a logical and formal-ontological perspective, although Quine turns it into an ontological criterion that sorts between philosophical methods of mathematics. From a formal standpoint, Spinoza’s normative nexus is identical to the Cartesian method, premised on the idea that objects have no independent existence, and are rather derived from a network of simple, clear, and distinguishable ideas which serve not only as a source of true knowledge, but also as a metaphysical-ontological source of existence itself. See especially:

    Descartes: “The Principles of Philosophy” in his *Philosophical Work* (English trans. by E.S. Holdane and G. R. T. Ross), I, New York, p. 204 ff. Cf. G. Buchdahl, *Metaphysics and Philosophy of Sciences*, Oxford 1969, note 3, p. 41 et passim.

    One can find another explication of the Cartesian outlook, according to the Kantian tradition. See, for example, E. W. Beth’s in E. W. Beth & J. Piaget, *Mathematical Epistemology & Psychology*, Dordrecht Holland 1966. [↑](#footnote-ref-31)
32. *Ma’amar `al Tiqun ha-Sekhel*, p. 19. [↑](#footnote-ref-32)
33. Of course, there exist certain differences between the approaches of Spinoza and Descartes to the system’s initial premises. The first difference is that Descartes is closer to the logisticians of our era, who demand a meta-language that discusses the rules for constructing axioms (for more on this, see, for example, A. Gewirth’s commentary in “Clearness and Distinctness,” in *Philosophy*, Vol. XVIII No. 69 (Apr. 1943), pp. 17-36). In the Treatise, however, Spinoza comes closer to the intuitionist approach, which denies the possibility of a meta-discussion on the method’s primary demands, focusing instead on the clarification of simple words (without defining them) and the given structural rules of natural language, and in so doing formulating the method’s primary demands (see, for example, Letter 36 from 1666). Any other procedure, per Spinoza, leads to an infinite regress or infinite hierarchy of super-languages (*Treatise* §§ 30-36, 107; see also: the close of Letter 37).

    The second difference is not methodological, but rather an ontological difference concerning the question of whether one must presuppose a being (or beings) outside the system of knowledge itself, or that of the world itself. In this Spinoza expresses a holistic perception, one that also characterizes, for example, Quine, and distinguishes him from Descartes and the logisticians and positivists of our era: there is no need to infer the existence of beings outside the method (or system); God is identical with the world. [↑](#footnote-ref-33)
34. The philosophy of K. Popper, particularly in his critique of problems in Kantian philosophy, demonstrates that there is no need to identify the deductive method with normative ontology. As I argue, Spinoza’s method in the *Treatise*, as expressed through the scientific nexus, demonstrates the same thing. [↑](#footnote-ref-34)
35. See also: § 21, Note 7 on the general theory of sensory perception and the size of the sun as an example of hypothetical deduction. I conclude from these clauses that Spinoza contended with two logical schema of hypothetical syllogism, or what we would nowadays call rules of deduction: *modus ponens* and *modus tollens*.

    The passage “…when something is inferred from some universal, which some property always accompanies” (*Treatise* § 19 III) can be rendered in modern logical notation as in A:

    A.

    Where Q indicates a certain property expressed in an existential conclusory statement: “therefore there is some power,” “therefore there is something.”

    A logical syllogism with a negative conclusion is *modus tollens*, expressed through schema B. The conclusion is, “Therefore this thing, or another, does not exist.”

    B.

    The argument in the example of the sun (*Treatise* § 21) can be expressed as in C.

    P – “we look at it from a great distance”

    Q – “it is larger than it appears to be”

    S – “the sun”

    C.

    [↑](#footnote-ref-35)
36. *Treatise* § 103, likewise in Letter 10. [↑](#footnote-ref-36)
37. This division reappears elsewhere, for example in § 37. [↑](#footnote-ref-37)
38. The word “tool” here, as in many similar contexts, is nothing more than a metaphorical usage drawn from the allegory of “corporeal tools” in § 30. From all the various contexts in which this word is used in the *Treatise*, we can conclude that the subject of discussion is a sentence or theorem or what it points to: an argument (and sometimes an idea). [↑](#footnote-ref-38)
39. Cf.: *Treatise* §§ 107, 108; Letter 37. [↑](#footnote-ref-39)
40. One could also add to the metaphysical realm the correct rules of logical structure: “notions they call *Second*,” *Ethics* Part II, Prop. 40, Schol. 1. [↑](#footnote-ref-40)
41. Operating, of course, from the premise that the structure of mathematics is normative and not descriptive; a premise accepted by Descartes in his time, and seemingly still accepted today. Following K. Gödel, I tend to disagree with it. [↑](#footnote-ref-41)
42. For more on the nature of the connection between science and metaphysics, whether logical or semantic, compare: Kant, *Critique of Pure Reason*; for more on transcendental derivation and the like, see: A. Pap, “Does Science Have Metaphysical Presuppositions” in *Reading in Philosophy of Science* (ed. by H. Feigl and M. Broadback), New York 1953. [↑](#footnote-ref-42)
43. Cf.: G. Buchdahl, *Metaphysics & The Philosophy of Science*, Oxford 1969, Chap. I (p. 82 ff.): “Descartes’ ‘Methods’ and its place in his Scientific and Philosophical Writings.” [↑](#footnote-ref-43)
44. *Ethics*, Part II, Prop. 38-39, Prop. 40 Schol. 2, Prop. 47 Schol. 1; Part V, Prop. 12. [↑](#footnote-ref-44)
45. See, for example, *Treatise* § 91. [↑](#footnote-ref-45)
46. In note 62 of the Hebrew translation of the *Treatise* (p. 31), editor Y. Ben-Shlomo identifies the “common notions” with universal notions, or with universal laws of nature. According to my interpretation, however, these notions express attributes of objects, and not general rules. H. A. Wolfson made a similar error. See, for example, H. A. Wolfson, *The Philosophy of Spinoza*, II, Cleveland 1934, pp. 125-140. [↑](#footnote-ref-46)
47. *Ethics*, Part II, Prop. 39. [↑](#footnote-ref-47)
48. *Ethics*, Part V, Prop. 28, Prop. 29 Schol. 1. [↑](#footnote-ref-48)
49. I. Scheffler calls such a categorical-conceptual framework a “basic set of categories.” See: I. Scheffler, *Science & Subjectivity*, Indianapolis: 1967, p. 43. [↑](#footnote-ref-49)
50. Logic within a normative system almost certainly belongs to metaphysics. Spinoza calls logical concepts and possibly fundamental logical laws as well “Second notions.” Compare the stances taken on this issue by C.S. Pierce and F. P. Ramsey brought forward by L. Wittgenstein (above, footnote 7) § 81. [↑](#footnote-ref-50)
51. Cf. W. V. Quine (above, footnote 13). [↑](#footnote-ref-51)
52. See K. R. Popper’s use of this concept: K. R. Popper, *The Logic of Scientific Discovery*, 1968, New York – Evanston, Chap. V. [↑](#footnote-ref-52)
53. From Edwin Curley’s translation. Edwin Curley, *The Collected Works of Spinoza*, Volume I (Princeton: Princeton University Press, 1985). [↑](#footnote-ref-53)
54. For more on Spinoza’s and his contemporaries’ relationship with Zeno’s paradoxes, and on the connection between the doctrine of numbers to the measurement of physical sizes, see: G. Galilei, *Dialogue* I; *Discourses* III; R. Descartes, *The Principles of Philosophy*, Part I P. XXVI; B. Spinoza, *The Principles of Descartes’ Philosophy*, Part II P. VI, sch. Cf: M. Guéroult (1968); M. Clavelin (1968: 282 ff.). [↑](#footnote-ref-54)
55. One could argue that this phrasing is in fact that of Boyle as presented in Letter 11. But it is also Spinoza’s opinion, as evidenced by Letter 6 and Letter 13. Spinoza’s debate with Boyle is to what extent one can know both the nature of niter and active causes in the context of a specific experiment, or, in other words, whether Boyle’s experiment can be considered a controlled experiment based on the mathematical quantification of those causes entering the experiment. Cf.: R. McKeon, 1928, Ch. III, “Spinoza and the Experimental Science,” esp. pp 151-2; see also Letter 16 from Oldenburg to Spinoza. [↑](#footnote-ref-55)