**Mathematical Thinking Styles and the features of Modeling Process**

**Abstract**: The current study investigated the relationship between students’ mathematical thinking style and their modeling processes and routes. Thirty-five eighth-grade students were examined. In the first stage, the students solved word problems, and according to their solutions, they were assigned to one of two groups: a visual thinking style group and an analytic thinking style group. The two groups engaged in three modeling activities. Findings indicated differences in the groups’ modeling processes in performing the three activities. The primary differences in the modeling processes were manifested in simplifying, mathematizing, and eliciting a mathematical model. In addition, the analytic thinking group skipped the real-model phase in the three activities, while the visual group built a real model for each activity.

**Key words**: Modeling, modeling process, modeling cycle, style thinking, visual thinking style, analytic thinking style

**Introduction**

Thinking style and cognitive methods strongly affect student performance in many areas largely determining significant differences in their performance, as demonstrated in empirical cognitive psychology studies (e.g., Dwyer & Moore, 1995; Cakan, 2000). Therefore, students’ different thinking styles should be taken into account when determining appropriate educational interventions (Sternberg & Zhang, 2005). Thus, teacher awareness of different thinking styles is particularly important, specifically when students are solving real-word problems. This is considered an important goal in mathematics education, as it was emphasized by the OECD. Such problems are the cornerstone of mathematical modeling approach; they offer students the opportunity to meet mathematical and everyday challenges and requirements (Lesh & Lehrer, 2003; Lesh & Doerr, 2003; Lesh, Hoover, Hole, Kelly & Post, 2000), and help them understand their world and critically view mathematical information in the sense of active citizenship (Niss, Blum & Galbraith, 2007). Mathematical modeling is the process of translating between the real world and mathematics (Blum & Borromeo-Ferri, 2009). Knowledge about students’ modeling processes can ameliorate their teachers’ interventions (Blum & Leiß, 2005). Given their potential, modeling processes have been studied widely (e.g. Chan, 2008; English & Fox, 2005; Stillman, Galbraith, Brown, & Edwards, 2007; Doerr & English, 2003; English & Watters, 2005; Shahbari & Daher, 2016; Shahbari & Peled, 2017). However, only a few scholars (e.g., Borromeo-Ferri, 2010, 2012) have examined the modeling process of individuals having different thinking styles. Furthermore, almost no studies have focused on modeling processes with respect to thinking styles characterizing groups where all modelers in each group have the same thinking style. We chose to examine students that worked in homogenous groups in order to emphasize the thinking style as the main variable. This study aims to shed light on the influence of group thinking style on their modeling process and route while engaged in modeling activities.

**Framework**

**Mathematical Thinking Style**

Thinking style is a way of thinking; it is not an ability, but rather a preferred way of using one’s abilities (Sternberg, 1997). Thus, mathematical thinking styles denote how individuals prefer to learn mathematics, not how their mathematical understanding is assessed (Borromeo Ferri, 2010). In addition, it is also indicative of how the individual prefers to proceed with the mathematical task (Sternberg, 1997). Klein (1892, cited in Borromeo-Ferri & Kaiser, 2003) suggested three different thinking styles: the philosopher, who constructs on the basis of concepts; the analyst, who operates within a formula; and the geometer, who has a visual starting point. Similarly, Borromeo-Ferri and Kaiser (2003) in their empirical study suggested three thinking styles: the analytic, the visual, and the integrated. In the current study, we will follow the latter classification, focusing on the visual and the analytic thinking styles. The visual thinking style has been defined as thinking based on the shapes, drawings, and images presented in real situations and relationships (Campbell, Collis, & Watson, 1995). Students with a visual thinking style are characterized by a strongly image-oriented way of thinking when solving mathematical problems; this facilitates their obtaining, representing, interpreting, perceiving, and memorizing of information, as well as expressing it (Borromeo-Ferri & Kaiser, 2003).

On the other hand, the analytic style of thinking is identified as thinking symbolically and formalistically (Burton, 2001). Individuals with an analytic style thinking tend to search for structures, patterns or formulas and their application (Borromeo-Ferri, 2003), or briefly operate with formulas, as Klein (1892, cited in Borromeo-Ferri & Kaiser, 2003) reported. Analytic thinking involves sorting and separating elements from context, a tendency to focus on the properties of objects and elements for classification into categories, and a preference for using rules about categories and predicting behavior (Monga & John, 2007). Presmeg (1986) treat to the dissimilar of visual thinking style as a nonvisual, one which involves no visual imagery as an essential part of the method of solution. Some studies reported that students with nonvisual thinking performed better than those with visual thinking (Lean & Clements, 1981); however, students typically used visual methods to solve difficult or novel problems, whereas nonvisual strategies were used in less difficult situations (Lowrie & Kay, 2001). Furthermore, some studies (e.g. Lowrie & Clements, 2001) indicated that students with a visual thinking style moved toward more nonvisual and analytic forms of reasoning when the familiarity of the tasks increased.

**Modeling**

Mathematical modeling means solving complex, realistic, and open problems with the help of mathematics; the process that students develop and use in solving such problems is termed modeling process. The modeling process is cyclic, whereby translating between the real world and mathematics transpires in both directions (Blum & Borromeo-Ferri, 2009). There are multiple modeling processes in the literature; in the current study, we chose the modeling processes suggested by Blum and Leiß (2005), who identified modeling processes from a cognitive perspective as phases and transitions. The phases comprise a situation model, a real model, and a mathematical model, as well as mathematical results and real results. The transitions include several actions: understanding the problem and simplifying a situation model; presenting a real model; mathematizing, which leads to the construction of a mathematical model; applying mathematical procedures; interpreting the mathematical results; and validating, whereby mathematical results are validated in a real-life task. Various visual descriptions of the cyclic process-modeling cycle have been reported in the literature. The current research is based on Blum and Leiß’s (2005) modeling cycle. Delineating the modeling process in detail, incorporating the various phases of the modeling cycle on an internal and external level, Borromeo-Ferri (2007) referred to it as the modeling route. It is important to state that the modeling cycle is considered as an idealized scheme, which does not describe the actual students’ process; the way through the modeling process of students’ actual modeling cycle is identified as the modeling route and it may be different from the modeling cycle (Borromeo-Ferri, 2007). The modeling route may not be linear and shift across levels (Maab, 2006).

**Modelers with Different Thinking Styles**

Borromeo-Ferri (2006) found that the modeling routes depend on students’ style of thinking. Reporting about two students with different thinking styles, her analyses indicated that students with an analytic thinking style tend to instantly use the mathematical model, and then comes again to the real model only if there is a need to understand the task better. On the other hand, students with a visual thinking style follow the modeling cycle mentioned by Blum and Leiß (2007). In general, Borromeo-Ferri (2012) also indicated that when analytic thinkers engage in modeling tasks, they prefer to change the real world model to a mathematical model and work in a formalistic way, while visual thinkers think more in terms of the real world rather than of formal solutions and tend to present their thinking through pictures and graphic drawings.

**Research aim and question**

Teachers have a central role while their students engage in modeling activities; their knowledge of students’ modeling activities will affect their intervention (Blum & Leiß, 2005). It is important to shed light about the modeling process and routes of students with different thinking styles. The aim of the research is to examine the relationship between two groups of eighth-grade students with different thinking styles and their modeling process and routes while they are engaged in modeling activities. More precisely, this research addressed the following question:

Do groups of students with different thinking styles (visual or analytic) differ in their modeling process and their modeling routes while working on a sequence of modeling activities, and how?

**Method**

The current study was qualitatively oriented, focusing on the interpretation of the data which emerged in students’ solving process of the tasks in the questionnaire. Students were observed while they worked on three modeling tasks, which were video recorded.

The research participants and procedure comprised two stages, as detailed below.

**Research participants, data sources and analysis in the first stage**

For the first stage of the study, 35 students in an eighth-grade class participated. The data source was a questionnaire for identifying participants’ thinking style.

*Questionnaire*: The study questionnaire comprised eight tasks for classifying students according to their thinking style. Some of these tasks were adapted from other studies (e.g., Lowrie & Clements, 2001), and some were designed by the researchers. The selected tasks were characterized by a variety of topic areas and of possible solution strategies. Below is an example of two tasks from the questionnaire:

(1) Turf Problem (Lowrie & Clements, 2001): A husband and wife want to turf their backyard (put grass squares down). Before purchasing the turf, they have a ground pool put in their backyard. The pool is 3m wide and 5m long. Sensibly, they also pave an area 1m wide around the pool. If turf costs $10 per square meter, how much would it cost to turf the backyard (150 m² in total) once the pool and the paving are finished?

(2) Handshakes task (Kaput & Blanton, 2001): Five people are at a party. If each person is to shake everybody else’s hand once, how many handshakes will take place at the party?

*Data analysis of the questionnaire*: We used the constant comparative method (Glaser & Strauss, 1967) to analyze the problem-solving processes for each task in the questionnaire for each student. We adopted the categories described by Borromeo-Ferri and Kaiser (2003): when illustrating and solving mathematical problems, the visual thinking style was characterized by sketches, drawings, or graphs, while the analytic thinking style was expressed in a formula-oriented way, i.e. the information from the text of a given problem is expressed by means of a formula. The integrative group was comprised by students who solved some tasks analytically and others visually. An example of students’ answers classification for the Turf Problem can be seen in Table 1.

Table 1: Samples of students’ solutions for the Turf Problem

|  |  |  |
| --- | --- | --- |
| Task | Visual style | Analytic |
| Turf Problem | 5\*7=35m2  150-35=115m2  115\*10=1150$ | 3+2=5m; 5+2=7m  7  5  7\*5=35;  150-35= 115m2;  115\*10= 1150$ |
| Handshakes | |  |  | | --- | --- | | Number of shakes | Person | | 4 | First | | 3 | Second | | 2 | Third | | 1 | Fourth | | 0 | Fifth | | The first person shakes hands four times; The second shakes hands three times; The third shakes hands twice and  The fourth shakes hands one time.  1 + 2 + 3 + 4= 10 |

Based on the styles reflected in solving the questionnaire’s tasks, students were then classified into three thinking style groups: analytic (14 students), visual (11 students), and integrated (10 students) thinking style groups.

**Research participants and data sources in the second stage**

The focus in the current study was the analytic and visual thinking style, so we did not focus on the integrative style thinking. From the analytic students we chose five students and so from the visual students we choose also five students (totaling 10 participants). We selected the 10 students with the assistance of their mathematics teacher in order to maximize matching variables (e.g., gender, mathematics abilities, socioeconomic status). The five analytic students together comprised the analytic group, and the visual students comprised the visual group. Both groups (analytic and visual) were assigned the same three modeling activities in the course of three weeks, with one activity per week. The modeling activities were adapted from the literature (Blum & Borromeo-Ferri, 2009).

*Video recordings:* Video recordings were made of the two groups working on the three modeling activities and were transcribed.

*Video recording analysis*: We used the constant comparison method (Glaser & Strauss, 1967) to analyze the students modeling processes in the three modeling activities, taking into account the cognitive aspect of the modelers’ modeling cycle (Blum & Leiß, 2005). The students’ modeling process was elaborated into phases and actions. The modeling processes were described visually, as in Blum and Borromeo-Ferri (2009), and were detailed later in Shahbari and Tabach’s study (e.g. Shahbari & Tabach, 2016a) in which the researchers visualized all the modeling phases and actions of the modelers while they engaged in modeling activities.

**Sequence of Modeling Activities**

The sequence includes three modeling activities; the context of these activities is not one of the foci of the current research. The first and the second activity (the Juice Activity and the Been Activity) are adapted from Ben-Chaim, Kerret and Ilany (2012). The third activity (the Giant’s Shoes) was designed by Blum and Borromeo-Ferri (2009). The main mathematical themes that related to the three activities are ratio and proportion, estimation, and average.

**Findings**

First, we will present the modeling processes for the two groups (visual thinking and analytic thinking) while working on the three modeling activities. Then we will focus on their modeling routes and present them visually.

**Modeling processes between the analytic and visual groups**

The analysis of the modeling processes of the two groups in the three activities revealed that each group (the analytic and visual groups) demonstrated similar features while working on the three modeling activities, but the two groups differed in their modeling processes (phases and actions). Table 2 presents the general findings regarding the two groups’ modeling processes as well as the phases and the actions that they went through while working on the activities.

Table 2: Modeling processes of the analytic and visual groups in the three activities

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Group | | Analytic | | | | | | | | | | Visual | | | | | | | | | |
| Modeling process | | Simplifying | Real model | Mathematizing | Mathematical model | Working mathematically | Mathematical results | Interpreting | Real results | Validating | Simplifying | | Real model | Mathematizing | Mathematical model | Working mathematically | Mathematical results | Interpreting | Real results | Validating |
| First activity Modeling cycle | 1 | √ | - | √ | √ | √ | √ | - | - | - | √ | | √ | √ | √ | √ | √ | - | - | - |
| 2 | - | - | √ | √ | √ | √ | √ | √ | √ | - | | √ | √ | √ | √ | √ | √ | √ | √ |
| 3 | - | - | √ | √ | √ | √ | √ | √ | √ | - | | - | - | - | - | - | - | - | - |
| Second activity Modeling cycle | 1 | √ | - | - | - | √ | √ | √ | √ | - | √ | | √ | √ | √ | √ | √ | √ | √ | √ |
| 2 | - | - | - | √ | √ | √ | √ | √ | √ | √ | | √ | √ | √ | √ | √ | √ | √ | √ |
| 3 | - | - | - | √ | √ | √ | √ | √ | √ | - | | - | - | - | - | - | - | - | - |
| Third activity Modeling cycle | 1 | √ | - | √ | - | √ | √ | - | - | √ | √ | | √ | √ | √ | √ | √ | - | - | √ |
| 2 | - | - | √ | √ | - | - | - | - | - | √ | | √ | √ | √ | √ | √ | √ | √ | √ |
| 3 | - | - | √ | √ | √ | √ | √ | √ | √ | - | | - | - | - | - | - | - | - | - |

Table 2 show that the major differences between the two groups were in the real model phase. However, finer analyses of the other phases and actions revealed three other main differences in the simplifying and mathematizing actions and in the mathematical model phase. Table 3 presents the differences between the two groups, illustrated by sample statements from the students’ discussions while working on modeling activities.

Table 3: Differences in modeling process between analytic and visual groups

|  |  |  |
| --- | --- | --- |
| Modeling process | Analytic group | Visual group |
| Simplifying | The simplifying actions occurred through mathematizing.  Students simplified the situations by mathematizing, skipping the real model for the situations. Ex.  [5] Student 2: We can calculate by ratio between width and length. | Students simplified the activities by drawing and illustrating.  [5] Student 1: I can explain the situation; we have information about... [they drew an illustration of shoes and body].  [6] Student 1: We can find the relation between us and the giants. |
| Mathe-  matization | Students mathematize the situation by searching for formulas. E.g.,  [9] Student 4: The ratio between the length and the width … length 32 and width 12 [length and width of their shoes].  [11] Student 2: We should simplify the ratio … 32:12. | Students mathematize the situation by working in tables and lists. E.g.,  [10] Student 3: Make a table  [16] Student 3: Your shoes are 26 cm, here I write 29 cm [in the column of the shoes’ length], your height  is 160.   |  |  | | --- | --- | | Height | Shoes” length | | 160 | 26 | | 163 | 30 | | 146 | 28 | |  |  | |
| Mathematical model | The mathematical model is presented through a formula.  H≈ shoes length\* 5 | |  |  |  | | --- | --- | --- | | Height | Shoes” length | Ratio | | 160 | 29 | 5.51 | | 163 | 30 | 5.43 | | 155 | 28 | 5.53 | |  |  |  | | X | 5.29 | 5.36 |   The mathematical model is illustrated by utilizing tables and lists. |

As presented in Table 3, the main difference between the two groups was the way of illustrating the mathematical ideas. In other words, the same idea was illustrated differently in the two groups. It is important to note that each group have different measures (shoes and height); because of that we have different numbers.

**Modeling cycles and routes in the analytic and visual groups**

Analysis of the modeling processes of the two groups in the three modeling activities indicated that the analytic group went through more modeling cycles in each activity to obtain the final model than the visual group, as presented in Table 2. In addition, the analysis indicated that the analytic group engaged in more skipping during the modeling phases than the visual group. Modeling routes among the two groups in the three modeling activities are presented in Table 4.

Table 4: Modeling routes of the two group in the three activities

|  |  |  |
| --- | --- | --- |
| Activity | Group | Modeling route |
| Juice Activity | Analytic group | Figure 1: Modeling routes of the analytic group in the Juice Activity |
| Visual group | Figure 2: Modeling cycle of the visual group in the Juice Activity |
| Been Activity | Analytic group | Figure 3: Modeling routes of the analytic group in the Been Activity |
| Visual group | Figure 4: Modeling cycle of the visual group in the Been Activity |
| Giant’s Shoes activity | Analytic group | Figure 5: Modeling routes of the analytic group in the Giant’s Shoes Activity |
| Visual group | Figure 6: Modeling cycle of the visual group in the Giant’s Shoes activity |

Table 4 indicates that the analytic group (Figures 1, 3 and 5) always has three modeling cycles, while the visual group (Figures 2, 4 and 6) has for the same activities two modeling cycles. In addition, the cycles in the visual group are more sequential than the analytic group. The figures above show that the analytic group always began with the action of mathematizing the activity and did not reach the phase of a real model, while the visual group always began with the action of simplifying the situation and illustrate it with a real model.

For more details, below are the groups’ modeling processes for one activity; to avoid repetition, we chose the ‘Giant’s Shoes’ activity:

The modeling process of the analytic group in the Giant’s Shoes activity can be split into three modeling cycles: the first cycle, identified by a straight line (C1.1, C1.2, C1.3, C1.4), the second cycle (dashed line) (C2.1, C2.B), and the third cycle, the dotted line (C3.1, C3.B, C3.3, C3.C, C3.4, C3.D, C3.5). Table 5 presents the modeling process and Figure 5 illustrates the modeling route of the analytic group.

Table 5: Modeling process of the analytic group in the Giant’s Shoes activity

|  |  |  |
| --- | --- | --- |
| Modeling cycle | Process | Explanation |
| First cycle | C1.1 | Understanding the situation, simplifying through mathematizing by thinking about the relation between the width and the length of shoes 5.29: 2.37 |
| C1.2 | Working mathematically: Finding the ratio between the width and the length of one student: 32:12 |
|  | C1.C | Mathematical result: The ratio 8:3 |
|  | C1.3 | Validating: Not helpful in solving the situation |
| Second cycle | C2.1 | Returning to the situation, simplifying through mathematizing: Finding the ratio between the length of student’s shoes and her height. |
| C2.B | Mathematical model: The height of person is four times the length of their shoes. |
| Third cycle | C3.1 | Returning to the situation, simplifying through mathematizing: Finding the ratio between the average length of their shoes. |
| C3.B | Mathematical model: The height of a person is five times the length of shoes |
| C3.2 | Applying the model: 5.29\*5 |
| C3.C | Mathematical result: the height of the giant is 26.45. |
| C3.3 | Interpreting to reality: it is almost 27 m |
| C3.D | Realistic result: 27 m |
| C3.4 | Validating the results in the situation: 27 m |

The visual group in the Giant’s Shoes activity engaged in two modeling cycles. The group began by simplifying the situation through the use of a drawing; they tried to draw an image of shoes through their simplification to yield a real model )A( and thought about the numerical relationship between the giant’s height and the length of his shoes, and how this relation would be equivalent for ordinary people (C1.1); they began mathematizing by ordering their own shoe length and height measurements, and the ratio between these measurements was recorded on a table they constructed (C1.2); they then elicited a mathematical model, indicating that the ratio between the length of the shoes and the height resembled the ratio of their own measures (C1.B); they applied the results (C1.3), and each student received mathematical results resembling his\her ratio. They received different results because each had a different ratio (C1.C); thus, these results did not resolve the problem (C1.4). The second cycle began with a mathematical model, comprising the average of the group’s ratio calculations (C2.B); they applied it (C2.3) and received the numerical result of 32 (C2.C); this result was then transformed into a realistic result, indicating the giant’s height as 32m (C2.D); they accepted this result (C2.5). Figure 6 illustrates the modeling route of the visual group.

**Discussion**

The aim of the current study was to examine the relationship between two group of eighth-grade students with different thinking styles (an analytic thinking style group and a visual thinking style group) and their modeling process and routes while they engaged in a sequence of modeling activities. The findings revealed that each group had similar features among the three modeling activities, while there were major differences in the two groups’ modeling processes. The main difference between the two groups was in the action directly after reading the situation, through the simplifying process and the accessibility to the activity. The analytic group tried to simplify the three activities by mathematizing them, while the visual group tried to simplify the activities by drawing and illustrating the situations. These findings are in line with Borromeo-Ferri’s (2012), who indicated that when analytic thinkers deal with a modeling activity they preferred to change the real-world situation to a mathematical model and operate in a formalistic manner, while visual thinkers think more in terms of the real world rather than of formal solutions, and thus tend to present their thinking by means of pictures and drawings. We can indicate that students with a visual thinking style make more connections between mathematics and the real world by starting with simplifying and using a real model; as reported by Huangs (2013), students with a visual thinking style make more connection between mathematics concepts and the physical world.

The findings also revealed differences in the mathematizing action and in the illustration of the mathematical model. The analytic group emphasized the use of formulas, while the visual group mathematized with the help of lists, tables and drawings; the same features of action were identified in mathematical models. The features of the mathematizing actions of the analytic group when they were engaging in modeling activities were found to be similar to features activated in solving routine world problems. In fact, as Klein (1892) (cited in Borromeo-Ferri & Kaiser, 2003) reported, students with an analytic thinking style are more likely to search for structures, patterns, or formulas and their application, or briefly operate with formulas.

According to the modeling cycles and routes, we identified that that the modeling routes of the analytic group are longer than the visual group’s. However, the analytic group engaged in more skipping of the modeling phases; in the three activities, they skipped the real model, while the visual group always addressed this phase. It is important to note that skipping modeling phases or actions did not relate to the effectiveness of the elicited models, as emphasized by Shahbari and Tabach (2016b).

The findings also indicated that each group had the same features in the three activities: we did not identify any difference in each group across the three activities. However, we did not have an indicator of whether the groups would have continued to work in a similar way and maintained the same features of the modeling routes, had the sequence of the modeling activities been longer. Researchers reported that changes can occur; for example, Kaiser (2007) reported that expert modelers control their solving strategies and therefore achieve their aim faster.

Finally, Stainberg (1997) proposed that understanding thinking styles help teachers to differentiate instruction to maximize the learning outcomes of all learners; in our case, teachers’ awareness of students’ thinking styles has an important role in designing effective interventions. The findings indicated that the two groups worked and used different modeling routes even when they obtained final mathematical models with parallel contents; therefore, it is important for the teachers to be aware of different preferences in simplifying and in mathematizing situations. Teachers should be aware that students with different thinking style prefer to simplify, mathematize and elicit mathematical models in a different way; being aware of the difference can make teachers aware to their intervention. We suggest expanding our work by examining more than a single group from each style in order to learn more about modeling processes and modeling routes of students with different thinking styles and the features of each thinker.

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