%% LyX 2.2.1 created this file. For more info, see http://www.lyx.org/.

%% Do not edit unless you really know what you are doing.

\documentclass[english]{article}

\usepackage[T1]{fontenc}

\usepackage[latin9]{inputenc}

\usepackage{geometry}

\geometry{verbose,lmargin=3cm,rmargin=3cm}

\usepackage{babel}

\usepackage{float}

\usepackage{amsmath}

\usepackage{amssymb}

\usepackage{graphicx}

\usepackage{setspace}

\PassOptionsToPackage{normalem}{ulem}

\usepackage{ulem}

\doublespacing

\usepackage[unicode=true,pdfusetitle,

 bookmarks=true,bookmarksnumbered=false,bookmarksopen=false,

 breaklinks=false,pdfborder={0 0 0},pdfborderstyle={},backref=false,colorlinks=false]

 {hyperref}

\makeatletter

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% LyX specific LaTeX commands.

%% Because html converters don't know tabularnewline

\providecommand{\tabularnewline}{\\}

%% A simple dot to overcome graphicx limitations

\newcommand{\lyxdot}{.}

\@ifundefined{showcaptionsetup}{}{%

 \PassOptionsToPackage{caption=false}{subfig}}

\usepackage{subfig}

\makeatother

\begin{document}

\thispagestyle{empty}

\begin{center}

{\Huge{}Quantum Computation with Ultracold Fermi Atoms}

\par\end{center}

\vspace{6cm}

\begin{center}

{\Huge{}Yanay Florshaim }

\par\end{center}{\Huge \par}

\vspace{6cm}

\begin{flushleft}

{\Large{}This work was performed under the supervision of Prof.

Yoav Sagi }

\par\end{flushleft}{\Large \par}

\newpage{}

\thispagestyle{empty}

\paragraph{{\Large{}Acknowledgments}\protect \\

{\Large{}}\protect \\

}

First and foremost, I would like to thank my supervisor, Prof. Yoav

Sagi, for his great support in every aspect of my M.Sc. Yoav\textquoteright s

contagious enthusiasm for research and his vast knowledge are all

a student can hope for from a supervisor. To work in a new lab is

not an easy task, with many difficulties and mistakes along the way,

but due to Yoav\textquoteright s optimistic vision and willingness to teach and share his wisdom, things always turned out for the best.\\

\\

I would like to thank Dr. Andrey Gandman, our research associate,

for his help in experimental issues. In addition, I would like to

thanks Dr. Jonathan Nemirovsky for his theoretical work, including

numerical simulations and calculations. \\

 \\

I would like to thank all my current lab colleagues: Kostya Sekadorov,

Iris Eitan, and Gal Ness, since much of work was done in collaboration

with them, I want to thank them for the professional help and good advice.\\

I would like to thank my parents for their support and

encouragement during my studies.\\

\\

Finally, I want to thank my wife Sara and my daughters Arbel, Be'eri,

and Yahav for their love, support, and assistance during my studies.

\\

\newpage{}

\cleardoublepage

\pagenumbering{Roman} % roman numbering for table of contents.

\cleardoublepage

\tableofcontents{}

\begin{doublespace}

\newpage{}

\end{doublespace}

\pagenumbering{arabic} % And moving back to arabic numbering (1,2,3,4) for the body.

\setcounter{page}{1}

\section{Introduction \label{sec:Introduction}}

In quantum mechanics, the dimension of the Hilbert space grows exponentially with system size. To represent a quantum state with $n$

particles in classical computation, we need an order of $C^{n}$ bits,

where $C$ is a constant. Therefore, the possibility of calculating many-body quantum states in classical computing becomes practically impossible. To overcome this problem, it was first proposed by Feynman to use a quantum computational machine (Quantum Computer)

\cite{feynman1982simulating}. A quantum computer is able to not only simulate quantum dynamics, but also solve complex mathematical

problems. In addition, the quantum computer is much faster than classical

computers in solving factorial problems \cite{shor1994algorithms} and in database searching \cite{grover1997quantum}. For two decades, researchers

have been trying to implement quantum computation using different

platforms, but all these platforms suffer from inherent experimental limits \cite{cirac1995quantum,home2009complete,opticalQC,weitenberg2011quantum,imamog1999quantum,barends2014superconducting}. Here, we present a new platform of a quantum computer system with ultracold fermionic atoms. We take advantage of the fermionic statistics and ultracold atom system benefits (Feshbach resonance and the ability to capture single atoms in optical traps) to perform a new protocol for quantum gate operators. \\

\\

Quantum computer system requirements, as stated by D.DiVincenzo \cite{divincenzo2000physical},

should comply with \label{sec:5 condition}five conditions:

\begin{itemize}

\begin{doublespace}

\item \textbf{Quantum state.} The quantum state encapsulates the quantum

information in a quantum computer. The state is usually spanned by

two basis vectors, $\left|0\right\rangle $ and $\left|1\right\rangle $,

and the qubit state is defined by

\[

\left|\psi\right\rangle =\alpha\left|0\right\rangle +\beta\left|1\right\rangle

\]

where $\alpha$ and $\beta$ are complex numbers. When the qubit is

measured, with the probability of $\left|\alpha\right|^{2}$, it is in a state $\left|0\right\rangle $ and with a probability of $\left|\beta\right|^{2}$

in a state $\left|1\right\rangle $, satisfying the following relation:

\[

\left|\alpha\right|^{2}+\left|\beta\right|^{2}=1

\]

since the probabilities must sum to one.

\item \textbf{Preparation of the Initial State.} The initial state of the qubit should be capable of being prepared. The particular initial state is of little importance, as we can transform it to any other state using

several quantum gates. However, it is important that the initial state

can be created with high fidelity.

\item \textbf{Quantum gates\label{Quantum-gates.-It}.} To perform

any quantum calculation, we need several unitary operations (“Quantum

Gates”) that form a universal set, namely, any other operation can

be decomposed to a series of gate operations taken from this set.\textbf{

} The quantum gates operate on one or two qubits. Examples of one-qubit

gates include the Hadamard gate, the phase gate, and the $\pi/8$ gate.

The two-qubit gate is a C-NOT gate. In place of a C-NOT gate, it is also possible to use a $\sqrt{SWAP}$ gate \cite{loss1998quantum}.

\end{doublespace}

\begin{enumerate}

\item \textbf{\uline{$\ $Hadamard gate}}\uline{.\label{enu:Hadamard-gate}}

The Hadamard gate is a one-qubit rotation. This gate maps the qubit

states $\left|0\right\rangle $ and $\left|1\right\rangle $ to two

superpositions with equal weight.

\[

U=\frac{\left|0\right\rangle +\left|1\right\rangle }{\sqrt{2}}\left\langle 0\right|+\frac{\left|0\right\rangle -\left|1\right\rangle }{\sqrt{2}}\left\langle 1\right|

\]

or in a matrix representation

\[

U=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}

1 & 1\\

1 & -1

\end{array}\right]

\]

In addition, Hadamard gate is essentially a \textquotedbl{}beam

splitter\textquotedbl{} for the two \textquotedbl{}modes\textquotedbl{}

$\left|0\right\rangle $ and $\left|1\right\rangle $, namely, $\left|0\right\rangle \rightarrow\frac{\left|0\right\rangle +\left|1\right\rangle }{\sqrt{2}}$

and $\left|1\right\rangle \rightarrow\frac{\left|0\right\rangle -\left|1\right\rangle }{\sqrt{2}}$.

\item \textbf{$\ $}\textbf{\uline{Phase gate.\label{enu:Phase-gate}}}\textbf{

}The phase gate is a one-qubit gate that leaves the basis $\left|0\right\rangle $

without a change while transforming $\left|1\right\rangle \rightarrow e^{i\phi}\left|1\right\rangle $.

\[

U=\left|0\right\rangle \left\langle 0\right|+e^{i\phi}\left|1\right\rangle \left\langle 1\right|

\]

or in a matrix representation

\[

U\_{\phi}=\left[\begin{array}{cc}

1 & 0\\

0 & e^{i\phi}

\end{array}\right]

\]

Where $\phi$ is the \textit{phase shift}. Some common examples are

the phase gate with $\phi=\pi/2$, the $\pi/8$ gate with $\phi=\pi/4$,

and the Pauli-Z gate with $\phi=\pi$. \\

\item $\ $\textbf{\uline{$\sqrt{SWAP}$ gate}}. A $\sqrt{SWAP}$ gate

is operated on the mixed states and swapped with them half way, namely,

$\left|1,0\right\rangle \rightarrow\frac{1}{2}\left[\left(1+i\right)\left|1,0\right\rangle +\left(1-i\right)\left|0,1\right\rangle \right]$

and $\left|0,1\right\rangle \rightarrow\frac{1}{2}\left[\left(1-i\right)\left|1,0\right\rangle +\left(1+i\right)\left|0,1\right\rangle \right]$.

In a matrix representation, the gate is defined by

\begin{equation}

U\_{\sqrt{swap}}=\begin{bmatrix}1 & 0 & 0 & 0\\

0 & \frac{1}{2}\left(1+i\right) & \frac{1}{2}\left(1-i\right) & 0\\

0 & \frac{1}{2}\left(1-i\right) & \frac{1}{2}\left(1+i\right) & 0\\

0 & 0 & 0 & 1

\end{bmatrix}\label{eq:swap}

\end{equation}

with respect to the basis $\left|00\right\rangle $, $\left|01\right\rangle $,

$\left|10\right\rangle $, $\left|11\right\rangle $.\\

\\

By using all these gates, we can reduce any unitary operation of $n$

qubits to a cumulative series of these gates \cite{Nielsen2000}.

\end{enumerate}

\begin{doublespace}

\item \textbf{Ability to Measure the Result.} The ability to measure the

final state of the system is required for all computation schemes.

\item \textbf{Scalability.} All physical resources (such as space, money, and number of components) should not scale as $X^{n}$, where $X$ is some

constant, and $n$ is the number of qubits. This requirement ensures that

the system is technically feasible.

\end{doublespace}

\end{itemize}

\begin{doublespace}

In quantum computing, the phase between states are determinate, and the system is coherent. However, in the real world, a quantum computer is not

completely isolated and suffers from gates fidelity being less than one.

Therefore, the coherence time decay of the state with time $T\_{D}$

(decoherence time). The time $T\_{D}$ is also indicated by the results

of the quantum-error correction algorithm that can find and correct the same errors in the quantum state \cite{divincenzo2000physical,preskill1998reliable}.

To implement error correction, we demeaned that the decoherence

time is much longer than the gate operation timescale $T\_{gate}$

times the typical number of operation $N$.

\[

\frac{N\cdot T\_{gate}}{T\_{D}}\ll1

\]

\\

To date, attempts have been made to use different physical systems

to meet these requirements and realize a quantum computer. For example,

in an optical system, the polarization of a photon is taken as a state

and optical component, such as polarizing beam splitters, and wavelength

plates are used to manipulate the state. Optical systems suffer from

photons not interacting; therefore, it is quite difficult

to implement two qubit gates \cite{opticalQC}. Another platform of

quantum calculations is ion traps \cite{cirac1995quantum,home2009complete}.

Ion traps use the internal state of the ion as the qubit, and quantum

gates are implemented using the coupling of the ions to lasers. These

systems are probably the closest to a successful implementation,

but there unsolved issues remain with the scalability and heating

from the electrodes. Another platform that has been investigated is based

on localized electron spins as qubits in quantum dots \cite{imamog1999quantum}

the interaction between the spins can realize the quantum gates. The

interaction and the detection are performed using lasers. The main problem in this platform is the strong coupling of the qubit to a noisy bath (i.e., phonons), which this limits the ratio of operation time (\textasciitilde{}10

psec) to decoherence gate-operation time (\textasciitilde{}1 nsec).

Another platform that could theoretically serve to perform quantum

computation is neutral atoms in a 1-Dimensional (1D) optical lattice \cite{weitenberg2011quantum}.

In this method, they used two sub-level $\left(m\_{f}\right)$ in the

ground state of an optical lattice and described a one-qubit gate

with Raman sidebant transition $\left(t\_{\pi/2}\sim150\;nsec\right)$

and with RF pulse $\left(t\_{\pi/2}\sim30\;\mu sec\right)$. In addition,

they use a movable optical tweezer for the two-qubit gate to transport

one qubit to another gate.\\

\\

In this thesis, we present a new platform of quantum computation that

is based on fermionic atoms in an optical microtrap. The basis for

this platform is the fermionic statistic of the qubits. In addition,

with ultracold atoms, we can control the interaction between atoms

by using Feshbach resonance. Furthermore, the depth of the micro-trap,

shape, and position can be controlled dynamically.\\

\\

In recent years, there has been substantial experimental progress with

preparation and measurement of individual atoms in the ground state

of an optical microtrap \cite{cheuk2015quantum,edge2015imaging}.

Several techniques have been used to accomplish this:\label{twoway}.

\end{doublespace}

\begin{enumerate}

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\item Light-Assisted Collisions (LAC) can reduce the number of atoms by

shining the atoms with a near resonant laser. By carefully tuning

the frequency, it is possible to increase the probability that one

of the atoms will leave the trap while the other will stay. Atoms

one by one by intensifying their interaction \cite{fung2016single}.

After the LAC has been used to remove all atoms other than one, it

is possible to use the Raman side-band cooling technique to cool this

single atom to the ground state of the trap \cite{kaufman2012cooling}.

\item By loading spin polarized atoms to a microtrap with one state or several atoms to a low optical microtrap (with several states) and then creating a linear potential that removes all bound states other than

one, it is possible to end with only a single atom in

one state\cite{Few-FermionSystem}.

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\end{enumerate}

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The measurement of a single fermion $^{40}K$ atom in a trap is clearly

not a simple task. In this field, there are few studies that have succeeded

in doing so \cite{cheuk2015quantum,edge2015imaging,kaufman2012cooling}.

In these studies, a sideband cooling technique was employed

to cool the atoms while measuring the fluorescence.

\\

\\

Our platform is based on ultracold fermion $\left(^{40}K\right)$

neutral atoms trapped in an optical micro-trap. There still remain

some questions regarding the experimental system that are discussed in the next chapters. Chapter \ref{sec:New-platform-of}

 presents the theory behind our proposed scheme. Chapter \ref{sec:ultracold-atomsback},

gives some relevant ultracold-atom background. Chapter \ref{sec:The-experimental-machines} presents the experimental work performed in route to implementing the

new computation scheme.

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\section{New platform of quantum computation\label{sec:New-platform-of}}

\end{doublespace}

This chapter explains how we fulfill the five principles mentioned

in the introduction. The $\sqrt{SWAP}$ gate was developed by Dr.

Jonathan Nemirovsky, and the numerical simulations were performed with a

code that was also developed by Dr. Nemirovsky.

\subsection{The new scheme}

Our new platform is based on neutral ultracold $^{40}K$ atoms. This chapter describes the five conditions for quantum computation

(\ref{sec:5 condition}) and how they are realized in our method.

\subsubsection{The Qubit }

Our quantum computer is based on two internal energy levels of a

single atom in a microtrap. We choose $\left|0\right\rangle =\left|9/2,-9/2\right\rangle $

and $\left|1\right\rangle =\left|9/2,-7/2\right\rangle $ with notation

$\left|f,m\_{f}\right\rangle $, where $f$ is the total atomic spin,

and $m\_{f}$ is the projection in z direction (set by external magnetic

field). We can choose any two $m\_{f}$ states, but we want to control

the interaction between the atoms by means of a Feshbach resonance

\cite{chin2010feshbach}. The Feshbach resonance between $m\_{f}=-9/2$

and $m\_{f}=-7/2$ is at $B=202.2$ G \cite{regal2006experimental}.

We can also work in spin states $\left|0\right\rangle =\left|9/2,-9/2\right\rangle $

and $\left|1\right\rangle =\left|9/2,-5/2\right\rangle $ or $\left|0\right\rangle =\left|9/2,-7/2\right\rangle $

and $\left|1\right\rangle =\left|9/2,-5/2\right\rangle $. Their Feshbach

resonance is $B\_{-\frac{9}{2},-\frac{5}{2}}=224.21$ G and $B\_{-\frac{7}{2},-\frac{5}{2}}=174$

G \cite{regal2006experimental}. However, with these states, there

is a possibility of spin-exchange collisions, which means that the qubit

can leave the designated Hilbert space.

\begin{doublespace}

\subsubsection{Preparation of the Initial State }

\end{doublespace}

\begin{doublespace}

In our method, the initial state requires a single atom state in each qubit.

As mentioned above, the preparation of one atom in a microtrap can

be performed in two ways.\\

The first method (Fast approach) is based on loading several

atoms $\left(\sim10-20\right)$ from a 3-Dimensional (3D) Magneto-Optical-Trap directly

to an optical microtrap and with a LAC \cite{fung2016single} with

a blue detuning laser from the $D\_{1}$ transition, reducing the atom's

number to one (this study has been performed with bosonic $^{85}Rb$).

When the trap contains a single atom, we can cool the atom to the

ground state with Raman sideband cooling \cite{kaufman2012cooling}.

This process grants two more features. We can measure the fluorescence

and calculate the atom number at the optical microtrap (zero, one,

or more). Additionally, we can know which qubit is empty and not

use it for the quantum calculation.\\

The second way (Degenerate fermi gas) is to reduce the trap

depth until there is only a single bound state left \cite{Few-FermionSystem}.

In ref \cite{Few-FermionSystem}, it was shown that by using a magnetic field with a gradient, the number of atoms up to single

atom trapped $\left(^{6}Li\right)$ can be controlled. To obtain high occupation

probability of the lowest state due to Fermi-Dirac statistics, such an

experiment must begin at very low temperatures $T/T\_{f}<0.5$ ,In

other words, $T\sim40$ nK. The time it takes to prepare atoms at

this temperature is about 80 seconds.\\

 In table \ref{tab:01}, we compare these two systems. The advantages

of the fast approach are rapid data acquisition, and it is experimentally

simpler, but there may be a higher final temperature of

the captured atom. Also, there are many unknowns with this method

that still need to be investigated before we can conclude that

this approach is viable. The advantages of the degenerate fermi gas

is low final temperature of the trapped atom, but the disadvantage

is long preparation time ($\sim$80 sec). Chapter \ref{sec:The-experimental-machines} presents the two systems in more detail.

\end{doublespace}

\begin{table}[H]

\begin{centering}

\begin{tabular}{|c|c|c|}

\hline

 & Fast approach & Degenerate fermi gas\tabularnewline

\hline

\hline

Number of Vacuum Cell & One or Two & Two or Three\tabularnewline

\hline

2-Dimenional (2D) and 3D Magneto Optical Trap (MOT) (15-40 sec) & maybe just 3D & $\checkmark$\tabularnewline

\hline

$D\_{1}$cooling (20 msec) & $\checkmark$ & $\checkmark$\tabularnewline

\hline

 Magnetic Trap \& RF Evaporation (30 sec) & X & $\checkmark$\tabularnewline

\hline

Optic or Magnetic Transfer (1-3 sec) & X & $\checkmark$\tabularnewline

\hline

Optic Evaporation & X & $\checkmark$\tabularnewline

\hline

Sideband Cooling (2 sec) & $\checkmark$ & X\tabularnewline

\hline

\end{tabular}

\par\end{centering}

\caption{\label{tab:01}A comparison between the two systems. The tabl shows that the preferable system in terms of time is the one using light-assisted

collisions. However, we are unsure if these will succeed. }

\end{table}

\subsubsection{Quantum gates }

After preparing one or two qubits made of single atoms, we need to

be able to perform quantum-gate operation. To call our system a

Quantum Computer, as was explained in paragraph one, we need to

adapt the Hadamard gate, the phase gate, $\pi/8$ gate, and the $\sqrt{SWAP}$

gate to our system.

\paragraph{Single qubit gates.\label{subsec:Single-qubit-gates}}

An arbitrary single qubit state can be written

\[

\left|\psi\right\rangle =e^{i\gamma}\left(\cos\frac{\theta}{2}\left|0\right\rangle +e^{i\phi}\sin\frac{\theta}{2}\left|1\right\rangle \right)

\]

where $\theta$, $\phi$, and $\gamma$ are real numbers. The numbers

$0\leq\theta\leq\pi$ and $0\leq\phi\leq2\pi$ define a point on a

unit three-dimensional sphere, which is commonly called the $Bloch\ sphere$.

A qubit state with an arbitrary value of $\gamma$ is represented

by a point on the Bloch sphere, as the factor of $e^{i\gamma}$ has no

observable effects. We can then write the following:

\[

\left|\psi\right\rangle =\cos\frac{\theta}{2}\left|0\right\rangle +e^{i\phi}\sin\frac{\theta}{2}\left|1\right\rangle

\]

\\

\begin{figure}

\begin{centering}

\includegraphics[scale=0.7]{qubit-the-bloch-sphere}

\par\end{centering}

\caption{$Bloch\ sphere$}

\end{figure}

The Bloch sphere is $S^{2}$, which can be embedded in $\mathbb{R}^{3}$

using the following map

\[

f:\left(r=1,\phi,\theta\right)\rightarrow\left(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta\right)

\]

The rotations of Bloch vectors can be generated by Pauli matrices

$\hat{\sigma\_{x}}=\begin{pmatrix}0 & 1\\

1 & 0

\end{pmatrix}$, $\hat{\sigma\_{y}}=\begin{pmatrix}0 & -i\\

i & 0

\end{pmatrix}$ and $\hat{\sigma\_{z}}=\begin{pmatrix}1 & 0\\

0 & -1

\end{pmatrix}$ . Therefore, the rotation around the axes is given by

\[

R\_{x}\left(\theta\right)\equiv e^{-i\frac{\theta}{2}\cdot\hat{\sigma\_{x}}}=\cos\frac{\theta}{2}\hat{\mathbb{I}}-i\sin\frac{\theta}{2}\hat{\sigma\_{x}}=\begin{bmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\

-i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}

\end{bmatrix}

\]

\[

R\_{y}\left(\theta\right)\equiv e^{-i\frac{\theta}{2}\cdot\hat{\sigma\_{y}}}=\cos\frac{\theta}{2}\hat{\mathbb{I}}-i\sin\frac{\theta}{2}\hat{\sigma\_{y}}=\begin{bmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\

-\sin\frac{\theta}{2} & \cos\frac{\theta}{2}

\end{bmatrix}

\]

\[

R\_{z}\left(\theta\right)\equiv e^{-i\frac{\theta}{2}\cdot\hat{\sigma\_{z}}}=\cos\frac{\theta}{2}\hat{\mathbb{I}}-i\sin\frac{\theta}{2}\hat{\sigma\_{z}}=\begin{bmatrix}\exp\left(-i\frac{\theta}{2}\right) & 0\\

0 & \exp\left(i\frac{\theta}{2}\right)

\end{bmatrix}

\]

Any unitary transformation on a single qubit can be decomposed into

a rotation in the Bloch sphere around some axis $\hat{n}$ by an angle

$\theta$ multiplied by a global phase $\phi$

\[

U=e^{i\phi}R\_{\hat{n}}\left(\theta\right)

\]

Next, we define the single-qubit gates using these terms.

\begin{itemize}

\item \textbf{\uline{$\ $Hadamard gate.}} A Hadamard gate operator can

be represented by rotations around the $\hat{x}$ and $\hat{z}$ axes.

We choose $\theta=\pi/2$, $\phi=\pi/2$, and $\hat{n}=\left(1,0,1\right)/\sqrt{2}$

\begin{align\*}

U\_{\mathrm{hadamard}} & =e^{i\frac{\pi}{2}}R\_{\hat{n}}\left(\pi\right)\\

 & =i\left[\cos\frac{\pi}{2}\hat{\mathbb{I}}-i\sin\frac{\pi}{2}\left(\frac{\hat{\sigma\_{x}}+\hat{\sigma\_{z}}}{\sqrt{2}}\right)\right]\\

 & =\frac{1}{\sqrt{2}}\left[\begin{array}{cc}

1 & 1\\

1 & -1

\end{array}\right]

\end{align\*}

\item \textbf{\uline{Phase gate.}} A Phase Gate Operator can be represented

by taking $\theta=\pi/2$ , $\phi=\pi/4$, and $\hat{n}=\left(0,0,1\right)$

\[

U\_{\pi/2}=e^{i\frac{\pi}{4}}R\_{z}\left(\frac{\pi}{2}\right)=e^{i\frac{\pi}{4}}\begin{bmatrix}\exp\left(-i\frac{\pi}{4}\right) & 0\\

0 & \exp\left(i\frac{\pi}{4}\right)

\end{bmatrix}=\begin{bmatrix}1 & 0\\

0 & \exp\left(i\frac{\pi}{2}\right)

\end{bmatrix}

\]

\[

U\_{\pi/2}=\begin{bmatrix}1 & 0\\

0 & i

\end{bmatrix}

\]

\item \textbf{\uline{$\boldsymbol{\pi/8}$ Gate.}} A $\pi/8$ Gate Operator

can be represented by using $\theta=\pi/4$, $\phi=\pi/8$, and $\hat{n}=\left(0,0,1\right)$

\[

U\_{\frac{\pi}{8}}=e^{i\frac{\pi}{8}}R\_{z}\left(\frac{\pi}{4}\right)=e^{i\frac{\pi}{8}}\begin{bmatrix}\exp\left(-i\frac{\pi}{8}\right) & 0\\

0 & \exp\left(i\frac{\pi}{8}\right)

\end{bmatrix}=\begin{bmatrix}1 & 0\\

0 & \exp\left(i\frac{\pi}{4}\right)

\end{bmatrix}

\]

We can realize these gates in our system by coupling a two-level

system to an external EM field \cite{rf\_onequbit,kuhr2003controlled}.

Let us write the state of the atom as follows:

\end{itemize}

\[

\psi\left(t\right)=C\_{0}\left(t\right)\left|\psi\_{0}\right\rangle +C\_{1}\left(t\right)\left|\psi\_{1}\right\rangle

\]

where $\left|\psi\_{n}\right\rangle $ are the energy eigenstates of

the atoms that are relevant to the computational scheme, $C\_{0}\left(t\right)=e^{-E\_{n}t/\hbar}C\_{0}\left(0\right)$

are the complex amplitude, and $E\_{n}=\hbar\omega\_{n}$ are the eigenvalues.

we write the Hamiltonian as

\[

H=H\_{0}+V\left(t\right)

\]

where $H\_{0}$ is the free Hamiltonian, and $V\left(t\right)$ is the

interaction between the electromagnetic field and the atom.

\[

V\left(t\right)=\mu\left[A\left(t\right)e^{-i\omega t}+A^{\*}\left(t\right)e^{i\omega t}\right]

\]

where $\mu$ is the electric or magnetic moment, $\omega$ is the

EM field frequency, and $A\left(t\right)$ represents the EM field

amplitude, which we can treat classically. We calculate the matrix

element as $\left\langle \psi\_{n}\right|V\left(t\right)\left|\psi\_{m}\right\rangle $

\[

V\left(t\right)=\begin{bmatrix}0 & V\_{0,1}\\

V\_{1,0} & 0

\end{bmatrix}

\]

where $V\_{n,m}=-\mu\_{n,m}\delta\_{n,m}\left[A\left(t\right)e^{-i\omega t}+A^{\*}\left(t\right)e^{i\omega t}\right]$.

Therefore, the Hamiltonian is

\[

H=\begin{bmatrix}E\_{0} & V\_{0,1}\\

V\_{1,0} & E\_{1}

\end{bmatrix}

\]

The time-dependent Schrodinger equation for the two-level system is

\[

i\hbar\frac{\partial\psi}{\partial t}=H\psi

\]

\[

i\frac{d}{dt}\begin{pmatrix}B\_{0}\left(t\right)\\

B\_{1}\left(t\right)

\end{pmatrix}=\begin{pmatrix}\omega\_{0} & V\_{0,1}/\hbar\\

V\_{1,0}/\hbar & \omega\_{1}

\end{pmatrix}\begin{pmatrix}B\_{0}\left(t\right)\\

B\_{1}\left(t\right)

\end{pmatrix}

\]

by transforming the amplitudes $B\_{i}(t)=C\_{i}\left(t\right)e^{-\omega\_{i}t}$

we can obtain

\[

i\hbar\frac{d}{dt}\begin{pmatrix}C\_{0}\left(t\right)\\

C\_{1}\left(t\right)

\end{pmatrix}=\begin{pmatrix}0 & -\mu\left[A\left(t\right)e^{-i\omega t}+A^{\*}\left(t\right)e^{i\omega t}\right]e^{-i\omega\_{10}t}\\

-\mu\left[A\left(t\right)e^{-i\omega t}+A^{\*}\left(t\right)e^{i\omega t}\right]e^{-i\omega\_{10}t} & 0

\end{pmatrix}\begin{pmatrix}C\_{0}\left(t\right)\\

C\_{1}\left(t\right)

\end{pmatrix}

\]

where $\omega\_{10}=\omega\_{1}-\omega\_{0}$. In the rotating wave

approximation, the terms that oscillate quickly are dropped, and the

term that rotate slowly remains.

\[

i\hbar\frac{d}{dt}\begin{pmatrix}C\_{0}\left(t\right)\\

C\_{1}\left(t\right)

\end{pmatrix}=\begin{pmatrix}0 & \frac{\Omega^{\*}}{2}e^{-i\delta t}\\

\frac{\Omega}{2}e^{-i\delta t} & 0

\end{pmatrix}\begin{pmatrix}C\_{0}\left(t\right)\\

C\_{1}\left(t\right)

\end{pmatrix}

\]

where $\delta=\omega-\omega\_{01}$ is the detuning of the EM field

from resonance, and $\Omega=2\mu A/\hbar$ is the Rabi frequency.

In the resonant case, the evolution of the Bloch vector in the presence

of an external pulse (Rabi pulse) can be described \cite{kuhr2003controlled}

\[

u\left(t\right)=\begin{pmatrix}1 & 0 & 0\\

0 & \cos\theta\left(t\right) & \sin\theta\left(t\right)\\

0 & -\sin\theta\left(t\right) & \cos\theta\left(t\right)

\end{pmatrix}u\_{0}

\]

where $\theta\left(t\right)=\int\_{0}^{t}\sqrt{\left|\Omega\left(t'\right)\right|^{2}+\delta^{2}}dt'$.

Namely, the Rabi pulse rotates the Bloch vector about the x axis. In

the state vector representation, a resonant pulse of duration t is

expressed by the application of a unitary operator $U(t)$ to the

state vector:

\[

\left|\psi\left(t\right)\right\rangle =\hat{U\left(t\right)}\left|\psi\_{0}\right\rangle

\]

\begin{equation}

\hat{U\left(t\right)}=\begin{pmatrix}\cos\frac{\theta\left(t\right)}{2} & i\sin\frac{\theta\left(t\right)}{2}\\

-i\sin\frac{\theta\left(t\right)}{2} & \cos\frac{\theta\left(t\right)}{2}

\end{pmatrix}\label{eq:onegateparameter}

\end{equation}

by setting the angle as $\theta\left(t\right)$, we can obtain the one-qubit gate.

This can be done with coils that create magnetic field with $\omega\_{1}$

and in this case, the Rabi frequency is given by $\Omega=\mu B/\hbar$,

and the detuning is $\delta=\omega\_{1}-\omega\_{0}$.

For example, by taking EM pulse as $\theta(t)=\pi/2$

\[

\hat{U\left(t\right)\_{\pi/2}}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\

-i & 1

\end{pmatrix}

\]

If the atom is initially prepared in one of the basis states, a $\pi/2$

pulse transforms it into a superposition state

\[

\left|0\right\rangle \rightarrow\frac{1}{\sqrt{2}}\left(\left|0\right\rangle +i\left|1\right\rangle \right)\qquad\left|1\right\rangle \rightarrow\frac{1}{\sqrt{2}}(\left|1\right\rangle -i\left|0\right\rangle )

\]

Therefore, by taking RF pulse with detuning relevant as $\left|0\right\rangle $

or $\left|1\right\rangle $, we can drive the atom state with phase

gate, $\pi/4$ gate, and Hadamard gate.

\paragraph{Two-qubit gate.}

To implement the two-qubit $\sqrt{SWAP}$ gate, we utilize two unique advantages of ultracold atoms.

\begin{itemize}

\item Ability to control the interaction between atoms around Feshbach resonance

\cite{chin2010feshbach}.

\item Ability to shape the potential landscape using far off resonance light,

controlling the atom tunneling between two traps \cite{Few-FermionSystem}.

\end{itemize}

These, together with fermionic statistics, are the basis for a new

protocol for $\sqrt{SWAP}$ gate. This protocol is original but similar

in some aspects to the gate first described in ref.\cite{hayes2007quantum}. We consider two optical microtraps with one atom at each site, with

a distance $d$ between them. Using second quantization and the Fermi-Hubbard

model \cite{hubbard1963electron}, the Hamiltonian is given by

\begin{align\*}

H\_{J,U} & =J\left(\hat{u\_{1}}^{\dagger}\hat{u\_{2}}+\hat{u\_{2}}^{\dagger}\hat{u\_{1}}+\hat{d\_{1}}^{\dagger}\hat{d\_{2}}+\hat{d\_{2}}^{\dagger}\hat{d\_{1}}\right)+2U\left(\hat{u\_{1}}^{\dagger}\hat{u\_{1}}\hat{d\_{1}}^{\dagger}\hat{d\_{1}}+\hat{u\_{2}}^{\dagger}\hat{u\_{2}}\hat{d\_{2}}^{\dagger}\hat{d\_{2}}\right)\\

 & \equiv J\cdot H\_{J}+U\cdot H\_{u}

\end{align\*}

Where $J$ is the tunneling energy, $U$ is on-site interaction energy,

$\hat{u\_{i}}$ and $\hat{u\_{i}}^{\dagger}$ are annihilation and creation

operators of particle $i$ in state “up”, i.e., $\left|1\right\rangle $, and

$\hat{d\_{i}}$ and $\hat{d\_{i}}^{\dagger}$ are annihilation and creation

operators of particle $i$ in state “down”, i.e., $\left|0\right\rangle $

with the usual fermionic commutation relations \cite{esslinger2010fermi}.

We assume only on-site interactions because of the short-range interaction

atoms e.g., $^{40}K$. \\

First, the operation of the $\sqrt{SWAP}$ gate in three

steps is as follows. At each step, the Hamiltonian is time independent and the unitary

evolution operator has the form

\[

\hat{U}=e^{\frac{-i}{\hbar}H\cdot t}

\]

The $\sqrt{SWAP}$ gate can be divided into

\begin{equation}

\hat{U}\_{\sqrt{SWAP}}=e^{-\frac{i}{\hbar}J\cdot H\_{J}\cdot t\_{1}}e^{-\frac{i}{\hbar}U\cdot H\_{u}\cdot t\_{2}}e^{-\frac{i}{\hbar}J\cdot H\_{J}\cdot t\_{1}}\label{eq:4}

\end{equation}

where $t\_{1}=\frac{\pi\hbar}{4J}$ and $t\_{2}=\frac{\pi\hbar}{4U}$.

To prove this relation, we need to calculate the time evolution of $H\_{J}$ and

$H\_{U}$.\\

For $H\_{J}$ we note that

\begin{align}

H\_{J}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle & =2\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle \label{eq:1}\\

H\_{J}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle & =2\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle \nonumber

\end{align}

\begin{align}

H\_{J}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle & =0\nonumber \\

H\_{J}\left(\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle & =0\label{eq:2}\\

H\_{J}\left(\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}\right)\left|0\right\rangle & =0\nonumber

\end{align}

Now we look at equations (\ref{eq:1}) and obtain a simple matrix

\begin{equation}

i\frac{d}{dt}\begin{bmatrix}A\_{1}\left(t\right)\\

A\_{2}\left(t\right)

\end{bmatrix}=\begin{bmatrix}0 & 2\\

2 & 0

\end{bmatrix}\begin{bmatrix}A\_{1}\left(t\right)\\

A\_{2}\left(t\right)

\end{bmatrix}\label{eq:3}

\end{equation}

Where $A\_{1}\left(t\right)$ and $A\_{2}\left(t\right)$ are the amplitude of the time-dependent

states, i.e., $\begin{pmatrix}\left|\psi\_{1}\right\rangle \\

\left|\psi\_{2}\right\rangle

\end{pmatrix}$. The solutions to eq.\ref{eq:3} are as follows:

\[

A\_{1}\left(t\right)=A\cos\left(2\left(t-t\_{0}\right)\right)\qquad A\_{2}\left(t\right)=A\sin\left(2\left(t-t\_{0}\right)\right)

\]

Now, we add the solution of equations eq.\ref{eq:2}(homogeneous solution),

and we get that the general solution as

\begin{align\*}

\left|\psi\right\rangle & =(C\_{00}\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}+C\_{11}\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+C\_{12}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)+\\

 & +A\_{12}\left[\cos\left(2\left(t-t\_{0}\right)\right)\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)-i\sin\left(2\left(t-t\_{0}\right)\right)\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\right]\left|0\right\rangle

\end{align\*}

Where $\left|\psi\right\rangle $ is a solution to the time-evolution

equation $i\frac{d}{dt}\left|\psi\right\rangle =H\_{J}\left|\psi\right\rangle $.

Now, we can choose, $C\_{11}=C\_{12}=C\_{00}=0$ and $t\_{0}=0$. This

means that the singlet $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}=\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{u\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}$

after $t\_{1}=\frac{\pi\hbar}{4J}$ into

\[

e^{-i\frac{\pi}{4}H\_{J}}\rightarrow-i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right).

\]

Now we find the solution for $H\_{U}$ .

\[

H\_{u}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle =2\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle

\]

\[

H\_{u}\left(\hat{u\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle =0\qquad H\_{u}\left(\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle =0

\]

\[

H\_{u}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle =0\qquad H\_{u}\left(\hat{d\_{2}}^{\dagger}\hat{d\_{1}}^{\dagger}\right)\left|0\right\rangle =0

\]

Here, the solution is simple, because all single-particle states are stationary,

while the solution for the state $\left|\psi\_{0}\right\rangle =\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)$

reads

\begin{equation}

\left|\psi\_{+}\right\rangle =e^{2it}\left|\psi\_{0}\right\rangle \label{eq:14}

\end{equation}

Now, we can calculate eq.(\ref{eq:4}). The first term with $t=t\_{1}$

is

\[

\left|0\_{1}\right\rangle =e^{-\frac{i\pi}{4}H\_{J}}\left|0\_{initial}\right\rangle

\]

We can see that due to the Pauli principle, all the three symmetric

states $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}$ , $\hat{u\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}$,

$\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)$

are stationary in time eq.(\ref{eq:2}). The singlet state, which

is anti-symmetric, evolves as follows: $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{u\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}\overset{H\_{\text{J}}}{\longrightarrow}\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}$. Therefore,

at the end of the first evolution, the symmetric states are unchanged,

while the anti-symmetric state becomes a state of “two particle”

(i.e., doublon $-i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)$)

. The second evolution with $t=t\_{2}$ (duo to eq. \ref{eq:14})

\[

\left|0\_{2}\right\rangle =e^{-\frac{i}{\hbar}U\cdot H\_{u}\cdot t\_{2}}\left|0\_{1}\right\rangle

\]

the “two particle” state obtains a phase of $e^{-i\pi/2}=-i$ and

transforms it into $-\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)$

.The three symmetric states does not change. Finally, by repeating

the first evolution with $t=t\_{1}$, the symmetric states are unchanged,

and the “doubloon” state gets a phase of $-i$. Now, it reverts

back to an anti-symmetric singlet state

\[

\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\overset{\sqrt{SWAP}}{\rightarrow}i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)

\]

In conclusion, the three steps give us

\begin{itemize}

\item $\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}\rightarrow\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}$

\item $\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}\rightarrow\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}$

\item $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\rightarrow\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}$

\item $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\rightarrow i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)$

\end{itemize}

Therefore, combing these three actions together is equivalent to a $\sqrt{SWAP}$

gate. In matrix notation,

\begin{itemize}

\item $\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}\rightarrow\hat{d\_{1}}^{\dagger}\hat{d\_{2}}^{\dagger}$

\item $\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}\rightarrow\hat{u\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}$

\item $\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}=\frac{1}{2}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\rightarrow$

$\frac{1}{2}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}+i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\right)=\frac{1+i}{2}\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\frac{1-i}{2}\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}$

\item $\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}=\frac{1}{2}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}-\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\rightarrow$

$\frac{1}{2}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}-i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\right)=\frac{1-i}{2}\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\frac{1+i}{2}\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}$

\end{itemize}

which is the same as the matrix form that I showed above in eq.(\ref{eq:swap})

We can simplify the gate in two additional steps or

one additional step. We note that

\begin{align\*}

H\_{J,U}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle & =2J\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle \\

H\_{J,U}\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle & =2J\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\left|0\right\rangle +U\left(\hat{d\_{1}}^{\dagger}\hat{u\_{1}}^{\dagger}+\hat{d\_{2}}^{\dagger}\hat{u\_{2}}^{\dagger}\right)\left|0\right\rangle

\end{align\*}

Now we can write these equations in a matrix form for the anti-symmetric state

\begin{equation}

i\frac{d}{dt}\begin{bmatrix}A\_{1}\left(t\right)\\

A\_{2}\left(t\right)

\end{bmatrix}=\begin{bmatrix}0 & 2J\\

2J & 2U

\end{bmatrix}\begin{bmatrix}A\_{1}\left(t\right)\\

A\_{2}\left(t\right)

\end{bmatrix}\label{eq:5}

\end{equation}

Where the matrix eigenvalues are

\[

\lambda\_{1,2}=U\pm\sqrt{4J^{2}+U^{2}}

\]

and the eigenvectors are as follows:

\[

V\_{1,2}=\frac{1}{2J}\begin{bmatrix}-\lambda\_{2,1}\\

2J

\end{bmatrix}

\]

Thus, the solution is given by $Ae^{-i\lambda\_{1}t}V\_{1}+Be^{-i\lambda\_{2}t}V\_{2}$

and $AV\_{1}+BV\_{2}=\begin{bmatrix}1\\

0

\end{bmatrix}$. The second-term solution is $A=-B$. Therefore, the solution for

the amplitude, eq.(\ref{eq:5}), is

\[

=\frac{Ae^{-iUt}}{2J}\left(e^{-it\sqrt{4J^{2}+U^{2}}}\begin{bmatrix}-U+\sqrt{4J^{2}+U^{2}}\\

2J

\end{bmatrix}-e^{it\sqrt{4J^{2}+U^{2}}}\begin{bmatrix}-U-\sqrt{4J^{2}+U^{2}}\\

2J

\end{bmatrix}\right)=

\]

\begin{equation}

=Ae^{-iUt}\begin{bmatrix}\frac{\sqrt{4J^{2}+U^{2}}}{J}\cos\left(t\sqrt{4J^{2}+U^{2}}\right)+i\frac{U}{J}\sin\left(t\sqrt{4J^{2}+U^{2}}\right)\\

-2i\sin\left(t\sqrt{4J^{2}+U^{2}}\right)

\end{bmatrix}\label{eq:7}

\end{equation}

We can find a specific solution if we choose the parameter correctly.

\begin{equation}

tU=\frac{\pi}{2}\left(4n-1\right)\qquad t\sqrt{4J^{2}+U^{2}}=\pi m\label{eq:8}

\end{equation}

Where $m$ is an odd integer and $n$ is any integer.

By using these choices, eq.(\ref{eq:8}) and $A=\frac{\sqrt{m^{2}-\left(2n-1/2\right)^{2}}}{2m}$

(the solution should be normalized), we obtain eq. (\ref{eq:7})

\begin{equation}

i\begin{bmatrix}1\\

0

\end{bmatrix}\label{eq:11}

\end{equation}

 From eq. (\ref{eq:11}), we obtain the $\sqrt{SWAP}$ gate.

\[

\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)\overset{\sqrt{SWAP}}{\rightarrow}i\left(\hat{d\_{1}}^{\dagger}\hat{u\_{2}}^{\dagger}-\hat{d\_{2}}^{\dagger}\hat{u\_{1}}^{\dagger}\right)

\]

From these two equations, (\ref{eq:8}), we can obtain the strength

of the interaction $U$ and the time $t$ for which the interaction

acts similar to a $\sqrt{SWAP}$gate

\begin{equation}

U=\pm\frac{2J\left(2n-\frac{1}{2}\right)}{\sqrt{m^{2}-\left(2n-\frac{1}{2}\right)^{2}}}\qquad t=\frac{\pi\sqrt{m^{2}-\left(2n-\frac{1}{2}\right)^{2}}}{2J}\label{eq:9}

\end{equation}

The last parameter, $J$, depends on the distance between the two qubits

, i.e., $d\left(t\right)$. One of our goals is to optimize $d\left(t\right)$.

\begin{doublespace}

\subsubsection{Ability to Measure the Results}

\end{doublespace}

In our system, we can detect the population of state $\left|0\right\rangle $

$\left(\left|-9/2,-9/2\right\rangle \right)$ in a fluorescence imaging

using the cycling transition $\left|-9/2,-9/2\right\rangle \rightarrow\left|11/2,-11/2\right\rangle $.

Unfortunately, we cannot detect the cycling transition $\left|-9/2,-9/2\right\rangle \rightarrow\left|11/2,-11/2\right\rangle $

in our platform. The typical trap depth is $\sim400$ nK, and the recoil

temperature in $^{40}K$ is $404\;\mu K$ \cite{tiecke2010properties};

therefore, our atom drives out from the trap (even when the direction

is random and the heating goes as $\sqrt{N\_{\mathrm{photon}}}$).

To overcome this problem, we can measure it with a Raman sideband

cooling technique \cite{sidebandcooling} (for more details, see section

\ref{subsec:Raman-Sideband-cooling}). Recent studies with $^{40}K$

sideband cooling have shown that single atoms release approximately

$60-80\;photom/sec$ \cite{cheuk2015quantum,edge2015imaging}. Consequently,

we can collect 10\% of the photons, and it uses the ability

to measure one-atom fluorescence with the EMCCD camera (depending on the

objective solid fraction angle and the laser detuning eq. \ref{eq:numberofatom}).

Shorter detection times can tune the probe laser frequency and

raise the microtraps depth. \\

\subsubsection{Scalability}

In our system, the scalability is straight forward. When one qubit can be initiated and controlled, by adding more microtraps,

you can obtain a larger number of qubits. The other microtraps are created

by other laser beams that reach the optical objective. These lasers

are then focused to different positions at the focal plane:

\[

d=f\cdot\theta

\]

\begin{figure}

\begin{centering}

\includegraphics[scale=0.5]{\string"focal lens\string".png}

\par\end{centering}

\caption{\label{fig:lens}The distance between two traps that reach the lens

with an angle $\theta.$ }

\end{figure}

Where $d$ is the distance between two microtraps, $f$ is the objective

focal length, and $\theta$ is the angle between the incoming beams

(see Figure \ref{fig:lens}). One way to do it dynamically is by employing

two Acousto-Optic-Modulators (AOM), one in x axis and one in y axis

\cite{lester2015rapid}. We can position the qubits with $d\gg\lambda$

and then $J\approx0$. Then, the qubits can be brought closer with

the optimal $d\left(t\right)$. For one-qubit gates, we can take one

qubit to a position where the RF field is optimal and far enough from

other qubits (Figure \ref{fig:Array-of-qubits}).

\begin{figure}[H]

\begin{centering}

\includegraphics[scale=0.7]{arrayofmicrotrap}

\par\end{centering}

\caption{\label{fig:Array-of-qubits}Array of qubits that are formed by AOM.

The qubits are moved to the $\sqrt{SWAP}$ region or to the one-qubit-gate region according to the quantum code.}

\end{figure}

 The qubit isolation depends on the lifetime in the optical microtrap.

We can reduce the laser power when the atom state is at the ground

state and obtain a lifetime of several minutes. Therefore, the decoherence in our system should be very slow. Furthermore, in our method, we can

find $m$ and $n$ (eq.\ref{eq:9}) such that $\mathcal{F}\rightarrow1$

(the fidelity is the overlap between the chosen target state and the

spin state as measured or calculated $\mathcal{F}=\left\langle \psi\_{target}\right|\hat{\rho}\left|\psi\_{target}\right\rangle $).

\subsection{Theoretical simulation and calculation}

To make a numerical calculation of a single atom in a microtrap,

we need to solve the time independent Schrodinger equation that is

given by

\begin{equation}

H\psi\left(r,\theta,z\right)=E\_{n}\psi\left(r,\theta,z\right)\label{eq:shrodinger eq.}

\end{equation}

where $E\_{n}$ is the state energy of state $n$ and $H$ is the system

Hamiltonian given by

\[

H=-\frac{\hbar^{2}}{2m}\nabla^{2}+V\left(r,\theta,z\right)

\]

where $V$ is the potential. In 3D, the potential of a single microtrap

is

\[

V\left(r,z\right)=-V\_{0}\frac{\omega\_{0}^{2}}{\omega\left(z\right)^{2}}e^{-2\frac{r^{2}}{\omega\left(z\right)^{2}}}

\]

where $\omega\left(z\right)=\omega\_{0}\sqrt{1+\left(\frac{z\lambda}{\pi\omega\_{0}^{2}}\right)^{2}}$.

The waist of a Gaussian beam is given by $\omega\_{0}=\frac{\lambda}{\pi\cdot NA}$,

where $NA$ is the numerical aperture. The trap parameters are laser

beams with $NA=0.9$ and $\lambda=1064\:nm$. We calculated the eigenenergies

and the eigenstates by solving numerically eq. \ref{eq:shrodinger eq}.

The numerical 2D calculation takes advantage of the cylindrical

symmetry with 112 divisions in the radial direction and 102 divisions

in the axial direction, and the accuracy of the results is better than

1\%. The result of the calculation is shown in Figure \ref{fig:singel numerical}.

We present calculations in low-optical trap $V\_{0}/k\_{b}=310\:nK$

\to obtain bound symmetric eigenstates. Figure \ref{fig:Calculations-of-bound} shows the plots of the bound states in a single Gaussian potential for $m=0,1$

($m$ is the azimuthal quantum number). For lower

NA, we need to lower the optical trap depth, and the lowest eigenenergy

depth is smaller. Figure \ref{fig:lowest-eigen-state-and} shows that for one bound symmetric eigenstate, we need a low-depth optical

trap ( NA=0.9).

\begin{figure}

\centering{}\subfloat[\label{fig:lowes}]{\includegraphics[width=4.5cm,height=6cm]{Atom\_in\_Trap\_Orbital\_1}

} \subfloat[\label{fig:second\_Eig}]{\includegraphics[width=4.3cm,height=6cm]{Atom\_in\_Trap\_Orbital\_2}

} \subfloat[\label{fig:third eig}]{\includegraphics[width=4.5cm,height=6cm]{Atom\_in\_Trap\_Orbital\_3}

}\caption{\label{fig:singel numerical}Calculations of bound states in a single

Gaussian potential. a) Lowest eigenstate (symmetric) with energy $E/k\_{b}\approx-40\:nK$.

b) Second eigenstate with energy $E/k\_{b}\approx-1.5\:nK$ (antisymmetric).

c) Third eigenstate with energy $E/k\_{b}\approx-0.316\:nK$ (symmetric).

Other states have $E/k\_{b}>0$ and are therefore not bound.}

\end{figure}

\begin{figure}[H]

\begin{centering}

\subfloat[]{\begin{centering}

\includegraphics[width=7.5cm,height=7cm]{temprature}

\par\end{centering}

} \subfloat[]{\begin{centering}

\includegraphics[width=8cm,height=7cm]{temprature\_NA\_0\lyxdot 75}

\par\end{centering}

}

\par\end{centering}

\caption{\label{fig:Calculations-of-bound}a) Calculations of bound states

in a single Gaussian potential with NA=0.9. The first 12 are with

$m=0$ and the next 12 are with $m=1$. There are three bound states

$\left(E<0\right)$ that are plotted in Figure \ref{fig:lowes}, Figure

\ref{fig:second\_Eig}, and Figure \ref{fig:third eig}. b) Calculations

of bound states in a single Gaussian potential with experimentally

condition NA=0.75. To obtain just two bound states, we need to

tune the optical trap depth to $V\_{0}/k\_{b}=175\:nK$. }

\end{figure}

\begin{figure}[H]

\begin{centering}

\includegraphics[width=11cm,height=7cm]{ground\_state\_vs\_p\_trap}

\par\end{centering}

\caption{\label{fig:lowest-eigen-state-and}Lowest and second eigenstates energy

(in terms of temperature) vs. optical trap depth. }

\end{figure}

There are many more numerical calculations that must be performed, e.g., the gates parameter $U$, $t$, $d\left(t\right)$ (eq. \ref{eq:9})

for two-qubit gate and one-qubit gate parameters that are given by eq.

\ref{eq:onegateparameter}. Another parameter is the transfer qubit

trajectory to obtain a fast transfer \cite{lewis1969exact}.

All these parameters need to be optimized with demand on the fidelity

$\mathit{\mathcal{F}>0.99}$.\\

\newpage{}

\section{ultracold atoms \label{sec:ultracold-atomsback}}

The field of ultracold atoms has seen rapid development during the

last 20 years. Many new experimental techniques have been introduced,

and the experimental toolbox has been vastly expanded. Cooling and

trapping of atoms is based on the use of forces acting on atoms in

laser fields or on the combination of laser fields and magnetic fields.

This chapter presents a brief background of cooling and trapping

techniques.

\subsection{Laser cooling technique }

\begin{doublespace}

\subsubsection{Doppler cooling\label{subsec:Doppler-Cooling}}

\end{doublespace}

Doppler cooling mechanism was experimentally described in 1978 \cite{first\_dooppler}

and is the basis of our cooling techniques. At low temperature,

kinetic energy sets the temperature by

\[

\left\langle E\_{k}\right\rangle =\frac{3}{2}k\_{b}T

\]

where $k\_{b}$ is a Boltzmann constant. Each time a photon is absorbed

by an atom, the atom receives the recoil momentum $\frac{h\nu}{c}$

in the laser propagation direction. When it emits a photon, it again

changes its momentum by the same value but in a random direction.

Accordingly, if the atom travels in the opposite direction to

the laser propagation direction, the atom slows. However, if

the atom moves in the same direction as the laser propagation direction,

it accelerates.\\

To slow down the atom, Doppler cooling

takes advantage of the Doppler effect, a shift in frequency for an

observer moving relative to its source. This means that as the atom

moves, it experiences a shift in laser-beam frequency. When the atom

moves towards the laser-beam propagation, it experiences a frequency

shift of $+\delta\nu\_{D}$, and if it moves in the opposite direction

to the laser propagation, the shift is $-\delta\nu\_{D}$. Thus,

if the laser frequency is lower than the resonance frequency $\nu\_{0}-\delta\nu$,

the atom that travels in the same direction as the laser experiences, per the Doppler effect, $\nu\_{0}-\delta\nu+\delta\nu\_{D}$.

In contrast, the atom that travels in the opposite direction experiences

$\nu\_{0}-\delta\nu-\delta\nu\_{D}$. \\

Accordingly, the atom that travels in the direction of the laser experiences a force corresponding to the resonance

frequency $\sim\nu\_{0}$, while an atom that travels in the opposite

direction of the laser experiences a force that corresponds to a frequency that is far of resonance

$\sim\nu\_{0}-2\delta\nu$. Changing the detuning is one way of controlling

the magnitude of this force and drastically affects the number of

trapped atoms. Therefore, Doppler cooling creates a velocity-dependent

force. It slows down atoms selectively based on the magnitude of their

velocity.

\begin{doublespace}

\subsubsection{Sisyphus Cooling\label{subsec:Sisyphus-Cooling.}}

\end{doublespace}

Sisyphus cooling (polarization-gradient cooling) is a laser-cooling

technique that was observed experimentally and later first given

a full explanation by Claude Cohen-Tannoudji \cite{dalibard1989laser}.

Sisyphus cooling is achieved by two orthogonal polarization laser

beams. The two lasers create a polarization lattice. When the atoms

move to the maximum of the potential (and the resonance frequency

is closer to the laser frequency), they lose kinetic energy and move

slower. As they reach the maximum, they are optically pumped to the

minimum, as shown in Figure \ref{fig:7a}.\textbf{ }In $^{40}K$, this

technique does not work due to the narrow and inverted hyperfine structure

of the $P\_{3/2}$ state \cite{landini2011sub}.

\begin{doublespace}

\subsubsection{Gray Molasses Cooling \label{subsec:Gray-Molasses}}

\end{doublespace}

Gray Molasses is a cooling technique similar to Sisyphus cooling.

The difference between them is that in Gray Molasses, the electromagnetic

field splits the energy levels into a dark state and bright states.

If the laser beam is blue detuned, the bright level is light-shifted

and the dark state does not change (since it is not coupled to the

light field). Similar to Sisyphus cooling, the atoms \textquotedbl{}climb\textquotedbl{}

to the maximum of the potential well and are then pumped to the dark level

(see Figure \ref{fig:cooling-scenme}). As a general principle, a

better-cooling scheme is where the coldest atoms are pumped to a

dark state and are not heated by spontaneous scattering events.

Recent studies \cite{D1coolingmechnisem,fernandes2012sub} have showed

that for $^{40}K$, Gray Molasses on the $D\_{1}$ line can reach a

temperature of $T\sim15\:\mu K$ .

\begin{figure}[H]

\begin{centering}

{\large{}}\subfloat[\label{fig:7a}]{\includegraphics[scale=0.7]{\string"Sisyphus cooling\string".JPG}

{\large{}}{\large \par}}{\large{} }\subfloat[]{\includegraphics[width=7cm,height=5cm]{\string"Sisyphus mechanism\string".JPG}{\large{}\label{fig:cooling-scenme}}{\large \par}

{\large{}}{\large \par}}

\par\end{centering}{\large \par}

\centering{}\caption{a) Sisyphus cooling scheme. Adopted from ref. \cite{sisyphus\_cooling}

b)Gray Molasses cooling scheme. With positive detuning, the ground

state splits to two states, $\left|\psi\_{D}\right\rangle $ and $\left|\psi\_{B}\right\rangle $.

These two states act similar to the states in Sisyphus cooling and were adopted

from ref.{\large{}\cite{fernandes2012sub}.}}

\end{figure}

\begin{doublespace}

\subsubsection{Magneto optical trap\label{subsec:MOT-1}}

\end{doublespace}

\begin{doublespace}

A MOT consists of laser-beam propagation and

retro-reflecting along three orthogonal directions and coils with

anti-Helmholtz configuration. The laser beams with red-detuning from

an energy transition in the potassium spectrum are sent to the atoms.

The main mechanism is the Doppler (\ref{subsec:Doppler-Cooling}) effect

\cite{letokhov1977cooling}. The red-detuned (light with a frequency

smaller than the resonance frequency) light is Doppler shifted in

the rest frame of a moving atom. This shift causes the atoms to interact

with the laser as if they are moving opposite to the laser-propagation

direction. We cool the atoms by lowering their velocities. However,

in this process, there is a limit \cite{letokhov1977cooling} to the

following temperature:

\[

T\_{D}=\frac{\hbar\Gamma}{2k\_{B}}

\]

where $k\_{B}$ is the Boltzmann's constant, $\hbar$ is the reduced

Plank's constant, and $\Gamma$ is the natural line-width. In $^{40}K$,

the Doppler limit is $T\_{D}\sim150\mu k$.

\end{doublespace}

\begin{doublespace}

\subsubsection{Magnetic field for MOT }

\end{doublespace}

Doppler cooling lowers the temperature of atoms but does not differentiate

between an atom far from the middle of the trap and an atom at the center.

A magnetic field takes advantage of the Zeeman effect to localize the

atoms and to increase the density. Atoms can have different angular

momentum $m\_{z}=-f,-f+1,..,f$ where $f$ is the total atomic spin.

In the presence of a magnetic field, the energy levels are split into

sub-levels. The energy change is given by the following:

\[

\Delta U=-\vec{\mu}\cdot\vec{B}

\]

where $\vec{\mu}$ is the magnetic dipole moment of the state, and

\emph{$\vec{B}$} is the magnetic field. Therefore, the energy difference

is proportional to the magnetic field and depends on its direction.

\begin{figure}

\begin{centering}

\includegraphics[scale=0.5]{\string"magnetic feild of anti-Helmholz\string".jpg}

\par\end{centering}

\caption{\label{fig:The-magnetic-field}The magnetic field created by anti-Helmholz

configuration }

\end{figure}

Coils with an anti-Helmholtz configuration produce a magnetic field that

switches its sign at the origin (see Figure \ref{fig:The-magnetic-field}

). This give two regions, positive and negative. At the origin, the

magnetic field is zero. Therefore, the energy shift is $\Delta U\approx0$. In the positive magnetic field, $m\_{z}<0$ and the photons have increased

energy, while in the negative magnetic field, $m\_{z}>0$ and the photons

have decreased energy. ($\Delta U$ is in the opposite direction as the magnetic field).

Therefore, as shown in Figure \ref{fig:Zeeman}, a photon with the

correct polarization is confined by the atoms, giving a spatially

dependent forces with zero force in the center. Figure \ref{fig:MOT-configuration} summarizes the laser directions and polarization in 3D due to the magnetic field from quadratic coils.

\begin{doublespace}

\begin{figure}[H]

\includegraphics[scale=0.7]{\string"sigma cunfigoration\string".png}

\centering{}\caption{\label{fig:Zeeman}Description of Zeeman split and polarization of

laser beams with detuning $\delta\nu$ in one dimension. The blue

line is the energy level at zero magnetic field. On the left, the magnetic field is negative; therefore, the atom interacts with $\sigma^{+}$

laser polarity. On the right side, the magnetic field is positive, so

the atom interacts with $\sigma^{-}$laser polarity. }

\end{figure}

\begin{figure}[H]

\begin{centering}

\includegraphics[width=8cm,height=7cm]{MOT}

\par\end{centering}

\caption{\label{fig:MOT-configuration}MOT configuration }

\end{figure}

\end{doublespace}

\subsection{Raman Sideband cooling \label{subsec:Raman-Sideband-cooling}}

To describe Raman sideband cooling, a Raman transition must be explained\cite{arimondo1976nonabsorbing}. A Raman transition

is a two-photon transition consisting of absorption and stimulated

emission. As shown in Figure \ref{fig:raman tra}, an atom moving with

velocity $v$ that absorbs a photon with frequency $\omega\_{1}$ is

excited to a virtual state $\left|\mathrm{c}\right\rangle $. Immediately,

another photon with frequency $\omega\_{2}$ traveling in the opposite

direction causes stimulated emission of the atom into state $\left|b\right\rangle $.

This allows for the precise selection of atoms with velocities that

satisfy the equation

\[

\frac{v}{c}=\frac{\omega\_{0}-\left(\omega\_{1}-\omega\_{2}\right)}{\omega\_{1}+\omega\_{2}}

\]

where $c$ is the speed of light and $\hbar\omega\_{0}$ is the transition

energy between $\left|a\right\rangle $ and $\left|b\right\rangle $.

We can use the Raman pulse to transfer an atom with velocity $v$

from $\left|a\right\rangle \rightarrow\left|b\right\rangle $, and

with another laser, we can excite the atom from $\left|b\right\rangle \rightarrow\left|c\right\rangle $.

At state $\left|c\right\rangle $, the width of the velocity distribution

is $\sigma\_{c}\left(v\right)\ll\sigma\_{a}\left(v\right)$. Therefore,

when the atom decays back to $\left|a\right\rangle $ with velocity

around $v-v\_{r}$, at the end of the cycle, we have more atoms with

lower velocity. In 1995, Wineland \textit{et al.}\cite{sidebandcooling},

proposed cooling an atom to the ground state in a 3D-optical trap

scheme that was based on Raman transition. Only recently and with

more sophistication was it performed with $^{40}K$ \cite{cheuk2015quantum}

in an optical lattice. By cooling with Raman sideband technique, we

gain two benefits. First, we can detect the number of atoms at each

site due to their fluorescence, and second, we can lower the atom

to the ground state.

\begin{figure}

\begin{centering}

\subfloat[\label{fig:raman tra}]{\includegraphics[scale=0.5]{\string"raman transition\string".png}

} \subfloat[]{\includegraphics{\string"sideband cooling\string".JPG}

}

\par\end{centering}

\caption{a) Raman transition between two atomic levels $\left|a\right\rangle $

and $\left|b\right\rangle $ b) Raman sideband cooling scheme in $^{40}K$

taken from \cite{cheuk2015quantum}. }

\end{figure}

\subsection{Magnetic trap - QUIC configuration}

One cooling technique in ultracold atoms experiments is RF evaporation

\cite{christensen19774}. In this technique, the atoms are loaded to

magnetic trap with $m\_{z}>0$, and by using a RF field, the atoms are

transferred to a state with $m\_{z}<0$, which is not magnetically confined; therefore, they leave the trap. In this technique, if the minimum of a magnetic field is zero, then the atoms that are closer to the minimum (with low temperature) can flip their spin and be ejected. A QUIC

configuration trap \cite{quiccoils} is formed by two quadrupole coils

and one Ioffe coil. The MOT uses the same coils as the quadrupole

trap, so the transfer of atoms from the MOT into the magnetic trap

is straightforward. Atoms are loaded into a quadrupole trap and subsequently

transferred to an Ioffe-type trap. Figure (\ref{fig:-Magnetic-field}) shows that the magnetic field goes from quadrupole with $\mathrm{min}\left(B\right)=0$

to quadrupole with $\mathrm{min}\left(B\right)=1\;\mathrm{G}$, and

the minimum is shifted around $17\:\mathrm{mm}$ towards the Ioffe

coil. The ratio between $\frac{I\_{I}}{I\_{Q}}$ depends on the exact

sizes of the coils and distance between the quadrupole coils and the

Ioffe coil.

\begin{center}

\begin{figure}[H]

\begin{centering}

\includegraphics[scale=0.35]{\string"QUIC\_calculation - Yanay\string".jpg}

\par\end{centering}

\caption{\label{fig:-Magnetic-field} Magnetic field calculations in $y$ direction

starts with quadrupole with $I\_{0}=210\:A$ and the addition of a

Ioffe coil with different current. The minimum is adiabatically moved

$\sim17\:mm$ towards the Ioffe coil. }

\end{figure}

\par\end{center}

\subsection{Optical trap}

Optical dipole force comes from the potential that an atom feels when

the oscillating electric dipole moment of the atom induced by the

oscillating electric field of the laser light interacts with the

field. Two important quantities for optical dipole traps are the depth

of the potential $U\_{dip}\left(r\right)$ and the scattering rate

$\Gamma\_{sc}\left(r\right)$. In terms of decay rate, they can be expressed

as \cite{optical\_trap\_theory}

\[

U\_{dip}(r)=\frac{3\pi c^{2}\Gamma}{2\hbar\omega\_{0}^{3}\delta}I\left(r\right)

\]

\[

\Gamma\_{sc}\left(r\right)=\frac{3\pi c^{2}\Gamma^{2}}{2\hbar\omega\_{0}^{3}\delta^{2}}I\left(r\right)

\]

where $I\left(r\right)$is the laser beam intensity and $\delta=\omega-\omega\_{0}$

is the frequency detuning of the laser from the frequency of the optical

transition $\omega\_{0}$. The dipole trap can be attractive for red

detuning $\left(\delta<0\right)$ or repulsive for blue detuning $\left(\delta>0\right)$.

\begin{table}[H]

\begin{centering}

\begin{tabular}{|c|c|c|}

\hline

$\lambda$ & $P\;[mW]$ & $lifetime\ [msec]$\tabularnewline

\hline

\hline

1064 & 75 & 220\tabularnewline

\hline

820 & 20 & 23\tabularnewline

\hline

\end{tabular}

\par\end{centering}

\caption{\label{tab:2}Comparison of the lifetime and the laser source power

in two commercial laser wavelengths. We required a $1\:mK$ trap that

is high enough for atoms at temperature following $D\_{1}$ cooling

$\left(T\_{D\_{1}}\sim30\mu K\right)$. }

\end{table}

The simple example is for $TEM\_{00}$ Gaussian mode far from resonance

frequency. Beam intensity is given by

\[

I\left(r,z\right)=\frac{2P}{\pi\omega^{2}\left(z\right)}e^{\frac{-2r}{\omega^{2}\left(z\right)}}

\]

Where $\omega\left(z\right)=\omega\_{0}\sqrt{1+\left(\frac{z}{z\_{R}}\right)^{2}}$.

The peak intensity is given by $I\_{0}=2P/\pi\omega\_{0}^{2}$. The

trap depth is defined as $U\_{0}=\left|U\left(0,0\right)\right|$ and

is linearly proportional to the beam intensity. Expanding around the

position of maximum intensity leads to a harmonic potential

\begin{equation}

U\_{dip}(r,z)=-U\_{0}\left[1-2\left(r/\omega\_{0}\right)^{2}-\left(z/z\_{R}\right)^{2}\right]\label{eq:20}

\end{equation}

\newpage{}

\section{The experimental machines \label{sec:The-experimental-machines}}

In our lab, we built an ultracold-atom system with $^{40}K$. This

Chapter describes the systems and concentrate on the parts I

constructed.

\begin{doublespace}

\subsection{The experimental systems}

\end{doublespace}

\begin{doublespace}

We considered two methods (see sketches in Figure \ref{fig:a)-First-System})

of creating a single atom trapped in an optical trap. Each of these

methods has advantages and disadvantages, and we have not yet decided which method to use. We plan to

advance in both before making the final decision. Considerations are

the preparation time of a single atom trap and the temperature of

the atom. The first system includes a machine producing as an initial resource

a quantum degenerate Fermi gas with $T/T\_{f}\ll1$. The second method

is characterized by loading from only a relatively ultracold cloud

after 3D MOT or $D\_{1}$cooling, removing all atoms other than one,

and then cooling inside the trap.

\end{doublespace}

\begin{enumerate}

\begin{doublespace}

\item \textbf{Degenerate fermi gas.} The first system (see Figure \ref{fig:first system })

is composed of three cells under ultrahigh vacuum $\sim10^{-11}\ torr$

. In the first cell (source), we release $^{40}K$ atoms from

homemade dispensers. The atoms are captured by a 2D MOT. On the third

axis, there is a mirror with a hole (nozzle) inside the chamber. The

atoms are cooled in two axes and pushed to the second cell by another

laser (with different detuning) in the third axis (reflected with

hole in the middle by a nozzle). In the second cell, they are captured

and trapped by a 3D MOT. At this point, the cloud temperature is around

$\sim220\mu K$ set by the Doppler limit. By using a Gray Molasses cooling

on the $D\_{1}$ atomic transition, the atomic cloud temperature is

reduced to $\sim15\mu K$. Next, we optically pump the atoms into

the states$\left|9/2,9/2\right\rangle $ and $\left|9/2,7/2\right\rangle $

and load the atoms to a magnetic trap with a QUIC configuration \cite{quiccoils}.

In this configuration, we obtain a magnetic trap without $B=0$. This

is important for a $RF$ evaporation. Following the evaporation, the

temperature is $T/T\_{f}\approx1-3$. Next, we load the atoms to a far-

from-resonance optical trap and move the optical trap adiabatically (with

air bearing stage ) to the science chamber. Then, we first confirm

that the cloud is spin polarized and then load it to a microtrap

and reduce the trap depth until there is only a single bound state

\cite{Few-FermionSystem}.\\

The advantages of this approach is that the process of cooling occurs

prior to loading, and there is a large spatial separation between

the source and the final trap (which ensures a long lifetime of the

trapped atom) and a greater density of atoms. The disadvantage is

that the process is rather complicated and takes around 80 seconds.

\item \textbf{Fast approach.} In the second system (see Figure \ref{fig:second system }), we

have one cell under high vacuum $\sim10^{-11}\ torr$. The $^{40}K$

atoms is released from a homemade dispenser by heating and trapped

with a 3D MOT. Then, by using a Gray Molasses cooling on the $D\_{1}$

atomic transition, we obtain a cloud with temperature of over hundreds

of micro-kelvin. Next, we can load directly to a micro-trap made of

a far-off-resonance light. Then, using light assisted collisions, only

a single atom remains trapped. This single atom is \textquotedbl{}hot\textquotedbl{}

in the sense that its spread over vibration states is large. To measure the atom and cool it to ground state, we plan to use

Raman side-band cooling \cite{sidebandcooling,cheuk2015quantum,edge2015imaging}.

 \\

The advantages of this approach are the simplicity of the apparatus and

the short duration of the experiment that allows for a fast data-accumulation

rate. The disadvantage is a shorter lifetime due to the residual ambient

gas. There is also a possibility to construct a system made of two

chambers where one chamber is used with 2D MOT to generate a source.

\end{doublespace}

\end{enumerate}

\begin{figure}

\begin{centering}

\subfloat[\label{fig:first system }]{\includegraphics[width=9cm,height=10cm]{lab1\_1}

}\subfloat[\label{fig:second system }]{\includegraphics[width=7cm,height=10cm]{vacuum\_1.PNG}

}

\par\end{centering}

\caption{\label{fig:a)-First-System}a) Degenerate fermi gas system description.

Atoms are released from the dispensers and are trapped by a 2D MOT

in the first cell. In the second cell (cooling),

atoms are trapped by a 3D MOT and cooled with $D\_{1}$ cooling and

RF evaporation. Then, they are loaded to an optical trap that transfers

the atoms to the third cell. b) Fast approach system

(with one cell) description. This chamber is similar to the 2D-MOT chamber

in the first system. Atoms are released from the dispensers and trapped

by 3D MOT. Then, the atoms are loaded into an optical microtrap.}

\end{figure}

We are currently building two experimental systems: the first system

is a degenerate fermi gas machine where we can proceed

with the first approach (this system is planned to be also used for

other experiments), and a second smaller system in which we are going

to explore the second approach. We started constructing the first system

two years ago, and in the meantime, we completed the 2D and 3D MOT,

$D\_{1}$ cooling, optical pumping, magnetic trapping in QUIC trap,

RF evaporation, loading into an optical trap, and transporting the atoms

to the science chamber. We started to build the second system eight

months ago (the vacuum chamber was actually evacuated two and a half

years ago); we have just one cell, and we have completed the 3D MOT and

are now working on loading atoms to the optical microtrap and detecting

them.

\begin{doublespace}

\subsection{MOT}

\end{doublespace}

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In both systems, the first stage is MOT. In the first method, we start

with a 2D MOT and continue to a 3D MOT, and in the fast approach method,

we start with a 3D MOT. In the first system, we first used a 2D MOT

as described in \cite{2dMOT}. For the 3D MOT, we needed, as explained

previously, two lasers (cooling and repump) and two coils with anti-Helmholtz

configuration. In this configuration, we cannot make RF evaporation,

as there is a zero-magnetic field at the bottom. Therefore, we added

a Ioffe coil in a QUIC configuration \cite{quiccoils}.

\end{doublespace}

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\subsubsection{Coils setup}

\end{doublespace}

Three coils were made from a $4.2x4.2\ mm$ square copper tube, which

is hollow to cool the coil at a high current by letting water

flow through it. To wrap this coil, we have designed a part made

of Teflon that connects to a rotating spindle (\ref{fig:coil twist}).

Teflon is use so the glue does not stick to the holder and to avoid harming the coating of

the coil. After each round, we smeared a layer of glue\textit{ (Araldite

2011)} and let it dry for 24 hours.\\

 Considering the dimensions of our system, two coils

(both for the 3d MOT and for the magnetic trap) were needed with \textit{7X5}

winding with $r=20\ mm$. Another coil with \textit{6X4} and an inner

most radius of $r=30\ mm$(\ref{fig:9-a}). The coil current is controlled

with a PID loop that measures the current by Hall probe. \\

\begin{figure}[H]

\begin{centering}

\subfloat[\label{fig:9-a}]{\includegraphics[scale=0.3]{QUIC}

} \subfloat[\label{fig:coil twist}]{\includegraphics[scale=0.3]{coil.PNG}

}

\par\end{centering}

\caption{\label{fig:coil } a) QUIC configuration. Atoms are loaded at point

a by two coils with anti-Helmholtz configuration with $U\_{min}=0$.

When the Ioffe current rises, the atoms are moved to the new minimum,

at point b $\left(d=\sim16.9\;mm\right)$, with $B\_{min}\approx1\:G$.

b) Picture of the part that twisted the coils. }

\end{figure}

\begin{doublespace}

\subsubsection{Lasers setup }

\end{doublespace}

For MOT, we need two lasers: one laser for cooling and the other laser as a repump

to return the atoms to the cooling transition if they end in the

other hyperfine state $m\_{f}=7/2$. In our setup, as shown in Figure

(\ref{fig:Laser-setup.-Cooling}), we used one laser as a reference

laser (DBR laser \textit{PH770DBR080T8 } from \emph{Photodigm} and

a current and temperature controller of \emph{LDC 501} from\emph{

Stanford Research System}). The reference laser is locked on the $\left|F=2\right\rangle \rightarrow\left|F'=3\right\rangle $

on the $D\_{2}$ transition in $^{39}K$. The reference laser is locked

to room temperature on the vapor cell with $^{39}K$ atoms; hence,

we need to use Saturated Absorption Spectroscopy (\ref{subsec:Saturated-absorption-spectroscop})

. The two other lasers are locked with Offset locking \cite{offsetlocking}

to the reference laser. That configuration was used because a wide

tunability range was needed for the lasers (we cannot obtain that configuration by using AOMs).

Theoretically, the shift between the reference laser to the cooling

laser, as described in Figure (\ref{fig:-Optical-transitions}), is

\[

f\_{\mathrm{cooling}}=f\_{\mathrm{reference}}+804.85\:MHz

\]

An AOM was placed as a switch before the fiber with $-100MHz$ shift,

and determined a red detuning of $3\Gamma\approx18\:MHz$. Therefore,

\[

\Delta f\_{cooling}=922\:MHz

\]

In addition, the theoretical shift between the reference laser to

the cooling laser is

\[

f\_{repump}=f\_{reference}-431\:MHz

\]

A AOM was placed as a switch before the fiber with $+110\:MHz$ shift,

and determined a red detuning of $3\Gamma\approx18\:MHz$. Therefore,

\[

\Delta f\_{cooling}=522\:MHz

\]

\begin{center}

\begin{figure}

\centering{}\includegraphics[scale=0.75]{mot\_script}\caption{Laser setup\label{fig:Laser-setup.-Cooling}. Cooling and repump are

locked by offset locking to the reference laser. The reference laser

locked on {\large{}$\left|F=2\right\rangle \rightarrow\left|F'=3\right\rangle $

in the $D\_{2}$ transition of $^{39}K$ with} SAS system. Most of

the power of the lasers (cooling and repump) goes through a AOM that

is used as a switch. After a 1:2 telescope, they are split, and the

most injected to one fiber leads to the MOT, while the other power

is injected to another fiber that lead to the probe. }

\end{figure}

\par\end{center}

\begin{figure}[H]

\begin{centering}

\includegraphics[width=6.5cm,height=10cm]{potassium\_propeties}

\par\end{centering}

\caption{ \label{fig:-Optical-transitions}Optical transitions of the $D\_{1}$

and $D\_{2}$-lines of $^{39}K\:\&\:^{40}K$. The blue arrow is the transition

that we lock to using the Saturated absorption spectroscopy for the

MOT. The orange arrow is a transition used for the $D\_{1}$ cooling. The green

arrow is the cooling transition, and the red arrow is the repump transition

for the MOT. The black arrow is the cooling transition, and the purple

arrow is the repump transition for the D1 cooling. The numbers are

in $MHz$. Adopted from \cite{tiecke2010properties}}

\end{figure}

\begin{doublespace}

\subsubsection{Saturated Absorption Spectroscopy (SAS)}

\end{doublespace}

\textbf{\label{subsec:Saturated-absorption-spectroscop}} In laser

cooling, we must lock the laser to the frequency of an atomic transition.

The atoms move with a random velocity distribution, so the laser comes

into resonance with different velocity groups of atoms. Therefore,

the laser interacts with atoms in different velocity groups of atoms.

Their velocities, according to Maxwell-Boltzmann distribution, are

\\

\[

\frac{dn}{dv}=n\_{0}\sqrt{\frac{m}{2\pi k\_{b}T}}exp\left(\frac{-mv^{2}}{2k\_{b}T}\right)

\]

\\

If the laser beam is at frequency $f\_{0}$ in the reference frame

of the lab, then in the atoms frame, the frequency is shifted due to the

Doppler effect:

\[

f=\left(1\pm\frac{v}{c}\right)f\_{0}

\]

 This means that each velocity group has a different resonance frequency

in their respective frame of reference. Therefore, the frequency assumes

a Gaussian shape

\begin{equation}

I\left(f\right)=I\_{0}exp\left[-\frac{mc^{2}\left(f\_{0}-f\right)^{2}}{2k\_{b}Tf^{2}}\right]\label{eq:15}

\end{equation}

 with a width of $\sigma=f\_{0}\sqrt{\frac{k\_{b}T}{mc^{2}}}$. In $^{39}K$

on temperature $T\approx340\:k$, the width approaches$\sigma=346\;MHz$\\.

However, Doppler broadening makes it impossible to determine the precise transition

frequency to within the natural linewidth $\left(\Gamma\sim6\:MHz\right)$.

To overcome this difficulty, we need to use an SAS system that is

a probe pump setup. \\

\\

Two counter-propagating probe and pump, laser beams derived from

a single laser beam are sent through an atomic vapor cell (in our

case, a vapor with $^{39}K$) at room temperature with same frequency

$f\_{0}$. A photodiode is placed after the vapor cell and measured

the probe beam. If the probe beam frequency is not at the resonance

frequency, $f\_{\mathrm{probe}}\neq f\_{0}$, then it interacts

with atoms that have velocity $v$ that satisfy the Doppler shift

$f\_{probe}=f\_{0}\left(1+v/c\right)$. In addition, the pump beam

interacts with atoms that have velocity $-V$. In this case, the signal

on the photodiode is a deep (eq. \ref{eq:15}) with width of

$\sigma$. However, when the beam is on resonance $f\_{probe}=f\_{0}$, the

atoms has zero velocity, and there is a sharp decrease in absorption

(seen as a sharp increase in the signal from the detector), since

many of these atoms have been pumped out of the ground state and

are not be able to absorb any photons from the resonant probe beam.

Figure \ref{fig:sas d2} shows the signal from the SAS system

with $^{39}K$ at room temperate for the $D\_{2}$ laser, and Figure

\ref{fig:sas d1} shows another SAS system result for the $D\_{1}$

transition. The system description in in Figure \ref{fig:Laser-setup.-Cooling}

for $D\_{2}$ and in Figure \ref{fig:D1-laser-setup.} for $D\_{1}$.

\begin{figure}[H]

\begin{centering}

\subfloat[\label{fig:sas d2}]{\begin{centering}

\includegraphics[width=15cm,height=8cm]{\string"SAS D2\string".jpg}

\par\end{centering}

}

\par\end{centering}

\begin{centering}

\subfloat[\label{fig:sas d1}]{\includegraphics[scale=0.38]{\string"sas d1\string".jpg}

}

\par\end{centering}

\caption{Saturated Absorption Spectroscopy in our system. Figure a) shows the $D\_{2}$ transition (fig.\ref{fig:-Optical-transitions}) where

zero frequency is for the repump transition in $^{40}K$, and Figure b) shows the $D\_{1}$ transition (fig.\ref{fig:-Optical-transitions}),

where zero frequency is the transition between $F=1\rightarrow F'=CO\left(1,2\right)$

in $^{39}K$.}

\end{figure}

\begin{doublespace}

\subsubsection{Offset locking\label{Offset-locking}}

\end{doublespace}

Offset locking is a technique to lock a laser to the reference laser

and give the ability of frequency tuning from tens of MHz to several

GHz. This technique is based on the frequency depended phase shift

experienced by the beat note of two laser frequencies, as shown in

\cite{offsetlocking}. The circuit and the locking signal are shown

in Figure \ref{fig:Offset-Locking}.

\begin{doublespace}

\begin{figure}[h]

\begin{centering}

\subfloat[]{\includegraphics[scale=0.35]{\string"offset circuit\string".png}

} \subfloat[]{\includegraphics[scale=0.35]{\string"offset locking\string".jpg}

}

\par\end{centering}

\caption{\label{fig:Offset-Locking}a) Offset Locking circuit. The signal

goes through a coupler (ZEDC-10-2B) to take a reference of the

signal (and measure the laser width) and amplify (zfl-1000+). Then,

it is mixed (zx-12MH-S+)with a voltage control oscillator (zx95-800A+).

It then splits to two lines (ZX10-2-12-S+), one short and another long

(0.1m \& 3.4m). Afterwards, the two lines are recombined on a phase

detector (ZRPD-1+). We use a low-pass filter of 1.9 KHz at the end.

b) offset signal. The “dot” position is controlled by the VCO}

\end{figure}

\end{doublespace}

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\subsubsection{Measurements of the number of atoms }

\end{doublespace}

\begin{doublespace}

To calculate the number of atoms, we measure their florescence

with a photodiode. We can calculate the number of atoms by the following equation:

\begin{equation}

N=\frac{V\tau}{g\_{1}g\_{2}S\cdot E\_{photon}\rho\_{6}}\label{eq:numberofatom}

\end{equation}

\\

where \emph{$V$} is the measured output voltage, $\tau$ is the excited

state life time of the atom, $g\_{1}$is the current to voltage photodiode

gain, $g\_{2}$ is the photodiode efficiency\emph{, $S$ }is the solid

angle fraction $\left(S=\arctan\left(\frac{d}{f}\right)\right)$,

$E\_{photon}$ is the photon energy, and $\rho\_{6}$ is the excited

state fraction that is calculated in \cite{sixlevelwilliamson1997magneto}

for a six-level model.

To calibrate the laser detuning, we first find the resonance.

We load the MOT for 15 seconds with cooling laser frequency at $f\_{0}$

optimized for MOT operation, change in 10 milliseconds the

cooling laser frequency to $f\_{1}$, and measure the fluorescence

fraction $\frac{V\left(f\_{1}\right)}{V\left(f\_{0}\right)}$. By performing

this sequence, we confirm that our signal does not depend on the number

of atoms and $f\_{0}$, but only on $f\_{1}$. The result is shown

in Figure(\ref{fig:Calibration-of-the}), and we repeat this measurement

for the repump laser (fig. \ref{fig:repump\_reson}). We optimized

detuning the lasers (cooling and repump) to obtain a high number

of atoms (see Figure \ref{fig:Number-of-atoms cooling} and Figure \ref{fig:Number-of-atoms repump}).

\begin{figure}

\begin{centering}

\subfloat[\label{fig:Calibration-of-the}]{\begin{centering}

\includegraphics[width=7.5cm,height=6cm]{\string"cooling resonance\string".jpg}

\par\end{centering}

}\subfloat[\label{fig:repump\_reson}]{\begin{centering}

\includegraphics[scale=0.4]{\string"repump resonance\string".jpg}

\par\end{centering}

}

\par\end{centering}

\caption{Calibration of the resonance frequencies. a) Cooling Laser Fluorescence

Fraction. b) Repump Laser Fluorescence Fraction }

\end{figure}

\begin{figure}

\begin{centering}

\subfloat[\label{fig:Number-of-atoms cooling}]{\begin{centering}

\includegraphics[scale=0.4]{\string"Number of atoms vs. cooling v tune\string".jpg}

\par\end{centering}

}\subfloat[\label{fig:Number-of-atoms repump}]{\begin{centering}

\includegraphics[scale=0.4]{\string"Number of atoms vs. repump v tune\string".jpg}

\par\end{centering}

}

\par\end{centering}

\caption{Number of atoms vs laser frequency. To know what are good

conditions for the MOT, we scan the laser frequency and calculate

the number of atoms. (a) Cooling Laser. (b) Repump Laser }

\end{figure}

The last parameter that is tunable is the dispenser current. The dispenser

current can shorten the loading time (Figure \ref{fig:a)-Example-of})

and increase the number of atoms.

\begin{figure}

\begin{centering}

\subfloat[]{\includegraphics[scale=0.4]{\string"loading time\string".jpg}

} \subfloat[]{\includegraphics[scale=0.37]{\string"getter current\string".jpg}

}

\par\end{centering}

\caption{\label{fig:a)-Example-of}a) Example of loading time measurement.

b) Number of atoms and loading time vs. dispenser current. High currents

release more potassium-40 and increase atom density in

the cell. Thus, the loading time decreases and the number

of atoms increases. However, a high current shortens the life of the

dispenser.}

\end{figure}

\end{doublespace}

\begin{doublespace}

\subsubsection{Temperature Measurement with Release \& Recapture Technique\label{subsec:Temperature-measurement-with} }

\end{doublespace}

\begin{doublespace}

To measure the MOT temperature, we use Release and Recapture

(R \& R) method \cite{R&Rmethod} described in Figure(\ref{fig:Release-=000026-Recapture-1}).

Assuming that the atoms in the MOT have a Maxwell Boltzmann distribution

\[

f\left(v\right)=4\pi v^{2}\left(\frac{m}{2\pi k\_{B}T}\right)^{3/2}e^{-\frac{mv^{2}}{2k\_{B}T}}

\]

\begin{figure}

\begin{centering}

\includegraphics[scale=0.5]{\string"R&R description\string".JPG}

\par\end{centering}

\caption{\label{fig:Release-=000026-Recapture-1}Release \& Recapture Experiment.

In a short time, most of the atoms do not escape from the area of the

MOT beams so they are trapped again. However, as time progresses, the

number of atoms that remain in the MOT beams decreases depending

on their velocity or, in other words, their temperature.}

\end{figure}

At some point, we immediately shut off the trap and let the atoms expand

ballistically for duration $\delta t$ and then open the lasers again

and recapture part of the atoms. The position of each atom after this

expansion is given by

\[

f\left(r,t\right)=\frac{4r^{2}}{\sqrt{\pi}\alpha^{3}t^{2}}e^{-\frac{r^{2}}{\alpha^{2}t^{2}}}

\]

Where $\alpha=\left(\frac{m}{2k\_{B}T}\right)^{-3/2}$. Now we can

use $v=r/t$ and obtain

\[

f\left(v\right)=\frac{4v^{2}}{\sqrt{\pi}\alpha^{3}}e^{-\frac{v^{2}}{\alpha^{2}}}

\]

Assuming that the MOT radius starts with $r\_{0}$ and captures with

the radius beam ($r\_{f}=\omega\_{0}$), we can calculate the number

of atoms that we trap

\[

N\left(t\right)=\int\_{r\_{0}}^{r\_{f}}N\_{0}f\left(v\right)dv=N\_{0}\frac{4}{\sqrt{\pi}\alpha^{3}t^{3}}\int\_{0}^{\omega\_{0}}r^{2}e^{-\frac{r^{2}}{\alpha^{2}t^{2}}}dr

\]

\[

\Rightarrow\frac{N\left(t\right)}{N\_{0}}=erf\left(\frac{\omega\_{0}}{\alpha\cdot\delta t}\right)-\frac{2\omega\_{0}e^{-\frac{\omega\_{0}^{2}}{\alpha^{2}\delta t^{2}}}}{\alpha\cdot\delta t\sqrt{\pi}}

\]

The fraction of the number of atoms in the MOT was measured after $\delta t$

without lasers divided by the number of atoms before closing the trap

(the results are shown in Figure (\ref{fig:Release-=000026-Recapture})).

We measured the MOT laser waist $\omega\_{0}=4.4\;mm$ and obtained $\alpha=0.01247\pm0.00258$.

Therefore, the temperature is $T=274\pm13\mu k$.

\begin{figure}

\subfloat[]{\includegraphics[width=7.5cm,height=6.5cm]{\string"R&R zoom\string".jpg}

} \subfloat[]{\includegraphics[scale=0.3]{\string"MOT Temprature\string".PNG}

}

\centering{}\caption{\label{fig:Release-=000026-Recapture}Release \& Recapture Measurement.

a) Example of sequence. We loaded the MOT and closed the lasers for

$\delta t$ and calculated the fraction of $\frac{N\_{\delta t}}{N\_{0}}$.

b) Fraction vs. $\delta t$. From the fit, the temperature was calcuated

and showed $T\approx247\mu k$. }

\end{figure}

\end{doublespace}

\begin{doublespace}

\subsection{$D\_{1}$ cooling }

\end{doublespace}

As explained in \ref{subsec:Gray-Molasses}, $D\_{1}$cooling can lower

the temperature to $T\approx15\mu K$ in $^{40}K$ without atom loss.

The following introduces our system and experimental results.

\begin{doublespace}

\subsubsection{Lasers setup}

\end{doublespace}

\begin{doublespace}

We used a DBR laser (\textit{photodigm PH770DBR080T8}) at $\lambda=770.1\;nm$

and a current and temperature controller (\emph{ Stanford Research

System LDC501}). We took a $\sim10mW$ towards an SAS system (\ref{subsec:Saturated-absorption-spectroscop})

. We locked the laser with the derivative signal by a PID loop on

the current of the laser.\\

\\

The $D\_{1}$ cooling transition is $\left|F=9/2\right\rangle \rightarrow\left|F'=7/2\right\rangle $.

However, we used a $^{39}K$ for locking the laser and the most obvious

line in locking signal is the crossover line $\left|F=co\left(1,2\right)\right\rangle \rightarrow\left|F'=2\right\rangle $.

\ref{fig:10 (b)} shows that to obtain the transition $\left|F=1\right\rangle \rightarrow\left|F'=2\right\rangle $

we need to add $230.85\:MHz$. Now we need to move to the energy level

of $^{40}K$. Therefore, the cooling resonance is the following:

\[

f\_{cooling}=f\_{lock}+704.85\:MHz

\]

\\

We manage this with a three Acousto-Optic-Modulator (AOM). The first

one is a double pass (\textit{ Gooch \& Housego -AOM AOMO 3200-124})

configuration with $230\:MHz$ on the $-1$ order. This configuration

gave the ability to change the frequency without changing the optic

system (Outgoing angle does not change when changing the frequency

of the AOM). The second AOM (\textit{ Gooch \& Housego -AOM AOMO 3200-124})

has a frequency of $200\:MHz$ (+1 order).\\

 The relation between the $f\_{lock}$ and $f\_{co(1,2)\rightarrow2}$

is:

\begin{eqnarray\*}

f\_{lock} & = & f\_{co(1,2)\rightarrow2}-\frac{f\_{AOM-SAS}}{2}-f\_{double-pass}

\end{eqnarray\*}

Therefore, the frequency shift is

\begin{eqnarray\*}

\Delta f & = & f\_{cooling(f=9/2\rightarrow f'=7/2)}-f\_{lock}\\

 & = & 704.85-60-230\times2\\

 & = & 202.55\:MHz

\end{eqnarray\*}

We added the third AOM (\textit{ Gooch \& Housego -AOM AOMO 3200-124})

at $+200MHz$ for the final frequency transition. Prior to the third

AOM, we added a homemade Tapered Amplifier (TA) to increase the laser

power. The beam after the TA diverges on an axis parallel to the

table. Therefore, we added a cylindrical lens with $f=75mm$. Afterwards, we added a telescope 4:1 to obtain a small beam for the third AOM.

We took the first positive order and made another telescope 1:2 to

match the beam mode to the fiber mode. \\

 For the repump laser, we used the cooling beam and added a sideband

by using home-made high frequency Electro-Optic-Modulator (EOM \ref{subsec:High-frequency-Electro-Optic-Mod}).

Then, the laser beam is injected to three optical fibers (the 3D MOT fibers)\\

The power beam is controlled by changing the RF AOM power

(with a voltage variable attenuator (\textit{Mini circuits ZX73-2500-s+})).

\begin{figure}

\begin{centering}

\includegraphics[angle=90,scale=0.6]{\string"d1 script\string".jpg}

\par\end{centering}

\caption{\label{fig:D1-laser-setup.}D1 laser setup. }

\end{figure}

\end{doublespace}

\begin{doublespace}

\subsubsection{High Frequency Electro-Optic-Modulator\label{subsec:High-frequency-Electro-Optic-Mod}}

\end{doublespace}

\begin{doublespace}

Cooling process requires two laser frequencies, one frequency for cooling and

one frequency for repumping (\ref{subsec:Sisyphus-Cooling.}). In $^{40}K$,

the $D\_{1}$ transition has a distance of $1.285\;GHz$. Therefore,

as in the MOT, we can take two different lasers locked by an offset

locking technique. However, in $D\_{1}$ cooling, the frequency shift

is the frequency shift between $\left|-9/2\right\rangle \rightarrow\left|-7/2\right\rangle $

in $^{2}S\_{1/2}$. In addition, in $D\_{1}$cooling, the magnetic field

is set to zero, and the state distances are not changed. Therefore,

we can use an Electro-Optic-Modulator (EOM) to add frequency side

band on the top of the cooling laser that are $\pm1.285Ghz$ apart

from the main laser frequency.\\

\\

EOMs are based on the linear Electro-Optic effect, which is the modification

of the refractive index of a nonlinear crystal by electric field

in proportion to the field strength. \\

The electric field at $\omega\_{0}$ enters the medium that operates

another electric field at $\omega\_{m}$. Thus, the equation of the field

is

\[

E(t)=E\_{0}\left(\sin\left(\omega\_{0}t+n\sin\left(\omega\_{m}t\right)\right)\right)

\]

\[

=E\_{0}\sum\_{n=0}^{\infty}J\_{n}(n)\sin\left(\left(\omega\_{0}+n\omega\_{m}\right)t\right)

\]

This new phase can be applied by sending the electric field through

a nonlinear crystal, resulting in a corresponding change in the refractive

index. To make a significant change in the crystal,

a high voltage needs to be produced with a frequency of $\omega\_{m}$ on the crystal.

There are electronics that can generate a high frequency voltage of

more than $1GHz$. Therefore, we needed to produce a resonant circuit

\cite{HighfrequencyEOM}. A circuit from copper foil was constructed

with thickness of 0.1 mm, and a loop with $3\;mm$ space was made for contact

with the crystal $\left(LiNbO\_{3}\right)$ (Figure \ref{fig:EOM cir}).

\begin{figure}

\subfloat[]{\includegraphics[scale=0.6]{EOM\_script\_27\_12\_2015.JPG}

} \subfloat[]{\includegraphics[scale=0.6]{hfEOM}

}

\centering{}\caption{\label{fig:EOM cir}a) High Frequency EOM prescription. The black

square with area of $A=w\*d$ is the crystal area cross section, and

the brown with radios r is the foil with a thickness of \emph{0.1mm}.

b) EOM picture where one loop is for antenna and anther is a pickup

coil for Q factor measurement. }

\end{figure}

The crystal could be described as an ideal capacitor. Therefore, $C=\epsilon wl/d$

, where $\epsilon$ is a dialectic constant at $\omega\_{m}$. Also

the accumulative inductance of the copper foil loop can be described

as an ideal cylinder current sheet ( because $2\pi r\gg d$ ) $L=\mu\_{0}\pi r^{2}/l$.

Therefore, the resonant frequency of this $CL$ circuit is given as

\[

f\_{0}=\frac{1}{2\pi}\left(\frac{c}{r}\right)\left(\frac{d}{\pi w\left(\epsilon\_{w}/\epsilon\_{0}\right)}\right)^{1/2}

\]

We used a crystal of dimensions $w=d$ , and $c$ is speed of light.

For our experiment, ($f\_{0}=1.285Ghz)$ $r\approx4.15\;mm$

(the value of $\epsilon\_{w}$ at this frequency is not known and

we assume that it is $\sim43$). \\

\\

In our lab, we used a $LiNbO\_{3}$ crystal with dimensions of $3\times3\times30\;mm$.

If the crystal would have been smaller than 3 by 3 mm, then the gap

would have been smaller, resulting in a larger electric field for

a given power. However, the laser beam must travel through the crystal,

and our laser beam is a $1.5\;mm$, Therefore, a crystal with dimensions

of $3\times3\;mm$ is well suited to our lab. \\

\\

In addition, the design for this EOM was constructed as follows. The holder of the crystal is

formed from Teflon to prevent unwanted changes to the resonator quality

due to inductance.\\

\\

 Copper foil with a thickness of $0.1\:mm$ was polished

to maximize the transmission of the foil. Then, the foil

on a drill was twisted with a diameter of $8.3\:mm$. Both sides of the copper

cylinder were bent so a surface of $3\:mm$ would fit the dimensions

of the crystal.\\

\\

A hole was made in the Teflon holder and threaded the RF antenna (end

loop). For good coupling, the antenna was located

as close to the copper foil cylinder as possible without

blocking the path of the optical crystal or touching the foil. The

antenna was connected to a Voltage Control Oscillator (\textit{Mini

Circuits ZX95-1410+}). \\

\\

Next, the quality of the resonator was measured. The $Q$ (quality)

factor describes how much energy is lost in the resonator, with a

large $Q$ meaning less energy lost. The $Q$ factor is defined as

\[

Q=\frac{f\_{0}}{\Delta f}

\]

where $\Delta f$ is the bandwidth (where the energy is reduced by

half the maximum value) and $f\_{0}$ is the resonance frequency.\\

\begin{figure}

\begin{centering}

\subfloat[]{.\includegraphics[scale=0.5]{\string"EOM HF Q factor\string".PNG}

} \subfloat[\label{fig:fabryperot}]{\includegraphics[scale=0.6]{D1\_fabty.JPG}}

\par\end{centering}

\caption{a) Measurement of Q factor $Q\approx150$ and $f\_{0}=1.285\;GHz$

. b) Measurement of EOM efficiency using Fabry Perot. The maximum

efficiency (at high RF power \textasciitilde{} 4W) of the EOM is $\frac{I\_{repump}}{I\_{cooling}}=0.19$.

The Fabry Perot scanning is $1.5\;GHz$, and the first-order peak distance

is $1500-216=1284\;MHz$ (this figure shows the sideband from

the next peak where the distance between them is $1.5\;GHz$ ). }

\end{figure}

The $Q$ factor was measured with an RF antenna and found that $Q\approx150$

and $f\_{0}=1.285GHz$. This gave us the possibility of adjusting the

device. The direction was made by a squeeze of the resonator, reducing

the radius and thus increasing the resonant frequency.

In addition, we studied the effect on the laser by measuring the laser

in a Fabry Perot. An RF power of $P=4\:W$ obtained $\frac{I\_{repump}}{I\_{cooling}}=7.5\%$

(Figure \ref{fig:fabryperot}), which should be sufficient for the $D\_{1}$cooling.

\end{doublespace}

\begin{doublespace}

\subsubsection{Measurement of the $D\_{1}$ Frequency Resonance}

\end{doublespace}

\begin{doublespace}

In the first measurement, we wanted to find the resonance frequency

of the cooling transition ($\left|F=9/2\right\rangle \rightarrow\left|F'=7/2\right\rangle $)

. For this measurement, a Photo Multiplier Tube (PMT) measured

the fluorescence of the atoms. We opened the PMT $3\:ms$ before opening

the $D\_{1}$laser (just cooling), as the PMT has an opening time of

$\sim2\:ms$. When we opened the cooling laser, the atoms are fluorescent

for $\sim100\mu s$. Therefore, the signal had an exponential decay.

We made a fit of $I=A\_{0}e^{-t/\tau}$ and took the $A\_{0}$ as the

intensity of the atoms fluorescence while scanning over a range of $f=25\:MHz$.

We cannot scan over more than $25\:MHz$, as we scan on the double-pas

AOM before the locking circuit, and any change in this AOM changes

the intensity on the locking signal and the laser would lock out.

We found that the cooling resonance is at $f\_{AOM-DP}=221.175\:MHz$

with a width of $10.02\:MHz$. We set the cooling frequency with blue

detuning at

\[

f\_{DP-AOM}=f\_{resonance}+3\Gamma=233.675\:MHz

\]

\begin{figure}

\begin{centering}

\includegraphics[scale=0.6]{D1\_cooling\_resonance\_freq.PNG}

\par\end{centering}

\caption{PMT signal Vs. DP-AOM frequency. The resonance is in $2\cdot f\_{AOM-DP}=443\:MHz$.

To cool with the blue detuning of $\sim3\Gamma$ on the

$D\_{1}$, $18\:MHz$ was added. Therefore, $2\cdot f\_{AOM-DP}=461\:MHz$.

The final parameter is set by the parameters of the atoms (temperature

and number of atoms), as shown in Figure \ref{fig:a)-cooling-tune}. }

\end{figure}

Next, we added the repump frequency to the cooling beam by using a

High Frequency EOM. This laser has two frequencies that are injected

to the three fiber of the 3D MOT (retro-reflection configuration). The

power at each axis is approximately $I=12I\_{sat}$ with $\frac{I\_{r}}{I\_{c}}\sim7.5\%$.

Before starting to reduce the temperature, the atoms must be compressed by adding a magnetic-trap for $2\;msec$ (which causes increased temperature).

\end{doublespace}

\begin{doublespace}

\subsubsection{Temperature and atoms number measurement by Time Of Flight (TOF )

technique}

\end{doublespace}

TOF measurements are performed by acquiring the absorption signal

of the probe laser beam through the falling and expanding atomic cloud.

There are several methods of measurement of temperature, R\&R \ref{subsec:Temperature-measurement-with}

, MOT fluorescence spectrum analysis\cite{MOTfluorescencespectrum},forced-oscillation\cite{forceOscillations}.

Another model that was suggested by Jerzy and Gawlik in \cite{brzozowski2002TOFmodel} shows

that the absorb signal from an atoms

\[

N\left(t\right)=\frac{P\_{0}}{2\pi\left(\sigma\_{I}^{2}+\sigma\_{t}^{2}\right)}\exp\left[-\left(\frac{g\left(t\_{0}^{2}-t^{2}\right)}{2\sqrt{2}\sqrt{\sigma\_{I}^{2}+\sigma\_{t}^{2}}}\right)^{2}\right]

\]

where $p\_{0}$ is the probe laser power, $t\_{0}$ is the arrival time

of atoms with no initial vertical velocity, $\sigma\_{I}$ are laser

beam waist along $x$ and $y$ axes, and $\sigma\_{t}=\sqrt{\sigma\_{0}^{2}+\sigma\_{v}^{2}t^{2}}$

is the Gaussian radius of the ballistic expanded cloud. The Gaussian

radius $\sigma\_{v}$ of the velocity distribution is associated with

the temperature $T$ of the atoms cloud by

\[

T=\frac{m}{k\_{B}}\sigma\_{v}^{2}

\]

After a loading time of $30\;sec$, we closed the coil current and

the $D\_{2}$ laser beam and opened the $D\_{1}$cooling for $t=4\:msec$.

We then closed the $D\_{1}$ laser, waited $18\;msec$, and then took

a TOF image. The parameters of the cooling and repump frequency were scanned,

and these parameters were optimized as described in Figure \ref{fig:a)-cooling-tune}.

At the end, the atoms parameters were $T=19\:\mu K$ and $N=2\times10^{8}$atoms

(where $f\_{AOM-DP}=461.3\:MHz$, $f\_{repump}=1287\;MHz$ and $D\_{1}$

duration $t=4\;msec$). The TOF image is shown in Figure \ref{fig:21}.

\begin{figure}[H]

\begin{centering}

\subfloat[]{\includegraphics[width=7.5cm,height=7cm]{\string"d1 cooling tune\string".jpg}

} \subfloat[]{\includegraphics[width=7.5cm,height=7cm]{\string"d1 repump tune\string".jpg}

}

\par\end{centering}

\begin{centering}

\subfloat[]{\includegraphics[scale=0.45]{\string"d1 duration\string".jpg}

} \subfloat[\label{fig:21}]{\begin{centering}

\includegraphics[scale=0.6]{Abs\_image\_30sec\_18msecTOF10msecD1cooling\_28uK2e8atoms\_14062016}

\par\end{centering}

}

\par\end{centering}

\caption{\label{fig:a)-cooling-tune}a) $D\_{1}$cooling tune vs temperature.

b) $D\_{1}$ repump tune vs temperature. c) $D\_{1}$ duration vs temperature

d) Absorb image of atoms after $D\_{1}$ cooling with Time Of Flight

$t=18\;msec$ }

\end{figure}

\begin{doublespace}

\subsection{Optical Trap}

\end{doublespace}

As shown above (\ref{eq:20}), in a microtrap, the potential and the

scattering rate depend on the beam $\omega\_{0}$

\[

U\_{dip}\propto\omega\_{0}^{-5}

\]

Therefore, we need precise measurement of the beam waist. In addition,

a laser with $\lambda=1064\:nm$ should be used to obtain

a long lifetime in the micro trap (\ref{tab:2}).

\begin{doublespace}

\subsubsection{Microtrap waist measurement}

\end{doublespace}

\begin{doublespace}

To know the optical trap's depth and size, we need to measure

the $\omega\_{0}$ of the beam. Each camera has a finite size of the

pixel that is greater than $7\,\mu m$; thus, we cannot use a camera

to measure the waist. We can use a knife edge measurement, but again,

we need a high resolution x-y-z stage ($<0.3\,\mu m$ for seven less

measurements at the waist). \\

We used two easy ways to measure the micro-trap waist. \\

A collimated laser beam with waist $\omega\_{1}=0.89\ mm$ and $\lambda=1064\:nm$

enters a 1:6 telescope. It then travels through an Aspheric lens with

$f=26\ mm$. The Numerical Aperture (NA) is given by

\[

NA=\frac{2\cdot6\cdot\omega\_{1}}{2f}=0.205

\]

\end{doublespace}

The NA of a Gaussian laser beam is then reduced to its minimum spot

size by

\[

NA=\frac{\lambda}{\pi\omega\_{0}}

\]

where $\lambda$ is the laser wavelength (in our trap, $\lambda=1064\:nm$)

and $\omega\_{0}$ is the laser beam waist at the focus. Therefore,

\[

\omega\_{0,theory}=\frac{\lambda}{\pi\cdot NA}=1.65\ \mu m

\]

\begin{doublespace}

\subsubsection{Measurement of a microtrap waist with an optical chopper }

\end{doublespace}

An optical chopper is a spinning wheel with holes at a constant frequency.

The holes are used as a knife for the knife edge measurement. A photodiode was placed after the chopper and measured power vs. time on a digital

scope. By knowing the frequency of the chopper and the distance between

the laser and the center of the chopper, we can calculate the

velocity of the knife. Therefore, we can translate the time to distance.

\begin{doublespace}

\subsubsection{Measurement of the microtrap waist with a piezoelectric actuator

and Michelson interferometer}

\end{doublespace}

In this measurement, a Piezoelectric actuator $\left(Thorlabs\,AE0203D08F\right)$

was inserted to a translation stage. In our labs, we only have an actuator that

can travel at $9.1\,\mu m$. The actuator receives a voltage of $0-150\;V$

from a ramp waveform. On the translation stage, a knife was set and

measured the power on the photodiode. We can assume that the actuator

travels linearly from $0\rightarrow9.1\;\mu m$, but we can calibrate

this with a Michelson interferometer (calculate the actuator traveling

distance). As described in Figure (\ref{fig:miclson int}), our

collimated laser beam $\lambda=1064\,nm$ and was split with a Non Polarizing

Beam Splitter (NPBS) to two mirrors. One mirror is moved with the

translation stage by the actuator, and the second mirror does not move.

The lasers from the two mirrors are combined on the NPBS and focused

on a photodiode. On the photodiode, we obtain a diffraction pattern

that is dependent on the difference between the optical paths \cite{fox1999reliable}.

\\

\[

\Delta L=\frac{\lambda m}{2}

\]

where $\Delta L$ is the distance that the mirror is moved, $m$ is

the number of maximums, and $\lambda$ is the wavelength of the laser.

As shown in Figure \ref{fig:measured-the-micro-trap}, we obtain $m=14.5$

in one waveform period; therefore,

\[

\Delta L=7.714\,\mu m

\]

Now, the distance in the knife edge measurement was measured as $\omega\_{0}=2.148\mu m$ . However, these measurements do not provide information regarding aberration or about $M^{2}$. To measure them,

there measurements of $\omega(z)$ are needed, but for this,

a long-travel Piezoelectric actuator is needed $\left(\Delta L>15\cdot\omega\_{0}\right)$.(\ref{fig:Calculation-of-the})

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\begin{center}

\begin{figure}[H]

\begin{centering}

\subfloat[\label{fig:miclson int}]{\includegraphics[scale=0.65]{\string"Michelson interferometer system\string".png}

}\textvisiblespace{}\subfloat[\label{fig:measured-the-micro-trap}]{\includegraphics[scale=0.7]{\string"Michelson Interferometer\string".jpg}

}

\par\end{centering}

\caption{Measuring the Microtrap Waist with a Piezoelectric Actuator and a

Michelson Interferometer. a) The system description. Collimated laser

beam split by NPBS and traveling to two mirror (mirror 1 is on the translation

state and mirror 2 is fixed). They reflected back and combined on

the NPBS and focused on a photodiode. b) Interferometer result. The figure shows that we obtain 14.5 maximum peaks, so the actuator travel is $7.714\,\mu m$, and that the travel path of the piezo actuator is not

linear (the frequency of the sin function is not the same).}

\end{figure}

\par\end{center}

\end{doublespace}

\begin{center}

\begin{figure}[H]

\includegraphics[scale=0.3]{\string"final waist (1)\string".jpg}

\begin{centering}

\caption{\label{fig:Calculation-of-the}Calculation of the beam waist with

Knife edge technique}

\par\end{centering}

\end{figure}

\par\end{center}

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\section{Summery and Future Plan}

\end{doublespace}

This study presents our new platform for quantum computation.

It is based on fermion statistics and the attributes of ultracold

atoms. Chapter \ref{sec:Introduction} introduces the fundamentals

of quantum computing and the features of ultracold atom. \\

\\

Chapter \ref{sec:New-platform-of} demonstrates the theory behind

quantum computation solutions for our system. In addition, the explained

one-qubit gates and two-qubit gates in ultracold fermion systems are presented.

Moreover, the chapter presents our indecision regarding the choice of system

from between the \textbf{Degenerate fermi gas system} (cooling to

low temperature and then loading to a micro trap) or the\textbf{ fast

approach System} (loading to an optical microtrap and then cooling

the atoms to ground state). \\

\\

Chapter \ref{sec:ultracold-atomsback} presents the relevant background

for ultracold atoms, and Chapter \ref{sec:The-experimental-machines}

describes our two systems that are in the middle of construction.

Additionally, it shows the MOT trapping and cooling stage and $D\_{1}$ cooling with

one laser. \\

\\

For future research, we need to perform a more theoretical study

on the system parameters, such as the velocity $d\left(t\right)$ of

one qubit without change the qubit state, defining $U$ and $t$

for a $\sqrt{SWAP}$ gate to obtain fidelity $\mathcal{F}=1$,

and more. \\

From an experimental perspective, we need to reach several

goals.

\begin{itemize}

\item Loading several atoms to a microtrap and developing the ability to

measure a single atom.

\item Reducing the number of atoms to one.

\item Construction of two tunable microtraps with the application of one

and two qubit gates.

\item Numerical calculation of the gates parameter (e.g., $U$, $t$, $d\left(t\right)$,

trap parameter,).

\end{itemize}

I hope that in a few years we will be able to provide answers to these

and other issues.

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\pagenumbering{Roman} % roman numbering for table of contents.

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\bibliographystyle{ieeetr}

\bibliography{yanaymaster}

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