# **Introduction**

The post-buckling behavior of an elastic fiber subjected to lateral constraints is of practical importance in a variety of fields that require access to enclosed spaces with an elastic fiber. They range from medical procedures (such as in vivo diagnosis) to engineering applications. In the field of oil drilling, the drilling accomplished by using a very long cylinder that serves to drill holes [1-4]. When the fiber is subjected to the load of the drill, it may buckled and touch the wall of the borehole. Examples of applications in the realm of medical procedures include inserting fibers for the purpose of medical imaging or cardiac catheterization. One of them is the treatment of artery disease patients in which a long catheter is inserted into the patient's blood vessel for the deploy stents [5-8]. When the leading end of the catheter encounters a narrow lesion, the catheter will bend too much and may touch the arterial wall. In invasive microsurgery, flexible catheters are manipulated in order to reach the organs that require treatment [9-10]. In these applications, there are interactions between the elastic fiber and the environment that is affected by the elastic fiber's behaviors. The simplest mechanical model for oil drilling is a thin winding fiber contained within a rigid cylinder with a circular cross-section. Research of this problem began as early as the 1950's and has accumulated many publications. Contrastingly, in applications involving an artery and human tissue as in the development of filopodia in living cells [11-14] it is not enough to assume that the cylinder is rigid. According to a review of articles about a fiber contained within an elastic cylinder, it appears that no relevant experimental research exists. This article aims to substitute a cylinder with an elastic wall for the cylinder with a rigid wall as in the experimental study [15]. As described in previous works regarding cases where the fiber is helix, it is possible to use an energy system to predict the relationship between the applied load and the size of the helix[16-19]. In inclined [20-25] and horizontal [26-31] cylinders, gravity pulls the fiber on the lower side of the cylinder. This stabilizes the fiber and allows it to withstand a significant amount of load. In curved cylinders [32-34] it was found that the load of buckling that causes the sinusoid shape and helix of the fiber at the time of buckling is usually much greater than those in straight cylinders. Using precise geometrical calculations, the theory of elasticity [35-40] was developed among other things to investigate the post-buckling behavior of the fiber moved along the cylinder wall. The end effects of the post-buckling behavior of a contained bent fiber that is subject to a load at the end are discussed [41]. While this work refers only to the final stage of deformation, which is a point-line-point connection.It has been reported in the literature[42-45] that before reaching a point-line-point deformation, the fiber passes through different deformation patterns, when all previous studies assumed a rigid cylinder wall. However, in the real world there is no such thing as a totally rigid cylinder.In certain applications, an elastic cylinder wall would be a better model, especially in biological and medical applications as previously noted. This article aims to abandon the assumption of a rigid cylinder as in previous studies [45] in favor of an elastic one. There are existing works in the literature [12, 46-50] that deal with the behavior of a fiber within an elastic wall but the subject has not yet examine the issue thoroughly empirically in three-dimensional and numerical investigation of the fiber's deformation within an elastic cylinder. The intention is to test the cylinder's sensitivity to failure after loads are applied to the wall by means of the fiber, to confirm the theoretical results in [46] empirically and numerically, and try to find an analytical model that will describe the problem.

# **Description of the problem**

In cases where the fiber is threaded into the cylinder and begins to have two-dimensional deformation up to its contact with the cylinder wall, it is assumed that the wall exerts a reactive force in the radial direction whose intensity is relative to the radial movement of the wall. We assume that the cylinder wall behaves similarly to the base of the spring as appears in the study [51], with the spring constant β. The spring model as it appears in the study [51] is very basic to a fiber on an elastic wall, but it simplifies our problem. The spring constant is not a value that is usually measured, but we assume that there is a good proportional relationship between the spring constant and Young's modulus of the material the elastic wall is made of and, as a close approximation, will be chosen as our spring constant. Similarly, to the possible scenario description [46], in the case of oil drilling, we assume that the outer and inner radii of the drill casing are 0.125 m and 0.04 m respectively. The length of the drill casing between the two stabilizers is 50 m [52]. Let us assume that the drill casing and the borehole walls are made of steel, with a Young's modulus of 200 GPa. According to these assumptions, β is approximately 1010. If the wall of the borehole is made of shale, with a Young's modulus of 20 GPa [53], then β is approximately 109. In these cases, it is likely that the elasticity of the cylinder wall is neglected. Now let us take the case of a stent deployment application, with one of the areas with the highest risk of the formation of a fatty layer in the blood vessels in heart disease patients being along the main left artery and the frontal left descending artery [54] with a total length of about 45 mm [55]. The Young's modulus of the artery is approximately 1.2 MPa [56-57]. For the fiber with which the stent is to be inserted, its radius is 0.18 mm and its Young's modulus is 200 GPa. With these rough estimates, the dimensionless spring constant β is on the order of 104. In this case, it will probably be necessary to take the elasticity of the arterial wall into account. In this research, we wish to carry out for the first time, a large number of experiments. In these experiments, thin fiber is subject to load, is exposed to the wall of an elastic cylinder, and continues to exert load on it, as in the case of the treatment of blood vessels and other tissues in the human body. Afterwards the results of these experiments are to be compared to numerical research of similar cases, in an attempt to find an analytical model that will describe the process of failure of the wall of the elastic cylinder, following the continued application of load by the fiber after its contact with the wall. With the help of the results of this research project, we will be able to better understand the characteristics of the failure and how it may be prevented.

# **Goal, Motivation and Achievement**

The goal of this study is to test the behavior of a fiber threaded into a flexible cylinder using axial force. The motivation for the study is to conduct a broad empirical, numerical investigation to find solutions for preventing damage/injury to a fiber in the wall of a flexible cylinder such as in the case of blood vessels, tissues, and similar engineering applications (deep drilling, etc.) Achievements resulting from this project would be: a comprehensive empirical investigation of the behavior of the fiber and the flexible cylinder, finite element simulations, comparisons with the literature, and adaptation to a dimensionless basic analytical model that may best help achieve the objective of this work. In the case of a fiber within a flexible cylinder, we expect to see two types of deformation of the external cylinder: Type 1 "Local" deformation, which causes changes in the shape of the cross-section in the area in which the fiber makes contact with the cylinder, and Type 2 "Global" deformation -- a global change in the shape of the cylinder, specifically: the axis of the cylinder that was straight becomes crooked. To date, no work has been published regarding a flexible cylinder, except for one [46] -- in which a limit case was studied, where only Type 1 deformation was involved. Specifically, the assumption there was that the cylinder was actually a two-dimensional deformation and not a three-dimensional one. In other words, when the fiber makes contact with the cylinder, the cylinder exerts reactive force in the area of contact that is proportional to the extent of "penetration" of the fiber beyond the surface of the cylinder. This is of course an unrealistic case, as the cylinder is not treated as a real elastic structure/body. That is, there is no effect/ coupling between the radial sliding only at point A and the sliding at point B, even if it is very close to it. However, this is a good limit case in the sense that it enables significant simplification of the analysis, focusing only on Type 1 (local) deformation. Another important point is that [46] carries out only numerical analysis (no finite elements, but rather the solution of a set of nonlinear differential equations). In addition, [46] has no element of experiment. In view of the great complexity of the general problem of a fiber within a flexible cylinder, we would also like to execute a "small step," one that would enable analysis of a problem that is a littler simpler than the most general case, but still allows for insights. Based on these insights, we would then be able to continue on to more general and complex cases. Therefore, for our study, we have selected a "reverse" limit case from that of [46]. We will examine the case in which only Type 2 deformation occurs, i.e. global deformation of the cylinder, while any local deformation is negligible. This is actually the case in which the circumferential/tangential stiffness of the cylinder is very large in relation to the lengthwise stiffness. This means that these cylinders do not allow a local change of shape/size of the cross-section, but do enable global deformations of the cylinder, where the cylinder changes from straight to curved. For the experiments, we use non-standard cylinders, which are actually commonly used for cladding/protection of another cylinder that is inserted into them. The cylinders to be used are made of polymeric material, but a thin rigid material fiber passes helically in their circumference. See Figure 1. In terms of simulations of finite elements -- to create a model of the cylinder, we use a composite material that is very rigid in the circumferential direction. Another benefit of taking on this problem (there are only global deformations), is that it is highly likely that a relatively simple mathematical model can be sketched, which may even allow analytical insights, at least regarding the first deformations. Specifically, the cylinder can be modeled as a beam. The fiber that is inside makes contact in its center and exerts force on it. This results in deformation in the cylinder like a beam that exerts force on it in the middle of the opening. It is possible to try to reach even the conditions of moving to a configuration of contact in a line, etc. Another practical advantage of performing experiments with the cylinders described above is the fact that they can be obtained without initial curvature--unlike other flexible cylinders that usually come with uniform initial curvature (rolled up) -- see Figure 2. Another important point, the numerical study in this case is not for the purpose of confirming the experiments. It is rather an inseparable part of the investigation: the process involves performing experiments. From these experiments, we can derive graphs of force versus shortening, and also information regarding the response of the cylinder. Contrary to experiments with a rigid cylinder, we cannot gather information regarding the behavior of the fiber inside. To obtain understanding of the behavior of the fiber, we use finite element simulations. In the first stage, we want to make sure that the response we receive indeed corresponds to the one we measured in the experiments with respect to the force-shortening graph and the response of the external cylinder. Once it seems that there is correspondence such that we can say that the simulations indeed model the behavior very well, we can complete the picture of the experiment. Using the simulations, we can represent the behavior of the fiber within the cylinder: where there is contact, what type of contact like point, linear, planar or three-dimensional deformation, etc. This research project is original in that there is no known publication/project that has dealt with the investigation of the combined behavior of a fiber within a flexible cylinder, except for [46], which only involved numerical research, based on assumptions that significantly simplified the subject, i.e. it involved only local deformation with no coupling. Our work is expected to be innovative in several ways: it is the first systematic empirical investigation, the first time there is treatment of global deformation of the cylinder, and the first time there is a combination of experiments, finite element simulations, and a simple analytical model, which enables the creation of a complete picture of the behavior of a fiber inside a flexible cylinder, the response of the flexible cylinder, and the interaction between them.