The Foundations of Rational Metaphysics

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Table of Contents

[Introduction 6](#_Toc61862917)

[An Informal Overview 13](#_Toc61862918)

[First Section: The Foundations of the Order of any World Whatsoever 22](#_Toc61862919)

[1. Objects: concepts and individua; capturing and predication 22](#_Toc61862920)

[1.1 The rule of basic existence 22](#_Toc61862921)

[1.2 Objects and concepts that capture them 23](#_Toc61862922)

[1.3 The rules of capturing 26](#_Toc61862923)

[1.4 Intermezzo: Some general second-order characteristics 33](#_Toc61862924)

[1.5 Object, concept, individuum 35](#_Toc61862925)

[1.6 Singular capturing 38](#_Toc61862926)

[1.7 Identity and nonidentity 39](#_Toc61862927)

[2. Topological foundations: boundary and closure 44](#_Toc61862928)

[2.1 Boundaries and what they bound 44](#_Toc61862929)

[2.2 Boundaries and identity 52](#_Toc61862930)

[3. Mereological foundations: part and whole 55](#_Toc61862931)

[3.1 Wholes and their parts 55](#_Toc61862932)

[3.2 Parthood and boundaries 62](#_Toc61862933)

[3.3 The world and its parts 65](#_Toc61862934)

[3.4 Complementation 68](#_Toc61862935)

[3.5 Free union and semi-free intersection 71](#_Toc61862936)

[4. Mereology of concepts 76](#_Toc61862937)

[4.1 On the modes of partition of concepts 76](#_Toc61862938)

[4.2 The ways concepts are partitioned 82](#_Toc61862939)

[4.3 The partition of concepts through the predicative structure 85](#_Toc61862940)

[4.4 Union and partition of concepts and their relation to propositional operators 86](#_Toc61862941)

[4.5 Identity and nonidentity in the perspective of the reduction rule 87](#_Toc61862942)

[4.6 Discrimination: The mereology of concepts and the determination of boundaries 89](#_Toc61862943)

[4.7 Bequeathing 91](#_Toc61862944)

[4.8 Bequeathing-to-parts 92](#_Toc61862945)

[4.9 Bequeathing-to-wholes 95](#_Toc61862946)

[4.10 Mereology of concepts and logic 96](#_Toc61862947)

[6. Between concepts and individua: Brentano's thesis, qualities and tropes 98](#_Toc61862948)

[6.1 Conceptual and propositional interpretations 98](#_Toc61862949)

[6.2 Qualities and their holding; Turning concepts to qualities (individua) and qualities to concepts 105](#_Toc61862950)

[6.3 Possessive and tropes 107](#_Toc61862951)

[6.4 Tropes and qualities: mereological aspects 112](#_Toc61862952)

[8. Manifold Theory 115](#_Toc61862953)

[8.1 Manifolds: wholes, collections and sets 115](#_Toc61862954)

[8.2 Bequeathing-to-items and collections, bequeathing-to-members and sets 120](#_Toc61862955)

[8.3 Foundations of manifold theory 121](#_Toc61862956)

[8.4 Extensions 130](#_Toc61862957)

[9. Number and quantification 131](#_Toc61862958)

[9.1 Numbered, number 131](#_Toc61862959)

[9.2 The essence of number 133](#_Toc61862960)

[9.3 Specific numbers 137](#_Toc61862961)

[9.4 Basic arithmetic operations 138](#_Toc61862962)

[9.5 Magnitude 140](#_Toc61862963)

[9.6 Comparative 146](#_Toc61862964)

[9.7 Quantification 154](#_Toc61862965)

[9.8 Laws and laws of logic 159](#_Toc61862966)

[9.9 Analyticity 161](#_Toc61862967)

[9.10 Modality 165](#_Toc61862968)

[10. A reminder: the basic signs of concept calculus 166](#_Toc61862969)

[11. The order of any world whatsoever, the primeval concept and the first partitioner 168](#_Toc61862970)

[11.1 The order of any world whatsoever 168](#_Toc61862971)

[11.2 The foundational concepts of any world whatsoever 170](#_Toc61862972)

[11.3 The general concept of the first metaphysic, or the Primeval Concept 173](#_Toc61862973)

[11.4 The First Partitioner 174](#_Toc61862974)

[Second Section: the Foundations of the Order of Our World 179](#_Toc61862975)

[12. Framework concepts: Forms of being, ontological entireties, pools, and ontological spheres 179](#_Toc61862976)

[12.1 Between the first and the second metaphysics 179](#_Toc61862977)

[12.2 Forms of being and ontological spheres 181](#_Toc61862978)

[12.3 Pools and ontological spheres 185](#_Toc61862979)

[12.4 The nature of pools of different forms of being 187](#_Toc61862980)

[12.5 Parallelism and sub-parallelism 191](#_Toc61862981)

[13. The sphere of thought: Foundational concepts 197](#_Toc61862982)

[13.1 The thinking subject 197](#_Toc61862983)

[13.2 The data-creating functions in the sphere of thought 198](#_Toc61862984)

[13.3 Introspection 213](#_Toc61862985)

[13.4 Decision 214](#_Toc61862986)

[13.5 Time and change 216](#_Toc61862987)

[13.6 Place 221](#_Toc61862988)

[13.7 Matter 222](#_Toc61862989)

[13.8 The different types of change 223](#_Toc61862990)

[13.9 Operation 226](#_Toc61862991)

[13.10 Crisp versus vague boundaries 230](#_Toc61862992)

[13.11 Useful and useless partition 231](#_Toc61862993)

[13.12 Causation 235](#_Toc61862994)

[13.13 Factor and action 239](#_Toc61862995)

[13.14 Body 245](#_Toc61862996)

[13.15 Tagging and continuous identity in time and change 248](#_Toc61862997)

[13.16 Retagging 255](#_Toc61862998)

[13.17 A subtotal: The foundational concepts of the sphere of thought 257](#_Toc61862999)

[14. The sphere of Language: Basic characteristics 259](#_Toc61863000)

[14.1 Language as a dependent form of being 259](#_Toc61863001)

[14.2 Language as a subsystem 260](#_Toc61863002)

[14.3 Language as pool 262](#_Toc61863003)

[14.4 The axiom of linguistic existence 264](#_Toc61863004)

[14.5 Between different languages: differences in the partitions of the world 266](#_Toc61863005)

[14.6 A subtotal: The foundational concepts of the sphere of language 269](#_Toc61863006)

[15. The sphere of reality: basic characteristics 270](#_Toc61863007)

[15.1 The forms of being not dependent on the ego: The noumenal and the real 270](#_Toc61863008)

[15.2 The double test of reality 274](#_Toc61863009)

[15.3 Space-time and the only pool of reality 281](#_Toc61863010)

[15.4 The realm of metaphysics and the realm of science 282](#_Toc61863011)

[15.5 Sub-total: The foundational concepts of the sphere of reality 284](#_Toc61863012)

[16. The sphere of the noumenon: its existence and some of its very basic characteristics 285](#_Toc61863013)

[16.1 The existence of the noumenal world 286](#_Toc61863014)

[16.2 On the "population" of the noumenal world 289](#_Toc61863015)

[16.3 Unity and plurality in the noumenal entirety 292](#_Toc61863016)

[16.4 The noumenal subject and its boundaries 295](#_Toc61863017)

[17. The order of our world and the Second Concept 299](#_Toc61863018)

[17.1 The foundational concepts of our world 299](#_Toc61863019)

[17.2 The general concept of the second metaphysic, or the Second Concept 303](#_Toc61863020)

[18. Overall summary 305](#_Toc61863021)

[Bibliography 308](#_Toc61863022)

## Introduction

Philosophy is Sisyphean work. While in most of the sciences there are theories that come to be accepted by the scientific community and serve as openings for further developments, in philosophy we cannot point to even one theory that enjoys unanimous consent. Furthermore, while in most of the sciences critical comments on a theory might bring forth its emendation and development to a more secure path, in philosophy critical comments most often bring forth the total collapse of the theory. Then, just as in the original Sisyphean myth, we return to the starting point.

 This state of affairs is probably one of the reasons for the path taken by philosophy in recent generations, a path that has discarded the attempt to present an overall picture of the world and instead is focused on articulate discussions that tend to pinpoint certain topics. However, by taking such a path, philosophy has turned her back on her purpose (and if the humanities of today are sunk in an unprecedented crisis, much of the responsibility for it must be ascribed to philosophy). If we assume that the world in which we live is one of ordered plurality, and if we believe that we can understand things rationally, we will never have the luxury of exempting ourselves from the task of understanding the foundations of the order of the world. This task is that of philosophy, and especially of one of its major branches – metaphysics.

 Rational metaphysics is in fact a semblance of two layers, which we will call simply first metaphysic and second metaphysic. The first metaphysic deals with the principles of *any* world qua unity of many beings (I do not say "any possible world" since this term implies modal logic, which I will employ very sparingly); the second metaphysic deals with the principles of *our* world.

 Put differently, the subject of metaphysics is the partition of the world. The world is one, but it has many parts. Those parts are the results of partitioning, which can be done in endless possible ways. The first metaphysic deals with the conditions of any partition whatsoever, while the second metaphysic deals with the partition of the world as conceived in our minds. It also deals with the question of the existence of a world outside our minds and its relation to the world within them, in terms of the principles of partition.

 The first section of this book will be dedicated to the first metaphysic. The first metaphysic needs the assistance of logic. This is not because logic serves as a foundation for metaphysics, but rather vice versa: All logic is built on a metaphysical infrastructure, whether explicit or implicit. Metaphysics and logic wish to describe the same thing: being *qua* being. Aristotle, who coined the term, thought that the concepts of "being" and "one" do not qualify for being a genus, for they are said of all objects and therefore do not say anything about the essence of any object (Aristotle, Metaphysics, 998b23, 1059b31). It seems that he saw those concepts as belonging to logic, the realm of "empty" forms, rather than metaphysics. Yet, these empty concepts are those that constitute the foundations for any "more essential" metaphysic. Therefore the discovery of these concepts should be accepted as the goal of the first metaphysic (note that a similar line was taken by Leibniz, Wolff, Kant and Hegel, each in his own way). What logic tries to attain via a technical tool, metaphysics tries to attain via systematic contemplation.

A good logic is one that is founded on a good metaphysic, and a bad logic is one that is founded on a bad metaphysic. When logic doesn’t "work," it is therefore necessary to examine its metaphysical foundations, which will equip us with the tools needed for its recovery. At the same time it will also bring us to refine our contemplation of the foundations of metaphysics. Indeed, even though metaphysics underlies logic, the progress advances from top to bottom: The problems "above," at the level of logic, are those that awaken the search of the deeper problems "below," at the level of metaphysics. In other words, the study of the foundations of logic is in fact the study of the foundations of the world, i.e. metaphysics. The endeavour to build a perfect logic is thus the endeavour to build the foundations of rational metaphysics.

 In view of the above, the starting point of the first section will be the problems of logic. The logic we employ most of the time is the predicate logic of Whitehead and Russell (partly preceded by Frege), together with set theory. Both predicate logic and set theory are doubtlessly brilliant works created by the best of human minds, but they do suffer from some undeniable deficiencies (for some of them, see Smith 2005). These deficiencies are not fallacies (Russell's paradox and similar challenges are indeed resolvable), but could be classified as "technical" problems. They actually relate to this system’s inability to formally express some of the simplest reasonable inferences that we all know how to make in our everyday lives. Thus, for example, in predicate calculus we cannot formalize the simple inference: "Mary is eating an apple, therefore Mary is eating." The inference "Mary is eating an apple, and an apple is a fruit, therefore Mary is eating a fruit" can be formalized only by an inelegant series of sentences, and even then will not be fully accurate. Instead of saying "an apple is a fruit" we will have to say "all apples are fruit," which is not quite the same: The latter relates to the extension of the concept, the former to its intension. In other words, “an apple is a fruit” tells us something about the concept of apple and not just about the overlap between the set of apples and the set of fruit (if all dinosaurs are dead, that doesn't mean that being dead is a part of the concept of dinosaur). For this purpose we need to develop a calculus that will also formalize internal relations within concepts, in what we will call (following Zalta 2020, 13.1.5, p. 588, and others) the *mereology of concepts,* which will be developed in the first section of this book. Similarly, set theory too fails to correspond with some of our everyday reasonable inferences: if all the tourists of a certain group get soaked in the rain, we will say that the group was soaked in the rain. In set theory, on the other hand, we will never be able to say that about the set of all those tourists, for a set is an abstract entity. For this purpose we will have to develop a new theory, which we will call *collection theory*, based on the collection-item relation. But even that will not be enough; there is something in common between the relations set-member, collection-item and whole-part, leading to the development of a more complex, overarching theory, which will be named *manifold theory*, which will also be expounded in the first section of the book. And no less important: we must make sure that all of these will be incorporated into a single logical system, that of Concept Calculus, which will also be done in the first section.

 To be sure, the problems enumerated above are only some examples; in fact, there are many more. To a great extent, we have come to terms with these problems, either because we have become used to them or because logic has accomplished incredible achievements despite them. Logicians did not confront those problems; at most, they offered some additions and emendations that turned modern logic into a patchwork of calculi. Philosophy, however, ought not succumb to this compliance. It has had to descend to the roots of those problems and re-contemplate the foundations of logic. Out of this contemplation, philosophers should have come up not only with additions and emendations, but rather with new foundations, better based and more sophisticated. They have declined to do so, among other reasons because they have focused on the technicalities of logic rather than its metaphysical depths. A part of the charge should be ascribed to Russell himself as well as some of his followers, who negated metaphysics altogether and saw logic as a means to freeing themselves from it. Yet, the more philosophers try to repress metaphysics, the more the allegedly technical problems, insofar as they are not resolved on the technical level, will come back and necessitate a return to that discipline, manifesting the necessity of building a first metaphysic through the study of the foundations of logic. As we will see, these foundations will be reduced to seven foundational concepts that are required for the order of any world whatsoever, and those concepts will be united into one. That single concept, the Primeval Concept, and the derivative concept of "concept," which enables the plurality of any world whatsoever, are the foundations of the rational first metaphysic. That, as stated above, is the purpose of the first section.

 The second section will be dedicated to the second metaphysic. Here we come closer to the enterprise of Aristotle, who sought the "higher kinds" – essences that are not said of any object. The second metaphysic is built on the analysis of the foundational concepts by which we perceive *our* world, the same world against which we examine our thoughts to judge them as true or false. The main goal of this endeavour is the aspiration to create a set of foundational concepts which any rational knowledge of nature, and in particular scientific knowledge, will have to take as its starting point. These concepts will eventually be reduced to one, which we will call theSecond Concept. These concepts will not be utilized in the building of a new calculus, but rather will be placed within the concept calculus of the first section. Indeed, the second metaphysic underlies the sciences, and therefore its exposition does not come from the examination of logical foundations, but neither does it emanate from the sciences. It will be derived from the articulation of the cognitive functions that work on the subject in the epistemic process and the study of their relation to reality. This is justifiable by the fact that our concept of reality, in particular that employed by the sciences, is not the philosophical concept of the "thing in itself" but a different concept, which is not detached from the world perceived by our cognitive functions. The precise character of this concept, as well as the "thing in itself" or "the noumenal", and its place among the various forms of being, will be discussed in the second section.

 The starting point for the creation of this set of concepts will thus be the Source Theory, which I expounded in *Thoughts and Ways of Thinking* (and which I will slightly amend in this book). However, soon enough we will sail away far beyond it. Through this theory we will understand our basic cognitive functions, which do not only transmit data of individual objects but also their general foundational concepts. These are not only the sense qualities of colour, sound, taste, smell and touch, but also the foundational concepts of the first metaphysic and other concepts. As we shall see, these functions are not only epistemological sources, but also functions that construct the ontological entirety of reality. Indeed, we will not be satisfied with a phenomenological description of the subjective human sphere, but will discuss four different forms of being, two subjective and two objective: the subjective (or "interior") ones being thought and language, the objective ("exterior") ones being reality and the noumenon (or the "in itself"). We will discuss the four ontological entireties in which these forms of being exist, as well as the relations between them. Through this we will discuss questions of their own importance. Thus, for instance, we will prove the existence of an exterior world and the normative character of the boundaries of the self. These are the tasks of the second section.

 The combination of the two sections will hopefully render a *Lingua Characteristica*, and so will bring us close to the fulfilment of Leibniz's age-old vision. It will not always be done in tight accordance with his substantive philosophical opinions, but in many cases it will be inspired by them (for a similar aspiration see Smith 1992). The theory of predication and the mereology of concepts are probably the most manifest ones.

The first section will be argued very rigorously, and I can hardly imagine it will be contested; the second section will also be argued in a rigorous method, but I assume it will be more open to criticism. Some will probably claim that it should have been written with greater involvement of current scientific theories. In that section, too, I have tried to state my arguments in the most abstract way, so that they will serve as foundations for any imaginable theory, even in the vicissitudes of contemporary science. If I have not succeeded in that, let others come to correct my work and complete it.

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This book owes much to many. However, due to the style I use I will not reference all who have preceded me. I therefore state here that from the great philosophers of the past my first metaphysic owes much to Leibniz, Brentano, Meinong, Frege and Couturat, and from those of the present, to Peter Simons, Barry Smith, Achile Varzi and Roderick Chisholm. I found a special propinquity to my work in the profound Uwe Maixner's *Axiomatic Formal Ontology*. Though there are differences between us on several issues, I learned much from it. The work of Edward Zalta, which I only discovered after the first section had already been completed, was nevertheless of some use as well. Studies by Giancarlo Guizzardi, Terence Parsons, Dale Jacquette and Maria E. Reicher helped me to refine some of the arguments.

 The concept calculus will employ, among other things, amended elements, to be developed through a discussion of predicate calculus, mereology, topology and set theory (which will be integrated into the broader Manifold Theory). To date, most of these have been discussed separately, and only seldom have they been seen as interrelated. Now we will hopefully weave them into a single, unified system. In each of these fields we will therefore need to borrow basic terms from the others. Since our work is about a system of foundational concepts, all of its basic terms are "coming together" (this is a literal translation of the Talmudic term *bain keehad,* but doesn't capture its full meaning; it approximately means "validating each other simultaneously," or, very roughly, "interdependent"). This is not circularity, but a consequence to be expected of the foundational character of those terms. We will encounter this phenomenon more than once in our forthcoming discussions, especially in the first section of this book. For this reason, from time to time I will use terms that have not been defined and will be defined only later on. There is no other way to conduct such a discussion, since in each minor discussion one finds terms from other minor discussions that are required before the start of that discussion, and one cannot put all of them first. I am not one of those tedious authors who ask their readers to read their books twice, one time for the details and then again for the whole picture (I will be grateful for even once), but I do ask the reader to be patient and use his or her memory when encountering the phenomenon of terms "coming together." When the end of each section is reached, we will all see how all of those terms are intertwined into a single whole.

Some of the arguments in this book are stated without formal proof, though in most of the cases they are easy to prove. Moreover, some of the readers might complain that several new ideas presented in this book do not receive proper articulation. Yet the developing of those ideas would not only change the style of the book, but will also undermine its goals. Therefore I sometimes shoot ideas out of the air into the book, and leave them for others to develop, presumably better than I would do.

 This book is replete with new directions, and I definitely know that readers often have difficulties in digesting many novel directions in one bite. In some respects it would be better to publish each of them separately and then compile them into a unified work. However, my essential goal in developing this theory is not to make innovations here and there, but to draw a comprehensive and coherent picture of the foundations of metaphysics. For this reason I could not release any mini-theory, not even one that could be read independently, until I could assemble all of them into a comprehensive and coherent theory. Consequently, all the innovations are now presented in a concentrated manner. This character of my argumentation gives another justification to my request from the reader to take a broader look at the entire section at the end of his or her reading. There is no use throwing away my entire argument because of a contestable statement or from questioning each premise I allegedly took for granted. The force of some of the arguments is not attained by being separately fully defendable against any possible criticism, but by joining together into a large, whole and comprehensive picture of the world. That picture can be seen only at the end of each section.

 Even though the book undermines predicate logic to a certain degree, throughout the first section, for the sake of convenience, I will use it. It is necessary to do so as long as the new concept calculus is not completely expounded. In the second section I will already be able to use concept calculus alone.

 The book is not written in the style that is accepted in today’s philosophy. I do not survey all the opinions of my predecessors, do not present all the possible alternatives, and do not endeavour to refute any differing view. Had I done so, I might have written a more complete work, but one that is less loyal to my goals and way of thinking. The force of the theory presented in this book is anyway not confined to one particular argument or another, but in the overall picture it draws. Furthermore, articulate discussions on each topic would have prolonged the completion of the entire book, on which I had already toiled for many years, if not make me lay it aside altogether. I therefore opted for the less complete extant work over the non-extant complete work. To confess the truth, I'm not even sure it's less complete this way. The path of contemporary philosophy often hinders more than helps the advancement of the discussions. I assume that most of the readers will not be troubled by this different path of mine, and to the rest – I apologize.

## An Informal Overview

A yellow box lies on the sidewalk. We can observe it because it is discriminated from its environs. What makes this discrimination is a collection of properties and relations, i.e. **concepts**, that apply to it and not to its environs. (Philosophers of today tend to distinguish between concepts and the properties and relations that constitute their contents, but I see no use in such subtle distinctions, that contradict Ockham's razor and contribute nothing to the subject matter, and therefore I will not use them here). For example colour, that also determines the shape of the box. Here we are talking about a yellow box, so the line of distinction between the object and its environs is the line in which the yellow block is ended, and this line forms the shape of a cube. The line is the **boundary** of the box. Here we are talking about a boundary in the simple, physical sense, but in fact any property of the object creates a boundary between it and the rest of the world.

 A **concept** is an entity that might (potentially) apply to many objects, or, in the terminology of this book, might capture many objects. Concepts are expressed, in logic as well as in language, by predicates (and at times I will use the two terms interchangeably). An object is an entity to which a concept or concepts apply. Therefore, a concept is also an object. If we wish to talk specifically about an object that is not a concept, we will call it an **individuum**.

 As I said, the concepts determine the boundaries of the box. This action is called **discrimination**. It should be said that the concept makes the object discriminated, but we will say, for short and somewhat ungrammatically, that the concept discriminates the objects. The boundaries give the box its identity and, in this respect – its existence. Something might have existed there without the discriminating properties, but then it was not *this* box, for it lacked properties that it now has. These properties are determined by its boundaries, and they are those that create the discrimination. Since that is the case, the boundaries are logically prior to the box, and the concepts are logically prior to the boundaries.

 We can describe metaphorically how things work: The idea of the concept "yellow" exists in our minds, and this idea can be said to "scan" the world. Whenever it "captures" ideas of yellow surfaces it determines their boundaries according to what it has captured. This way the idea of the box was captured as well. Indeed, the applying of a concept to an object will hereinafter be called **capturing**.

In this manner, yellow objects enter our minds as having a certain shape, and consequently as having a certain identity. However, they might change. How will we know that it is one and the same object that is changing and not a new object, time and again? If we want to follow the development of an object through time, we have to **tag** it, i.e. to fix its identity based on the concept by which it was captured in our minds, and see how the lines of its boundaries change continuously from the boundaries that were tagged.

 Thus far I have described the issue in the subjective context of mind, and that is why I mentioned the **idea** of the concept and the **idea** of the object captured by it, but what about the objective world? In fact, the same things apply, *mutatis mutandis*, to any world whatsoever ("world" being ordered plurality). In the above example I referred to a three-dimensional item, whose boundaries are determined by a concept of colour (yellow), which gives it its shape (cube), that is also perceived by sight. It is not at all necessary, however, that the concept of boundary be restricted to such objects alone. Thus, for example, if we take a world in which all that exists is one atom hovering in the void, even that atom will have to be discriminated from its environs by something. This discrimination will be done by a boundary, and the boundary will be determined by a certain property that this atom has and its environs does not have (note this point: its environs is also an object!), i.e. by a concept. And the same is true, *mutatis mutandis*, even for non-spatial objects.

 In other words: All that was said above about the idea of a concept capturing the idea of an object is also true for the objective level. Thus, the discrimination between two objects in reality is made by an objective boundary, and that boundary is determined because an objective concept (a "Fregean" one, as contrasted to an idea of a concept) captures objects (real ones, as contrasted to ideas of objects). In any world whatsoever, be its form of being as it be, the concepts precede – logically as well as ontologically – the individua, and discriminate them. By this discrimination they also give them their existence qua what they are. By this we actually accept the realistic thesis (with regard to concepts) in ontology, the one often named "Platonist."

 Concepts must go through a process of demystification. Leibniz started it, Frege continued, and now it is time to complete the move. A concept is not a mysterious entity. Every object has boundaries that determine its identity. These boundaries are determined by concepts. In short, concepts are boundary-determining entities.

 Each object is captured by many concepts, and each one of these determines a boundary of a different sort: colour-boundaries, sound-boundaries, as well as boundaries of more complex or abstract character. Without any boundaries at all, the object is not what it is; without the concepts that capture it, and I mean all of those concepts, the object is different from what it is.

 Between the concepts there are relations that we will have to examine profoundly: Some concepts are parts of others, some have common boundaries with others, and some are disjoint (or alien) to others, i.e. do not share any common part with them. All these relations can be described by the metaphor of the grand "map of concepts," on which all the concepts are displayed in their proper relations. (Later on we will see that it is not just a metaphor and the map is none other than "the Big Concept," i.e. the concept of being, as in Aristotle, Metaphysics, 1059b31, or "something," of which all the concepts are parts. At this point, however, this might seem just empty words or mysticism, so we will suffice, for the moment with the metaphor of the map.)

 When I say that many concepts capture every object, I mean that many concepts overlap each other, and the object is a singular "intersection" of this multiple capture. This implies that the entirety of the object’s boundaries are determined by a single "concentrated" property, which is a concept that constitutes an intersection of other general properties (while those, in their turn, are also concepts).

 Some concepts are **inverse** to one another: An inverse concept denotes the relation existing between two objects when they "change places" with regard to it. Thus, for example, the concept of part is the inverse of the concept of whole, for if the hand is a part of the body the body is the whole of the hand. But inverse concepts are not necessarily opposites: parent is an inverse concept of child, because if John is the parent of Mary, Mary is the child of John. What is clear is that inverse concepts are inferred from one another.

 Objects of all types might be unified or partitioned. Restrictions might be imposed on both partition and unification, but every such restriction will be a result of a decision, i.e., a determination on the normative level. From a metaphysical point of view, we can take any two (or more) objects and see them as one, or any one object, and partition it however we wish. In fact, all those unifications and partitions already exist, and the question is only whether we treat them as such or not. This, of course, depends on our decision. All of this is said not only about individua, but also about concepts.

 The concept of decision will be discussed in detail below, but it should be said now that a decision is a choice of the subject – whether conscious or unconscious, voluntary or involuntary – to treat an object in a certain way that is not logically necessitated, i.e., that we could imagine ourselves treating it otherwise without being considered as logically mistaken. Thus, when our sight differentiates certain hues and ignores subtler ones, we make a decision, and when we start following an object after we have tagged it in this manner and not another, we make a decision. The term "decision" as used in this book overlaps to a great extent the currently accepted term "normative determination."

 The decision about the determination of boundaries is actually a decision concerning the **partition of the world**. In our discussion below we will see that we have a variety of options regarding the modes of that partition, i.e. the concepts we should use in order to determine the "place" of the boundaries. This way, for example, we can employ a larger concept or a smaller one, a concept built through unification of other concepts or by an intersection of concepts, and so on. However, in spite of this apparent unrestricted freedom, we are not really free. The world is organized in a general order – and we will prove that this order is a logical (!) necessity – and if we decide to employ concepts that do capture many objects, we will fail in our endeavour to understand the world properly, i.e. in a way that enables us to conduct ourselves efficiently therein. However, if we decide to employ only concepts that capture one or two objects, neither will that lead us to efficient conduct in our world. We therefore tend to do our partitions of the world in a **useful** manner. There is an evolutionary postulate that we decide on the more useful concepts, since otherwise we will not know the world and will not survive. But here we must remember: surviving necessity is not a logical necessity, and even in the state of affairs described here it is all about a decision, even if unconscious and involuntary. Remembering that a decision is standing behind all of this will help us cope with a few philosophical problems.

 Above the ordinary concepts there are a few concepts that capture all the objects. These are the **foundational concepts** of the first metaphysic. (I could use the traditional term "categories" but it might be misleading in some respects). It is not by chance that these concepts apply to all objects; they capture all the objects because there can be no world of ordered plurality without them. After carrying out all the necessary logical moves, it will be possible to reduce the list of foundational concepts required for any world whatsoever to seven. I will just enumerate them here, without explaining their meanings: (1) object; (2) identical; (3) nonidentical; (4) manifold; (5) bounded; (6) sized; (7) numbered.

 Since all the foundational concepts capture all the objects, they do not contradict each other, and therefore we can build out of them a supreme intersection-concept. That is **the Primeval Concept**.

 But at this point we are facing a problem: I have talked about two types of objects, concepts and individua, and asserted that these two types of entities are objects of concepts. I emphasized that concepts, too, are objects. In order to discriminate a certain concept from other objects, it must have boundaries. However, in order to have boundaries there must be concepts that capture it. But those concepts are again objects, and therefore have boundaries too, so they need concepts to capture them. And so on, and so forth. Until where?

 As we will see, at the head of the chain we have to assume that there is a concept that does not need other concepts in order to be captured; in other words, a self-capturing concept. This requirement is met by all the foundational concepts and, above all, the Primeval Concept. However, in this way the select concept might capture only itself, actually leaving us in a world of one entity without discriminations. Such a world is not a world of ordered plurality, and therefore cannot be the object of a rational metaphysic. It is a Parmenidean world that might only be an object for the experiences of mystics, if at all. We must therefore find among the foundational concepts one that is not only self-capturing but also exclusive, i.e. one that discriminates the object (in this case: itself) from its environs, thus creating a distinction between two objects, or plurality. At first sight the requirement of exclusivity seems contradictory to the requirement presented above that the concept be a foundational concept, for the latter is all-capturing. Yet, out of a concept one may infer its inverse, so that the inverse is, so to speak, contained in it. The inverse of the concept of "object" – which is one of the foundational concepts – is the concept of "concept." The concept of concept is actually the only foundational concept that meets both the requirement of reflexivity and that of exclusivity. Since this is the concept that creates the most foundational partition of the world as plurality, it should be called **the First Partitioner**.

So far I have mentioned two **forms of being**: thought and reality. Every form of being exists in a **pool**, or pools. Thus, for example, thought exists in subjects, while reality exists in space-time. The sum of all the pools of the same form of being will be called an **ontological sphere**. However, we have to distinguish between two senses of the word "reality": On the one hand there is the sphere of reality about which we all talk in our everyday lives, which is not totally detached from the sphere of thought. It is the sphere that is common to other subjects and myself, and the one we wish to attain when we say we seek truth. In contrast there is the reality of the philosophers, who talk about a sphere totally "external" and unreachable by the subject. Some of them, the idealists, in their various factions, even negated its existence. In order to distinguish between these two senses of reality, I will give them different names. The sphere of reality in the ordinary sense will continue to be called "reality," while the other, unattainable "external" sphere will be named, following Kant, the "noumenon" or "thing in itself," or the "in-itself" for short.

 Reality is indefinable, but for the sake of its identification (and characterization) we should use two accumulative tests, long discussed in philosophy: The coherence test and the inter-subjective test. The former was developed mainly by recent philosophers but already appears in Locke (1999, book 4 Chap.1); the latter was developed mainly by Husserl and his fellow phenomenologists (Husserl 1970, vol.1, §39, §48; Dupre 1964, p.346). According to the coherence test, the sphere of reality is the one that consists of the collection of ideas that appear in my mind in a coherent, ordered, long-lasting enduring and stable manner. According to the inter-subjective test, the sphere of reality is the one that consists of the collection of ideas that considerably resembles those of other subjects who share the same truth system with me. However, this resemblance does not have to be actually given, but is rather the common arena that all the activators of the truth system aspire to join. When both of these tests are satisfied with regard to a certain object, that object will be considered as real, and the collection of all the objects in a given truth system is the sphere of reality for that system.

 And what about the world in-itself? Does it exist at all? I answer this in the positive. The proof thereof is in fact a proof against solipsism. Once we prove that something exists that is not the "I," or the ego, nor an idea within it, we prove *in adjecto* that it is "external" to the ego. The main proof for this (having put aside some proofs that will be considered unprofessional) is the proof from the boundaries of the ego. If the ego has boundaries, these boundaries discriminate it from its environs, and hence it is clear that such (external) environs exist. If the ego does not have boundaries, then it becomes identical with the Parmenidean world, whose existence we negated from the outset. Furthermore, it contrasts with our immediate and most basic experience of ourselves, provided that we are not mystics. Does the sphere of the in-itself have plurality and discriminations in it? That – we cannot know. Is there any relation between it and the sphere of reality? Yes, such a relation exists by virtue of a logical (!) necessity, but we cannot characterize it.

 Between the sphere of thought and the sphere of reality there is another ontological sphere, that of language. Language expresses itself by auditory or graphic signs (and if we add the Braille writing, tangible ones as well) that exist in the sphere of reality, but transmit data to, and create contents within, subjects in the sphere of thought. I will argue that this double-faceted character of language does not have to mislead us, and this sphere is primarily subjective, but nevertheless there are some important differences between it and the "pure" sphere of thought, that justify its enumeration as a separate ontological sphere.

 We may therefore summarize that there are at least four forms of being, each having its own ontological sphere (provided that we will succeed in proving the existence of the in-itself): **thought, language, reality** and the **in-itself**.

 Our world also has foundational concepts. The Primeval Concept, with the foundational concepts that constitute it and the First Partitioner inferred from one of them, may be useful for the understanding of any world whatsoever, but to understand *our* world we have to build another level of foundational concepts upon it. These will be the foundational concepts of the second metaphysic. When I say "our world" I mean primarily the sphere of reality, but not only that. Thought and language are also worlds in which we all live. The sphere of reality is also closely linked with both of them, in particular the sphere of thought. After all, the foundational concepts of the first metaphysic do not include colours or sounds or space or time. The foundational concepts of our world will not express these concepts in their raw form, but will give them a generalized form. We know our world through **cognitive functions** (senses, intellect, and others), and each of them transmits its data by function-concepts (senses – by colours, sounds etc.; intellect –the rules of logic, etc.). Admittedly, scientific research teaches us that reality is constructed by elements that are not captured by these concepts (quarks, elementary particles), but any knowledge of reality, including these elements, will always start from the data received in these function-concepts. The general concept of function-concept is therefore a foundational concept of our world.

 These concepts work in our minds as a result of a decision of the subject. Once again, this term does not denote a voluntary or even conscious act, but only the fact that the subject could use other concepts and partition the world in a different way. This implies that all the boundaries of objects in our minds are all determined by decisions. Even the boundaries of the subject itself are determined by decision. How is it possible that a subject not yet existing as such performs an act of decision? The answer is that it does exist even before it had its boundaries, engulfed in a larger whole, but is not yet discriminated there in the identity it will acquire after the decision. Indeed, sometimes we ascribe an action to a larger object only because we don't know which of its parts performed it. And only after we have attained a better discrimination of its parts can we point at the exact identity of the performer.

 The list of the foundational concepts of the second metaphysic contains six items, which will also be presented at this stage without further explanation: (1) having a form being; (2,3) content, pool or ontological sphere (the latter is actually a collection of pools and therefore does not constitute an independent foundational concept); (4) function-concept; (5,6) decided or inherent.

The general concept of the second metaphysic, which concentrates all of these together, will be named the **Second Concept**. It is built as a union of the items within each article and an intersection of the articles themselves. Of course, this concept is assembled on the Primeval Concept, for the latter constitutes a general foundational concept of any world whatsoever, including our world.

These are the foundational concepts of the sphere of reality. They also all serve as foundational concepts of the subjective spheres – thought and language – but there they will be joined by another foundational concept: "transmits that…" This concept denotes the delivery of data from source to subject. Due to its purely epistemological character, it does not qualify for entering the list of foundational concepts of reality, and thus remains pertinent only to the above-mentioned spheres.

Once we establish these two general foundational concepts, of both the first and the second metaphysic, we will be able to state with satisfaction that we have displayed the foundations of rational metaphysics. But before we reach that stage we will have to go through a long journey with many elaborate discussions, sometimes even entire sub-theories, about each of these foundational concepts and issues related to them. So far I have sketched them in an introductory, somewhat inaccurate, manner but now they should be established in the fullest and most rigorous way, with the help of logic. So let us not waste time, and get down to work.

## First Section: The Foundations of the Order of any World Whatsoever

## Objects: concepts and individua; capturing and predication

### The rule of basic existence

The starting point of this book will be called **the rule of basic existence**. It is actually a combination of three premises. In a philosophical phrasing it will go as follows:

P1) Something exists.

P2) This something is a plurality.

P3) This plurality is united by being subject to common predicate(s).

In the language of predicate logic we may formalize it as following:

P1) ∃x

P2) ∃y(y≠x)

P3) Fx˄Fy

In a less elegant way we can put the three premises in one:

P1-3) There are at least two parts of the world.

It means as following: Once there are two beings, something exists; once there are *two* beings, there exists a plurality. Any two beings that exist together, be the distance between them whatever it is, are the parts of a common whole, that will hereinafter be called "the world," and therefore at least one common predicate is attributed to both of them: "…is a part of the world."

The meaning of this premise that the world – whatever world we will discuss hereinafter – is not Parmenidean. It is a whole that has parts.

 The first premise (P1) speaks about existence. Later, below, I will prove the existence of an external world, but for the moment I will take its existence as a working premise. I might say that this is the existence referred to in P1, but in truth we don't have to assume that premise. Since this section discusses any world whatsoever, it also discusses any relevant form of being. However, for the sake of convenience, the existence mentioned here and in the rest of this section should be understood, unless otherwise stated, as the existence in reality.

 Our second premise is that the world under discussion, be it whichever it be, is a world of plurality. Taken that an external world does exist, there are two possibilities: Either the world is a Parmenidean unity without discriminations, or it contains plurality and discriminations. If it is a Parmenidean unity, all that is said below about the world should be shelved or regarded as a description of an illusory or hypothetical world, while for the true description, if there is any such, we have to turn to mystics. If the world contains plurality of entities, we have to discuss the discrimination between different objects. Since this premise states that the plurality is not illusory but real, we have to discuss the boundaries between objects.

 Even though objects are discriminated from one another, they are, after all, parts of one world. That is our third premise (P3). The fact that (at least) one predicate applies to them already makes them subject to one overarching order. The existence of (at least) one such predicate has already been implied above: All the objects share the predicate "… is a part of the world." The concept of parthood is thus revealed to be one of the first principles of the order of the world. As we will soon see, however, that order has some more principles. The study of the foundations of the order of any world whatsoever is the purpose of this section.

###  Objects and concepts that capture them

One of the main purposes of this book is the presentation of a new calculus aimed at replacing the predicate calculus. It will be named **concept calculus**. This calculus embodies to a great extent Leibniz's view on predication, and one might reckon that Leibniz himself would have built a similar calculus according to his vision had he had the modern algebraic tools for building logical calculi.

 What there is in the world are objects. Any entity is an **object**. Objects are objects *of* concepts, being discriminated by concepts.

 A **concept** is an object embodying a property (including relation) existing in an object. The concepts create the **discrimination** between objects. In principle, every concept might apply to more than one object, but actually many apply to one alone. As Frege taught, a concept works as a function, being an "unsaturated" objective entity that might apply to multiple terms, one term or zero terms.

 I will note, parenthetically, that in the mathematics of today the convention is to define a function as a set of ordered pairs. This definition implies the assumption that the basic terminology of set theory is prior to the concept of function. Such an assumption might satisfy the mathematicians, since in science it is acceptable to adopt certain concepts as primitives without contemplating their metaphysical essence, as long as they are useful as foundational building blocks for the theories at stake. In philosophy, however, we may not act that way, for the analysis of the metaphysical essence of concepts is the very core of our work. From a metaphysical point of view, it is clear that the terms of set theory cannot precede the concept of function: (a) because the most basic term of set theory is the membership predicate "… is a member of …," (∊) and that term, just as every other predicate, is a function (and the same is true for the less basic "…is a subset of…," i.e.⊃); (b). The term "ordered pairs" cannot serve as a definition for a function, since the very action of the ordering of the pairs is made by a function.

 All the concepts are objects, and therefore they, too, are discriminated by concepts. There are objects that are not concepts. An object of this type will be named an **individuum**.

 The existence of a concept in an object will be named **capturing**. Namely, when a concept applies to an object we will say that former **captures** the latter.

A **predicate** is a grammatical expression denoting a concept. In predicate logic we distinguish between one-place predicate, denoting a property, and multi-place predicate, denoting a relation. Below we will see that this distinction is misleading, and in fact those are various partitions of concepts.

Variables of objects will be denoted by the letters x,y,z,w, sometimes with indexes. Usually, uppercase will denote concepts, lowercase – objects. However, I will not be strict about this distinction, since concepts might sometimes be dealt as objects, and flexibility is required for that purpose. These letters will therefore signify variables of objects of all types, including concepts, when discussed as being captured by other concepts (the latter serving as second order predicates).

Constants of objects of all types will be denoted by the letters attached to them throughout the discussion or, when unspecified, by a,b,c,d, sometimes with indexes. Here, too, I will not be strict about the lowercase-uppercase distinction. These letters will therefore signify constants of objects of all types, including concepts, when discussed as being captured by other concepts (the latter serving as second order predicates).

Variables of sentences will be denoted by Φ,Ψ, sometimes with indexes.

Constants of sentences will be denoted by p,q,r, sometimes with indexes.

 Concepts capture objects. **Capturing mark** will hereinafter be ⮯. The sentence "(concept) y captures (object) x)" will therefore be written:

yΦcx

 Objects are captured by concepts. When x is captured by y we encounter the condition we know from traditional logic as predication, i.e. y is attributed as predicate to x. **Predication mark** will hereinafter be⮭. The sentence with the ordinary structure "(Object) x is such that (concept) y applies to it" or "(Object) x 'falls under' (concept) y" will therefore be written:

x⮭y

We will often read the mark ⮭ as "captured by."

Predication can be defined using the capturing mark:

x⮭y =def y⮯x

The form x⮭y is the one known to us from predicate logic as Yx. Thus, for instance, if we agree that d1 denotes Danny and WISE denotes the predicate "wise," the sentence "Danny is wise" will be now written as follows:

d1⮭WISE

Since our topic is metaphysics, we discuss assertoric sentences. The premise taken here is that all the assertoric sentences (at least those relevant to our discussion) are of the form of predicate attributed to object. Therefore we can now state that every sentence is of the form "x is Y" (again: without taking too strictly the conventional denotation, in which a predicate is uppercased and an object is lowercased).

 As we will see below, I will take the concept of concept or the concept of object as a primitive (no need to take both, since the one can be defined through the other). I do not take the concept of capturing as a primitive. Allegedly, if we were to be strict about Ockham's razor we should have taken capturing as a primitive and decide that the concepts of concept and object are determined according to their placement in relation to the capturing sign. Such a convention would follow in the footsteps of axiomatic set theory, in which the membership sign is a primitive, and the concepts of set and member are determined according to their placement in relation to it. However, as we will see below, in concept calculus there is no need to go that way, since the concept of concept already includes the fact that it captures objects (at least potentially), and the concept of object includes its being captured by concepts.

### The rules of capturing

The **axiom of universal capturing** (or, for short, the **axiom of capturing**):

Every object has a concept that captures it.

∀x∃y y⮯x

This implies:

∀x∃y x⮭y

That is, every object has certain properties (including relations) by which it is characterized and by which its identity is determined. These are the concepts by which it is captured.

This is true for concepts as well, since they are objects, too. As stated, concepts are both captured and capturing.

 This statement is far-reaching, certainly more than it looks. This will be revealed when we discuss more thoroughly the concept of boundary, but already now I can say this: We might have assumed that we ought to distinguish between an object with a concept-dependent boundary and one with non-concept-dependent boundary. According to this assumption, if I draw a boundary line around a box by the line that discriminates its colour (yellow) from that of its environs, then this line is concept-dependent (and the concept is "yellow," while if I draw a line of no specific form around some part of the world – that, allegedly, would be considered as a non-concept-dependent line, one that we might call "arbitrary." Yet, the axiom of capturing states that nothing in the world is "arbitrary," since every object is discriminated by a concept, for otherwise it would not have a discriminated existence, and therefore its boundary is never determined arbitrarily. Furthermore, since the boundary is itself an object, it, too, is discriminated through a concept. Therefore even the boundary itself is not arbitrary.

 The basis for this argument is the identification of concept as function, as established and developed mainly by Frege (Frege, 1960, pp. 21-41). If we take any finite collection of points on a coordinate system, and draw a graph that connects between them (and there is an infinite number of possible graphs which could do that), that graph will immediately become a general function as soon as we decide that it can be an infinitely repeated on further points of the system. This is true, *mutatis mutandis*, for any finite collection of objects. We may state that between any two objects there is a unifying concept (actually, numberless unifying concepts) that determines a relation between them on "the map of the world" (or, in the case of concepts, on the concept map, which will be expounded below), and this concept might work periodically on other objects, even those that do not actually exist in the world. Of course, this generalizing concept is not necessarily useful, let alone extant in language; however, it does exist on the concept map.

 Following this thesis we can now state the **rule of multiple capturing**:

For any given collection of objects there is a single concept that captures them all:

∀,x1,x2…xn ∃Y Y⮯x1 ˄ Y⮯x2… Y⮯xn

At its basic level, this principle is simple and trivial, since every object is an object, i.e. "something," and therefore it is captured by the Big Concept (marked UK below), that constitutes the concept of "something" (as expounded below). However, I wish to argue for a stronger principle, which I will name the **rule of universal conceptualization**:

∀,x1,x2…xn ∃Y (Y≠UK) Y⮯x1 ˄ z⮯x2… z⮯xn

This principle states that any collection of objects (in a finite number) has a concept that captures them in addition to the Big Concept. For the time being we will accept this rule as an axiom, but later on (Chap. ###) I will prove it.

 Is this also the case with infinite collections? Presumably yes, since the graph could have been extended cyclically a finite number of times; but this requires further study, which is not to be done here. Lacking a compelling proof that it is infinite, I will hold the default that it is not.

 In predicate logic we mark the predicate with a capital letter and the terms in small letters, usually after the predicate, with a comma separating between multiple terms. In the older system of notation they used to write the first term before the predicate and the others after it. In our concept calculus, when we want to build a sentence of the traditional form we will mark the first term, "the subject term," before the predicate (at its left side), then we will write the predication mark ⮭, then will come the predicate, and if there are other terms we will add indexes after the commas, with upper indexes that mark the order of the coming terms within the predicate. Thus, the mark ,¹ will be named "first comma"; the mark ,² - "second comma"; ,³ - "third comma", and so on.

 However, this notation is a little cumbersome and a little confusing; not always is it easy to see if the number of the index refers to the comma or to the letter that precedes it. Therefore when we know that the number of terms is not higher than three – as is usually the case – we may, for the purpose of convenience, replace the above notation with a simpler one: Instead of ,¹ - we will write a semicolon (;), instead of ,² - we will write a double comma (,,), and instead of ,³ - a triple comma (,,,). (Admittedly, I could use a single comma for the first comma, but I would like to preserve the single comma's traditional role as denoting alternatives). Since this is a different notation for the very same thing, we will keep calling these alternate marks first, second and third comma, respectively. When we want to denote variables of comma numbers we will mark the indexes with tiny n or m, according to the mathematical convention.

 In the spirit of the above we are now able to state the **Rule of Predicative Structure**. According to this rule, every sentence in concept calculus has the following structure or an equivalent thereof:

x⮭y,¹…,²…,³…

One such equivalent that we already know is the capturing:

x⮭y,¹z1,²z2,³...zn ≡ (Y,¹z1,²z2,³...zn)⮯x

Let us take, for example, the sentence "Danny gave Dina an apple." If we agree that "d1" denotes Danny, "d2" denotes Dina and "a" denotes the apple, then in predicate logic we would formalize the above sentence as follows:

Gd1,d2,a

While in concept calculus we will formalize it this way:

d1⮭G,¹d2,²a

Or this way:

d1⮭G;d2,,a

One might think that this is all about technicalities, but it is not so. This way of writing will make room for new logical possibilities. If Danny ate an apple, then Danny ate. And if Danny gave Dina an apple, then Danny gave an apple. And also: Danny gave. (Even if in natural languages "gave" is usually a transitive verb that requires an object after it, this is one of the places in which logic does not have to be chained by the restrictions of natural language but rather has to surmount them). Consequently, in this issue we will have to divert from the old predicate logic (and I will add parenthetically that this diversion will also help us to work with second-order predicates, as anyone trying to work with this new tool will notice). While in predicate logic one could not make an inference from n-place predicate to (n-1)-place predicate, in concept calculus that is made possible, and is even required.

 The way in which it will be done will be named the rule of predicate reduction, or, for short, the **reduction rule**, that states as follows:

x⮭Y,¹z1,...znx⮭Y;zm|(m≥n)

(The term "reduction" refers to the graphic aspect; in terms of intension, it enlarges the predicate, as will be expounded below).

This means that in concept calculus the three following inferences are valid:

d1⮭G;d2,,a d1⮭G;d2

d1⮭G;d2,,a d1⮭G,,a

d1⮭G;d2,,ad1⮭G

It is noteworthy that in concept calculus the commas do not only serve for separation. Rather, they denote the nature of the relation between the predicate and each object. Thus, the first comma denotes a certain relation of the predicate to the first object, the second comma a different relation to the second object, and so on. In fact these relations were supposed to be denoted by letters, but such a notation would encumber our writing too much. We will therefore agree that the commas denote variables of these relations, and when we wish to mark them as constants – a rare event in our discussions here – we will use the Hebrew letters כ,ל,מ. Thus, for example, if in the above sentence we wanted to refer to known relations we would write it as follows:

d1⮭Gכd2לa

In accordance with that, the predicate might come either alone (without an adjacent object) or together with the appropriate comma and the object following it (or the appropriate Hebrew letter and the following object), be it whichever it be, without necessarily mentioning all the objects.

 Now, here we may wonder: Suppose we have a predicate denoting eating and another one denoting refraining from eating. We will be able to say: "Danny ate an apple" and "Danny refrained from eating a pear," and the two sentences will not contradict each other. However, if we apply the reduction rule, we will supposedly face a problem, since "Danny ate" and "Danny refrained from eating" do, prima facie, contradict each other. Yet the contradiction is only a seeming one, since the meaning of "Danny ate" is that "Danny ate *something*" (even if that "something" is not specified in the place of the predicate), and in the same way "refrained from eating" means "refrained from eating *something***.**" So when we say that Danny "refrained from eating something" we utter a true sentence. He refrained from eating a pear, just as he refrained from eating a schnitzel and refrained from eating seaweed, and many more things; and therefore he certainly refrained from eating *something*. "Refrained from eating something" does not mean "refrained from eating everything." Of course "refrained from eating everything" is in stark contradiction to "ate (something)." However, "refrained from eating something" does not contradict it at all. (Indeed, maybe we should build a "square of opposition" for this purpose, inspired by the Aristotelian one).

 Admittedly, sometimes language misleads us, and therefore we should employ the reduction rule cautiously. An example similar to the above: Iron melts at a temperature of 1,532 Celsius, and therefore, according to the reduction rule, iron melts in a temperature. The listener might understand that iron melts in *any* temperature. That, however, is a linguistic fallacy, not a logical one. The sentence wants to say that iron melts at a *certain* temperature that is not specified, not that it melts in any temperature. All this will be discussed in the proper place below (in Chap. ### mereology of concepts).

For the time being, let us take the reduction rule as a given. Below, it will be expounded by mereology of concepts. I am well aware that in certain places this rule will pose some difficulty to some readers' "intuition" (or rather their deep-rooted customs). Thus, for example, we will see that two-place predicates like identity and nonidentity may become one-place predicates, and so we will be allowed to say: "x is identical" without specifying to what, and "x is different" without specifying from what. Further below I will suggest a didactic method to overcome this difficulty; but, in truth, the whole thing should not be seen as a problem, and one can simply become accustomed to this rule.

 I want to emphasize that we may treat, without hesitation, anything that follows the predication mark (⮭) as a full predicate, i.e. an expression representing a full concept, that stands on its own even outside the sentence. In the above example, the expression

Gכd2לa will be considered as a full-fledged concept, even with regard to mereology of concepts (below, at ###). I will soon elaborate on this matter.

 By this we can put an end to the hard distinction between one-place, two-place, three-place predicates etc., that weighed heavily on logic for over a century, quite needlessly. From now on, the predicate does not have to have a fixed number of places.

 This issue takes us back to the controversy (or seeming controversy) between Russell (1937: 12-15) and Leibniz (1890: 266-7), or in fact between Russell and Scholastic logic. Russell criticized Leibniz for recognizing only properties, i.e. one-place predicates, and not relations, that is, multi-place predicates. (But he also showed that Leibniz had a more complex position on this matter than the one often attributed to him). Russell argued that, by this, Leibniz put the world into an unnecessary straitjacket; a world that contains a variety of objects with a network of relations between them, and not only isolated objects with "interior" properties. Russell's argument seems to imply that the determination of whether we speak about property or relation is dependent on reality. In truth, it is more often dependent on language or other conventions. The predicate "loves French [people]" will be considered as a two-place predicate, while its synonym "Francophile," as a one-place predicate; "hates Jews" as a two-place predicate, while "anti-Semite" as a one-place; "eats meat" as a two-place predicate, while "carnivore" as a one-place; and so on. This is a linguistic distinction that makes logic dependent on arbitrary conventions.

 In the light of this we can understand the sense of the Leibnizian theory of predicates, or rather rephrase it. This theory will not argue that there are no relations, or multi-place predicates; in fact, Leibniz himself never claimed it, but that those predicates may be *reduced* to one-place predicates. Furthermore, this reduction will not impede the logical fertility of the concept, but rather increase it by enabling us to make new types of inference that we could not make hitherto, through the use of the reduction rule and other means that will be elaborated below.

 In the spirit of Leibniz we may now state that all the expression following the predication mark ⮭ will be considered as one concept, so that all the objects that appear in it will be considered as relations that are within it, just as the concept "carnivore" contains the concept "meat." And the same is true, *mutatis mutandis*, for the capturing mark ⮯.

 If we take the structure given above, x⮭y;z1,,z2,,,...zn, then according to Russell only y is a predicate while according to Leibniz all the expression x⮭y;z1,,z2,,,...zn, is a predicate. From my point of view, they are all predicates that maintain relations between their constituents. As far as terminology is concerned, "Russell's predicate" will be called **nuclear predicate** and "Leibniz's predicate" will be called **articulated predicate**. The objects added to the nuclear predicate in order to build the articulated predicate will be called **predicative constituents**.

 Between the nuclear predicate and the objects attached to it – both the object captured by it and the predicative constituents – there is a relation which we will call **sidehood**. The captured object and the predicative constituents are **sides** of the nuclear predicate. The relation "… are sides of …" will be denoted by SD and will be defined:

x1,x2,x3….⮭SD;Y ≡def x1⮭Y;x2,x3…˅ x2⮭Y;x1,x3,... ˅x1,x2⮭Y;x3….

The reduction rule applies also to objects bound to quantifiers. Thus, for example, we may state the following auxiliary sentences, inferred from the reduction rule:

∀x x⮭Y;z1,...zn∀x x⮭Y;zm|(m≥n)

∃x x⮭Y;z1,...zn ∃x x⮭Y;zm|(m≥n)

∀x∃(z1, ….zn) x⮭Y;z1,...zn∀x x⮭Y;zm|(m≥n)

This sentence is also true for m=0, and so we may infer:

∀x∃(z1, ….zn) x⮭Y;z1,...zn∀x x⮭Y

And given the accepted assumption that the universal quantifier contains the existential quantifier, we may also add:

∀x∃(z1, ….zn) x⮭Y;z1,...zn∃x x⮭Y;zm|(m≥n)

The reduction rule with its auxiliary sentences will be proven useful in our next discussions. I believe their bearings can be far-reaching for many other disciplines as well, and might also have practical applications.

### Intermezzo: Some general second-order characteristics

Before we continue the main discussion I will present some definitions of second-order relations, i.e. concepts that capture concepts. Most of the definitions are traditional and well known, and we need them for the sake of notation and the completeness of presentation.

**Reflexivity**. The predicate *reflexive* will be denoted by RFLX and will be defined as follows:

X⮭RFLX ≡def ∀y,y⮭X;y

**Symmetry**. The predicate *symmetric* will be denoted by SMTR and will be defined as follows:

X⮭SMTR≡def ∀y,z z⮭X;y↔y⮭X;z

**Transitivity.** The predicate *transitive* will be denoted by TRNT and will be defined as follows:

X⮭TRNT≡def ∀y,z,w y⮭X;z˄z⮭X;wy⮭X;w

The negated forms of these characteristics will not interest us at this stage, but we will define them for future use:

x⮭NRFLX ≡def ¬∀y (y⮭x;y)

x⮭NSMTR ≡def ¬∀y,z (y⮭x;z↔z⮭x;y)

x⮭NTRNT ≡def ¬∀y,z,w ((y⮭x;z˄z⮭x;w)y⮭x;w)

The three classical relations presented above – reflexivity, symmetry and transitivity – will be named, for short, **RST characteristics**. I will use them often in this section to characterize new concepts that I will introduce.

**Self-capture**. The predicate *self-capturing* will be denoted by the letters SCP and defined as follows:

X⮭SCP =def X⮯X

**Universal captur**e. The predicate *all-capturing* will be denoted by the letters ACP and will be defined as follows:

X⮭ACP =def ∀y X⮯y

**Exclusivity**. The predicate *exclusive* is the negation of all-capturing, i.e. it refers to a predicate stating that there is at least one object which is not captured by it. This predicate will be demoted by the letters NACP and may be defined in each of the following ways:

X⮭NACP =def ¬X⮭ACP

X⮭NACP =def ¬∀y X⮯y

X⮭NACP =def ∃y ¬X⮯y

Now I wish to present another general characteristic, very important for our discussions:

**Inverse** will be marked INV and will be defined as follows:

X⮭INV;Y ≡def ∀X,Y,z,w (X≠Y), (z⮭X;w≡w⮭Y;z)

(X≠Y is required for differentiating between inverse and symmetry).

And at this stage we can already also state that INV is symmetric:

X⮭INV;Y ≡ Y⮭INV;X

Which means that:

INV⮭SMTR

Since INV is a dyadic concept by its very essence, reflexivity and transitivity are not relevant.

In view of the definition of the inverse it is important to note: every concept that is a relation – i.e. sided by two objects – has an inverse.

I must emphasize that the relation between the concept and its inverse is purely formal, and therefore from the logical and ontological perspectives we may define the one through the other or vice versa, without ascribing advantage or primacy to one over the other.

Now we can continue the discussion on our foundational concepts.

### Object, concept, individuum

We should now discuss the concept of capturing more profoundly: the concept "captured by," denoted by the predication mark ⮭, is itself a predicate (and the same is true for "capturing," denoted by the capturing mark ⮯). If we take as example the sentence x⮭Y, which means "x is captured by Y," then "captured by Y" is a predicate that is attributed to x, i.e. that captures it. Being a predicate, let us denote it with a special sign – the letter O. The concept O will not be defined by other concepts and will therefore be taken as a primitive. Now we may say that O;Y is the predicate attributed to x. However, in order to state this predication – x⮭O;Y – we will once again need the copula mark ⮭, to denote the predication. The sentence will therefore now mean: "x is captured by 'captured by Y’." This sentence is identical with "x is captured by Y."

We will therefore state the rule of the reduction of the capturing predicate:

x⮭O;Y ≡ x⮭Y

And if so, the following is also true:

x⮭O;(O;Y) ≡ x⮭O;Y ≡ x⮭Y

… and so on, to infinity. The same is true for the inverse concept of O, which we will name K. We will now define this concept through concept O:

X⮭K;y ≡def y⮭O;X

Now we can learn what O and K are: O is the predicate denoting "object" and K is the predicate denoting "concept."

The meaning of the above sentence is therefore: If X is a concept of y, then y is an object of K. It entails that K and O are inverse:

O⮭INV;K

But in this case we won't be able to state that

X⮭K;(K;y) ≡ X⮭K;y

Since the concept of the concept of y is not necessarily the concept of y.

But now we will be able to apply the reduction rule to O and K:

x⮭O;y  x⮭O

x⮭K;y  x⮭K

And this is very intuitive: If O is an object of something, then O is an object; and if K is the concept of something, then K is a concept.

Now we need to define individuum. For this we need the connective of negation, which has not yet been defined in the concept calculus, so at the moment we will use its notation in predicate logic. The predicate **individuum** will be denoted by the letter I and will be defined as follows:

x⮭I≡def x⮭O˄¬x⮭K

In other words, an individuum is an object that is not a concept.

Now we can state:

∀x x⮭O

Namely, everything is an object.

When we wish to refer in a non-sentential way to a concept that captures object x we will name it a **capturing concept**. A capturing concept of x will be denoted by the letter K followed by brackets flanking the object which it captures:

K(x) =def y|y⮯x

It is needless to add that every object has many capturing concepts.

K⮭K means: K is a concept (and Russell's paradox has nothing to do with this).

But O and I, in themselves, are also concepts, and therefore we may state:

O⮭K

I⮭K

But according to concept calculus predicates K, O and I may also be two-place predicates. Therefore:

x⮭O;K means: x is an object of the concept K.

x⮭I;K means: x is an individuum of the concept K.

x⮭K;K means: x is a concept of the concept K (i.e. captures it).

K⮭O;K means: the concept K (the concept of Concept) is an object of the concept K.

And also:

X⮭K;y means: X is a concept that captures y.

K⮭K;K means: K is a concept that captures K.

And since O and I are concepts we may also state:

K⮭K;O

K⮭K;I

Usually we won’t need these complex structures since the shorter form is simpler:

x⮭O;Y ≡ x⮭Y

X⮭K;y ≡ X⮯y

The longer form will be needed only when we will have to stress ontological aspects of the fact that K, O, and I are concepts.

I will now introduce the arrows of **plain conceptualization** and **plain objectification**.

**The arrow of plain (or multiextensional) concept**: The sign of a double arrow directed upwards at the right side of an object’s sign will designate a concept which captures that object (even if it captures other objects as well):

x↑↑ =def Y|Y⮯x

**The arrow of plain (non-singular) object**: The sign of the double arrow directed downwards at the right side of an concept sign will designate an object which is captured by that concept (even if that concept captures other objects as well, and even if other concepts capture that object):

X↓↓ =def y|y⮭X

Now we can turn to discuss singular capturing.

### Singular capturing

Now we will present the operator of definite description. It will be denoted, according to the common convention, by iota (ɩ), and will be defined as follows:

ɩx(x⮭y…) =df ∃x(x⮭y)˄∀z(z⮭y)z=x

When we have a concept that captures only one object we will say that that concept has a **singular capture.** We will name the captured object the **singular object** or the **singular instance** of that concept. We will denote that object by the mark of **singular objectification** ↓, but, in contrast to the iota, the definite description arrow will appear after the sign of the concept. We will define:

X↓=def ɩy|X⮯y

Taken that the sign n represents number (as will be clarified below), and taken that the number 1 is defined (it will be below), we may also phrase it this way:

X↓=def y|X⮯y˄nX=1

When we have a given object x and we wish to denote a concept (any concept, even if it is one of many), that captures it singularly, we will name it the **uniextentional concept** of x. It will be denoted by the letters TK mark followed by brackets flanking the object which this concept individualizes:

TK(x)=def Y|(Y⮯x˄∃z y⮯zz=x)

For purposes of brevity, the uniextensional concept will often be denoted by the sign of singular conceptualization ↑.

x↑=TK(x)

I must emphasize again, that the uniextensional concept of is a concept of a singular object, but is not singular itself. x↑ means "*a* concept that captures x" and not "*the* object that captures x," since one object might have multiple uniextensional concepts. Thus, for example, Isaac might be defined as "the son of Sarah," as "the father of Jacob" or as "the second Patriarch," and all the three are uniextensional concepts. In fact, the term "uniextensional concept" is quite close to Frege's *Sinn* (sense). The sign ↑ is therefore an open sign that denotes *any* of the above possible concepts. Therefore, when X↓=Y↓ is true, it is *not* necessarily true that X=Y.

Furthermore: It is necessarily true that

∀x (x↑)↓=x

But the opposite is not necessarily true and so -

¬∀X (X↓)↑=X

In the light of all this we will see below that we may talk about the set (or the collection, in the sense defined below) of uniextensional concepts that every object has. The expression x↑ denotes, therefore, x qua a member – *any* member! – of that set (or any item of that collection).

For the moment we can state the **axiom of singular capture**:

∀x∃y y=x↑

This means that every object has at least one uniextensional concept that captures it – a concept that captures only it. The rationale of this axiom will be expounded below, but I will already note briefly here that every object has a boundary, and the boundary is determined by the concept that captures it singularly. Hence, every object has a concept that captures it singularly.

### Identity and nonidentity

Before we continue to the next chapter, let us discuss the most foundational concepts of every metaphysical analysis: identity and nonidentity.

**Identity:**

The relation **identical** (... is identical with ...) will be taken as a primitive. It will be denoted by the letters ID, and sometimes, for short, by the sign =.

RST characteristics:

x=x

x=y ↔y=x

x=y ˄ y=z  x=z

**Nonidentity**:

The relation **nonidentical** (… is different from …) will be denoted by NID, and sometimes by the sign ≠. It will be defined as follows:

x≠y≡def¬(x=y)

RST characteristics:

¬(x≠x)

x≠y ↔y≠x

Already at this stage we can state the **axiom of identity**:

∀x x⮭ID;x

This axiom rises from the RST characteristics of the concept of identity, as presented above, but what will be revealed as more important for us is the rule of identity inferred from it through the reduction rule:

∀x x⮭ID

To state: Every object is identical. As I have noted above, according to the reduction rule any two-place predicate can be reduced to a one-place predicate. The same is true for the predicate of identity. One might argue that the predicate ID is by its very essence a two-place predicate. However there is no such thing as a two-place predicate "by its very essence." Just as any other predicate, one may reduce ID to a one-place predicate, so that the sentence x⮭ID is a WFF. I am fully aware, of course, that due to the limitations of language it is difficult for us to think of the word "identical" as a one-place predicate, because in language an object always has to be identical *to something*. Logic, however, obliges us to surmount those limitations. In order to facilitate the process, the reader may imagine it as if it were a predicate with an "open" object: "identical to something." But this advice is useful only for the stage of adaptation, and in truth we are talking about a full-fledged one-place predicate. One should get used, therefore, to the reduction as such, even when it has no parallel in natural language. Below (at ###) I will explain this in greater precision.

Similar to the axiom of identity we can also state the **axiom of nonidentity**:

∀x,∃y x⮭NID;y

Thus the axiom stems from the understanding that an object is determined by a boundary, and that boundary discriminates between it and its "environs," i.e. "the rest of the world." And here, too, if we use one of the auxiliary sentences of the reduction rule we can infer the **rule of nonidentity**:

∀x x⮭NID

Here, too, we have to cope with the stridency of this expression from the perspective of natural language, being presented as a one-place predicate, and here, too, this stridency must be overcome. The problems arising from this type of use of the reduction rule will be broadly discussed below (at ###).

 I would like to add parenthetically that I have had some deliberations with regard to the relation of identity, and similarly also with the concepts of negation, union, parthood and intersection, that will follow. By this I do not mean the familiar deliberations of the logicians (especially from Quine onwards) regarding the nature of the relation of identity and whether it is needed for predicate logic. The arguments of those deniers of identity are not convincing, even prima facie, and I will not argue with them here. The problem is different. First, we should remember that the above-mentioned relations have parallels in the connectives of propositional logic, and it would be proper to set for these connectives formal parallels relating to non-propositional relations (and I will do so below). Second, we have to remember that according to Ockham's razor we should aspire to reduce those connectives (and their non-propositional parallels) to the lowest possible number, i.e. to the most foundational concepts. And herein lies the problem: Which are the most foundational concepts? From the perspective of our intuition, the concepts of identity and negation, for example, are foundational in the first degree; from the perspective of formal analysis, however, they are not: Truth tables teach us that the connectives NOR and NAND are more elementary than equivalence and negation, which are their approximate propositional parallels (actually, equivalence is a weaker connective in the propositional plane than identity is in the object plane, but particularly because of that it is prior to it and more elementary than it). That is because one can express equivalence and negation, in fact all the connectives, by NOR and NAND. Eventually, I opted in this case for natural intuition, in spite of my principled position that logic must surmount "natural" intuitions. Put simply, without identity and negation there are no objects, no propositions, no sentences and consequently there are no relations and no connectives. Therefore even NOR and NAND will not be able to be applied to anything until an object is determined to be identical to itself and nonidentical to all the rest. The concepts of identity and negation are therefore a precondition to the existence, and also to the notions, of NOR and NAND, and therefore precede them from the metaphysical point of view. In the light of this, it is not sheer natural intuition that decided the dilemma, but a solid metaphysical justification, even if not a formal one.

 In the margin I will add that if I went in another way, seeing NOR and NAND (and their parallel non-propositional relations) as primitives and defined identity and negation with them, it would cause a strange entanglement: As we know from the truth tables, both NOR alone and NAND alone might serve as primitives by which all the other connectives can be constructed. Hence, a logical approach that would prefer to lean on formal analysis rather than natural intuition would face the strange condition in which the logician might build his foundations out of two alternative concepts, and therefore build two alternative metaphysics of the same validity. This strangeness ought not to justify the rejection of such a line, and a "metaphysic of relativity" should not be excluded on the threshold, even if it does not fit the spirit of traditional metaphysics. However, as I noted, it is not this consideration that led me to favour natural intuition over formal analysis in this matter, but rather the metaphysical consideration regarding the primacy of the concepts of identity and negation over any possibility of having a relation to anything at all. Thanks to this consideration I am exempted from the need to entertain such an unusual line of thought.

 For the very same reason we cannot reduce the concept of identity to "double nonidentity." From the formal logical perspective we could take the concept NID as a primitive and define ID with it: x⮭ID;y ≡def x⮭¬NID;y. Yet here, too, the metaphysical consideration must precede the formal one. Before anything else that we can say about an object it has to have an identity, and therefore it is the concept of identity that has to be taken as a primitive.

Having clarified all that, we can now turn to another issue and state the **axiom of relation**:

∀x,y∃Z x⮭Z;y

In other words: Between any two objects there is some relation. Note, that this is true even when x and y are not different (nonidentical) objects, since in that case there is at least one known relation between them – that of identity.

This axiom might be phrased this way, too:

∀x,y∃Z x,y⮭SD;Z

At first glance, the axiom of relation seems trivial, since between any two objects there is a relation of the identity-nonidentity category: they are either identical or nonidentical. Yet, we have to assume that between any two nonidentical objects there must be another relation, beyond that of nonidentity. That relation stems from the fact that the concept which discriminates the one and the concept which discriminates the other both occupy "places" on "the map of concepts," and the determination of the relations of their "placing" is the determination of the relations between those concepts. In view of this, we may now assume the **strong axiom of relation**:

∀x,y (x≠y)∃Z x⮭Z (Z≠NID);y

Leibniz's principle of the identity of the indiscernibles entails that object x is discerned from object y by the very fact that one of x's properties is that of being "not-y." In contrast to what is understood at first glance, this difference is not merely formal. The property "not-y" actually means that x and y have different boundaries, and that is the case because different concepts capture them. This brings us to the concept of boundary and to the concepts related to it.

## Topological foundations: boundary and closure

### Boundaries and what they bound

The identity of an object is determined by its boundaries. The boundaries of an object are what discriminates it from its "environs," whether in the simple physical sense or in the metaphoric one. If a yellow box lies in the street, there is a line that marks the boundary between it and the pavement and the air that surrounds it. This line is determined by virtue of a different colour (that entails a different shape), different degree of solidity, and other differences of property. If in the desert there is a type of snake that has perfect camouflage colours that make it impossible to discern it from its environs in terms of colour (nor, consequently, in terms of the shape that the colour determines), then it is discriminated from its environs by its motion. If the yellow colour differs from the green one by its wavelength, it is those wavelengths that determine the boundaries. In fact, anything that discriminates one object from others "draws" a boundary between that object and "the rest of the world." In other words, any predicate, even the most abstract one, makes a boundary. These boundaries are the subject matter of this chapter.

The key concept of this chapter is that of boundary, which is a necessary condition for the partition of the world. A boundary is an object that discriminates the object enclosed within it, and in this way creates its identity. In this context we will talk, unless otherwise stated, about the *full* boundary that surrounds the object on all sides ("maximal boundary" as it is called in contemporary scholarship), and the discussion will focus on physical objects, but applies, *mutatis mutandis*, to all other objects, including concepts. Let me emphasize that boundaries, too, are objects, and therefore have boundaries. Being objects, their boundaries, too, are determined by concepts.

 Some of the contemporary scholars contend that one should conceive objects, and most of all physical objects, as having the property of self-connectedness, i.e., that all their parts belong to one continuum. (It is considered, for example in: Varzi, 1994; Casati and Varzi, 1999, pp. 12-16, 54-62, 80-83; Smith and Varzi, 2000; Guizzadri, 2005, p. 184.) They are well aware that some objects do not satisfy this property, but we nevertheless treat them as unified objects. Their favourite examples are bikini swimming suits and the letters i and j. Some of these scholars see them as exceptions and endeavour to explain why each of these objects should be seen as unified objects while other separated objects should not. I do not see them as exceptions at all, and furthermore, many other objects of similar character might and should be considered as wholes. To say it in short, a boundary does not require self-connectedness. The reason is that from a logical perspective, anything that might be predicated is an object. When we say "the Smith Family went on vacation" we treat that family as one object even though it numbers six persons (plus a dog). When we say that the total landmass of planet Earth is 148,939,063.133 square kilometres, we refer to an object that includes over 180,000 land units that have no territorial continuity. In the very same way we could talk about "the big yellow," i.e. the sum total of all the yellow objects in the world, as one object. Furthermore, from the perspective of logic there is no restriction on the "creation" of objects that are unions of other objects that are not connected to each other and share no intuitive common denominator. The object combined from the dark side of the moon and the tip of Cleopatra's nose is one whole object, and those two are its parts just as much as the two hemispheres are parts of Earth (See, for instance, Varzi, 1998; Tsai and Varzi, 2016). This readiness to recognize an object as a single object even when its parts are not connected and there is no conceptual linking between them will be named the **principle of free union** (which might be considered as a mereological parallel of the axiom of extensionality in set theory). It will be discussed below (at ### this chapter) after I will have presented some preliminary statements, and then later in other parts of this book.

 I am sure that some potential readers will blame me for being counter-intuitive and will contend that objects have "natural boundaries," and only that which they enclose might be considered as a whole object. Yet, the "naturalness" of those boundaries is determined by our cognitive tools and has nothing to do with logic or metaphysics, which are our concern here. I will allow myself here some general advice for any person who engages in philosophy, and which maybe relevant for life as a whole. Whenever someone mentions to you the words "intuitive," "natural" or "commonsensical" – sharpen your sense of criticism, for very often they cover our most deep-rooted customs and biases. Indeed, the sense of criticism must also be awakened by the complex structures of the professionals, including professional philosophers, when these structures go too far from robust intuition and common sense; but even then we should be cautious not to fall from the frying pan into the fire. In summary, intellectual caution is a commendable virtue at any case.

 In truth, there is no canonical definition and of the two definitions I presented here – the one that requires self-connectedness and the one that embraces the principle of free union – are equally legitimate, as long as we follow them consistently. In other words, the choice between the approach that recognizes any two objects as parts of a single whole and the one that recognizes only self-connected objects as wholes is a matter of decision. The decision I have made here is to adopt the more open definition, since it fits the interests of logic better, and consequently also those of metaphysics. This fitting stems, among other reasons, from the fact that it does not follow blindly after the cognitive tools' partition of the world, which is influenced in its turn by human needs and other subjective elements. This is actually another case where the metaphysical perspective has to surmount common intuition and the "natural" way of looking at things. We will thus remain loyal to the principle of free union (and the broader principle of free composition, to be presented below, at ###), according to which any two objects might be considered as parts of a single object and the boundaries of these parts will be considered as boundaries of the unified object even if there is no continuity between them.

 By the way, it should be noted parenthetically that as we have seen in the discussion above, the concept of boundary is closely linked with the concepts of part and whole. On the one side, the concept of boundary – a topological concept in essence – is prior, in some respects, to the concepts of part and whole, but on the other hand the concepts of part and whole are required, and will be further required to properly treat the concept of boundary. This is another example of a case in which the foundational concepts of metaphysics *bain keehad* ("come together").

 The status of boundary is controversial in philosophy. Is the boundary a part of the object? Is it a part of its complement? (The concept of complement will be defined formally below, but for the moment will be taken intuitively as "the rest of the world" that surrounds the object at stake. Brentano argued that a boundary is a dependent object, since its existence depends on the existence of the bounded object (Brentano, 2010). He also claimed that the boundary of an object is a part of it and the boundary of the complement is a part of that complement, and that the two parts "coincide." Chisholm follows in his footsteps (Chisholm, 1983, 1989, 1992/93 and 1994). Bolzano, in contrast, rejected this view, mainly on the grounds of the principle of spatio-temporal uniqueness, i.e. that no two objects can occupy the same place at the same time, and contended that a boundary is a part of the object but not of its complement (Bolzano, 1950). According to this doctrine, therefore, the complement remains boundaryless. Brentano objected to this view and called it a "monstrous doctrine" (Brentano, 2010, p. 105). Husserl emphasized, even more than Brentano, the fact that the boundary is a part of the object and regarded the dependence as the main characteristic of the part in relation to the whole (Husserl, 1970). If they were right, we would have to discuss mereology before topology. However, I think these positions are all wrong. Even though I have not yet defined part-whole relation formally, it is already necessary to clarify at this stage what a boundary is and to negate the misleading options regarding it. I will do so based on the simple intuitive meaning of this relation.

 Of course we cannot accept Bolzano's position, since we acknowledge the complement as a full-fledged object, and there is no ontological difference between it and the object it complements. People's tendency to see the smaller object as an object and its complement, as “what remains,” lacking properties of its own, is not a result of a logical consideration but of practical utility. Therefore we should not take it into account in this discussion. Nor can we accept Brentano's position, in spite of its symmetry, for two reasons:

1. It is contradicted by the premise that an object without "breadth" might be a part of something. I will give a concrete example: Suppose we have a sheet divided into two halves, black and white. The boundary line between them is neither black nor white, because a coloured object must be two-dimensional while a line is one-dimensional (By this I answer a question already raised by Suarez, 1861, Disputation 40, Sect. V, §58 and Peirce, 1893, 7.127, who used the example of a black spot on a white sheet). And above all:
2. The boundary is logically prior to the bounded object, and therefore cannot be a part of it.

Note: we could supposedly add Bolzano's argument that Brentano's position contradicts the principle of spatio-temporal uniqueness (according to which no two objects can occupy the same place at the same time). Yet, this argument is doubtful, for according to Brentano we don't talk about two objects occupying the same place but rather about one object serving in two "roles," which is a common phenomenon in geometry.

 The postulated conclusion is, therefore, that the boundary is neither a part of the bounded object nor a part of its complement. It is an object on its own (and therefore has boundaries, too), by which both the object and the complement are discriminated (and in this respect even created). Moreover, the boundary is not dependent on the bounded object, but rather the bounded object is dependent on the boundary. That object cannot be discriminated from its environs without a boundary, and that discrimination gives it its identity qua what it is.

The predicate of boundary will be taken as a primitive and will be denoted by the letters BD.

x⮭BD;y,,z will denote: x is a boundary between y and z.

x⮭BD;y will denote: x is a boundary of y.

x⮭BD will denote: x is a boundary.

The relation of boundary is neither reflexive, nor symmetric nor transitive, but is subject to the **axiom of boundary reciprocity**:

x⮭BD;y,,z ↔ x⮭BD;z,,y

We may certainly infer the following, using the reduction rule:

x⮭BD;y,,z  x⮭BD;y

x⮭BD;y,,z ↔ x⮭BD,,y

x⮭BD;y,,z  x⮭BD

x⮭BD;y  x⮭BD

But also the following (using the axiom of boundary reciprocity and the reduction rule):

x⮭BD;y,,z ↔ x⮭BD;z

 Since contemporary mereotopology treats boundary as part of the object, it distinguishes between that part and the other, bounded part of the object, which it named the **closure**. But for us, who do not take the boundary as a part of the object, the closure is nothing but the object itself, and whatever is in the former is in the latter. We will therefore continue to use the term "closure," but not as denoting a part of the object but rather the object itself, in the aspect of being bounded. We will denote the relation "**is a closure of**" (or "**is bounded by**") by the letters CL and will define:

x⮭BD;y =def y⮭CL;x

This means that BD and CL are inverse concepts:

BD⮭INV;CL

Now we may state the **axiom of boundedness**:

∀x∃y x⮭CL;y

This means: every object is a closure. It is inferred from the fact that every object exists by virtue of its boundaries, and if it has boundaries it is a closure. But according to one of the auxiliary sentences of the reduction rule we may state this, too:

∀x x⮭CL

Furthermore, the boundaries determine the identity of the object. Therefore, if two objects are identical, they share the same boundary:

x=y x⮭BD;z˄y⮭BD;z

x=y x⮭CL;z˄y⮭CL;z

This is a conditional and not a biconditional because that boundary is shared not only by identical objects but also by complementary objects (as will be demonstrated below).

I would like to clarify a few issues in which I will depart from prevalent trends in the mereology and mereotopology of the last few decades.

 A boundary is determined by a concept. The concept discriminates that which is within the boundary (the closure) and that which is outside it (the complement). If we express it in visual terms, the boundary is "drawn," by the concept that determines it. In this sense, the concept actually works as a boundary-determining function, and therefore we will name it, in this context, **boundary function**. If we take, for example, a black round spot painted on a white sheet, we might say that the black colour serves as a boundary function for the spot. Even if we assume that the spot does not yet exist and now we are supposed to paint it, our painting will also be done by a boundary function: that of a circle. Here I mean a function in the pure mathematical sense, but even when we talk about non-mathematical concepts they work as boundary functions just as much as the mathematical ones, and we will borrow the mathematical terminology to describe them.

 Hence comes another point. One of the distinctions made in modern mereotopology is between crisp and vague boundaries. This distinction has developed, particularly following the invention of Fuzzy Logic, and it is relevant to our discussion especially once we strip it of its epistemological elements (i.e. those concerning vagueness caused by doubt). From our perspective, the correct distinction is between different forms of boundary function: A crisp boundary is a boundary determined by a one-dimensional function, or, for short, a **linear function**, while a vague boundary is one that is determined by a two-dimensional function, or, for short, an **areal function**. If on a sheet we have a black part and a white part and between them passes a vague boundary of a grey part that gets darker and darker (or lighter and lighter, depending from which side we are looking), the existence of the gradual greyness does not negate the existence of the black and the white; rather, there is a black area, a white area and a middle area with varying hues of grey. (I find it important to stress this for the main issue we are discussing here, and also as a basis for ramification of other contexts, especially those regarding fashionable postmodern methodologies that tend to discover vague boundaries and then are quick to declare the abolition of all distinctions). One may claim, against this argument: The object enclosed in the grey area might be considered as a third object separating between the black and the white, and therefore any areal boundary might actually be described as an object standing in between two other objects. However, this claim should be rejected, for *any* boundary, including a linear one, is a third object that does not constitute a part of the objects it bounds.

 Sometimes areal boundaries are particularly useful, as in the above example, in which the areal boundary is not just a separating object but one that creates continuity between the two objects that flank it (and hence it is clear why areal boundaries are suitable for the resolution of sorites paradoxes). This way or the other, the question of whether to treat the separating area as an areal boundary or as an "ordinary" object on its own is a matter of decision, and from a purely metaphysical perspective the two options are equally correct. We will return to this issue in the second section of this book, in our discussion on the realm of thought.

 Now we turn to another issue. The concept of boundary as constructed in recent mereotopology sometimes explicitly refers to "medium size" objects. This restriction enabled philosophers not to see the complements of objects as objects. Put differently: In the eyes of current philosophers, if we take the yellow box described above, its boundary is its external surface. If, however, we take the complementary object of that box ("the rest of the world"), that surface is not its boundary, since it is "too big" an object, and it has to be taken as a boundaryless or "open" object, in contrast to the box itself, which is a "closed" object (see Smith, 1996; Smith and Varzi, 1997). If not seen as such, it may be regarded as an object that formal ontology doesn't have to take into any account, being outside the "medium size" that this discipline studies. From a logical perspective, however, as well as from an ontological one, there is no difference between the object and its complement. Hence, the above distinction lacks philosophical justification and ought to be discarded. Consequently we also ought to discard the distinction between "open" and "closed" objects in terms of their boundaries. All objects are "closed" in the sense of being bounded throughout the entire line by which they are discriminated from their environs.

 Even the world, which is the entirety of being (and will be formally defined below), has boundaries, but not in the physical sense, for it does not have surrounding "environs" to be discriminated from, but in the sense that it is discriminated from its objects. I will clarify what I mean.

Instead of the distinction between "open" and "closed" (bounded on all their sides) objects, let me propose another distinction: between objects with **circumferential** and **non-circumferential boundaries.** There is a temptation to call them "finite" and "infinite" boundaries, but we will resist this temptation, for reasons I will present immediately). We stated above that the boundary is needed in order to discriminate the object from its environs. However, some objects have parts that do not have any environs at all. Let's take as an example the complement of the yellow box lying on the pavement. One part of this complement's boundary is the boundary it shares with the box; but what about the rest? Since the complement is "the rest of the world" (beyond the box) we may say that its remaining boundary is the boundary of the world, but what is *that* boundary? One might suppose that by this we are drawn into the question of whether or not the world has an end in space (or in time, in the case that the object discussed is a time unit). But in truth we do not need to get into that here. Whether the world is finite or infinite in space, it does not have an "external" boundary. If it is infinite, the answer is obvious, but even if the world is finite, beyond "the end of the world" there are not any environs from which it has to be discriminated. A boundary, as we have said, is an object that discriminates the object from its environs. Consequently, the complement of the box has a boundary only along the line by which it is tangential to the box, while on its "external" side, where it is not discriminated, it does not have any boundary. In contrast to the prevailing distinction between "open" and "closed" objects, that applies even to areas in which objects are tangential to their environs, the new distinction between objects with circumferential or non-circumferential boundaries relates only to areas in which objects do not have any environs.

 Once we discard the distinction between open and closed objects, we are also driven to discard the concept of "surface," as understood in current literature (see: Chisholm, 1992/3; Smith, 1994). From a logical perspective, the external surface of an object is nothing but the boundary that separates between it and its complement, and therefore it belongs both to it and to its complement. Whoever thinks that the external surface belongs to the box but not to its complement looks at things from the box's "point of view," because the box is a useful thing in our everyday life and so we have, as subjects, a special attitude. If, however, the complement of the box were to be chracterized by similar advantages, and we looked at the world from *its* "point of view" we would see the very same place as a surface belonging to it. Logic, as a discipline tasked to overcome psychological and other subjective considerations, is supposed to ignore such differences.

 Another distinction proposed by current philosophers is between bona-fide versus fiat boundaries (Smith, 2001; Varzi, 2011). This distinction, too, is not logic-based, and therefore should be discarded. Instead of it we could supposedly adopt another distinction, close to the former one yet more logic-based than it, between a **concept-dependent** and **arbitrary** (i.e., non-concept-dependent) boundary. The term concept-dependent boundary will refer to a boundary determined by a concept, i.e. by a general function. In contrast, an arbitrary boundary is one that supposedly was determined by a "scribbled line" lacking any general function. Yet, this distinction does not hold, either. As I noted above, an arbitrary boundary does not exist: (a) because a boundary is itself an object, and as any other object it is discriminated by a concept; (b) because every line (in both the literal and the metaphoric senses of the word) that looks arbitrary may be conceived as a part of regularity of the same pattern, and therefore as an application of a general function. In terms of reason (a) if we characterize a certain area as "the area enclosed in the line scribbled by Smith" it will not be logically different from its characterization as "the yellow area" because "scribbled by Smith" is as much a concept as is "yellow." In terms of reason (b) we will be able to find in the accidental line scribbled by Smith a certain function, presumably a complex one, even though Smith himself was not aware of it when scribbling his line. In view of these, all the boundaries are dependent on concepts, whether simple or complex, and the term "arbitrary boundary" should be put aside. In fact, the full boundary of the object is determined by the sum total of concepts capturing that object; but this will be clarified only after we discuss the mereology of concepts (at ###). +++

### Boundaries and identity

As we have seen, boundaries determine the identity of the object, and they are determined through concepts that capture the object. We have also seen that every object might have a few uni-extensional concepts. These are closely equivalent so Frege's *Sinn* (sense). Here we need to emphasize that the fact that an object has a few different concepts that capture it does not mean that it has a few different (full) boundaries. Rather, it means that the same boundary might itself be created by different intersections of concepts (the term "intersection concept" will be explained below). In the example given above, Isaac might be defined as "the son of Sarah" or as "the Father of Jacob" or as "the second Patriarch," and all the three are uni-extensional concepts, built from different intersections of concepts; and still, the boundary they determine is the very same one – the boundary of Isaac.

 The principle stating that the boundary is the basis of the object’s identity will be named the principle of boundary and identity, and will be phrased as follows:

∀x,y,z,w (x⮭NID;y˄z⮭BD;x˄w⮭BD;y)z⮭NID;w

This means: If two objects are different, their boundaries are different. In less formal language we may say that the boundaries of the object grant it its identity, and consequently its existence. One might suggest that this principle be phrased in an even stronger manner, with biconditional connectives:

∀x,y,z,w (z⮭NID;x↔y⮭NID;y) ↔ x⮭BD;y˄z⮭BD;w

Which means: If two objects have different boundaries, they are nonidentical and if they are nonidentical they have different boundaries.

 However, this stronger version of the principle pushes us into problems we are not supposed to discuss at present, (even if they are beloved by current philosophers). In particular: change of identity, identity through possible worlds, and identity in time. We will therefore put this stronger version aside for the time being.

 In mereological calculi accepted today (such as Simons, 1987; Casati and Varzi, 1999) the common practice is to take the relation of parthood, most often denoted by the letter P, as a primitive and through it define identity and proper-parthood. The authors who went this way usually did it because of Ockhamian parsimony. I did not go this way because in this matter I am loyal to the doctrine of the ancients that identity is the most foundational relation, and if you don't presume that an object is identical with itself, you cannot say anything about it. That includes anything about part-whole relations that it has with other objects. As I noted above (in the previous chapter), any assertion about an object presumes the relation of identity, and for this reason identity must be a primitive. That is why I began with the relations of identity and nonidentity, and continued to the objects that create them – boundaries. Now, however, having clarified all those, we may turn to parthood and related concepts.

## Mereological foundations: part and whole

### Wholes and their parts

Rational metaphysics is about the partition of the world. We have seen that already at the primary level, that of determining the identity and nonidentity of the object, we create a boundary between the object and "the rest of the world." This is in fact an act of partition of the world. This partition requires a systematic presentation of the concept of part and its inverse – the concept of whole. That is the role of mereology. This branch has not yet received the status it deserves in the discipline of logic, and its linkages to other disciplines have not yet been worked out. In this book we will only be able to join this endeavour in a small segment, that which is related to metaphysics. I will present the main predicates and connectives of mereology, and will examine each of them by their RST characteristics.

 Beyond the RST characteristics I will not dwell at the present stage on the characterization of the concept of part. It is noteworthy that some authors tried to do that, not always successfully. Husserl, for instance, characterized the part as "dependent" on the whole. This characterization is incorrect, if not diametrically the opposite of reality. If the concept of dependency is relevant to our discussion, it is the whole that depends on the part, not vice versa. The wall can exist without the house but the house cannot exist without the wall, at least not in its present shape. Admittedly, the failures of my predecessors do not have to deter me from suggesting better characterizations, but I will be able to construct the rigorous logical definition of part only later on, after having developed the manifold theory (below, at ###). In that chapter the concept of part will be defined by the use of a more foundational concept – that of component. That definition, in turn, will be possible only after I have introduced some other new concepts in the forthcoming chapters. As long as we have not reached that stage, the concept of part will be taken as a primitive. It is important to remember, therefore, that this status is temporary. Nevertheless, all that I will say about the concept of part will remain in force after I reduce it to the concept of component.

**Proper parthood:**

The relation of (ordinary) parthood, or proper parthood, is that of the predicate "… is a part of …." It will be denoted by the letters PP, and for the moment will be taken as a primitive.

RST characteristics:

¬(x ⮭PPx)

¬(x ⮭PPy↔y⮭PPx)

x ⮭PPy˄y⮭PPzx⮭PPz

**Parthood-identity:**

The relation part-identical is that of the predicate "… is a part of … or identical to it." It will be denoted by the letters IP and will be defined as follows:

 x⮭IP;y ≡def x⮭ID;y˅x⮭PP;y

RST characteristics:

x⮭IPx

x⮭IPy↔y⮭IPx

x⮭IPy˄y⮭IPzx⮭IPz

I deliberated about whether I should follow in the way trodden by most of the recent works on mereology, that took parthood-identity as a primitive and defined proper parthood with it (for example: Simons, 1987; Casati and Varzi, 1999), or rather do it the opposite way. Since this choice has metaphysical implications, the decision may not be arbitrary nor based on parsimony considerations. I chose the relation of proper parthood as a primitive for purposes of symmetry with other component-manifold relations (the terms component and manifold will be expounded below).

 The inverse relation of proper parthood is wholeness, which will be read: "… is the whole of …" It will be denoted by the letters WH and will be defined as follows:

x⮭WH;y =def y⮭PP;x

Which implies that:

PP⮭INV;WH

The inverse relation of parthood-identity is wholeness-identity, which will be read: "… is the whole-identical of …" It will be denoted by the letters IW and will be defined as follows:

x⮭IW;y =def y⮭IP;x

Which implies that:

IP⮭INV;IW

According to the reduction rule, every two-place predicate might be reduced to a one-place predicate. This applies to the parthood concepts as well. One might say that the predicates IP, PP, WH and IW are by their very essence two-place predicates; but one can definitely reduce them to one-place predicates so that the sentences x⮭PP, x⮭IP, etc., are all WFFs. I am well aware, of course, that due to limitations caused by natural language we cannot imagine the concept of part as a one-place predicate; but logic obliges us to overcome such limitations. In order to make it easier for the ear, the reader might take here, too, the advice I gave above regarding the concepts of identity and nonidentity, and imagine them as having open objects: "… is a part of something," "… is a whole of something…" etc. But this advice is meant only for the adaptation stage, while in truth these two predicates are full-fledged one-place predicates, lacking any objects. Later below (at ###) I will explain it more accurately.

 The authors of axiomatic set theory took the relation denoted by the membership sign ∊ , i.e. the predicate "… is a member of…," as a primitive and believed they could thereby avoid the essential characterization of the foundational terms "set" and "member." Those were defined according to their placements at the sides of the membership sign. It was also presumed to be a parsimonious move, in the spirit of Ockham's razor. However, as we know, in continuation of what we have seen above, the premise underlying this move is wrong, since the two-place predicate"… is a member of …" is a logical enlargement of the one-place predicate "… is a member." The same is true for the predicate "… is a part of…." The mereologists believed they could avoid the essential characterizations of the predicates "part" and "whole," and attain the advantages of formality and parsimony. However, the two-place relation of parthood is nothing but an enlargement of the one-place concept of parthood, that does not have a parallel in natural language.

**Intersection:**

The operator of **intersection** will be denoted by the sign ∩ and will be defined as follows:

x∩y=def ɩz∀w((w⮭IP;x˄w⮭IP;y↔w⮭IP;z)

When serving as a predicate, this relation is often named **overlap**. It will be denoted by the letters OL and will be defined as follows:

x⮭OL;y≡def ∃z(z=x∩y)

RST characteristics:

∀x x⮭OL;x

∀x,y x⮭OL;y↔y⮭OLx

¬∀x,y,z x⮭OL;y˄y⮭OL;zx⮭OL;z

The intersection of a whole and its part renders the part:

x⮭IP;y  x∩y=x

Union:

The operator of **union** will be denoted by the sign ∪ and will be defined as follows:

x∪y=def ɩz|∀w((w⮭PP;x˅w⮭PP;y↔w⮭IP;z)

We can also denote it by the conventional sign of sum, the (ordinary) sigma, σ. Let us define:

σ(x,y,….) =def x∪y∪…

When serving as predicate, this relation is often named **underlap.** It will be denoted by the letters UL and will be defined as follows:

x⮭UL;y≡def ∃z (z=x∪y)

or:

x⮭UL;y=def ∃z(x⮭PPz˄y⮭PPz)

RST characteristics:

∀x x⮭UL;x

∀x,y x⮭UL;y↔y⮭UL;x

∀x x⮭UL;y˄y⮭UL;zx⮭UL;z

Note: some might argue that union ought to be a primitive and parthood ought to be inferred from it. But that would create a problem, as it is difficult to define union informally. If we were to define it as a combination of any pair of objects, we could confuse it with other forms of manifold, and especially with the relation of itemhood (explained below).

The union of a whole and its part renders the whole:

x⮭IP;y  x∪y=y

Difference:

The operator of difference will be denoted by the sign - (minus) and will be defined:

x-y=def ɩz(y∪z=x)

or alternately:

x-y=def ɩz(z⮭PPx˄∀w(w⮭IPy¬w⮭IPz))

While serving as predicate it will be denoted by the letters DF and will be defined as follows:

zDF;x,y df z(z=x-y)

RST characteristics: Regarding the RST characteristics of the relation of difference we need to cope with the question of the existence of an "empty object." In mathematics we affirm the existence of such an object: = the zero. We do so too in set theory – the empty set; in mereology, however, the notion of empty object has not been accepted, and I see no good reason to divert from this position. Since different disciplines apply different laws, this asymmetry is not a grave deficiency. Anyway, we should not resolutely embrace any position before we study this topic extensively.

**Complement**:

The complement of an object to another object is the difference between the latter and the former. We will denote the complement with the letter C accompanied by the object to which it complements, put in brackets, and then the object from which it complements. Thus we can define the complement as follows:

C(x)y=def x-y

The predicate of complement will be denoted by the letters CM. We can define as follows:

z⮭CM;x,,y ≡ def z=x-y

The concept of complement will be discussed in greater depth below (at ###).

**Disjointness**:

Two objects are **disjoint** when they do not have any part in common, i.e. when there is no overlap between them. The predicate of disjointness will be denoted by NOL and will be defined as follows:

x⮭NOL;y≡def ¬(x⮭OL;y)

RST characteristics:

∀x¬(x⮭NOL;x)

∀x,y x⮭NOL;y↔y⮭NOL;x

¬∀x x⮭NOL;y ˄ y⮭NOL;zx⮭NOL;z

Now we can define the operator of **discrete union**, which will be denoted by the sign + and will be defined as follows:

x+y =def x∪y|x⮭NOL;y

The sum of x+y will be named the **discrete** sum and, in the arithmetic context, the **arithmetic sum**, or just **sum**.

We often find the following as the axioms of classical mereology:

The **axiom of union**:

∀x∀y∃z z=x∪y

The **theorem of complementation**:

∀x∀y∃z x⮭PP;yz⮭PP;y˄z⮭NOL;x)

Which implies that:

∀x∀y x⮭PP;y ∃z(z≠x) z⮭PP;y

In current literature we often find the mereological axiom of extensionality:

∀x∀y∀z x=y ↔ (z⮭IP;x↔z⮭IP;y)

I would be happy to accept this axiom, that could be helpful for our discussion below, but unfortunately it is wrong. One can build, for example, different buildings from the same collection of Lego bricks. Similarly, the broken statue and the built statue are made of the same parts, and one cannot say that they are identical. We can mitigate the force of this axiom and so make it true by either of the following ways: (1) to suffice with material implication instead of biconditional or (b) to state that it is true only when another condition is added, i.e. that the boundaries between the parts are the same boundaries. This means:

(1)

∀x∀y∀z x=y  (z⮭IP;x↔z⮭IP;y)

(2)

∀x∀y∀z x=y ↔ (z⮭IP;x↔z⮭IP;y˄(∀w∀v (v⮭IP;x˄w⮭BDz,v)↔(v⮭IP;y˄w⮭BD;z,v))

Simons (1987) suggests as an axiom the Proper Parts Principle, according to which:

∃z(z⮭PP;x)˄∀z(z⮭PP;xz⮭PP;y)z⮭IP;y

Namely, if two objects have common parts, either they are identical or one is a part of the other, However, we have to reject this principle on the very same grounds we rejected the axiom of extensionality.

 Simons (1987) also suggests the Strong Supplementation Principle, according to which: x⮭IP;y↔∃zz⮭IP;x˄z⮭NOL;y)). This principle, too, we will be able to accept only after we make a small change in it:

x⮭PP;y↔∃zz⮭PP;x˄z⮭NOL;y))

Namely, if an object is not a proper part of another object, it has a part that is disjoint from that object.

### Parthood and boundaries

Having clarified the foundational concepts of part-whole relations, we can now return to the issue of boundaries. What is the boundary that passes between the whole and its part? Let's take again the sheet divided into two halves, black and white, and discuss the relation between the whole sheet (the whole) and its black half (the part). Let's suppose that the entire sheet is a rectangle delineated between the points A,B,C,D, while the boundary line between the black and the white parts crosses it along the line stretched between points E,F. What is, then, the boundary between the whole and the black part?

 At first glance, the more plausible answer is that the boundary is the line E-F. However, this line separates between the black half and the non-black part of the sheet, i.e. the white one, while we are looking for the line separating between the black half and the whole, that contains both the black and the white halves. Any other attempt to answer this question will lead us to absurdities, even greater than the above. What, then, is the boundary line?

 Here we have to return to the broader and more principled definition of boundary. A boundary is not just the physical "border" that separates between the object and its environs, but any property that discriminates it from "the rest of the world." In this case, lacking a physical line of separation, we return to the properties of the objects, which are the concepts that capture them. The black is captured by the concept "proper part of the sheet," and the sheet is not; the sheet is captured by the concept "proper whole of the black half," and the black half is not. Furthermore, the sheet is captured by the concept "half black half white," while the black half is captured by the concept "all black." The dimensions of the sheet are 210mm x197mm, while those of the black half are 210mm x 148.5mm. These are all boundary-determining concepts.

 Now we will combine the concepts of parthood with those of boundary. The **partial boundary** of an object is simply a part of its full boundary. It will be denoted by the letters PBD and will be defined as follows:

x⮭PBD;y =df (x⮭PP;z˄z⮭BD;y)

A partial closure of a given boundary is a part of the object bounded by that boundary. It will be denoted by the letters PCL and will be defined as follows:

x⮭PCL;y =df (x⮭PP;z˄z⮭CL;y)

No let us turn to the predicates of common boundaries. Two objects x and y will be described as **co-bounded** if they have a full or partial boundary in common. Of course, we can talk about more than two objects, but for our present purposes two will suffice. We will denote the concept fully co-bounded with the letters CBD and the concept partially co-bounded with the letters CPBD, and will define them as follows:

x⮭CBD;y ≡def ∃z x⮭BD;x˄z⮭BD;y

x⮭CPBD;y ≡def ∃z x⮭PBD;x˄z⮭PBD;y

CBD and CPBD are symmetric predicates:

∀x∀y x⮭CPBD;y↔y⮭CPBD;x

It is proper to distinguish between objects that are co-bounded "from within," i.e. when one object is a part of the other, and those that are co-bounded "from without," i.e. the two objects are disjoint. This distinction, however, is not required for our discussion so I will not formalize it.

 Now we can introduce the **axiom of common partial boundary** (the theorem of common full boundary will be presented below):

∀x∃y x⮭CPBD;y

We may also state that:

∀x∀y x⮭PP;yx⮭CPBD;y

It should be emphasized that the boundary discussed here is the boundary mentioned above, i.e. not necessarily the simple physical boundary. For if we were dealing with the physical boundaries alone, we could imagine a state of affairs in which there is no common boundary at all between the whole and its part, for example when the part is located at the midst of the whole and has no tangency with the whole's environs. (If someone seeks a truly concrete example: My navel does not have any common physical boundary with the boundary of my entire body, but only with other parts of my body.) On the metaphysical level, in contrast, it is clear that the whole and its part have common properties that discriminate both of them in the same way from the rest of the world. The first and simplest common concept to capture both of them, even before we get into more specific properties, is the fact that both of them constitute part-identicals of the whole. If x⮭PP;y is true then so are both x⮭IP;y and y⮭IP;y. In this sense, indeed, every two objects where one is the part of the other share a common capturing concept, and consequently a common boundary.

 Of course, the opposite cannot be said: It is clear that not all the objects that have common boundaries are engaged in part-whole relations. It should be emphasized that from a logical perspective it is the boundary that discriminates the object from "the rest of the world," and in this issue there is no difference between a physical boundary and a concept-dependent one. Furthermore, the physical boundary is nothing but a particular case of concept-dependent discrimination.

In view of this we may recall the axiom of boundedness:

∀x∃y x⮭CL;y

Namely, every object is the closure of some boundary (the world is no exception because I mean any sort of boundary, not just the physical one). In other words, every object has a boundary that discriminates it:

∀x∃y y⮭BD;x

Put differently:

∀y∃x y⮭CL;x

And, as I have noted above, according to one of the auxiliary theorems of the reduction rule we may also state that every object is a closure (of something):

∀x x⮭CL

Before closing this subchapter I would like to present the concept of continuity. An object will be characterized as continuous iff each of its parts has a common boundary with some other part of it. The predicate "continuous" will be denoted by the letters SCN and will be defined as follows:

x⮭SCN ≡def ∀y y⮭PP;x∃z(z≠y) z⮭PP;x˄z⮭CPBD;y

The concept of continuous should be distinguished from another concept, that of predicative continuity, which will be defined below.

We will return to some of these definitions in our further discussions.

### The world and its parts

Having defined the relation of parthood we can now define three entireties of objects:

The first is the world. **The world** is the entirety of all objects. We will denote it by the letter u and will formally define it as follows:

u= def ɩx|∀y x⮭IW;y

**The Big Concept** is the entirety of all the concepts. We will denote is by the letters UK and will formally define it as follows:

UK= def ɩx|∀y y⮭Kx⮭IW;y

**The Big Individuum** is the entirety of all the individua. We will denote it by the letters ui and will formally define it as well (even though it won't be very useful):

ui= def ɩx|∀y y⮭Ix⮭IW;y

The following definitions might also be offered for the Big Concept and the Big Individuum:

UK=x (x⮭K)

ui=x (x⮭I)

And the world is in fact the sum of the Big Concept and the Big Individuum:

 u = UK+ui

It should be noted that the world is also an individuum, and therefore constitutes a (singular) instance of the concept u↑.

Note, that all the above definitions are purely formal, even though they denote particular objects.

Allegedly, the Big Concept might also be defined in a different way:

UK=def ɩx|∀y y⮭x

This is so because the Big Concept is the "something" or the "being" (in Aristotle's terms), and therefore every object is captured by it. However, as we will see below, there are some other concepts that capture all the objects, and therefore that definition might be misleading. (It should be noted that x⮭UK does not read "x is the Big Concept," but "x is something"; in the sentence "x is the Big Concept" x is not an object captured by the Big Concept but a concept identified with the Big Concept, and therefore that sentence should be formalized as: x=UK). In fact, the Big Concept UK fully overlaps the concept O, since being something and being an object are one and the same thing, and if we replace one with the other we won't lose anything. We can therefore state the identity:

UK=O

Yet, for now we will keep using these two concepts separately according to our needs, until we will be called upon to make necessary reductions.

The Big Concept serves us also as **the map of concepts**. The map of concepts is a metaphoric expression aimed at describing the entirety of the concepts in their mutual relations. If the Big Concept covers the entire area of the map, all the other concepts are its parts. These parts are discriminated by numerous lines crossing each other's paths in different ways. These intersections create overlaps, disjointnesses and other relations. Those relations actually map the relations between the various concepts in the world. It should be emphasized that the concept of the real ontological sphere (in contrast to those of other ontological spheres, as explained in the second section), contains all the possible concepts. That is, to include all the properties and relations that might exist in the world, but not the impossible ones, such as those that are self-contradictory, since the map cannot express the intersection of disjoint parts. Following Hume we can state that the ability to have in mind a certain concept is a sufficient condition for acknowledging it as a possible concept, but not necessarily vice versa. Therefore, if some concept exists in our thoughts, it has a parallel in the real map of concepts. This topic, obscure as it may seem at this stage, will be clarified in the second section.

Now that we have defined the concepts presented above we can prove that:

∀x x⮭K↔ x⮭IP;UK

∀x x⮭I↔x⮭IP;ui

And therefore we can also state that:

∀x (x≠u) x⮭PP;u

Or

∀x x⮭IP;u

We will now turn to two important axioms: the axiom of the part and the axiom of the whole.

The **axiom of the part**: Every object has a part.

∀x∃y y**⮭**PP;x

Hence we can infer the **principle of infinite partition**, known in contemporary mereology as the theory of atomless gunk. This book embraces this theory, not in the physical sense but in the logical one (as for the physical sense, I will not take sides here). Even if physically there is an atomic object that is indivisible, from a logical perspective one can still talk about its upper part in contrast to its lower part, its left side in contrast to its right side, and similar differentiations. Namely:

:

∀x∃y x⮭WH;y

Using the reduction rule we may infer:

∀x x⮭WH

This theorem, stating that each being is in fact a whole, could also be taken in a poetical or even theological sense, but we will focus here on its logical and metaphysical meanings: Every object is a whole of another object.

Since the way of partition is not restricted, the principle of infinite partition entails*, in adjecto*, the **principle of free partition**.

**The axiom of the whole**: every object has a whole.

∀x∃y y⮭IW;y

Namely:

∀x∃y x⮭IP;y

And, according to the reduction rule,

∀x x⮭IP

Or:

∀x(x≠u)∃y y⮭WH;y

Namely:

∀x(x≠u)∃y x⮭PP;y

And, according to the reduction rule:

∀x(x≠u) x⮭PP

This theorem, too, might be taken poetically, to say that except the world – the entirety of being – "no man is an island," and no object is an island; we all are parts of the "continent" which is the world or one of its parts. But here, too, we will suffice with the logical and metaphysical meanings of this theorem.

### Complementation

So far we have discussed the relation of complementation between objects in a general way. Now it is time to address particular forms of complementation.

**Universal complement**: The universal complement of an object is the entire "rest of the world" outside of it. In principle it should be denoted by C(u)x, bot for the sake of brevity we will allow ourselves to omit the sign of the world and to write: Cx

Even though the predicate CM (complement) is a three-place predicate, there is no problem with the *universal* complement being a two-place predicate. The latter will be denoted by the letters UCM and will be defined as follows:

x⮭UCM;y ≡def x=Cy

It might also be defined in the following ways:

x⮭UCM;y ≡def σx (¬x⮭IP;y)

x⮭UCM;y ≡def σx (x⮭NOL;y)

RST characteristics:

¬x⮭UCM;x

x⮭UCM;y↔y⮭UCM;x

¬(x⮭UCM;y˄y⮭UCM;zx⮭UCM;z)

This is the case since if x⮭UCM;y˄y⮭UCM;z then x=z; and since x cannot be the complement of itself, the entire sentence is false.

Now we can state the **theorem of universal complement**: Every object has a universal complement.

∀x∃y x⮭UCM;y

And if we apply an auxiliary theorem of the reduction rule we will have:

∀x x⮭UCM

In words; Every object is a complement. And now we can formulate a theorem that has previously been presented informally: Every boundary of an object is also the boundary of its complement:

∀x x⮭BD;y↔x⮭BD;Cy

**The theorem of the additional part**: If an object has a (proper) part, it has at least one more part that is disjoint to that part:

∀x,y x⮭PP;y∃z(z⮭PP;y˄z⮭NOL;x)

Proof: Every object except the world is a proper part of the world (above, ###), and therefore has a complement (by the theorem of universal complement). This complement is not the world, since the world cannot complement itself.

Theorem:

∀y¬∃x x⮭CM;y,y

Theorem:

¬∃x x⮭UCM;u

Proof: This follows from the previous theorem.

In addition we may state the following:

∀x,y x⮭UCM;yx⮭CBD;y

And therefore we can state the **theorem of common full boundary.** Every object has a full common boundary with another object:

∀x∃y x⮭CBD;y

Theorem:

∀x∃y x⮭CBD;y˄x⮭NID;yx⮭UCM;y

We can also prove that if two objects are fully co-bounded then they are either identical or complementary to each other:

x⮭CBD;y↔ x⮭ID;y⊕x⮭UCM;y

Hence we can re-define the universal complement:

x⮭UCM;y ≡ x⮭CBD;y˄x⮭NID;y

Namely, the universal complement of an object is the object that is fully co-bounded with it and is nonidentical to it.

Now we will define two other important types of complement: the individual and the conceptual.

We have already defined the Big Individuum: ui=x (x⮭I). Now let us define the **individual complement**. The individual complement of a given individuum is the complement of that given individuum to the Big Individuum. For the sake of brevity we will denote it by CI and will define it as follows:

CIx=def C(ui)x= ui-x

We have also defined the Big Concept: UK=x (x⮭K). Now let us define the conceptual complement. The conceptual complement of a given concept is the complement of that given concept to the Big Concept. For the sake of brevity we will denote it by CK and will define it as follows:

CKx=def C(UK)x= UK-x

Some of these definitions will serve us in later discussions.

### Free union and semi-free intersection

The time has come to discuss the **principle of free union**. This principle has been dealt with extensively in scholarly literature. Van Inwagen (1990, p.74) and Van Cleve (2008) named it the Principle of Universality, Chisholm (Van Cleve, 2008, p. 335) called it conjunctivism and David Lewis (1986, p. 211-213) preferred Unrestricted Composition. (The reason I prefer to coin a new term is that the relation of union is a particular case of a broader category that will be presented below, the category of composition, and therefore, for the purposes of further analyses, should appear in the name of the principle). This principle says that any two objects may be considered as parts of a third object:

∀x,y ∃z z=x∪y

Namely:

∀x,y ∃z x,y⮭IP;z

But if we exclude the world from being x or y we can make an even stronger statement:

∀x,y (x,y≠u) ∃z x,y⮭PP;z

It should be emphasized: We exempted ourselves from the requirement of self-connectedness (or continuity) and so all of the above applies to any two objects, including those that have no connection (or tangency) between them. Thus, the tip of Cleopatra's nose and the dark side of the moon might be considered as two parts of some whole.

Hence we can infer that any two objects entertain a relation of underlap:

∀x,y x⮭UL;y

And hence we can infer:

∀x,y,z x=y ↔ z⮭PP;x↔z⮭PP;y

This might be considered as a mereological parallel of the axiom of extensionality in set theory. We will see further below that this linkage is not coincidental, since both of the principles are subject to a higher axiom: The principle of free composition.

 I would like to emphasize that all these unions exist in the world. The fact that we see only some combinations as "wholes" is nothing but a matter of decision. Let us take the example given above, of the tip of Cleopatra's nose and the dark side of the moon. There is no justification for seeing them as two separate objects any more than there is for seeing then as one object with those parts. I have already given the examples of the Smith family, whom we can see as one object for some purposes or as a few objects for others, as well as the landmass of planet Earth. There are many other examples: We can treat a pair of shoes as one object or as two, and the same is true for nations, archipelagos, sheep herds and numerous other things. From the logical and metaphysical perspectives there is no difference between these examples and two objects that are more remote and more different, less tied by spatial or functional links.

 Admittedly, this is not the common intuition: If we ask several people from the street whether the tip of Cleopatra's nose and the dark side of the moon are two objects or one, we will probably get a hundred percent of answers supporting the former option. However, if I present those people with a wall and ask them how many objects they see, most of them will answer one, even though the wall can be divided into two halves and an even greater number of bricks. Here, too, from a logical or metaphysical perspective there is no justification for seeing the upper and the lower halves of the wall as parts of one object rather than seeing them as two different objects. If we were to ask our respondents to account for their answers, and if they could give such accounts, we may assume that they would suggest criteria based on sensory qualities (colours, texture) or functional qualities (if they are parts of the same device designed to separate between rooms in the house or between the house and its exterior). Yet, all those criteria could be superseded by others if our sensory mechanisms or our functional needs were different. Both our sensory mechanisms and our functional needs do not belong to the realms of logic and metaphysics, but to the realm of decision.

 Consequently, the decision to see two objects as parts of one object will be named **unifying view** and the decision to see them as two separate objects – **dividing** **view**. The principle of free union is the ontological principle that enables both of these views.

 I think that all this is beyond any doubt, and indeed any two objects might be taken as parts of a single whole. Even if there could still be found someone who disagrees with that (Simons, for instance), we will certainly be able to agree that there can be two meanings of the unity of the object: one that requires continuity or other linkage between the parts and one that does not. In view of this, the whole argument is about words, and if so I can simply determine that I will use the term "one" in its latter sense.

**The principle of semi-free intersection**:

∀x∀y (¬(x⮭NOL;y)∃z(z=x∩y))

Every two non-disjoint objects can have an intersection. All those intersections exist in the world. The fact that we view only some objects as intersections of several objects and do not act this way with others is a matter of decision.

Now we can make another statement:

**The theorem of universal dual conceptualization**: Any two objects have a concept that captures them.

∀x,y∃Z Z⮯(x∪y)

Proof: Any two objects are one (by the principle of free union); but every object has a concept that captures it (by the axiom of universal capturing); hence, any two objects have a concept that captures them. QED.

Hence we can easily prove the broader argument, **the theorem of universal conceptualization**:

∀,x1,x2…xn ∃Y (Y≠UK) Y⮯x1 ˄ z⮯x2… z⮯xn

The proof for this theorem is the same as that of the dual conceptualization, with the change of the number: Just as two objects can be considered as parts of a single whole, so can be any other number of objects.

On this occasion I would like to present another proof for the theorem of universal dual conceptualization, which proves something beyond itself.

Above (###), I presented the axiom of relation, stating that between any two objects there exists a relation.

∀x,y∃Z x⮭Z;y

But a relation is an object (of concept type), and therefore between the relations there exist relations. Hence we may state:

∀x1x2,y1,y2,Z1,z2,∃W1 x1⮭Z1;y1˄x2⮭Z2;y2 Z1⮭W1;Z2

But if so, then this is also true:

∀x2x3,y2,y3,Z2,z3,∃W2 x2⮭Z2;y2˄x3⮭Z3;y3 Z2⮭W2;Z3

Now, taken that the consequents of the two above sentences hold, the consequent of the following is also true:

Z1⮭W1;Z2˄Z2⮭W2;Z3  ∃X'1 W1⮭X'1;W2

Namely, between the relation concepts that stand between the relation concepts – there is a relation; and so on and so forth with regard to any number of relation concepts.

 Hence we may infer that in any collection of objects, whether finite or infinite, there is one concept that overarches them all, whether directly or indirectly. This is the case because for any number n of objects, there exists a (minimal) number of n-1 two-place relations between them; and between those n-1 relations there exists (according to the same rule) a (minimal) number of n-2 relation concepts, and so on, until we reach the top of the pyramid, where there is always one uppermost relation between the two almost-uppermost relations. QED.

 Indeed, this statement has far-reaching metaphysical bearing. A concept is, in essence, general, as it may apply to multiple objects, and therefore the existence of one unifying concept for all the objects in the world entails the existence of a single principle of order for all the world. While philosophers and theologians were taken by the unitary order that prevails in the world, we have proven that this order is nothing but a logical necessity. Furthermore, the implicit premise of those philosophers and theologians was that disorder is the "basic state," so to speak, while the order is something innovative that requires explanation and that has to be created by some deliberate act. However, here we have proved that order is a logical necessity that cannot be non-existent, and therefore nothing is more "basic state" than it.

 This book is not about theology, but since we have touched upon the issue of order I will add parenthetically that the theorem of universal conceptualization also has bearings on the concept of miracles. A miracle in its simple popular sense, that of an event that breaks the *physical* order of the world, requires a separate discussion. A possible argument, however, that a miracle breaks any order whatsoever should be ruled out, since any breach of order will immediately entail the conceptualization of another order, in which the seeming breach be also be integrated into the realm of the extension of some single general concept.

## Mereology of concepts

### On the modes of partition of concepts

The starting point of this chapter will be that the main principles of mereology apply to concepts as well. I have already embodied this premise in our discussion by the very fact that I dealt with mereology as applying to "objects" in general, without distinguishing between individua and concepts. Since this is not an altogether new discipline, we do not need a new terminology, and our terms will be based on those presented above. (I will add parenthetically that mereology might also serve as a very good basis for a new set theory, intensional, but its development will lead us far beyond the scope and purposes of this book).

 Mereology, just as the topology of boundaries, applies to concepts just as much as it applies to individua. This applying is not artificial, nor is it metaphoric, and its justification is not that "it works great," but is postulated by the simple fact that concepts are also objects.

 The partition of concepts has been already discussed by David Armstrong (1978, 1989) and others, but has received an excellent formal development by Uwe Meixner (1997) and recently by Edward Zalta (2020). I follow in their footsteps, while integrating their insights into the area of discourse of this book. In my following words I will try to discuss this issue using "professional" terminology, but from time to time I will allow myself to work with the metaphor of the map of concepts. As we have seen, according to that metaphor, one may imagine the entirety of all the concepts as a two-dimensional map divided by numberless lines crossing each other in all directions: length, breadth, diagonal and crooked. (The normal eye has difficulty to grasp all this, but here we don't have to see but only to imagine – and that is supposed to be easier). Every such line cuts the map and creates a geometric shape, and every such shape is crossed by other lines, that belong, in their turn, to other geometric shapes. This map represents the concepts. The entirety of the area of the map represents the Big Concept, and all the rest are the partial concepts, their parts and the parts of their parts, created by the partition of the Big Concept.

 A concept, just as any other object, might be partitioned, even infinitely:

∀x∃y x⮭K y⮭PP;x

This is inferred from the axiom of partition.

Thus, for instance, the concepts of parthood themselves maintain a mereological relation between them. Since the concept of identity-parthood includes the concept of proper parthood, we may state:

PP⮭;PP;IP

Let us begin with a presenting of the relations already known to us in ordinary mereology, and putting of them in the context of concepts.

The complementary concept of x renders the concept of all that is not x:

y⮭CKx ≡ ¬y⮭x

The union of concept x with concept y renders the concept of that which is x or y:

z⮭(x∪y) ≡ z⮭x˅z⮭y

The intersection of concept x with concept y renders the concept of that which is both x and y:

z⮭(x∩y) ≡z⮭x˄z⮭y

The principles of free union and semi-free intersection apply to concepts just as they do to any other objects. Yet, here we should beware of mistakes: The union of two individua x and y is the individuum z, which constitutes the sum of both of them. But the union of the concept of x with the concept of y does not render the concept of c, but the concept of "that which is x or y." Thus, for example, the kitchen knife is composed of two parts: the blade and the handle. But the union concept of the concept of blade and the concept of handle renders the disjunctive concept of "that which is a blade or a handle." This concept does not capture the (whole) knife, since a knife is neither a blade nor a handle. And the same rule applies to partition, intersection, complementation etc.

For that matter, propositions "behave" like concepts, not like individua.

From a metaphysical perspective, all the parts of a concept are within it; however, in order to discriminate all those parts we have to carry put an act of partition. The "line" that divides the concept into its parts will be named **conceptual partitioner**, or, for short, a **partitioner**. This dividing "line" is itself an object, and so is captured by a concept, i.e. an object that might be expressed as a general function. We will mention three possible types of partitioners (and maybe there are more): Arbitrary partitioner (the existence of which I will negate, following our discussions above), intersection-forming partitioner and relational partitioner. But beforehand I will say a few words not about partition but about complementation and union.

I have already noted that the complement is virtually the negation:

x⮭CKy¬(x⮭y)

But we can make a stronger statement:

x⮭IP;CKy¬(x⮭y)

This sentence is also true in the other direction:

¬(x⮭y) x⮭IP;CKy

Therefore we can present the two material implications as one equivalence:

¬(x⮭y)≡ x⮭IP;CKy

This sentence does not contradict Brentano's thesis (to be addressed below); on the contrary, the two converge beautifully. Brentano's thesis says that:

x⮭y∃z z⮭x↑∩y

Adding the rule of basic existence, we may now state:

¬x⮭y ≡ ∃z z⮭(x↑∩IP;C(UK)y)

But we could also state that:

¬x⮭y ≡ ∃z z⮭((IP;C(UK)x)↑∩y)

This assertion might puzzle some of the readers. Yet, this is the place to remind ourselves that I do not attribute y to x but only assert that they cut into each other (i.e. they have a common intersection). Furthermore, I do not deal with the complement of individuum x but with the concept of that complement (as expressed by the sign ↑).

Since the sign IP;CK – that may be named the **partial conceptual complement** – is supposed to be used frequently, and since its present form is quite long, we will agree on an abbreviated sign for it: C'K. We will therefore define it as follows:

x⮭C'K;y ≡def x⮭IP;CKy

In a similar manner we may also abbreviate the sign IP;CI. It will be named the **partial individual complement**. It will be denoted by C'I, and defined as follows:

x⮭C'I;y ≡def x⮭IP;CIy

Allegedly, the meaning of this concept is nothing but disjointness, but this is not so. An object is disjoint to another when it is a part of the latter's universal complement, while here we are talking about a part of its individual complement. Therefore we cannot argue here for equivalence, but only for material implication:

x⮭C'I;y  x⮭NOL;y

Of course, these relations are symmetric:

x⮭C'K;y≡y⮭C'K;x

x⮭C'I;y≡y⮭C'I;x

And now we can state the following:

∀x,y x⮭C'K;y↔¬∃z z=x∩y

And hence we can infer the **intersection rule**:

∀x,y ¬x⮭C'K;y↔∃z z=x∩y

This is trivial, since it actually says that if two objects are not disjoint to one another they intersect with one another.

This is also true in the other direction: If two concepts capture the same object, then they are not disjoint to one another, i.e., there is an overlap between them:

∀x,y,z x,y⮯zx⮭OL;y

An object that is disjoint to another is necessarily captured by at least one concept that is disjoint to the concept that captures its fellow object. Therefore we may formulate a rule:

∀x x⮭C'I;y  ∃z x⮭z ˄ y⮭C'K;z

However, we may formulate it in an even stronger way, which we will name the **rule of partial complements**:

x⮭C'I;y  y⮭C'K;(x↑)

Where x↑ is denoting any possible uniextensional concept.

The simple assumption in the mereology of concepts is that if some concept captures an object, its whole will also capture that object. We will name this assumption the **principle of enlarged capture**.

y⮯x˄(y⮭IP;z)→z⮯x

x⮭y˄(y⮭IP;z)x⮭z

Hence we can easily infer that if a concept captures an object, any union of that concept with any other concept will also capture that object. This is so because the union is actually the whole of both these objects. It is noteworthy that above we distinguished between the nuclear predicate, which parallels the predicate of the predicate calculus, and the articulated predicate, which also includes all the objects that relate to it in different ways ("predicative constituents"). What I said here applies also to the enlargement of the nuclear predicate:

x⮭y;z,,w,,,….x⮭(y∪v);z,,w,,,…

Now that we accepted the principle of enlarged capture, we can take it "to its end" and state:

∀x x⮭UK

I have already written this sentence above, but now we see it in its broader context. Since UK is the whole of all the objects, it captures every object. This way we have proven Aristotle's healthy intuition that the most general and most neutral property of "being" (or "something") is the property of every object. However, above we said the same thing about the predicate "object" (in the sense of a correlate of a concept), so we may conclude: Everything is an object and everything is something.

∀x x⮭O

∀x x⮭UK

∀x x⮭O≡x⮭UK

This equivalence will be named the **foundational equivalence**, and soon further constituents will be added to it below and in our following discussions.

 In fact we could go on to a stronger statement, asserting the identity UK=O. For the moment we will leave them separately, mainly for reasons of convenience, but later on we will make this move.

In this case we are talking about an intensional identity, since "being something" and "being an object" have the very same sense, and there is nothing in one of them that is not in the other. We should beware, however, not to mix up this condition with the condition in which two concepts are in extensional overlap that does not stem from intensional identity. By this I mean an overlap of the type given by Quine (1953) between chordates (creatures with heart) and renates (creatures with kidneys). A similar overlap might apply even to universal concepts, i.e., such that capture all objects. Thus, for instance, we stated above that every object is a whole and that every object is a part of the world. In this case it will be even more difficult to claim that the two concepts are of identical intension. Let us examine these statements:

∀x x⮭WH

∀x x⮭IP;u

Yet, according to the reduction rule we may simply say that:

∀x x⮭IP

There is no identity between the (Fregean) sense of "being a part of the world" and "being a part [of something]." Nor there is identity between any of these and the concepts of "being an object" or "being something." Therefore in this case we cannot state the existence of an intensional identity, but can definitely continue our foundational equivalence and state:

∀x x⮭O≡x⮭UK≡x⮭WH≡ x⮭IP

This equivalence can corroborate further what I wrote above, i.e. that there is no intensional identity between all of its constituents (i.e., "O=UK=WH=IP" is false). That is because if we add, for example, an enlargement called "a" to O and WH, then in the case of O the full predicate will become "an object of O" while in the case of WH it will become "a whole of a." Needless to say that these are not the same.

 Four other concepts that ought to enter the foundational equivalence are the concepts of identity, nonidentity, closure and universal complement. As we have seen above, the one-place predicates "identical," "nonidentical," "bounded" and "is a universal complement" capture every object. Consequently, we may broaden the foundational equivalence and state:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭WH≡x⮭IP≡x⮭CL≡x⮭UCM

This broadened version of the foundational equivalence is not the final one. We will broaden it even more, below. This equivalence leads us to the main purpose of this book.

### The ways concepts are partitioned

Now lets us examine the three modes of partition presented above:

The arbitrary partitioner: This mode of partition is not done by means of any object other than the "line" of partition itself. This "line" is what will become the boundary between the two parts of the partitioned concept. It is difficult to give examples for such a case, because, being patently useless, they are not extant in natural languages. But if we nevertheless wish to imagine an example, we might take the concept of "yellow" and decide that we artificially create a partition between two types of yellow, ungoverned by any general function whatsoever, and simply determine that one part of the "area" of the yellow (on the map of concepts) will be taken as "yellow A" and the other as "yellow B."

 However, in the light of our above discussions, it is clear that such a partitioner does not truly exist. As stated above, every part of a concept is an object, and every object is captured by a concept. A boundary is also an object, and so it is also captured by a concept. Consequently, it is no longer arbitrary.

The intersection-forming partitioner: One of the most widespread modes of concept partition is creating intersections with another concept. In the example given above, the concept "light yellow" is actually an intersection of ”yellow" and "light." Similarly, "yellow square" is an intersection of "yellow" and "square." Below we will see that even in a sentence like "the sun is yellow" there is an intersection of "sun" and "yellow" and the propositional structure affirms that the rendered concept captures an object.

A concept rendered by the intersection of two concepts will be named their **intersection concept**.

An intersection concept will be denoted by the letters CC followed by brackets in which the two concepts cut across each other will be written:

CC(x,y) =def x∩y

An intersection concept is always a part of the concepts of which it is an intersection:

x∩y⮭IP;x,y

The **full concept** of an object is the intersection concept of all the concepts that capture (in part, present or future) that object. Full concept will be denoted by the letters FK followed by the brackets flanking the object to which it relates. We will define it as follows:

FK(x)=def ∀y,z… CC(y,z…)|y,z,…⮯x

And we can state as a rule:

∀x FK(x)⮭IP;x↑

And therefore:

FK(x)⮭IP;TK(x)

But this is also true:

TK(x)⮭IP;K(x)

And therefore, according to the transitivity of IP, this is also true:

FK(x)⮭IP;K(x)

A concept might capture one object or many objects. When it captures many objects, we will wish to discriminate "this" (singular) object from other objects. The discrimination will be made by adding another property, i.e., by intersecting the capturing concept with another concept. If, for example, the concept "yellow" captures three objects, then in order to discriminate one of them from another we will have to intersect it with another concept, say "standing in the north-eastern corner of the room." By this, the object that is standing n the north-eastern corner of the room will become a singular object, and the concept that captures it will become uniextensional. The intersection concept of the concepts "Greek," "philosopher" and "lived in the 5th century BC" may probably capture a few personalities. We have to intersect it with more concepts in order to render the singular object of Socrates. Only by such an intersection we will get the uniextensional concept that Quine (1960) calls "socratizes," which is the predicate that captures only Socrates.

 It might seem that a full concept is a dynamic thing. It is dynamic because we take the term predication in its broadest sense. A predicate that applies to object x is not just one that relates to its essence, but to anything that might be attributed to it. Thus, for instance, Napoleon died in 1821 and since then no new "essential" predicates applied to him, except, maybe such that relate to the condition of his corpse. But if anyone wrote something about Napoleon, thought something about him, or otherwise related to him – any such action adds a new predicate to Napoleon as well. Therefore the full concept of Napoleon is a dynamic thing, and if so it is also relative to the point in time in which the predication is examined. Yet, we refer to the full concept in an a-temporal way, i.e., from the imagined perspective of an "infinite mind." By this, our understanding of the full concept is very close to that of Leibniz (and some might add Hegel, too).

 However, if this seems to entail that the full concept of an object x includes all the concepts that capture all the objects that entertain any relation with x, then the list is endless, since all the objects in the world entertain the relation "… is a part of the complement of…" with x. Indeed, this means that every full concept of every object actually includes all the full concepts of all the objects in the world (and Leibniz had already understood that). If we follow this line consistently, we will find that the full concept includes predications regarding the entire world. We should not be deterred from this conclusion. This is indeed the meaning of a full concept.

 In view of this, we seem to need a special term for the intersection concept of all the concepts that capture an object, except for those that are attributed to it via its partial conceptual complement. This concept might be named the **rich** **concept** of concept x. However, even if such a concept might be useful on the practical level, it has no logical justification, and therefore no ontological justification either. That is why it is difficult, if not impossible, to formalize it. The decision to include such and such concepts in some intersection concept and to exclude others is a legitimate decision under the principle of free union, but it is still a *decision* (in the sense we will ascribe to it below), and therefore is not more justifiable on the metaphysical level than any other decision. The full concept, in contrast, is a term of clear metaphysical justification, being independent of any subjective decision. Therefore we will not address the rich concept very often, and do not have a burning need to define it here formally.

**The relational partitioner**: Apart from intersection, there is another way of partitioning a concept, and that is the way of adding predicative constituents. By this I mean an object with which the nuclear predicate has some relation, usually in the status of objects (in the grammatical sense of this word). The verb "ate" in the sentence "Danny ate an apple" is a concept, but if so, then "ate an apple" is a part of this concept, since Danny could also eat an orange, a sandwich or a herring, and still remain under the concept "ate." The addition of such a constituent is not different, metaphysically speaking, from the addition of an intersection, as presented above. Indeed we have already stated (at ###), that the predicate "ate an apple" is a partial concept of the predicate "ate." Now we can discuss it more systematically.

### The partition of concepts through the predicative structure

The predicative structure provides us with a full-fledged concept. Now we can apply the mereology of concepts to it.

Suppose we have a sentence in the form

x⮭Y,¹z,²w,³v

According to what I wrote above (###), we can infer from it all the following sentences:

x⮭Y,¹z,²w

x⮭Y,¹z,³v

x⮭Y,²w,³v

x⮭Y,¹z

x⮭Y,²w

x⮭Y,³v

Every concept that is comprised of a nuclear predicate and a given number n of predicative constituents is a part of any concept that is comprised of the same predicate and a smaller number of those predicative constituents. This means that any addition of any predicative constituent in the above way is in fact an intersection of the predicate with the other constituents. In other words, an n-place predicate is a partial concept of an (n-1)-place predicate. Two different predicates containing the same nuclear predicate intersect each other and create the n-place predicate. (Admittedly, we have addressed here the concept of number before having discussed it systematically, but, as I noted several times above, this is typical of discussions on foundational concepts, since many of them "come together.")

We may set this as a rule, the **rule of the partition of the articulated predicate**:

X;y1….yn⮭PP;Xy1…ym(m<n).

I took here the predicate in its metaphysical form of a concept. Thanks to this perspective, the metaphysical one, we could view X;y as the whole of all the articulated predicates in this sentence.

 In truth, the partition through the addition of predicative constituents is nothing but another form of partition through intersection: If we only allowed ourselves to break out a little bit from the chains of natural language we could definitely characterize Y,²z (and not z), Y,³w (and not w), etc. as concepts, and their "combination" with x, as well as with one another, as intersections. Indeed, that is a desirable direction for the further developing of the concept calculus, but for the present discussion it is not a vital need.

### Union and partition of concepts and their relation to propositional operators

We will now turn to the "translation" of unions and partitions of concepts to propositional operators.

The rule of conjunctive distribution:

∀x,Y,Z x⮭Y˄x⮭Z↔x⮭(Y∩Z)

The rule of disjunctive distribution:

∀x,Y,Z x⮭Y˅x⮭Z↔x⮭(Y∪Z)

But here we can say something stronger.

**The rule of enlargement**:

∀x,Y,Z x⮭Yx⮭(Y∪Z)

It can be stated in a different way, still arguing a very similar thing:

∀x,Y,Z (x⮭Y˄ x⮭IP;Z x⮭Z

However, it is important to preserve the form of disjunctive distribution which (unlike the enlargement) is symmetric.

Hence:

∀X,y,Z (X⮯y˄X⮭PP;Z)Z⮯y

∀x,Y,Z x⮭(Y∩Z)x⮭Y

But since intersection is symmetric, the following is also true:

∀x,Y,Z x⮭(Y∩Z)x⮭Z

Before we continue, we should return to a topic discussed too briefly above.

### Identity and nonidentity in the perspective of the reduction rule

I wrote above (###) that even predicates such as identity and nonidentity, or parthood and wholeness, might be reduced to one-place predicates. Yet, only once we have formulated it systematically can we return to this topic and see its implications with regard to the mereology of concepts.

 In natural language, sentences like "The Eiffel Tower is identical" or "The Eiffel Tower is nonidentical" are taken as senseless sentences, maybe even not qualifying to be WFFs. The same is true for "Danny's head is a part" and "Danny's head is a whole." The Eiffel Tower is expected to be identical or nonidentical *to something*, and Danny's head is supposed to be a part *of something*. Only "Danny's head is [a] whole" might be conceived as meaningful, but that happens thanks to the double meaning of "whole" in natural language, where it denotes both the technical "union of parts" and "not broken." In its former sense, the one used in this book, "whole" is the other side of "part," and therefore every whole must be the whole *of something*, i.e. of some parts. The problem seems to deepen when we apply the reduction rule to both of the contradictory predicates: "Danny is identical to himself," therefore "Danny is identical"; but "Danny is nonidentical to Dina," and therefore "Danny is nonidentical." The outcome is that Danny is both identical and nonidentical at the same time. The same is true for Danny's head: "Danny's head is a part of Danny," and therefore "Danny's head is a part"; But "Danny's head is the whole of Danny's nose," and therefore "Danny's head is a whole."

 In truth, however, this problem is not real. It relates to the gap between natural language and the language of logic, which is also the language of metaphysics. As for the alleged contradiction: it may happen to many predicates: "Danny is building" and "Danny is demolishing" look like contradictory, and indeed they are, but if we add objects to them – say, "Danny is building his new house" and "Danny is demolishing his old dog kennel" – they do not contradict anymore (provided that Danny is able to carry out both of them at once).

 Now let's try to see what we said here in terms of the mereology of concepts, and draw the right conclusions. Let us take the above examples of "building" and "demolishing." If we examine the concept "demolishing" we will see, supposedly, that it is a part of the complement of "building" and is therefore disjoint to it. We might continue the same line and say that "building his new house" is a part of "building" and "demolishing his old dog kennel" is a part of "demolishing," and therefore we remain with two disjoint concepts that cannot intersect. In reality, however, we know that there is no contradiction between the fact that a man builds his house and demolishes his dog kennel, even at the same time. What, then, is the solution?

 The solution is found in the fact that "building" and "demolishing" are not truly disjoint concepts. They become disjoint only as soon as a common object is added to them. If we take Danny's new house as that object, then the problem starts when we talk about: "Danny's house is being built" and "Danny's house is being demolished" and indeed have two contradictory predicates for the very same subject. That is not the case when the common subject is absent.

 That is why in the map of concepts "building" and "demolishing" have an area of overlap. As soon as a common object is added to them, that object serves as a partitioner both for the concept of "building" and to that of "demolishing." It partitions the concept of "building" into two parts – let's call them "building1" and "building2" – and does the same to the concept of "demolishing," which is partitioned into "demolishing1" and "demolishng2." After the partition, building1 and demolishing1 become disjoint to one another; not so the rest.

 The same is true for the concepts of identical/nonidentical and part/whole. When on their own – as one-place predicates (that do not have parallels in natural language, but that does not negate their existence) – they are not disjoint, i.e. the sentences in which they both appear as predicates are not necessarily self-contradictory. Only when an object, serving as a partitioner, is added to them, some (but not all) of their parts become disjoint.

### Discrimination: The mereology of concepts and the determination of boundaries

In light of the mereology of concepts we can now return to discuss in greater depth the role of the boundary, i.e. the discrimination of the object. As we remember, the function of the boundary is that it separates two objects: the one from this side of the boundary and the one on the other. Let us imagine a world that is composed of two complementary objects, one white and the other black. In this case we will say that the discriminating function is black/white. If we now return to our world and suppose that the property discriminating the yellow box from the rest of the world is its yellow colour, we will be prone to say that the discriminating function is yellow/not-yellow. This, however, will not be true, since this function discriminates all the yellow objects in the world. We will need to say, therefore, that the intersection of all the properties that discriminate our box will be denoted by w, the discriminating function will be w/not-w. Supposedly, w is the full concept of the box, but in truth the full concept is too strong a tool for this purpose, and we may suffice with a uniextensional concept of this object. As we already know, it is not necessary that there be only one uniextensional concept for each object and objects regularly have many uniextensional concepts. Therefore w will be one of the uniextensional concepts of x. In our case – the box on the sidewalk – it will be convenient to take the properties discriminating the individuum in space. This concept has no logical advantage over other uniextensional concept; its only advantage is its usefulness.

 For our continued discussion on boundaries, let us improve our notation. We will continue to see the concept of boundary as primitive and keep the letters BD to denote it, but the function of (spatial) boundary will be denoted in brackets after those letters. The sign of negation will be denoted by the conceptual complement CK. If we wish to state that this function separates between x and y we will write:

x (w/CKw)↓⮭BD;y,,z

This function will be defined as follows:

x (w/CKw)↓⮭BD;y,,z ≡df ∃y,∃z(x⮭BD;y˄x⮭BD;z˄y⮭w ˄¬z⮭w)

While it is taken that:

x(w/CKw)=x(w)=x(CKw)

Now we can better formulate the axiom of boundedness:

∀y∃x∃w x⮭BD(w/CKw);y

As noted above, this boundary discriminates (or delineates) two objects: y and z. The inverse relation of discrimination is boundedness, as defined above (###). If x discriminates y and z, then y and z are bounded by x and will be called closures of x.

Now we can state:

x (w/CKw)↓⮭BD;y,,z ↔ y⮭CL;x(w/CKw)˄z⮭CL;x(w/CKw)

This notation, however, is too general, since it does not specify which object is bounded by virtue of which property. We will therefore accept as a graphic convention that from now on we will mark one closure by the property that discriminates it, and the other closure (the complementary one) by the (complementary) property that discriminates it. Thus, for example, if the uniextensional concept of y is w, then the conceptual complement of y, presented here as z, is CKw. When we wish to denote it we will write:

y⮭CL;x(w)

z⮭CL;x(CKw)

All this is under the semantic premise, agreed above, that x(w/CKw)=x(w)=x(CKw).

### Bequeathing

We now turn to discuss a new logical term: bequeathing. We will start with a preliminary definition, but we will allow ourselves to change it as the discussion develops.

A concept X will be characterized as **bequeathing-to-Ws** iff whenever it captures an object y it also captures any object z that maintains the relation W with y. Such a concept will be denoted by the letters BQ(W) and formally defined as follows:

X⮭BQ(W) ≡def ∀x∀y∀z (y⮭W;z(z⮭Xy⮭X))

Thus, for instance, the property "wise" will be considered as "bequeathing to granparents" iff whenever a person is wise his/her grandparents are also wise.

We will also define the opposite concept. **Non-bequeathing-to-Ws** means as follows:

X⮭NBQ(W) ≡def ¬x⮭BQ(W)

A concept X will be characterized as **weakly-bequeathing-to-Ws** iff whenever it captures an object y it also captures at least one object z that maintains the relation W with y. Such a concept will be denoted by the letters WKBQ(W) and formally defined as follows:

X⮭WKBQ(W) ≡def ∀x∃y∀z (y⮭W;z(z⮭Xy⮭X))

We will now define the opposite **Non-weakly-bequeathing-to-Ws** as follows:

X⮭NWKBQW) ≡def ¬x⮭WKBQ(W)

When we mention bequeathing without any addition, we will refer to the ordinary ("strong") bequeathing (in contrast to the weak one).

It is needless to say that the above mentioned concept W must be a two-pace predicate, but concept X is of any number of places. In this book we will mainly discuss the issue of bequeathing-to-components and manifolds (according to the manifold theory developed below). For the moment we will discuss the most basic type of component: part.

### Bequeathing-to-parts

A concept X will be characterized as **bequeathing-to-parts** iff whenever it captures an object it also captures all of its parts. Such a concept will be denoted by the letters BQ(PP) and formally defined as follows:

X⮭BQ(PP) ≡def ∀x∀y∀z (y⮭PP;z(z⮭Xy⮭X)

Thus, for instance, all the concepts of colours are bequeathing-to-parts: if an object is yellow, all its parts are also yellow. The concepts "smaller than the Eiffel Tower" and "absorbed with water" are bequeathing-to-parts. In contrast, the concept "is five centimeters long" is not bequeathing-to-parts, since the (proper) parts of a five centimeter-long object will always be shorter than five centimeters. Similarly, if the whole is square, there is no necessity that all its parts be square.

 We will also define the opposite concept. **Non-bequeathing-to-parts** means as follows:

X⮭NBQ(PP) ≡def ¬x⮭BQ(PP)

The next concept is weak bequeathing-to-parts. This concept will not be very useful for our further discussions, but nevertheless it is worthy of introduction.

 A concept X will be characterized as **weakly bequeathing-to-parts** (or: **bequeathing-to-a-part**) iff whenever it captures an object it captures at least one of its parts (and not necessarily all of them, as is the case with the ordinary bequeathing-to-parts). This concept will be denoted by the letters WKBQ(PP) and will be defined formally as follows:

X⮭WKBQ(PP) ≡def ∀x∃y∀z (y⮭PP;z(z⮭Xy⮭X)

When we mention bequeathing-to-parts without any addition, we will refer to the ordinary ("strong") bequeathing (in contrast to the weak one).

Can we say that every object has at least one property (i.e. one concept that captures it) which it bequeaths to all of its parts? Of course, when we talk about universal properties, that capture every object, the answer is positive. We have already seen at least four such concepts (and will see more below): O, UK, WH and IP;u. But I am asking the question about non-universal properties.

 If we were talking about bequeathing to-part-identical, the answer would be flat positive, but we are talking about proper part. Yet, in this case the answer is also positive. If the concept that captures the object is the one that determines the boundary, the whole and the part necessarily have a common boundary-part (see above, ###), and by this boundary they also share a common concept (they are both captured by the concept "having boundary x"). In a formalaic language:

∀x∀y x⮭PP;yx⮭CPBD;y

∀x∀y (x⮭PP;y(x⮭CPBDy˄y⮭CPBD;x)

∀x∀y (x⮭PP;y(x⮭CPBD˄y⮭CPBD)

And we can also prove that:

∀x∀y (x⮭PP;y(x⮭CPBDy↔y⮭CPBD;x)

It is therefore proper to conclude that "having a common partial boundary" is a property bequeathing-to-part.

CPBD⮭BQ(PP)

When a concept is bequeathing-to-parts, might we also say the opposite, that if an object is captured by that concept, its whole will also be captured by it. This state of things is called bequeathing-to-wholes, and will be discussed below, but now we can already answer the question in the negative: If a brick is captured by the concept "yellow," it does not necessitate that the entire wall (or the entire building, or the entire world) be yellow, and the same is true for the concept "up to thirty centimeters high."

 And what about the concepts themselves? If a concept X is bequeathing-to-parts, will its part also be bequeathing-to-parts? Will its whole also be? As for the part, we can answer negatively. Instead of delving into explanations, we can give an example: The concept UK – the Big Concept – is certainly bequeathing-to-parts. If X is a being, then any part of it is also a being. But can we say that any part of UK is bequeathing-to-parts? This is absurd, since all the concepts in the world are its parts, and that would imply that all the concepts in the world are bequeathing-to-parts. This conclusion is, of course, unacceptable. We can therefore conclude that the part of a concept that is bequeathing-to-parts is not necessarily bequeathing-to-parts. The meaning of this assertion is that the concept of bequeathing-to-parts itself is not bequeathing-to-parts, i.e. BQ(PP)⮭NBQ(PP).

 In contrast, with regard to the whole the answer is more complex. Suppose we have a certain concept X1 that is bequeathing-to-parts. Now, by the principle of free union, any additional concept – say X2 – may join it to become a larger concept. If we take the stricter assumption that X2 is not bequeathing-to-parts is X1+X2 bequeathing-to-parts? The answer is negative. Let us take for example the (bequeathing) concept "yellow" and the (non-bequeathing) concept "higher than 300 meters," unite them and so build the concept "yellow or higher than 300 meters," The Eiffel Tower is captured by this concept, being higher than 300 meters, but many of its parts are not, since they are neither higher than 300 meters nor yellow.

 However, the unification of two concepts is only one way of constructing a larger whole, thus creating parthood relations between concepts. Above we saw another one: an articulated predicate is a part of a nuclear predicate. If an articulated predicate is a concept bequeathing-to-parts, will the nuclear predicate also be such? Here the answer seems to be positive. As we remember, the meaning of the reduction rule is not opening the predicate to *any* object whatsoever but stating that *some* objects, that are not specified explicitly, maintain a relation with the nuclear predicate. Therefore when we use the articulated predicate, where these objects are specified, we only provide fuller information, but the relation itself is also preserved when we omit some of the information. We may conclude that if the reduced predicate is bequeathing-to-parts so is the articulated predicate. And so we may state:

(X;w1,,…wn)⮭BQ(PP)(X;wm|(m≥n))⮭BQ(PP)

Hence we can ask: is the concept "…is a part of w" bequesthing-to-parts? The answer is positive:

∀y∀z (y⮭PP;z(z⮭PP;wy⮭PP;w))

Therefore we can state that for any w:

PP;w⮭BQ(PP)

But in view of the above rule we may conclude that:

PP⮭BQ(PP)

This is even more so when w is not a specific object but any object whatsoever. This means that for *every* w it is true that PP;w⮭BQ(PP), and therefore it is evident that PP⮭BQ(PP). The concept of parthood is therefore bequeathing-to-parts.

 At the margins of the above discussions it should be noted:

1. We should be cautious not to confuse different things with regard to bequeathing-to-parts, esp. when it comes to mereology of objects as opposed to mereology of concepts as opposed to that of individua. Let us take, for example, the concept of "good for health." This concept is not bequeathing-to-parts: If the *object* apple is good for health this does not mean that every smallest grain of an apple is also good for health. However, if we take the *concept* of "apple" it is obvious that each of its parts will also be good for health: Honey Crisp. Golden Delicious, Granny Smith, Ambrosia and the rest.
2. Concepts bequeathing-to-parts raise an interesting point regarding their extension. Any object captured by such concepts might be partitioned into an infinite number of parts and parts-of-parts, which are all included in its extension. Since the size of the extension of a concept is measured by the number of the objects that it captures, with regard to concepts bequeathing-to-parts we may say, sweepingly, that all of them have an infinite extension. We thus have here an illuminating case in which we can infer from the intension of a concept to the size of its extension, in contrast to Kant's opinion in his critique of the ontological proof.

### Bequeathing-to-wholes

Just as we have bequeathing-to-parts, so we also have bequeathing-to-wholes. The latter is the opposite of the former. A concept X will be characterized as bequeathing-to-wholes iff whenever it captures an object it also captures all of its wholes. Such a concept will be denoted by the letters BQ(WH) and formally defined as follows:

X⮭BQ(WH) ≡def ∀x∀y∀z (y⮭WH;z(z⮭Xy⮭X)

Thus, for instance, the concept "weighing more than 5 kilograms" is bequeathing-to-wholes (taken that its complement is also a weighted object). The concept "contains sugar" is also bequeathing-to-wholes. I do of course understand that the inference "My drink contains sugar; My drink is a part of the cosmos; Therefore, the cosmos contains sugar" looks somewhat weird at first glance, but it is logically and ontologically valid, and the conclusion is correct. In contrast, the concept "weighing 5 kilograms" is not bequeathing-to-wholes, since any (proper) whole of an object weighing 5 kilograms weighs more than 5 kilograms (taken that its complement is also a weighted object). Similarly, if the whole is square there is no necessity that all of its wholes be square, and if it is yellow,that they be yellow.

As far as we can judge at this stage, there is no logical linkage between bequeathing-to-parts and bequeathing-to-wholes, either in this direction or in the opposite one. Furthermore, it is clear that bequeathing-to-wholes is far rarer than bequeathing-to-parts. Therefore, if I write here "bequeathing" alone I will mean bequeathing-to-parts, while when I will wish to refer to bequeathing-to-wholes I will say it explicitly. As a rule we may say that most of the concepts which are bequeathing-to-wholes relate to size and containment, just as in the examples presented above.

 Having defined bequeathing-to-wholes and understood its nature, we can now define the concept of non-bequeathing-to-wholes NBQ(WH) as follows:

X⮭NBQ(WH) ≡def ¬x⮭BQ(WH)

We have seen from the above examples that there is no necessity that a concept bequeathing-to-parts will be bequeathing-to-wholes or vice versa. In the same way there is no visible logical linkage between non-bequeathing-to-parts and non-bequeathing-to-wholes.

A concept X will be characterized as **weakly bequeathing-to-wholes** (or: **bequeathing-to-a-whole**) iff whenever it captures an object it captures at least one of its wholes (and not necessarily all of them, as is the case with the ordinary bequeathing-to-wholes). This concept will be denoted by the letters WKBQ(WH) and will be defined formally as follows:

X⮭WKBQ(WH) ≡def ∀x∃y∀z (y⮭WH;z(z⮭Xy⮭X)

At this stage we should be entering a complex discussion about the logical relations between the various types of bequeathing, but I will not weary the reader and will leave that for others to complete.

### Mereology of concepts and logic

Towards the end of the chapter on the mereology of concepts I would like to stress its importance. Inspired by Aristotle, I stated above that a concept is an entity that might (potentially) apply to many objects, i.e. to capture them. As we will see in the next chapter, there is no principled difference between an object and a proposition. Hence we will be able to say that there is no principled difference between a general object and a general proposition. A general object is a concept, while a general proposition is a law. A law is therefore the propositional parallel of a concept. The laws pertaining to the relations between objects qua objects and concepts qua concepts, regardless of their concrete intension, are the laws of logic. However, treating objects qua objects and propositions qua propositions virtually means treating them on the general level. Hence, logic always treats objects as concepts and propositions as laws. Logic, therefore, seeks to discover the laws applying to concepts and laws. Yet, the relations between concepts (and consequently laws) are the subject matter of the mereology of concepts. This means that ultimately any foundational logic must be based on the mereology of concepts.

##

## Between concepts and individua: Brentano's thesis, qualities and tropes

### Conceptual and propositional interpretations

Logical relations can be formulated either in the form of sentence or in that of (non-sentential) object-term. To use Couturat's terminology, they have either conceptual or propositional interpretations (Couturat, 1914). The conceptual interpretation does not require assertion, and it exists between different constituents of a concept; while the propositional interpretation is presented as assertion.

 This assumption takes us to a further-reaching statement: One can reduce any propositional interpretation to conceptual interpretation accompanied by the assertion that this concept has an extension larger than 0, i.e. that it captures something. By this we actually adopt Brentano's claim that any sentence may be reduced to an existential sentence (Brentano, 1973). We will therefore name it **Brentano's thesis**.

 Let me clarify this. From the perspective of syntactic structure we can discern two types of sentence: Sentences of the form x⮭y or x⮯y will be named **capture sentences**, while sentences of the form ∃x x⮭y or ∃x x⮯y will be named **existential sentences**. When we deal with constants rather than variables, the equivalence x⮭y≡y⮯x is metaphysically important. It implies that every capture sentence might be replaced with an existential sentence, as Brentano argued (based on different reasons). This is the case because x⮭y is an ordinary predicative sentence while y⮯x is a sentence that speaks about the extension of y, and by that it resembles the sentence ∃x x⮭y, with the difference that the former specifies the captured object. In other words, every capture sentence speaks about the extension of a concept, and asserts that such an extension is larger than 0. (From this point of view, the equivalence x⮭y≡y⮯x is at odds with free logics). Hence it is prone to reformulation as existential sentence.

 Even though the argument is clear, I will add here a discussion I presented elsewhere, with a change in the example:

 Suppose there is a group of people that sees a ripe orange falling down from a tree. Each member of the group is asked to tell someone about what he or she saw. Suppose that the vocabulary of the speakers includes only the words "object," "orange," "ripe," "fell down" and the words required for the basic conjugations of the spoken language. Each of the members described the event by a sentence from series P1-P6:

P1: A ripe orange fell.

P2: The ripe orange fell.

P3: The orange that fell was ripe.

P4: The ripe object that fell was an orange.

P5: The object that fell was a ripe orange.

P6: The fall was of a ripe orange.

It is obvious that each of these sentences describes the same event; what, then, is the difference between them? The answer is clear: Each of them addresses the hearer based on a different assumption regarding the state of his or her knowledge, as follows:

In P1 the speaker assumes that the hearer does not know the orange that is being spoken about;

In P2 the speaker assumes that the hearer knows the orange, but does not know that it fell;

In P3 the speaker assumes that the hearer knows that some orange fell, but didn't know if it's ripe;

In P4 the speaker knows that the hearer knows that some ripe object fell (for instance if he/she heard the noise from afar) but doesn't know it was an orange;

In P5 and P6 the speaker assumes that the hearer knows about some object that fell, but doesn't know if was ripe and not even that it was an orange.

 From a linguistic point of view these sentences, due to the above differences, say different things. In truth, however, they all describe the very same event with no sentence containing more information than the other with regard to that event (in contrast to what concerns the subjective conditions of the interlocutors). Frege, as being very strict in distinguishing between the "physics" and the "psychology" of our accounts of the world, must have characterized the differences between them as "psychological," i.e. as a difference in which the language does not only inform us about the world, but also about the mental states of the parties to the flow of information. Since Frege saw logic as dealing with "physics" and not with "psychology," he should have insisted that these differences not be reflected in the logical formalization of these sentences, and that they all have the same truth value. However, he only abolished the ancient distinction between subject and object but preserved the distinction between "function" and "argument," or between objects and predicate. However, predicates (concepts) are also a type of object, and a better-fitting logic could overcome this distinction too.

 A good logic should therefore accept the premise that all the above sentences are nothing but verbal expressions for the very same **datum**. If we borrow a term from Wittgenstein (slightly off the meaning he attached to it), datum is a mental object that presumes to *depict* the world that is "outside" of it. (It seems that this datum is the *meaning* of the sentence, but I will not delve into the imbroglio of the definitions of meaning, that has already attracted too much attention in the philosophy of today.)

The world-segment described by all the above sentences P1-P6 is after all one and the same. This is the reason for the statement that all those sentences are verbal expressions of the same datum. One datum might appear in different sentential guises, so that all of these verbal expressions are not the datum itself but different means of its verbal formulation. Therefore, the truth value of all these sentences must be identical.

 The above discussion provides a proof for Russell's claim that sentences like "The present king of France is bald" are false (Russell, 1905), but this proof goes in a different path from the one taken by Russell himself. Russell proved his claim by dismantling the sentence into a conjunction of sentences that supposedly underlie it. This argument was criticized by Strawson (1950), Geach (1950) and others. In fact, Russell was right in his conclusion, but the proof should be different from his. Put simply, the truth or falsity must be determined by anything but the question: Does the datum depicted in the sentence have a parallel in reality (or in another ontological sphere, if the proposition relates to such). This is the only question that makes truth and falsity an interesting issue from a genuine philosophical standpoint. Insofar as logic decided on different concepts of truth and falsity, dependent on the grammatical structure of the sentence and the like, it opted for philosophically uninteresting concepts, and should change them as soon as possible.

 Hence we can also justify Brentano's thesis, i.e. Brentano's argument that one can "translate" any sentence into an existential sentence, including a sentence asserting non-existence (under one reservation explained below, that the said existence does not necessarily relate to the sphere of reality). In the above example: All the sentences P1-P6 could have been "translated," according to Brentano, to a sentence like

P7: The ripe orange that fell exists.

Or

P8: The fall of the ripe orange exists.

We may assume that Brentano understood very well that this "translation" does not preserve the full meaning of the sentence in natural language, since there is a considerable difference between P1-P6 and P7-P8. Rather, he meant to say that from the aspect of the truth values of the sentences – which is the interesting issue from the perspective of logic and philosophy – each sentence "draws" a certain object into the ontological sphere of language and asserts that it "exists," i.e., that it has a parallel in the ontological sphere of reality. Now, the "picture" drawn by P1-P8 is a single picture. The only question that now arises is whether it does have such a parallel (and then the sentence is true) or not (and then the sentence is false). In this sense, which is the only sense that interests philosophy (or has to interest it), Brentano's thesis is correct.

 Parenthetically, I will add that Brentano's thesis swept away the ground from under the concept of "state of things" (*Sachverhalt*), which served important philosophers such as Husserl, Wittgenstein and Meixner, since states of things are just properties of the object whose existence we claim.

 We can therefore summarize this discussion by stating that from a logical perspective (and maybe from other perspectives as well), we can replace every sentence with an existential sentence without having its truth value changed. For technical reasons, this rule is correct only in one direction, and therefore it is not an equivalence but a material implication. I have already mentioned that we will name it Brentano's thesis, but its official name should be the rule of reduction to existential sentences. It should be formulated as follows:

x⮭Y;…,,… ∃x(x⮭(x↑∩Y;…,,…))

Or

x⮭Y;…,,… ∃z z=(x↑∩Y;…,,…)↓

As I said, this rule goes in one direction, so we cannot state that:

∃x(x⮭(x↑∩Y;…,,…)) x⮭Y

The proof thereof: The bracketed expression is an intersection, and as such it is commutative. Therefore we will not be able to decide whether x⮭Y or rather Y↓⮭x↑. And these two sentences are obviously different.

 In other words, in order to make any assertion we first have to construct a concept Y, which constitutes an intersection of all the concepts about which we wish to assert, and then add the assertion of existence to that concept, i.e., that there is some x that is Y. The same rationale applies also to negative sentences, but with those sentences there seems to be some complication, as we cannot attribute existence to negation. In truth, however, there is no problem here at all, and I will address the way negative sentences are reduced to existential sentences below (at ###).

 When we reduce a sentence to an existential sentence we intersect all the concepts capturing the object and assert that the rendered concept has an extension larger than 0. But when we create a conjunction of such existential sentences, is it possible to combine their concepts? Basically, conjunction parallels intersection, but since in this case we affirm the existence of an object, it has to be seen discussed as a combination of objects, i.e. union. This means that if we have the sentences x⮭y and w⮭z, and if we reduce both of them to existential sentences we will get:

∃x x⮭(z1↑∩z2) ˄ ∃y y⮭(z3↑∩z4)

Hence we can infer:

∃x x⮭(z1↑∩z2) ˄ ∃y y⮭(z3↑∩z4)  ∃w w⮭((z1↑∩z2)∪(z3↑∩z4))

But we can infer nothing with regard to the unified object: (z1↑∩z2)↓ ∪ (z3↑∩z4)↓

It is also noteworthy that Brentano's thesis is based on the acceptance of the premise of existence in a particularly strong version. This premise (meaning the ordinary premise of existence, as different from the rule of basic existence formulated above and the strong premise of existence to be presented below) states that in order to attribute a predicate to a subject one must premise the existence of that subject:

x⮭Y∃x

As we know, logicians disagreed about the acceptance of this premise in ordinary assertion sentences, and, in fact, this is a convention that one may embrace or give up. However, when we accept Brentano's thesis we decide *in adjecto* to accept that premise, not just the simple one, but a stronger version that actually includes four premises together:

1. x⮭Y ∃x
2. x⮭Y∃Y
3. x⮭Y∃x↑
4. x⮭Y∃Y↓

(In this case we may bind a predicate to a quantifier, since the quantifier serves here as an object).

We can formulate these four premises as a conjunction that will turn them into one, which we will call **the strong premise of existence**:

x⮭Y ∃x˄∃Y˄∃x↑˄∃Y↓

It is noteworthy that the premising of the existence of concepts Y and x↑ is not only unproblematic, but even postulated. Their existence in our thought is a sufficient and necessary condition for their existence in reality as well (I have already noted this above, at ###, and will dwell on it below, in the second section). Thus, for instance, a gryphon, being an animal with half its body that of a lion and half of an eagle, exists on the map of concepts – as I said, the very fact that we can think about it is a proof for it – but this concept captures nothing. In fact, every concept that is not an intersection concept that is combined of contradictory (i.e. disjoint) concepts – exists in reality. The existence of those concepts is something we take as a general and universal premise, and therefore the fact that it serves as a premise with regard to this or that concept in this or that sentence is not problematic at all. The innovation in some, even if not all, the sentences lies in the assertion that those concepts do not only exist but also capture objects. In the series of conjuncts listed above the innovative point is therefore in the individual ones, numbered (1) and (4). These concepts are those whose existence is not taken as premises throughout the entire system but stated as an "innovation" by the sentence at stake. The sentence "A small gryphon is resting at the cave" premises not only the existence of the concept of gryphon, whose existence is taken as a part of a general premise, but also that of an object captured by it, i.e. of the individual gryphon described in the sentence.

 The premise stating that every concept that is not an intersection of disjoint concepts exists in reality should be asserted as an axiom. In contrast to the strong premise of existence, which implicitly underlies any sentence relation to concepts, this premise is a postulate standing in its own right, and we need to add it to any sentence dealing with concepts. Therefore we will call it the **rule of conceptual existence**, and may formulate it as follows:

x⮭K˄¬∃y,z (y,z⮭WH;x˄y⮭NOL;z)∃x

The very fact that a concept – a property or a relation – might capture some object, even if it doesn't actually do so, makes it real. This is not about an "ontological proof" of the existence of each and every possible concept, since the concept does not necessarily contain existence, but actually a concept is an object that necessarily exists. In a way it is like a "performative utterance," but it is not an "utterance" in the sphere of language which creates an object in the sphere of reality, but it is all in all in the sphere of reality.

We can also accept the premise of the almost opposite direction, i.e. that any concept is "consistent," which means that it is not an intersection of disjoint concepts. The word "consistent" is not completely appropriate here, being usually used with regard to sentences, but for the lack of better wording we will call this premise the **rule of concept consistency** and will formulate it as follows:

∃x x⮭K¬∃y,z (y,z⮭WH;x˄y⮭NOL;z)

This premise, too, will be accepted on a general basis as underlying any statement referring to a concept. And here a question might arise: If all the concepts are consistent, how can we attribute a predicate to a concept that is not consistent? In fact, the only predicate that might be attributed to such a concept is the predicate "does not exist"; yet it is still to be asked: If the proposition that x does not exist means that the complement of x does exist, what is the complement of x when x does not and cannot exist? Furthermore, the proposition that x does not exist means that the concept that captures it - x↑ - is of zero extension; but how can we talk of a concept that captures it when it does not and cannot exist? The full answer to these questions will be given in the second section. Here I will only say this: Indeed, in reality there is not, and cannot be, an inconsistent concept. In contrast to Meinong (1960), I think we should distinguish between the assertion of the non-existence of consistent and inconsistent concepts. The assertion of the non-existence of a consistent concept says: (a) that the linguistic expression of that concept does not have a parallel in the sphere of reality; (b) its complement does exist in the sphere of reality. In contrast, the assertion of the non-existence of an inconsistent concept asserts only (a). In respect to reality it is indeed impossible to assert anything with regard to an inconsistent concept, and in respect to language it ought to be judged as meaningless. It is even a mistake to refer to it as a "concept," since in truth it is not one. Therefore any concept to which we will refer as such is one that could, at least in principle, be meaningful, namely, a consistent one.

It is also noteworthy that, even though the reducibility of sentences to existential sentences is a sweeping principle, many times the reduction creates cumbersome and inconvenient structures, and those might hinder the reader's ability to catch the arguments and argumentations. Since the importance of this principle does not lie in its practical applications but rather in its metaphysical significance, we will often use sentences in their normal forms, and only when we need to address their metaphysical implications will we return to the form of existential sentences.

### Qualities and their holding; Turning concepts to qualities (individua) and qualities to concepts

A quality is a noun, i.e. it is an individuum. "wisdom," "stupidity," "beauty," "ugliness," "the height of 20 centimeters" and "the weight of 3 kilograms" all function, from a logical perspective, as individua. In contrast, "wise," "stupid," "beautiful," "ugly," "20 centimeters high" and "3 kilograms heavy" all function as predicates. We need, therefore, to create a new predicate form and a new operator to link between the two types of objects.

 The new predicate will be denoted by the letter H and it will signify the holding, or having, of some qualities. (Of course, we use the concept of "holding" in its metaphoric sense.) If King Solomon is "wise" then he "holds/has wisdom." If the brick is 20 centimeters high, it "holds/has the height of 20 centimeters."

 The new operator will signify "…is the quality of…" It will be denoted by the sign q followed by the concept at stake, flanked by brackets. This is the concept that the quality embodies.

We can therefore state:

x⮭Y ≡ x⮭H;q(Y)

And consequently we can state the identity between the concepts:

Y=H;q(Y)

But if that is the case, we can proceed a step further: The same way that every object is captured by a concept (by its very nature as an object) so, as well, every object is a holder of a quality. We can state, therefore:

∀x∃Y x⮭H;q(Y)

And if we apply the reduction rule we can state:

∀x x⮭H

(Of course, this sentence has no parallel in natural language, but we have already announced our will to release ourselves from its bonds.)

In principle, we could add this sentence to the foundational equivalence, but since the sentence x⮭H is of absolutely identical content (and not just logically equivalent) with the sentence x⮭O, there is no use in iterating the same proposition.

In its grammatical status, a quality is a singular individuum. I must emphasize that in creating the term "quality" I do not enter into the (slightly boring) discussion that has erupted among some of today's philosophers regarding the ontological status of qualities, or universals. Our use of q is purely technical: Anything that could have been said in the way of simple predication ⮭ can be said by the way of holding ⮭H;q. For this matter there is no difference between "wise" and "goes to the beach every Friday," and so there is no difference between the quality of "wisdom" and that of "going to the beach every Friday." From this technical logical perspective, since predicates exist, the qualities to which they are translated also exist, and the question of whether to use this form or the other is merely a linguistic matter.

 Thus we have attached a new logical meaning to the term "quality," far broader than it has in the philosophy of today. While in this matter philosophy followed blindly after the spoken language, and recognizes qualities by mysterious intuition, in this book the term "quality" will have a much broader sense: *Every* predicate means "holding a quality," and *every* predicate has a quality that it holds.

 Indeed, we could imagine a historical development of logic in which we would not speak of objects and predicates but of objects and other type of objects (i.e. qualities). But Aristotle, not Plato, constructed the first logic, and probably because of that our logic was constructed as it was, and we have become accustomed to it. Since there is no real advantage to one way over the other, we can continue to use the one that is more convenient to us, but we must keep in mind that, after all, this is a technical choice.

 Hence we can define the operator inverse to q, i.e. the fact that something is a predicate, or the existence of a quality in an object. In natural language we only seldom find a convenient expression for such a logical relation. We will denote this operator by the sign of direct predicate, p, and will define it as follows:

x⮭p(y) ≡ x⮭H;y

when y is an individuum (for example: wisdom), while p(y) serves here as a one-place predicate (for example: wise). According to the above, we can apply the same rules to multi-place predicates, even though it looks somewhat complex.

 It is almost redundant to note:

q(p(x))=x

p(q(X))=X

And of course:

p(q(p(x)))=p(x)

q(p(q(X)))=q(X)

and so on.

The reduction rule, which we established with regard to predicates, can now be applied to qualities as well:

(X,Y⮭K˄X⮭PP;Y)↔qX)⮭PP;q(Y)

Namely: If one concept is a part of another, the quality linked to that concept is also a part of the quality linked to the other one.

### Possessive and tropes

Our next discussion will focus on the objects known as tropes. First we have to clarify what we mean by that term. In contemporary formal ontology it has a narrow sense. Usually it denotes a quality in the way it appears in an individual object. The latter is often called *quale*. As examples of tropes we can give "the redness of this rose," "the wisdom of King Solomon" or "the beauty of France." With the help of the concepts I presented in the previous subchapter we can describe the trope as a quasi-inverse of the concept of holding H(y). If it is given that x⮭H;y then there is something which is the y *of* the x, and that is the trope.

 From a mereological perspective there is no distinction between this form and other forms of the same structure, i.e. the structure of "the y of the x." Therefore I find no ontological justification for the existence of independent denotations for these forms. Consequently, in this book we will use the term "trope" for any expression that will have the logical form of "the object of type y of the object x." In these expressions the preposition "of" denotes the existence of one object in another. In other words it expresses the relation of the *possessive* (in the linguistic sense, not in the legal-proprietary one, such as in "the house of Danny and Dina"), and therefore also as a certain form of parthood. Therefore the term "trope" will not include only expressions like the three mentioned above, but also "the sepals of this rose," "the right hand of King Solomon" and "the capital of France." In view of this, even though at the formal level we will not use existing signs to define the possessive relation, from a metaphysical perspective we may not see it as a primitive, but rather as another aspect of the relation of parthood.

 Do I imply by this that the relation "of" in the expressions "the beauty of France" and "the capital of France" is one and the same? The answer is negative. The word "of" functions in the same way from mereological perspective, but is not identical from semantic perspective.

 The relation "of" in this meaning will be denoted by the asterisk sign \* between the two objects, namely:

X↓\*y

The asterisk will sometimes be named **the possessive sign**. In the order of reading, it precedes all the other signs and therefore requires no brackets. In the above notation, the X will be called the **possessor** and the y – the **possessed**.

But the y, that functions here as an individuum, is a singular instance of a concept, and therefore we can see the possessive connection as one of the following form:

X↓\*Y↓

Additionally, the same sign is also appropriate for a possessive connection between concepts, without any individuum. Thus, for example, when I say "the sound of sparrow" I don't necessarily mean the sound of an individual sparrow, but that of the general kind of sparrows, and therefore referring to the *concept* of sparrow. The same is true for rose-red, which is the red of roses in general, and not this or that individual rose. These expressions will be denoted by the form:

X\*Y

Another form of a trope is the one that appears in the sentence: "These pants are navy blue." The concept navy blue is combined of a concept ("blue"), a possessive sign and an individuum ("navy"), since it alludes to the British Navy, and the latter is an individual object. Therefore the expression navy blue will be written in the ordinary trope form: X\*Y↓.

Now what about the expression (X\*Y)↓ ? This is a singular instance of a concept. This means, that if the concepts "sparrow sound" or "rose-red" had only one instance, this expression would befit them. But, as we know, they have more than one instance. The sparrow sound can be heard from this sparrow and from the other ones alike. The same is true for the rose-red. However, "the left hand of King Solomon has only one instance, and therefore the expression (X\*Y)↓ is appropriate for the representation of such an object, and nevertheless "the right hand of King Solomon" was denoted above not by (X\*Y)↓ but by (X↓\*Y↓). Yet, this is not a real problem, since from the discussion above we may conclude that those two forms are equivalent. And so we can formulate the rule of possessive distribution:

X↓\*Y↓ =(X\*Y)↓

Here we have arrived at an important point. We have seen that a trope designates an individuum, marked by the arrow of singular objectification (↓). If we name it, the name will be a linguistic expression of the fact that this individuum is the singular instance of some concept, i.e. another expression that includes (whether explicitly or implicitly) an arrow of singular objectification. Hence, the sentence expressing the relation between the two expressions is actually a sentence arguing for the identity of the two expressions of singular instances (two "senses," in Frege's terminology). Therefore a sentence like "The capital of France is Paris" will receive a form of an identity sentence:

X↓\*Y↓=a.

Here a warning is required: Due to the singular nature of the trope, we may not say that if concept X serves as a possessor for object y and captures object a, then X's whole can replace X. A capital city is a city, and yet, even though we can say that "The capital of France is Paris" we cannot say that "The city of France is Paris." The opposite, however, can be said: If concept X is replaced by a concept that is its part, it will capture the same object. Thus, for example, if it is true that "The pet of the Smiths is Maxy," then "The dog of the Smiths is Maxy" is also be true. Let us formalize it:

∀X,Y,Z, (X↓\*Y↓=a)˄(Z⮭PP;x) (Z↓\*Y↓=a)

¬∀X,Y,Z, (X↓\*Y↓=a)˄(Z⮭WH;x) (Z↓\*Y↓=a)

The sentence that does remain true when we substitute a whole concept for its part is one that asserts ordinary predication rather than identity. If we wish to say "Paris is a city (indefinite) of France" or "Maxy is a pet (indefinite) of the Smiths," the proper formulation for it will be:

 (X↓\*Y↓)⮯a

In a sentence in this form we may say that if "the capital of France" captures Paris, then so will "a city of France" as well:

∀X,Y,Z, (X↓\*Y↓⮯a)˄(Z⮭WH;x) (Z↓\*Y↓⮯a)

The predicate of holding, H, is appropriate for expressing the possessive by the way of propositional interpretation. We may therefore state:

∃X,y X\*y ≡ y⮭H;X

Note, that the sentence in the opposite direction is, according to Brentano's thesis, obvious:

y⮭H;X∃X,y X\*y

Yet, Brentano's thesis works in one direction, in material implication from the ordinary sentence to the existential sentence, and not vice versa, while here there is a material implication in the other direction as well, and that means: equivalence.

And this brings us to another point. Suppose a man has a broken leg. Suppose a woman has a wise daughter. Suppose France has a beautiful capital. Even though the predicates mentioned here (broken, wise, beautiful) relate to the part, not to the whole, they certainly tell us something that provides us with more information about the whole: The whole holds a part of such and such properties. We should be able to express this point on the formal level as well. We will do it by using the form H;(Y;Z), which will read: "has a Y that is Z," or, in a more precise manner: "holds a Y that is Z." Thus, we can define it as follows:

x⮭H;(Y;Z) ≡def ∃w (w⮭Y\*x˄w⮭Z)

My main use of this form will be when I substitute Y for a predicate of parthood, whether identity-parthood (ID) or proper parthood (PP). Indeed, the fact that an object has a part with certain properties says something about it.

 The classical trope, discussed by most of the philosophers hitherto, is an object, not a concept. Expressions like "the colour of the sky" or "the taste of watermelon" denote objects. And since the colour of the sky is azure and the taste of watermelon is sweet, in this context "azure" and "sweet" are also objects. In many the natural languages, we cannot say "The shirt is the colour of the sky" or "The ice cream is the taste of watermelon," but we have to add or change a word in order to turn the expression into a predicate: "The shirt *has* (*holds*) the colour of the sky" or "The ice cream *has* (*holds*) the taste of watermelon." In this context we can even talk about "*this* azure" and "*this* sweet," and that is a sign of individua, as the scholastics pointed out. This must be emphasized since some of the trope theorists view the tropes as a twilight ontological entity that bridges over the gap between concept and individuum; and that is not the case.

 Yet, as we have seen above, we can take "sky-azure" and "watermelon-sweet" as properties, and build an expression with a structure similar to that of a trope, but one that will be a concept. This may be done, of course, through the use of the operator p. A predicate created this way will be named a **trope-based predicate**.

 Moreover, even if the entire expression (X\*y)↓ is an individuum, its first bracketed constituent, the X, is a concept. By adding the second constituent we do not change the ontological status of the first, but only limit its extension. Therefore the entire bracketed expression is of a concept, and only the sign of singular extension gives it its status as an individuum.

 From this point we can advance another step forward, as I mentioned above. It seems to me, and I say it with some caution, that the possessive relation might be defined as a certain form of parthood. If so, then the sign of the asterisk is a sign of a conceptual partitioner, in the sense attached to it above (###). Let us take the watermelon as an example. We may see the watermelon as a whole whose parts are the skin and the flesh, or the upper half and the lower half, etc. But we may also see the watermelon as a whole whose parts are the taste of the watermelon, the volume of the watermelon, the weight of the watermelon, the shape of the watermelon, etc. All of these constitute, apparently, the whole watermelon. Taken that tropes are objects, as I have proven, these are all objects that constitute parts of the object watermelon. Furthermore, as I suggested in the beginning, the trope that relates to properties resembles the trope that relates to objects, such as the right hand of King Solomon and the capital of France, where it is easy to see the parthood. Indeed, if this understanding is correct, we may formulate the rule:

∀X,Y (X\*Y)↓⮭PP;Y↓

But in view of the above the following is also inferred:

x⮭H;y y⮭PP;x

In other words, holding is also a type of parthood, and consequently in the mereology of concepts the concept of holding is a part of the concept of parthood:

H⮭PP;PP

And here we should emphasize again that "possession" is only a metaphor, and has nothing to do with the legal sense of the word. It only means that the thing possessed "rests within" the thing possessing, and thus is a part thereof even on some intuitive level. The wisdom "rests within" the wise and the colour of the watermelon "rests within" the watermelon, and therefore is a part of it, but the house of Danny and Dina does not "rest within" this couple, and therefore is not a part of them in any sense (save, maybe, according to the Hegelian approach, which views a person's property as a certain way by which he realizes his self). Indeed, from this aspect as well it is important to discern properly between the marking of a singular object and that of a concept which might capture some object.

### Tropes and qualities: mereological aspects

The very same line, without any difference, applies to trope-based predicates, in the sense attached to this term above (in the previous subchapter). There we stated that the trope expression "the wisdom of Solomon" – which may be formulated as q(wise)\*ks – can become the two-place predicate "holding the wisdom of King Solomon," which can become, in its turn, a one-place predicate "wise in the wisdom of King Solomon": p(q(WISE)\*ks)↓ . I have also written, however, that p and q cancel out each other, and therefore the expression "wise in the wisdom of King Solomon" will simply be WISE\*ks. From here we can move another step forward: We have shown that from the perspective of the mereology of concepts whoever is wise in King Solomon's wisdom is wise (see above), but not vice versa. Hence, the concept "wise in the wisdom of King Solomon" is a part of the concept "wise." We can thus state more generally:

p(x)⮭WH;p(x\*y)

Here we should return to the mereology of individua. If the concept "wise in the wisdom of King Solomon" is a part of the concept "wise," might we also say that the individuum "the wisdom of King Solomon" is a part of the individuum "wisdom"? A rather alluring intuition pulls us to answer positively. Presumably, "wisdom" is the whole of all the wisdoms, and therefore includes all of them as parts, be it that of King Solomon, or Aristotle, or Einstein or all the other wise people, in the same way that it includes (in a different partition) both theoretical wisdom and practical wisdom. Yet, in truth, we should resist this temptation, since it leads us to an incorrect conclusion. If the wisdom of King Solomon were a part of wisdom in the sense that we ordinarily mean by the word "part," then whoever holds wisdom must hold the wisdom of Solomon; and we know that this is not true, but rather the opposite. (Remember: In the mereology of individua a union is not like a disjunction but rather like conjunction!) The entailed conclusion is therefore that the wisdom of Solomon is not a part of wisdom, but, in contrast, the uniextensional concept of the wisdom of Solomon is a part of the uniextensional concept of wisdom. (Remember: In the mereology of concepts a union is not like a conjunction but rather like a disjunction!):

(x\*y)↑⮭PP;x↑

Or better:

x↑⮭WH;(x\*y)↑

Having equated between trope-based predicates and articulate predicates, we can now state that the same rule also applies to qualities built from articulate predicates. I have already emphasized that articulate predicates are full-fledged predicates, and so we can build qualities from them. If "ate an apple" is a predicate, we can build from it, using the operator q, the quality of "eating an apple." About qualities, just as about concepts, we may state, on mereological grounds, that if eating is healthy then eating apples is also healthy. Since, by the reduction rule, it is true that:

(x\*y)↑⮭PP;x↑

Then it is also true that:

X;y⮭PP;X

And therefore it is also true that:

q (X;y)⮭PP;q(X)

At this point (and perhaps even earlier), one might comment that in fact qualities function to great extent in a similar way to that of concept (the above example about the partition of wisdom can show it); but since we have seen above that with regard to parthood relations they function as individua, it may be correct to determine a special ontological status for them. This is an interesting idea, that converges with similar ideas in contemporary ontology, which seek to grant qualities a special status, of interim entities between concepts and individua. In this spirit, one might suggest refining this demand and building a systematic mechanism that would reflect the aspects in which qualities are closer to individua and those in which they are closer to concepts. In my opinion, however, there is no use in that, since we have seen that the passage formula from concept to object (i.e. from quality to holding) is simple and easy, technical for the most part, which demonstrates that the entire difference between the two stems from the grammatical form in which the sentence is phrased. And we have already learned above that this form belongs to the "psychology" of logic and not to its "physics" (in Frege's terms), and therefore we should not build logical constructs on it.

##

## Manifold Theory

In this chapter I am about to introduce two innovations: The first is a new relation, item-collection, or, for short, itemhood, that establishes a new subject – Collection Theory. The second is an overarching theory that seeks to reduce three subjects, mereology, collection theory and set theory, to a single infrastructure of an enhanced mereology. I am confident that these are very fruitful ideas and may also be (as some might claim) presumptuous, but here I will discuss them quite briefly. This brevity is not caused by a cursory or perfunctory attitude to the subjects, but from the need to focus on the basic matters required for the metaphysical discussion. The desired elaboration will remain for another work – or another author.

### Manifolds: wholes, collections and sets

Set theory – undoubtedly one of the central theories in mathematics and logic of today – is based on the relation member-set. Mereology, also a very fruitful theory, even if less widespread, is based on the relation part-whole. On the intuitive level, there is some proximity between the two relations. In the first lesson on set theory there is always some student (and often more than one) who mistakes membership for parthood. The teacher would patiently explain that a member is not a part and will point at the differences between them (usually disregarding the fact that the mistake of so many might reflect a deep intuition). Parthood relations exist between a subset and a set, as we were told by Leśniewski (1983), Lewis (1991) and our commonsense (Until not long ago, in the non-professional Hebrew terminology subset was often named "partial set.") Even though set-subset relations are not identical to whole-part relations, as I will prove below, they are still parthood relations. They do not exist between a member and a set. In the following lines I will examine the nature and sources of this difference and in the course of this discussion I will present the new relation that will serve as a bridging link between part-whole and member-set, i.e. the relation of item-collection. But even before presenting it I will say that all these relations belong to one general kind, which I will name **consolidated pluralities**. In all of these relations we can find on the one side a "big object" and on the other side a "small object" that is contained in this way or the other in the big one. This "big object" will be named a **manifold** (and sometimes a **consolidation**) and the small object will be named a **component**. The relation between them will be named **composition**. It seems that Leśniewski (1983) treated them as part-whole relations, but he understood these terms in much broader a sense than the one given to them in mereology. This implies that he might have meant to develop a manifold theory, which he actually never did, but saw it, rightfully, as the cornerstone of metaphysics. But first we have to clarify what this is all about and what pushes us to develop a manifold theory, and only then turn to the formal and systematic presentation of the its main concepts.

 Already at this stage, I will state the notation of manifold and component. **Component** will be taken as a primitive and will be denoted by the letters CP, while manifold will be denoted by the letters MF. We can now define it:

x⮭MF;y ≡def y⮭CP;x

Hence, manifold and component are inverse concepts:

CP⮭INV;MF

We will now turn to the different types of component-manifold relations. As mentioned before, membership is not parthood. If x is a member of set B, it is not a part of that set. Lewis (1991) enumerated the differences between them. One of them is that the part-whole relation is transitive, not so the member-set relation: If x is a member of B and B is a member of C – x is not necessarily a member of C. For example, if Smith is a member of the set of engineers, and the set of engineers is a member of the set of sets – Smith is not necessarily a set.

 Furthermore, a set is an abstract entity, and therefore if a predicate applies to all of its members, it does not necessarily apply to the set as a whole. Thus, if all the passengers of the Titanic drowned in the ocean, the set of the passengers of the Titanic did not drown, since an abstract entity cannot drown.

 But what if we wish to say that all the passengers of the Titanic drowned while still looking at them as a collective entity and not as separate individua? If we ask ourselves what is the extension of the concept "Titanic passenger" we will be able to enumerate 2,200 names, but these are all separate individua, and do not constitute a set until we build such a set for them and declare them as its members. However, whatever we will say about that set will already treat it as a new object and will not refer to those individua, as I demonstrated by the above example of "The passengers of the Titanic drowned."

 In order to enable us to assert a sentence conveying the above message on the passengers of the Titanic as a unified entity that preserves the identity of its components we therefore have to build a new relation, different from that of membership, in which the passengers of the Titanic will be conceived as belonging to a concrete object rather than an abstract one. This object will be named hereafter a **collection**, and the objects belonging to it will be called **items**. Thus we will be able to say that the *set* of the passengers of the Titanic did not drown, but the *collection* of the passengers of the Titanic did. A collection is thus an object that unifies other objects while letting them preserve their separate identity. The items are those separate objects. But all this verbosity does not provide rigorous definitions as it ought to, and cannot replace them. We therefore have to formally develop the collection theory, which will formulate the item-collection relation, the same way that we developed set theory, which formulates the member-set relation (as well as mereology, which formulates the part-whole relation).

 As we develop the collection theory we must bear in mind another difference, which will also be discussed at greater length below: In set theory there is a clear distinction between the member x and the singleton that contains x. Not so in the collection theory, where the manifold is not abstract and therefore we can state that a collection which contains only x is identical with x.

 Parenthetically, I will comment that the above elucidation of the term collection and its contrast to the term set raise another highly important point (indeed I wonder why it was not raised thus far). Any use of set theory in topology and infinitesimal calculus seems to be mistaken. Graphs, geometric forms and segments are not *sets* of points, since sets are abstract entities. Rather, all these are *collections* of points.

 The term item will be taken for the present as a primitive and will be denoted by the letters IT. A collection will be denoted by CN so we can define it:

x⮭CN;y ≡def y⮭IT;x

Namely, CN and IT are inverse concepts.

CN⮭INV;IT

One could argue that the collection of the passengers of the Titanic is nothing but a sum, or a union, of the passengers of the ship, i.e. it is the whole, whose parts are the passengers. According to this argument, there is no room for collection theory, and the appropriate theory to deal with item-collection relations is mereology, which will reduce them to part-whole relations. Yet, a collection is not a sum. A sum is a concept that is indifferent to its internal parts, while in a collection the items that constitute it are those items in particular, and not their parts. In our case: The *sum* of the passengers of the Titanic are a huge mass, among the parts of which we can find the tip of the nose of Captain Edward Smith and the right eye of Molly Brown, while the collection of the passengers of the Titanic will not recognize these as items, but only the whole Captain Smith and the whole Molly Brown. The same is true for any other example. The sum of all the yellow balls in the world constitutes the huge unified mass of all those balls together, but its parts are not only the balls, but any part of this mass, such as half a ball and even a ball and a half or a ball and a quarter. In contrast, the collection of the yellow balls contains, as items, only whole yellow balls. This makes a great difference with regard to extension. If we ask how many items of the collection of the Titanic passengers there are, the answer will be 2,200. While if we ask how many parts of the sum of the Titanic passengers exist, the answer will be infinity. From this point of view, a collection resembles a set: the relation item-collection is determined by a holistic composition, i.e. the manifold can be composed only from objects that were determined as its components, they and not their parts (and below I will dwell on this issue and formalize this characteristic).

 Set theory is a recognized and established discipline, and is widely used in contemporary mathematics. The early attempts to reduce it to mereology failed. The more recent attempts by David Lewis (1970, 1991) and Harry Bunt (1985) are not compelling either, in my opinion. Lewis tried to reduce membership to a more basic concept, but mistakenly identified object with the singleton containing that object, while Bunt took the membership relation as a primitive, and therefore did not contribute much to the reductionist endeavour.

 Lewis argued that the relation between a set and a subset is that of parthood, in the mereological sense of the word. Thus, for instance, the set of Siamese cats is a part of the set of cats. Similarly, if we take all the Siamese cats, not as a set but as a sum, the sum of the Siamese cats will be a part of the sum of all cats. However, as I demonstrated above, here lies the difference: The object which is the sum of the tail of Siamese cat A and the head of Siamese cat B is a part of the sum of cats, but this object can by no means be considered as a subset of the set of cats. The same is true for the sum of the heads and the tails of all the Siamese cats or that of all the cats in general. The reason: According to the principle of extensionality, that which determines the identity of a set is its members, and since the limbs of the cats are not cats but new objects, they cannot be contained in the set of cats.

 The principle of extensionality is a normative principle. In order to manifest this I will allow myself another illustration taken from the game of Lego. Let us imagine a competition of building with Lego blocks, in which a few children take part. All of them receive a pack of Lego blocks, and are told to build a Lego house with them. One of the children, Sammy, started building, but in one place he needed a relatively short block, while all the blocks he had were long. Sammy found a quick solution: he pulled out a hacksaw, cut one of the longer blocks in two and put one of the halves where he needed it. Of course, the judges of the competition disqualified his house, since the competitors are expected to use only the Lego blocks given to them, and, they accused, the shortened block which Sammy used was not such. Sammy, on his part, contended that he did build only from the original blocks, and the shortened piece he used was also made of such a block. The judges rejected that argument, as the rules of the game which refer to the "Lego blocks" given to the participants mean only whole ones. In other words, this is a normative restriction without the fulfilment of which the conditions for creating new structures are not satisfied and they will therefore be annulled. Similarly, a manifold is always a normative entity, about which the relevant theory (mereology, collection theory or set theory) tells us how to look at a few objects together. The relations of set and subset cannot therefore be discussed as simple parthood but rather as normative parthood, subject to the definitions and characterizations presented below.

We can now start the formal discussion. For the next steps we will use a notation different from the conventional one, in order to integrate set theory into the language of concept calculus. The relation of membership will be taken, for the present stage, as a primitive and will be denoted by the letters MM ("… is a member of …"). The inverse relation is ST ("… is a set containing …"), so we can define it as follows:

x⮭ST;y ≡def y⮭MM;x

And let us state explicitly that these two concepts are inverse:

MM⮭INV;ST

Let us summarize the three composition relations we have presented:

Parthood relation (x is a proper part of y) has been marked PP.

Itemhood relation (x is an item of y) has been marked IT.

Membership relation (x is a member of y) has been marked MM.

Composition relation (x is a component of y) has been marked CP.

According to the mereology of concepts we can state that

PP,IT,MM⮭IP;CP

One might suggest that we could make an even stronger argument, that CP=PP∪IT∪MM. But, as we will see below, there can be other types of manifolds. However, we could certainly state that all of them are parts (and not the only parts) of the whole CP, as in the above. Consequently we can also state that:

WH,CN,ST⮭IP;MF

I said above that in some deep intuitive way we feel that there is a linkage between these different types of relations between component and manifold. The very need to explain and emphasize the difference between them to students is a sign of their basic conceptual proximity. We will now try, using formal tools as well, to understand the common and the differentiating elements of these relations. But before that we will study the instance of the concept of bequeathing-to-items, collections, members and sets.

### Bequeathing-to-items and collections, bequeathing-to-members and sets

Above (###) we have shown that there might be bequeathing from a whole to its parts (in weak bequeathing, to only one of its parts) and bequeathing from part to whole. The same rule can also apply to other components and manifolds. Therefore there might be bequeathing from item to collection and from collection to item. I will now present a few definitions, even though they will not serve us very often, because they will help us in the formation of a general concept of bequeathing-to-components or manifolds.

A concept X will be characterized as **bequeathing-to-items** iff whenever it captures a collection it also captures all of its items. We will denote such a concept by BQ(IT) and will formally define it as follows:

X⮭BQ(IT) ≡def ∀x∀y∀z (y⮭IT;z(z⮭Xy⮭X)

A concept X will be characterized as **bequeathing-to-collections** iff whenever it captures an item it also captures all the collections to which it belongs. We will denote such a concept by BQ(CN) and will formally define it as follows:

X⮭BQ(CN) ≡def ∀x∀y∀z (y⮭CN;z(z⮭Xy⮭X)

A concept X will be characterized as **bequeathing-to-members** iff whenever it captures a set it also captures all the members of that set. We will denote such a concept by BQ(MM) and will formally define it as follows:

X⮭BQ(MM) ≡def ∀x∀y∀z (y⮭MM;z(z⮭Xy⮭X)

A concept X will be characterized as **bequeathing-to-sets** iff whenever it captures a member of a set it also captures all the sets to which that member belongs. We will denote such a concept by BQ(ST) and will formally define it as follows:

X⮭BQ(ST) ≡def ∀x∀y∀z (y⮭ST;z(z⮭Xy⮭X)

We can go on developing these concepts in accordance with the above concepts of bequeathing, i.e. by adding concepts of non-bequeathing and weak bequeathing, but the intelligent reader will be able to do it for himself, and I see no use in wearying him with that. All these definitions will be temporary anyway, because now we turn to a general theory of manifolds, into which a general theory of bequeathing will be integrated.

### Foundations of manifold theory

We have already stated above that the concept of component will be taken by us as a primitive and will be denoted by the letters CP. We also declared that the concept of manifold will be defined as its inverse and will be denoted by the letters MF. Now we can move on. The types of components known to us will be marked by ordinals and from now on will be considered as levels of composition:

PP=CP1

IT=CP2

MM=CP3

The types of manifolds known to us will be denoted in parallel by their levels:

WH=MF1

CN=MF2

ST=MF3

There is no substantial mathematical issue in the numbers, and we could just as well decide on other discerning marks.

A component/manifold of a given but unspecified level will be denoted respectively as CPn, CPm, MFn, MFm.

Since we have already stated in general that ∀x,y x⮭MF;y ≡ y⮭CP;x, we can now state it with regard to each of the levels:

∀x,y x⮭MFn;y ≡ y⮭CPn;x

Thus we can also create a general model of bequeathing-to-components and bequeathing-to-manifolds. Instead of talking about bequeathing-to-parts, items or members separately, we can now talk generally about bequeathing-to-component n. We can substitute the values 1-2, which determine the level of the component, for n. Let us denote **bequeathing-to-component n** by the letters BQ(CPn), and define it as follows:

X⮭BQ(CPn) ≡def ∀x∀y∀z (y⮭CPn;z(z⮭Xy⮭X)

Let us denote bequeathing-to-component n by BQ(MFn) and define it as follows:

X⮭BQ(MFn) ≡def ∀x∀y∀z (y⮭MFn;z(z⮭Xy⮭X)

Now let us define identity-composition. The concept identical-manifold will be denoted by the letters IMF and will be defined as follows:

⮭IMF;y ≡ def x⮭ID;y˅x⮭MF;y

x⮭IMFn;y ≡ def x⮭ID;y˅x⮭MFn;y

Similarly, the concept identical-component will be denoted by ICP and defined as follows:

x⮭ICP;y ≡ def x⮭ID;y˅x⮭CP;y

x⮭ICPn;y ≡ def x⮭ID;y˅x⮭CPn;y

Even though we haven't so far defined formally the various types of composition, we can state already now that:

WH,CN,ST⮭PP;MF

PP,IT,MM⮭PP;CP

Do not wonder about the circularity in stating that the concept of part is a part of the concept of manifold, for foundations, by their nature, often "come together."

No we turn to introduce a few rules concerning manifolds, which will help us formalize the differences between them.

1. Identity of manifolds (extensionality in composition): If two manifolds are identical, their components are also identical:

X=Y∀z(z⮭CP;X≡z⮭CP;Y)

Note, that I switched the places of the antecedent and the consequent since, as we have seen above, in part-whole relations it is not correct that if all the parts are identical then so are the wholes (Remember that one can build different structures from the same Lego blocks); yet the opposite material implication is correct.

 This principle resembles the axiom of extensionality in set theory. The principle of the identity of manifolds applies to all the types of composition, and we can state it as an axiom of our manifold theory.

1. Free composition

∀x,y∃z x⮭CP;z˄y⮭CP;z

From the manifold's side we will say:

∀x,y∃z z⮭MF;x˄z⮭MF;y

1. Holistic (free) composition

Transitivity in manifold-component relations in a manifold of level n will be denoted by MFTRNS:

Xn⮭MFTRNS ≡def Xn⮭PP;MF˄ ∀w,y,z w⮭Xn;y˄y⮭Xn;zw⮭Xn;z

Intransitivity in manifold-component relations in a manifold of level n will be denoted by NMFTRNS and will be defined as expected:

Xn⮭NMFTRNS ≡def Xn⮭PP;MF˄¬∀w,y,z w⮭Xn;y˄y⮭Xn;zw⮭Xn;z

What I called above "holistic composition" is, ultimately, a particular form of intransitivity in composition relations.

Transitivity in component-manifold relations in a component of level n will be denoted by CPTRNS:

Xn⮭CPTRNS ≡def Xn⮭PP:CP˄ ∀w,y,z w⮭Xn;y˄y⮭Xn;zw⮭Xn;z

Intransitivity in component-manifold relations in a component of level n will be denoted by NCPTRNS and will be defined as follows:

Xn⮭NCPTRNS ≡def Xn⮭PP;CP˄¬∀w,y,z w⮭Xn;y˄y⮭Xn;zw⮭Xn;z

It is provable that in any n-level relation, when the manifold-component relation is transitive, then so is the component-manifold relation, and vice versa:

∀ x,y,Z,W (Z⮭PP;CP˄W⮭PP;MF˄x⮭Zn;y=y⮭Wx)(Z⮭CPTRNS≡W⮭MFTRNS)

∀ x,y,Z,W (Z⮭PP;CP˄W⮭PP;MF˄x⮭Z;y=y⮭Wx)(Z⮭NCPTRNS≡W⮭NMFTRNS)

1. Holistic manifoldhood

Submanifold-identical will be denoted by IPNF and will be defined as follows:

x⮭IPMF;y ≡def ∀z z⮭CP;yz⮭CP;x

Submanifold will be denoted by PMF and will be defined as follows:

x⮭PMF;y≡def x,y⮭MF˄x⮭NID;y

Submanifold in an n level will be denoted PMFn and will be defined as follows:

x⮭PMFn;y ≡def ∀z z⮭CPn;yz⮭CPn;x

Holistic level n submanifold will be denoted HPMFn and will mean a submanifold without transitivity to the components of its components:

x⮭HPMFn;y =def x⮭PMFn;y ˄¬∀z x⮭CPn;xz⮭CPn;y

The relation of manifold MFn will be considered holistic – a property denoted by HMF iff all its submanifolds are holistic, as follows:

X⮭HMF ≡def X⮭PP;MFn˄∀y (y⮭X;zy⮭HPMF;z)

But MFn is the inverse of CFn, and therefore we can speak about CFn whose manifolds are holistic. We will denote it by CPHMF and will define it as follows:

X⮭CPHMP ≡def X⮭PP;CP ˄∀y,z,w y⮭X;z(z⮭PMF;wz⮭HPMF;w).

Now we can state the axiom of transitivity in submanifolds:

∀x,y,z x⮭PMF;y˄y⮭PMF;zx⮭PMF;z

There is no formal proof to it, but PMF functions like a quasi-part, and therefore transitivity is preserved in it. (We haven't yet defined part formally in the framework of manifold theory, but mereology was presented above in its basic level, and in that framework we saw that transitivitiy is a charcteristic of the parthood relation) Indeed, since PMF functions as a quasi-part, it is also a type of component, and regarding it we can state that:

PMF⮭PP;CP

1. The identity of the sum of the components and the manifold will be named, for short, sum-manifold-identity (abbr. SMI). It will be denoted IDSMF and will be defined as follows:

X⮭IDSMF ≡def X⮭PP;MF˄∀X,y(σy(y⮭CP;X↓)=X↓)

In other words, there are cases in which the sum of all of the manifold's components is identical with the manifold itself. The simplest of these cases is obviously that of the whole: the sum of all of its parts is identical with it. This is true by definition.

1. The identity of a single component with a manifold with a single component will be denoted by IMFSCP and will be defined as follows:

X⮭IMFSCP≡def X⮭PP;MF˄∀y,z (y⮭CP↓zy=z)z⮭X

In other words, there are types of manifolds regarding which we say that when the manifold has a single component, that component is identical with the manifold. The property IMFSCP captures those manifolds.

1. The identity of a single submanifold with the manifold will be denoted by IMFSPMF and will be defined as follows:

X⮭IMFSPMF=def X⮭PP;MF˄∀x,z (y⮭PMF↓;zy=z)z⮭X

Namely, there are cases where we say that if the manifold has only one submanifold, then that submanifold is identical with the entire manifold. In these cases, the property IMFSPMF will capture the manifold.

We can also present a couple of weaker properties:

The identity of a component-identical with a one-component manifold will be denoted by IMFSICP:

X⮭IMFSCP≡def X⮭PP;MF˄∀y,z (y⮭ICP↓zy=z)z⮭X

The identity of a submanifold-identical with the entire manifold will be denoted by IMFSIPMF and defined as follows:

X⮭IMFSPMF=def X⮭PP;MF˄∀x,z (y⮭IPMF↓;zy=z)z⮭X

1. Bequeathing-to-component n:

X⮭BQ(CPn) ≡ ∀y,z y⮭X˄z⮭CPn;yz⮭X

Permanent bequeath:

A property is **permanently bequeathing** when it bequeaths from any component to any manifold. It will be denoted by UBQ(CP) and will be defined as follows:

X⮭UBQ(CP) ≡def ∀y,z y⮭X˄z⮭CP;yz⮭X

Much as I tried to think about this property, the only concepts captured by it are those that are common to all objects whatsoever (I found no formal proof to this fact, though). These are concepts that tell us nothing about the essence of particular objects. Therefore it is necessary to emphasize the partial bequeath:

A property is **partly bequeathing** when it bequeaths to some n(s) but not to all ns. It will be denoted by PBQ(CPn) and will be defined as follows:

 X⮭PBQ(CPn)≡def X⮭BQ(CPn)˄¬∀X X⮭UBQ(CP)

As examples for all those properties we can take the concepts presented above as bequeathing-to-parts and to items, such as "yellow" and "drowned in the ocean." They truly do bequeath-to-parts and to items, but not to members.

We can phrase it also in a different way:

Let n be the variable of the number of the level of the component, so that BQ(CPn) will denoted as concept bequeathing to level n. Namely,

X⮭BQ(CPn) ≡ ∀y,z y⮭X˄z⮭CPn;yz⮭X

A permanently bequeathing property will now be defined:

X⮭UBQ(CP) ≡def ∀n X⮭BQ(CPn)

While a partly bequeathing property will be defined:

X⮭PBQ(CP) ≡def ∃n X⮭BQ(CPn) ˄¬∀n X⮭BQ(CPn)

Now we will be able to formulate the above as characteristics of composition relations.

A manifold-component relation in which there is a bequeathing only for permanently bequeathing properties will be denoted by HUBQ(CP) and will be defined as follows:

X⮭HUBQ(CP) ≡def X⮭PP;MFn ˄∀Y (Y⮭BQ(CPn)Y⮭UBQ(CP))

In contrast, a composition relation in which there is also a bequeathing of partly bequeathing properties will be denoted by HPBQ(CP) and will be defined as follows:

X⮭HPBQ(CP) ≡def X⮭PP;MFn ˄∃Y Y⮭PBQ(CPn)

Indeed, unlike what might seem to be implied, the partial bequeathing is the stronger relation, while the permanent one is weaker. In view of the above, the relation of membership is HYBQ(CP) while the relations of parthood and itemhood are HPBQ(CP).

(One might contend that there could always be found some properties bequeathing to components, and that is due to the rule of universal conceptualization, which states that any collection of objects has a concept that captures them. However, even if this is agreed, we will still find in this collection some properties that are not common to all of its items, so that the bequeathing will not be permanent after all.)

Now we can redefine all the composition relations with the parameters we have set: Holistic composition; identity of a single component with the manifold, identity of a single submanifold with the manifold; and bequeathing.

Wholeness (whole-part):

MF1 =def FMF∩TRNS∩IDSMF∩IMFSICP∩IMFSIPMF∩HPBQ(CP)

Itemhood (collection-item):

MF2 =def FMF∩NMFTRNS∩HMF∩IDSMF∩IMFSCP∩IMFSPMF∩HPBQ(CP)

Set (set-member):

MF3 = def FMF∩NMFTRNS∩HMF∩HUBQ(CP)

Parenthetically we may add that in wholeness we could use the stronger characteristics IMFSCP and IMFSPMF, which also hold for it formally.

As we remember:

X⮭IMFSCP≡def X⮭PP;CP˄∀y,z y⮭X↓;zy=z

And since the consequent and the antecedent are both false the entire phrase is true and the definition is satisfied. The same is true with submanifolds:

X⮭IMFSPMF=def X⮭PP;CP˄∀x,z y⮭X↓;zy=z

Still, I preferred the broader and weaker definition, because I wanted to base the description of wholeness on conditions that are satisfied rather than on those that are not.

Now we can return to the foundational equivalence which I presented above (chap. ###). There we stated that:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭WH≡x⮭IP≡x⮭CL≡x⮭UCM

However, since we have just proved that WH is nothing but a partial concept of MP and therefore that IP is but a partial concept of ICP, we can now reduce this equivalence to more basic elements and state:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭MF≡x⮭ICP≡x⮭CL≡x⮭UCM

By this we brought forth the formalization of the three main types of manifolds – whole, collection and set – to a successful conclusion, reducing them all to the concept of manifold. *In adjecto*, this formalization is applicable to their inverses – part, item and member. While the two better-known components of those – part and member – have been so far considered as primitives in the theories relating to them (mereology and set theory, respectively), now they have become derivatives, thanks to the new primitive: component, together with the eight parameters we applied to it (or rather to its inverse) as intersections. Of course, the seven parameters I introduced here, or at least the last five, may be used for the creation of a crop of new types of manifolds, less intuitive than those we discussed. But at this stage we are not interested in those.

 In spite of the elegant uniformity we have reached above, I find it important to emphasize that from all the types of manifolds, the relation of part-whole has some priority. This priority stems above all from the fact that every discrimination of object is actually an act of partition of the world – to that part and to its complement – but also from reasons related to manifold theory itself. In manifold theory we defined the concepts of whole, collection and set as **parts** of the concept of manifold, even if occasionally it is subject to the requirement of holism. Indeed, from this aspect the use of the relation of partiality is circular, but here, too, we should not be puzzled by that, since we are discussing a foundational concept, and foundational concepts often "come together."

 Manifold theory completes our discussion of mereology. In the above discussion I turned mereology, that had been sometimes conceived as a step-daughter of logic, into one of its parents. Now I have reduced both mereology and logic to a more elementary foundation – and a sterner one – and by the way also helped them find a lost sister: collection theory.

### Extensions

Having built the different types of manifold, we can now define the extension of a concept. So far, extension has been considered as the set of all objects that "fall under" (i.e. are captured by) a given concept. We can keep using this definition, and will sometimes call the extension of this sense "the extensional set." This traditional extension will be denoted by ST(X), so that the letter X denotes the capturing concept. We will define it as follows:

ST(X) = Y|∀z z⮭X↔z⮭MM;Y

If we use the traditional notation of set theory the definition will go as follows:

ST(X) ≡def {z}|X⮯z

Yet extension may also be the *collection* of all the items captured by a given concept. When that is the case, we will call it "the extensional collection" of that concept. As we will see below, this term is not less useful that that of the extensional set. An extensional set will thus be denoted by CN(X), the letter X denoting the capturing concept. Let us define:

CN(X) = Y|∀z z⮭X↔z⮭IT;Y

We can now turn to the concept of number and its derivatives.

## Number and quantification

### Numbered, number

Once there is a concept there is a number. That is because every concept captures a certain *number* of objects. Even a concept that doesn't capture anything might be described as capturing a 0 number of objects. (Might a concept capture a negative number of objects? That is a nice question. In contrast to the mathematicians, who ingeniously developed entire branches of "pure mathematics," the logicians had less imagination and did not develop even one branch of "pure logic." once there is such, and I hope this will come true sometime in the near future, the above question will find its place within it). Furthermore, the size of the extension of a concept is one of the properties of that concept (i.e., it is a predicate that might be attributed to it, a second-order concept that captures it).

 The concept of number has been widely discussed in philosophy (though we could expect that it be discussed even more), and has often been taken as a primitive. I will not take the concept of number as a primitive, but rather the concept "[is] numbered [at]," from which the concept of number will be inferred. "Numbered" is a two-place predicate which means "having the extension whose components (members or items) are numbered at …" Namely, we are talking about a concept which captures only concepts. As I noted, this term which is in fact the predicate of quantification, is foundational, and will thus be taken as a primitive. From an Ockhamean point of view we earned nothing from this reduction (or replacement) of number to numbering, but, as we will see pretty soon, from the logical and metaphysical points of view it will help us understand the nature of number and attain the coherence required for our metaphysic. I will therefore state my above words more precisely: Once there is a concept there is also the concept of "numbered." The concept and the concept of numbered come together.

"Numbered" will be denoted by the letters NMBRD. Therefore, when we wish to say that the number of objects holding property X is y (and we will substitute numbers for y, once these are defined), we will write:

X⮭NMBRD;y

But according to the above conclusions, we may also say:

X⮭H;q(NMBRD),y

This sentence actually means: X is of the number y.

But according to the reduction rule we may also infer that

X⮭H;q(NMBRD)

This sentence means: X is of a number, i.e. X is a concept whose extension is a numeral magnitude (a NMBRD that is true for every concept).

But hence we get the definition of number. We will denote **number** by n and will define it as follows:

 n =def q(NMBRD)

It is important to emphasize that a quality is an individuum, and hence every number is an individuum. We will return to this below and will prove this proposition in another way.

When we want to use a different notation for the number of members/items in the extension of X we will denote it by n\*X. Let us define it:

n\*X=y ≡def X⮭NMBRD;y

For the sake of brevity we may sometimes give up the asterisk (but will keep it in mind for the metaphysical discussion), so that we may rephrase the above definition as follows:

nX=y ≡def X⮭NMBRD;y

In fact, I have described quantification (the "size" of the extension), that was expressed above by NMBRD, as a primitive and the number, denoted by n, as a term inferred from it. We could, of course, do it in the other direction. If we took n as a primitive, we would define NMBRD by it:

X⮭NMBRD;y ≡def X⮭p(n);y

How will we express the number of individua? As Aristotle has already taught us, the number of every individuum is 1, since every individuum exists only once (the number 1 will be defined shortly). There was only one Samson, there is only one Western Wall, and only one Friday 15.4.2016. We should be very precise in this issue. When I say "There is only one Western Wall" I seem to attribute a number to the individuum itself, while we said that the concept "numbered" might be only attributed to concepts. However, when we say that "There is only one Western Wall" we mean that the *uniextensional concept* of the Western Wall is numbered at one (i.e. it has an extension of only one member/item). If, for instance, we denote the Western Wall by the letters ww, and the number one by the conventional sign 1, we can therefore state:

ww↑⮭NMBRD;1

However, even if in essence the predicate NMBRD relates only to the concept of the Western Wall, it obviously tells us something about the Western Wall itself, as an individuum. Furthermore: When I say "Joanne, Jean and John are numbered at three" I essentially relate to the extension of a concept which captures all of these three persons, but I am also saying something about each one of them qua individuum. We should therefore formulate a predicate which describes this relation. We will call it "individually numbered," denote it by ND and define it as follows:

(x,y…)⮭ND;z ≡def (x,y…)↑⮭NMBRD;z

Note: The difference between NMBRD and ND is crucial. Let us take, for instance, the concept 'dog', and denote it by DOG. The sentence DOG⮭ND;1 is true, since the concept 'dog' is one; but the sentence DOG⮭NMBRD;1, which states that the extension of 'dog' includes only one item, is obviously false.

Everything has a number. We should therefore state as follows:

∀x∃y x⮭ND;y

And according to the reduction rule we may infer:

∀x x⮭ND

And if so, we may add ND to the foundational equivalence presented above (###), and now state:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭MF≡x⮭ICP≡x⮭CL≡x⮭UCM≡x⮭ND

We added another constituent to that equivalence. And it is not yet the last one.

### The essence of number

Number exists only in a world of plurality. In a Parmenidean world there are no numbers. Supposedly, the number one does exist in it, but when there is no number from which this "one" can be discriminated, that one itself is not discriminated, and consequently does not exist.

 Number is indefinable. The definitions suggested by Frege, Russell and Zermelo-Fraenkel seem, on the face of it, as presuming certain numbers from the outset, and therefore as circular, even more so in the light of the fact that Frege himself admitted his failure to define number and Gödel has proved that the definitions offered by his predecessors were insufficient. Hence comes the need to take concept of number as a primitive. Above I have defined it through another concept, NMBRD, but I also noted that there is no principled difference between taking NMBRD as a primitive and taking n as such.

 The concept of number is often conceived as logically related to the action of counting, but the concept of this action presumes the primacy of the natural numbers, a presumption that is postulated neither by logic nor (consequently) by metaphysics. In truth, the nature of number in its "pure" sense is conceived only through the metaphor of the number line (and I will corroborate this claim below). As we know, this line is described (metaphorically) as straight, i.e. as a one-dimensional object, lacking points of starting or ending, and the points on it are the numbers. In order to get the natural numbers we have to carry out an action of partition of this line. Someone, whom we may call "the partitioning agent," supposedly takes a certain point as a starting point, and from it skips to the next points on the line in certain length units, while keeping three rules: equality, disjointness and tangency. Equality – the units have to be of equal size; disjointness – the units have to be disjoint from one another (i.e. not overlapping); tangency – the units have to be tangential to one another, i.e. having a common boundary, so that the ending point of one unit is the starting point of the next one. The points accepted from this skipping are those that are determined to be the natural numbers, and each skip unit is determined to be one numeral unit (the question to what degree these numbers are more "natural" than others will not be discussed here). These numbers serve us for counting, while keeping other maxims such as: generality (one must encompass all the objects of the set of the counted objects); non-repetition (one mustn't count the same object more than once); and formality (one must ignore the character of the counted objects, whether they are big or small, beautiful or ugly, etc.). Taken all these we can know the number of objects in a given set. (Amd parenthetically we may add that in a Wittgensteinian language the above descriptions manifest that counting is not "a state of things" but rather "obeying of rules.")

 The early Frege deliberated about whether numbers are concepts or individua ("objects" in his terminology) and concluded that the latter is the correct answer (Frege, 2007, section 62). He based this conclusion on a line of argumentation that centered on rephrasing sentences containing numbers in order to isolate the role of the numbers in them (and indeed, Frege himself backed down from some of his arguments; Klement, 2012). I agree with the conclusion, but not in the method of its proof. If we continue this line of argumentation, we will not get very far. On the one hand we can argue: the sentence "The number of the oranges on the table is 4" is built, grammatically, the same as "the colour of the sky is blue"; "Blue" is a concept and therefore "4" is also a concept. On the other hand, we can also argue that the very same sentence "The number of the oranges on the table is 4" is built in the same grammatical structure as "The capital of France is Paris"; "Paris" is an individuum, and therefore "4" is also an individuum. This is another example for the various uses, and the confusing character, of the copula.

 Supposedly, it would be more correct to argue as follows: When I say "The number of the oranges on the table is 4" it is applying the concept of "the number of the oranges on the table" to an ontological sphere, while "4" is the object captured in that sphere. This means, allegedly, that it is an object. This argument is strong, but not preponderant, since a concept is also an object (of another concept). Therefore, we could propose the possibility that "4" is a first order concept while "the number of the oranges on the table" is a second order concept, i.e. a concept that captures a concept. Yet, at this very point we should see that Frege's argument was based on a wrong premise. The core of the problem lies in the fact that Frege created a sharp dichotomy between concept and object. We, in contrast, recognized the fact that an concept might be an object, and furthermore, that all concepts are objects, and therefore the fact that an expression serves in the sentence as denoting an object does not exclude it from denoting a concept. Thus, for instance, in the sentence "The colour of the sky is blue" the word blue might denote an object even though we usually use it as a concept (see above ### tropes).

 We could supposedly see numbers as concepts and arithmetical operations as relations between concepts. According to the test of Frege himself, a number ought to be considered as a concept, since it appears to be an "unsaturated" function. If we take the number two as an example, we may say all of the following sentences: "The number of the tablets is two"; "The number of a person's biological parents is two"; "The number of the world wars (so far) is two." "Two" functions here as a predicate. For this reason, the old Aristotelian test, which is not so very different from Frege's, leads to the conclusion that a number is a concept, since it might be attributed to many objects. However, the same Frege that gave us the test of the unsaturated function is the Frege that gave us the distinction between sense and reference. In the same way that we can see all the above occurrences of the number two as unsaturated functions, we may see them as different senses of the same referenced object. Thus, for example, we may say that "The third king of the Israelites is Solomon," "The initiator of the First Temple is Solomon" and "The traditionally attributed author of the book of Proverbs is Solomon." In all of these sentences the words "… is Solomon" appeared, in terms of the grammatical structure of the sentence, in the same place that "… is two" appeared in the above series of sentences. Will this be a reason for us to say that Solomon is a concept and not an object?!

 The truth is that numbers as such are indeed objects (in Frege's terms), which means individua (in the terms of this book), but every such individuum has a uniextensional concept. The numbers are individua because if they were concepts the sum (i.e. union) of 1 and 2 would not be 3 but the concept of "1or 2" (as written above, at ###). However, every number has a uniextensional concept, since every object has such a concept.

 Consequently: the union of 1 and 2 is 3, but the union of the uniextensional concepts of "1" and "2" is the concept of "1 or 2." In fact, the union of all the existing numbers is the concept of "number" per se. We might have thought it to be an infinite concept, since it is the union of infinity of finite numbers, but the concept of number is a finite concept, since it has boundaries just as any other concept (there are objects that are not numbers, and it is discriminated from them); yet it is prone, in principle, to infinite partition. (By this it does not differ from other concepts: The concept of colour, for instance, is finite, since there objects that are not the concept of colour, but can be partitioned into an infinite number of colours and hues.) Instead of saying that the concept of number is infinite it will be correct to say another thing, that is that the concept of number is partitioned into an infinity of parts, which are the concepts of the specific numbers. This understanding fully complies with the conclusions I have reached above.

 According to the above discussion we can reach the conclusion that number is a quality. It qualifies on all the criteria of a quality: on the one hand it is general, since many objects "take part" in it, but on the other hand it's an individuum, because of the two arguments we presented above – because it is a quality, and every quality is an individuum, and because the operation of union between two numbers functions like a union of two individua (as explained above).

### Specific numbers

From the concept of number we can now construct the entire Peano system with the help of mereology. As we remember, it is also possible to construct it through set theory, as was demonstrated by von Neumann, but in that I see a logical problem, since the axioms of set theory are formulated in the language of predicate logic, and the latter contains quantifiers, which, in turn, are built on the concept of number.

I will add parenthetically that in contrast to Peano (1967), the concept of number 1 will not be inferred from the concept of the subsequent, but rather the opposite is true: the subsequent of x will defined as the sum of x and 1. In Peano's system the concept of the subsequent requires a new primitive, while in truth that concept might be inferred from the basic concepts of mereology (as shown above). Embracing this latter path enables us, therefore, to have the advantage of Ockhamean parsimony. (Below, when we discuss the concept of the comparative we will suggest another way to define the subsequent, through the concept "greater than … by …")

The number 1 will be defined as follows:

1 =def nu↑

Namely, this is the number of the objects captured by the concept of "world," or, in a less accurate wording, the number of the worlds. Theoretically, we could define the number 1 in plenty of other ways: The number of the kings of the dynasty of Saul, the number of Denmark's capital cities, the number of Brahms' violin concertos, the number of the suns in our solar system, and so on. However, when we discuss metaphysics we should adopt a formal rather than a concrete definition, and this is the case about u, for it was defined by the predicate of parthood alone. Such a definition is guaranteed to be true in every possible world.

The number of every concept is 1, since every concept exists only once: There is only one concept of "yellow," only one concept of "beautiful" and only one concept of "the first king of the Israelites." (Here, too, we must be precise and emphasize that we relate to the concept qua object, and not to the size of its extension, and only in its existence qua object it is an individuum as all the rest, and, as such, exists only once, as Aristotle has taught us.) The only thing related to concepts in which we can speak of a number greater or smaller than 1 is the number of members/items in the extensions of a given concept. When we state the number of certain objects, using the predicate NMBRD, we actually state the number of items in the extension of the concept which captures those items.

Now let us return to some of the above statements and formalize them:

∀x x⮭Ix↑⮭NMBRD;1

∀x x⮭Kx↑⮭NMBRD;1

But since I∪K=O, we can state that:

∀x x⮭Ox↑⮭NMBRD;1

Having clarified the nature of numbers we can now turn to arithmetic operations.

### Basic arithmetic operations

In the course of our discussions on numbers we will need to use the basic arithmetic operations. The concepts of sum and difference will remain as defined above (in Chapter ### mereology). Let us recall:

Arithmetic sum is nothing but a discrete union, and is defined as follows:

x+y =def x∪y|x⮭NOL;y

While the definition of difference is:

x-y=def ɩz(y∪z=x)

We will denote the concept of subsequent by the letters SSQ and will define it as follows:

x⮭SSQ;y = def x=y+1

From this point our way to the developing of the entire Peano system is paved, and I will not do it here. It is important to me, however, to formulate a few definitions that will be required for our further discussions.

The concept of predecessor, denoted by PRD, is the inverse of the concept of subsequent:

x⮭PRD;y =def y⮭SSQ;x

PRD⮭INV;SSQ

The number 0 might be defined in the way it was defined by Frege, i.e. the size of the extension of a concept that contains a contradiction. That is a fine definition. However, we do not necessarily have to enter into such a relatively complex definition and may instead define it simply as follows:

0 = x| x⮭PRD;1

Or as follows:

0=x-x

Some philosophers had troubles with zero. Already medieval and Renaissance thinkers objected to it (Kaplan 1999) and Husserl contended that zero is not a number just as "nowhere" is not a place and "never" is not a time (Husserl, 2003, p. 138). But it seems that he was wrong. If we know about some temporal point right after which time began, or even if we only wanted to describe such a point theoretically, we could consider it as "never," and say that events which did not happen that they "happened," so to speak, at that point. As for space, it is a little more difficult to imagine such a point since it, in contrast to time and the number line, is not one-dimensional, but the above argument is pertinent, *mutatis mutandis*, to it as well.

The number 2 will, obviously, be defined as follows:

2=def 1+1

In the same way we can define all the natural numbers.

In continuation to the above, the number infinity is also definable:

∞ =def ∀x ny(y⮭PP;x)

This definition lies on the assertion that every object might be infinitely divided. Another option, perhaps even more elegant and parsimonious than the previous one, is the following:

∞ =def nx(x=x)

Namely, infinity is the number of objects that are identical to themselves, which means the number of the objects in the world. This definition relies on the same premise underlying the previous one: The number of the parts of every object is infinity. Since that is the case, the number of the objects in the world is also infinity.

The concept of natural number will be denoted by NN and for its definition I will embrace, with a slight change, Peano's definition:

x⮭NN =def (x=0˅ x=y(y⮭SSQ;0)˅ x=z(z⮭SSQ;y)˅…)

The concept of negative integer will be denoted by NINT and will be defined as follows:

x⮭NI =def (x=0˅ x=y(y⮭PRD;0)˅ x=z(z⮭PRD;y)˅…)

The concept of integer will be denoted by INT and will (as expected) be defined as follows:

x⮭INT =def x⮭NN˅x⮭NINT

We could go on in the same way and define similarly all the arithmetic operations, but that work has been well done long since, and for our purposes we can stop here.

### Magnitude

Now we can introduce the concept of **magnitude**. Magnitude is the quantity of the closure, i.e. the quantity of the entity that is enclosed within the confines of a given boundary. (This is also true in non-physical contexts, such as concepts, as explicated below). While this characterization looks like a definition, it is not a proper definition but a circular one, since it requires the concept of quantity, and quantity is a magnitude. Admittedly, quantity is often related to number, but in truth magnitude is a quantity that does not necessarily require a number. Numbers measure quantities only if we determine measuring units according to the criteria mentioned above with regard to the number line: equality, disjointness and tangency. But these units are units of magnitude, and the concept of equality also refers to magnitude, and thus it is impossible to determine them without presuming the concept of magnitude. In other words, even if a magnitude is *measured* by numbers, its essence might not be fully captured by those numbers, and any attempt to reduce magnitude to numbers is doomed to failure, since whenever we try to characterize a magnitude through a certain number of measuring units, those units will have to be equal, disjoint and tangent, and so we will find ourselves going in a vicious circle.

Magnitude ought to be a primitive, but since we need a concept and not an individuum, we will take the concept **having the magnitude …**, or just **sized**, as primitive, and will denote it by MG. The sentence "x is sized y" we will therefore write: x⮭MG;y. Consequently, magnitude will be written as q(MG).

According to the above, any object has a magnitude. Since every object is determined by its boundaries, and within those boundaries there is a closure of certain magnitude (regardless whether finite or infinite), that closure, which is actually the object "itself," has a certain magnitude. Therefore we may state:

∀x,∃y x⮭MG;y

But according to the reduction rule we may deduce that:

∀x x⮭MG

It is worth emphasizing this point, since the reader might ask: Does any object have magnitude? Do qualities have magnitudes? Do thoughts and emotions have magnitudes? The answer is yes, and not because thoughts, feelings and emotions should be treated as "lines, planes, and bodies" (Spinoza), but simply because we refer to magnitude in its logico-metaphysical sense. As I said, every object is discriminated by its boundaries, and every boundary determines the closure, which is the object "itself." Whatever has a beginning and an end has a magnitude, and even what doesn't have a beginning or an end has an infinite magnitude. This does not imply that we have fixed standards for measuring those magnitudes, and therefore we won't be able to devise instruments for their measuring, but this fact does not rebut the claim that they do have magnitudes. Therefore the concept "has magnitude" has to be taken as a primitive that is required for any logic and any metaphysic to be built on it. Soon I will return to this issue.

 Since above we stated that the concept "has magnitude" would be denoted by the letters MG, we will use it as a predicate which captures the object whose magnitude we will wish to assert. Here, however we might encounter a problem. How is magnitude measured? If we talked about length, width or height – we would use length units; if volume – volume units; if weight – weight units. But now, might we not say that the magnitude of the Eiffel Tower is 10,100 tons and that the magnitude of the Eiffel Tower is 324 meters, and so create an identity between 10,100 tons and 324 meters? The answer is negative, of course, in light of what I wrote above (### tropes). If we wish to talk about a uniextensional trope we will have to talk only about the measuring units relevant to that trope; and if we talk about a trope of larger extension, we will not be able to claim identity but only capturing. I will explain this.

 Let us denote "has weight" by WEG and "has height" by HGT. It is clear that the following sentence is true:

WEG,HGT⮭PP;MG

The measurement unit ton will be denoted by "ton" following the number, and the measurement unit meter will be denoted by "m" following the number. If we denote the Eiffel Tower by "te" we can now state (with some approximation):

te⮭WEG;10,100ton

te⮭HGT;324m

This can also be formulated as an identity between two objects:

Q(WEG)\*te↓=10,100ton

q(HGT)\*te↓=324m

However, from these sentences it is impossible to deduce that Q(MG)\*te↓=10,100ton or that Q(MG)\*te↓=324m. All we can state is only by the describing the capturing:

Q(MG)\*te⮯10,100ton

q(MG)\*te⮯324m

Namely, that *a* magnitude (*some* magnitude, and not *the* magnitude) of the Eiffel Tower is 10,100 tons, and that *a* magnitude of the Eiffel Towers is 324 meters.

 The concept of magnitude necessitates adding measuring units to the numbers. However, it applies to different objects in different manners. It is obvious that physical objects have magnitudes that are measured in physical measuring units (meters, square meters, cubic meters, kilograms, etc.) and it is obvious that concepts have intensional magnitudes, unrelated to their extension. Today we don't have the means to measure these magnitudes, but I am confident that such measures will be developed in the future. Note, that I do not refer to the measuring of sensations – an issue discussed since the time of the Pythagoreans and recently the subject of impressive achievements – but the measuring of concepts qua concepts. Thus, for instance, the concepts "part" and "whole" must be of the same magnitude, due to the symmetry between them, even though the extension of "whole" is larger by one item that the extension of "part" (the item being the world). The concepts "good" and "bad" are probably of the same magnitude – again due to their symmetry – but there is no reason to think their extensions are of the same magnitude. (Whose is larger? Depends if you're an optimist or a pessimist!). The concept of pink centaur is smaller than the concept of centaur, although their extensions are equal (both size zero, at least in the sphere of reality). That is because "pink centaur" is a (proper) part of "centaur," and the part is smaller than its whole. Furthermore, it appears that the size of the extension of a concept might change through time (I am not completely sure of that, but we'll leave this at present), while the magnitude of the concept itself is a-temporal.

 However, the magnitudes of concepts have no measuring units, and as long as those are not developed, the use of magnitude in relation to them can be done in a very limited scope. Needless to say that this type of magnitude cannot be measured by the units we use to measure physical objects. But this should not stop us, in my opinion, from using the term "magnitude" with regard to both types of objects. The same might be true also for other types of objects to which this term can be relevant, insofar as the rules applying to magnitudes work for all the types of objects.

 Nevertheless, in some cases we can measure magnitudes of concepts, or at least relations of magnitude between them. Here are some examples:

First, as I said, the whole is larger than its part in concepts as well. Usually we cannot determine the relation between the magnitude of concepts and the size of their extensions, but when parthood relations are concerned it is possible to a certain degree: If we have two concepts X and Y, and Y is a part of X, it is impossible that the extension of Y is larger than that of X; it is either of the same size or smaller than it. We can formally state it as the rule of extensions:

∀X,Y Y⮭PP;YnY≤nX

Second, let us presume that the distinction between colours on the colour spectrum is made by the length of their waves, and that a difference of 5 nanometers causes a difference in colour perception, i.e. a hue. In such a case, we have a tool for measuring the distance between the boundaries of the concepts of hues regardless of the extensions of those concepts. Consequently, we can measure relations of magnitude of colour concepts. If, for instance, colour A is defined as ranging between wavelengths of n and n+5 nanometers, and colour B is defined as ranging between wavelengths of n+5 and n+20 nanometers, the concept of B is 3 times larger than the concept of A (regardless of how many objects each of them captures).

Third: Even in other types of concepts, less prone to scientific standardization, it is possible to determine intuitively some relations of equality or inequality of concepts. Thus, for example, we may assess that two opposite concepts are of equal magnitude. Above, I gave the ethical example of good versus evil, but the same is true for aesthetic concepts (beautiful versus ugly) and other evaluative concepts (smart versus dumb), and also of metaphysical ones (being versus naught).

 If we could measure concepts, we could create a quasi-mathematical calculus for them. The problem, of course, is that we do not have a way to measure them. But maybe there is nevertheless some use in creating such a calculus after all. First, because there are measurable concepts; second, because it has a theoretical importance even without applicability. I do not intend to do it here, but in the following lines I will present some directions for further thinking.

 When I am talking about measurable concepts I am talking, for example, about colours and sounds. In the past, the sensations of colour and sound were purely qualitative, and not measurable. Today we can say that the difference between colours is measured by their wavelengths and sounds by the difference between the quotients of their frequencies. Consequently we can measure if the concept of red is (intensionally) larger, smaller or equal to the concept of blue, and if the difference between sol and la is equal to that of la and si.

 The desired calculus can be built by units of conceptual magnitude. In principle, we could try to attach accepted numerical values to every concept. For this purpose we wouldn't need to define the good, the beautiful, the wise or the existent, since the attached numerical value would not express the meaning of the concept, only its intensional magnitude. However, it is quite clear to me that such a suggestion will be judged as impossible, not to say weird, and therefore at this stage we can suffice with attaching only a schematic numerical value, by using variables (n, m etc.), for the only purpose of expressing magnitude relations (equality or inequality to its various types), not the magnitudes themselves. In fact, the expression of magnitude relations does not even require variables, and it is enough to decide that one concept is larger than the other by the tools we will develop below, in the subchapter on the comparative. This way we can treat the concepts of conceptual magnitude more rigorously. Still, there is no doubt that there is a long way between these poor tools of measurement and the ideal condition, in which we would have measurements for concepts just as we have for physical objects.

 Above I noted that the concept "having magnitude' must be a foundational concept of logic and metaphysics, since every object qua object has some magnitude. Beyond that, we have seen that the concept of "having magnitude" has to be a primitive for other reasons as well. First, every concept has a magnitude (as explained above), and since every object is captured by a concept, we must see the concept of "having magnitude" as required by every logic and every metaphysic. Second, the concept of number necessitates the preliminary determining of the concept of magnitude, in order to keep the equal distance between one number and the other on the number line. Therefore we must see the concept of "having magnitude" as required for the concept of number which is, in its turn, required for the building of every logic and every metaphysic. One way or the other, the fact that every object qua object has a magnitude leads us to include the concept of "having magnitude" in our foundational equivalence, that will now look as follows:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭MF≡x⮭ICP≡x⮭CL≡x⮭UCM≡x⮭ND≡x⮭MG

By this we have completed the foundational equivalence. All of these concepts are the concepts that capture every object qua object. As far as I can see, there are no such concepts beyond this list. And if so, then we have attained here, for the first time in the history of philosophy, a rigorous and formalized expression for the concept "being qua being." The attainment of this rigorous characterization is, as we know, the great aspiration of metaphysics since Aristotle. And here we deciphered the mystery.

 For the sake of accuracy I should add that this is not the plain "being qua being" but rather "being qua plurality." In a Parmenidean world there would be neither concepts nor captured objects, neither manifolds nor components, neither numbers (not even the number 1, since it is discriminated only thanks to the fact that there exists other numbers) nor magnitude. However, our world is not Parmenidean (as wrought in the rule of basic existence). And since our world is of plurality and discriminations, these are the foundational tools required for their creation. Yet, we have a few more miles to go before we extract from this knowledge what has to be extracted.

As marginalia to the discussions above I would like to note that the logical tools which I presented here included some basic terms of mathematics. Does this mean that I am committed to the logicist thesis? I find it almost obvious that this thesis should be embraced, but in an upgraded version. Any person who is trained in metaphysical thinking (and philosophical thinking in general) seeks the elementary system of tools from which he/she will be able to deduce the foundational concepts of all the systems in the world. (I use the word "system" in its plain meaning, not its epistemological one, such as in Source Theory). Since logic seeks to formulate a similar system by formal means, the aspirations of logic and metaphysics overlap. Therefore, logic, too, wishes to create the groundwork for mathematics. If it succeeds in doing it by a certain logical calculus, which we can name A, then that calculus has been proven successful, and all is fine; if not – we have to create another calculus for the foundations of mathematics, which we may name B. But this latter calculus, as soon as we develop it, will immediately become a **part** of logic, which **complements** the ordinary calculus, and hence what we will call "logic" will have to be a third system C, which will include A+B. And the same is true for any additional elementary system that will fail to be deduced from the existing system. Logicism, in the way we know it, sought to deduce the foundations of mathematics from the logical calculus of Frege or that of Whitehead-Russell, and this thesis, on which I will not dwell here, might indeed be problematic; but the question of whether the foundations of mathematics might be deduced from logic *per se* – not one or another calculus – must be beyond debate, and the answer to it must *necessarily* be positive. And if that is the case about logic, the same is true for metaphysics as well, the two being closely linked as I have written above.

All of this is argued from the principled point of view. But, as I have demonstrated above, even on the concrete level there is no reason for separating the foundations of mathematics from logic: Every logic includes its quantifiers, and those, in turn, presume the concept of number. This is even more so with our logic, which has in its foundations the concept of concept (K). Any concept has, by its very essence, a numerable extension, and the number is therefore an inseparable part of it.

### Comparative

The discussion of logic will not be complete without a proper formalization of the comparative and the superlative. This began to interest some 20th century logicians, but the yield was not rich and the suggestions raised were not always impressive. Some of the suggestions took those expressions as primitives, in contrast to Ockham's razor, and others needlessly relied on mathematical patterns, sometimes circular. I will propose here a new way for definition, based purely on logic (the latter including mereology, as we remember).

 Let μ be the operator denoting "more … than," when it comes before a multi-place predicate.

 MG denotes "… is large," because for our purposes "… is large," insofar as we do not mention how large it is, is the same as "having magnitude."

 Now we can define "… is larger than …," which will be denoted by μMG:

x⮭μMG;y ≡def q(MG)\*x⮭WH;(q(MG)\*y)

Let LT denote "small." We can now define "… is smaller than…" which will be denoted by μLT:

x⮭μLT;y ≡def q(MG)\*x⮭PP;(q(MG)\*y)

Or:

x⮭μLT;y ≡def y⮭μq(MG);x

In other words, μMG and μLT are inverse concepts:

μMG⮭INV;μLT

And from this we can deduce, as a tautology, that the whole is larger than its part:

x⮭WH;y → x⮭μMG;y

Parenthetically, I will add that according to the reduction rule the predicates μMG and μLT also have meanings as one-place predicates, as in the sentences x⮭μMG or x⮭μLT. We might suppose that these expressions are meaningless, since in order to be "greater than…" the greater object must have a smaller object. However, as I noted above, these are constraints of natural language which must not enchain logical language, which surmounts it. If we nevertheless wish to facilitate this issue, the reader may, at the adaptation phase, think of these predicates as "greater than something" or "smaller than something." This hints that these predicates are relative in their basic essence, and maybe this is the essence of the predicates "large" and "small" on the whole.

 Equality, which will be denoted by EQ, will also be defined:

x⮭EQ;y =def q(MG)\*x⮭ID;q(MG)\*y

And now we can adopt the common mathematical notation:

x>y =def x⮭μMG;y

x<y =def x⮭μLT;y

x=y =def x⮭EQ;y

(The sign "=" was adopted for the sign of identity, and obviously identity and mathematical equality are not the same, but for our purposes we can take them as such).

x≥y =def x=y˅x>y

x≤y=def x=y˅x<y

It is also clear that in this way we can also express relations of magnitude on the number line: 2 is greater than 1, and so on. However, in the mathematical notation we cannot express sentences such as "… is greater than … by …" unless we transform this sentence to a sentence dealing with subtraction. In contrast, with the verbal predicate we can express it as follows:

5⮭μMG;3,,2

The concept "plural" will be denoted by PLR, and we will define it as follows:

nX⮯PLR =def X⮭Y)Y≥2)

And now we can define other mathematical concepts as well:

The concept "positive number" will be denoted by PN and will be defined as follows:

x⮭PSN =def x≥o

The concept "negative number" will be denoted by NGN and will be defined as follows:

x⮭NGN =def x≤o

 Now we will address the basic comparative concepts, "more" and "less." As we have seen above, we have defined the concepts "greater than" and "smaller than" without turning to "more" and "less," relying solely on the concepts of parthood and magnitude. However, it seems that other comparative concepts do not enjoy this advantage: "better," "smarter," "nicer," "richer," "more crowded" and other comparative concepts expressing "more" as well as their opposites expressing "less" – all of these cannot be defined through the concepts of parthood and magnitude. Yet, once we have defined the "more …" by concepts we had in our toolkit – those of parthood and magnitude – it stands by itself and can be applied to other predicates as well.

 However, before we do that we had better dwell on the character of these concepts. What is the character of "smarter" for instance? A predicate of this type requires a scale of degrees: "slightly smart," "medium smart," "very smart" and other degrees of smartness. Only after such a scale exists can we say that the "very smart" is smarter than the "medium smart" and so on. This scale can be conceived in two different ways:

**One way** views the function of capturing as non-binary but rather gradual. The concept "wise" might capture Smith to a greater degree and Jones to a lesser one. Theoretically, we could try to quantify, at least schematically, the degree of capturing in each case. This approach is reminiscent of Plato's "taking part" of the objects in the ideas, but not less so the premises of Fuzzy Logic, which is much younger than Plato. If we adopt this approach, the concepts of "more" and "less" will refer to the degree of capturing.

**Another way** will view all those predicates as parts of the predicate "wise." Just as beforehand (in Chap. ###) we stated that "light yellow," "middle yellow" and "dark yellow" are all parts of the union-concept "yellow," so we can talk about "very wise," "middle wise" and "slightly wise" as parts of the concept "wise." If this approach is correct, the concepts of "more" and "less" will be taken as predicative constituents within the partitioned concepts.

 The metaphysic proposed in this book does not postulate a decision between the two ways, but accepts both of them together. As we have seen, the concept "yellow," just as many other concepts, is a spectrum-like concept, i.e. is built as a continuous scale starting from the light yellow and ending with the dark one. If we take the famous colour spectrum as presenting the parts of the concept "colour," the determining of the boundaries of "yellow" in the place they were determined is a matter of decision, and the same way we could determine them to be more "to the right" or more "to the left" of that place. However, once we have determined that boundary, it contains within it all the continuous range from the most left to the most right sides of the concept "yellow." This range, too, can be partitioned by boundary lines that will discriminate between light, medium and dark hues of the colour, or by any other boundary lines. The function that will determine those boundary lines will be, whether exclusively or not, the fact that one side is "darker" and the other is "less dark," or "lighter." And the same is true for other concepts, such as "wise."

 Let us state, then:

 The operator "more" will be denoted by μ before the concept to which it refers. Thus, for instance, if BTFL denotes "beautiful," then "more beautiful" will be denoted by μBTFL. We have already used this operator above in the expression μMG.

RST characteristics of the operator μ when adjacent to some predicate Y resemble those of the sign > in mathematics:

¬(x⮭μY;x)

¬(x⮭μY;zz⮭μY;x)

(x⮭μY;z˄z⮭μY;w)x⮭μY;w

The operator "less" will be denoted by λ before the concept to which it refers. We can define it easily:

x⮭λY;z =def z⮭μY;x

RST characteristics of the operator λ when followed by some predicate Y resemble those of the sign < in mathematics :

¬(x⮭λY;x)

¬(x⮭λY;zz⮭λY;x)

(x⮭λY;z˄z⮭λY;w)x⮭λY;w

We will also present a third operator, which will denote the concept "equally-sharing." x is equally-sharing concept Y with z iff both of them are captured by the same concept Y to the same degree. For instance: x is exactly as tall as z or even x is just as wise as z. We will denote it by the sign ν (the Greek letter Nu, not to be confused with the Latin v) followed by the concept shared by the two objects to an equal degree. There might be some more elegant formulation for the definition of this operator, but for the moment I will suffice with a definition by elimination:

x⮭νY;z =def ¬ (x⮭λY;z) ˄ ¬(x⮭μY;z)

Note, that I did not mention in the definition the postulation that the concept Y actually captures x and z (x,z⮭Y), since x and z are equally-sharing even if they both are not captured by Y, i.e. when the degree of capturing is 0.

RST characteristics:

x⮭νY;x

x⮭νY;z↔z⮭νY;x

(x⮭νY;z˄z⮭νy;w)x⮭νY;w

Now we will present the operators equal/more sharing, denoted by νμ, and equal/less sharing, denoted by νλ, and will define them as follows:

x⮭νμY;z =def (x⮭νY;z) ˅(x⮭μY;z)

x⮭νλY;z =def (x⮭νY;z) ˅(x⮭λY;z)

The RST characteristics of the operator νμ followed by some predicate Y resemble those of the sign ≥ in mathematics

x⮭νμY;x

x⮭νμY;zz⮭νμY;x

(x⮭νμY;z˄z⮭νμY;w)x⮭νμY;w

The RST characteristics of the operator νλ followed by some predicate Y resemble those of the sign ≤ in mathematics

x⮭νλY;x

x⮭νλY;zz⮭νλY;x

(x⮭νλY;z˄z⮭νλY;w)x⮭νλY;w

We can now turn to defining the superlative operators.

The operator most will always be followed to a predicate and will be denoted by μμ and will be defined as follows:

x⮭μμY =∃x˄¬∃z z⮭μY;x

We might suppose that it could be followed only by one-place predicates, but in truth it can also be followed by two-place predicates ("loves Danny most"), three-place predicates, and even more.

The operator least is the opposite of μμ. It wll be denoted by λλ and will be defined as follows:

x⮭λλY =∃x˄¬∃z z⮭λY;x

It should be emphasized that the superlative mentioned here is an absolute one, i.e. a universal one: the wisest in the world, or the least wise. If we wished to express a relative superlative – someone or something that is "the most something" among some extension – we would face a problem. Let us take for example the concept "the wisest Pygmy." If we tried to build this concept by the intersection of the concepts "Pygmy" (PGM) and "wisest" (μμWISE) – i.e. PGM∩μμWISE – it would render a concept which means "the Pygmy who is the wisest [person] in the world,", and not "the wisest Pygmy." To build the latter concept we will have to create a formal construction reflecting a relative superlative, such as a limitation on the universe of discourse, as it is done in set theory. If we take the letter w as denoting the property whose holders are the extension in which the superlative is observed, we may also propose to build it this way:

x⮭μμY(w) =∃x x⮭w˄¬∃z (z⮭w˄z⮭μY;x)

Hence we can turn to redefine the world, the big individuum and the Big Concept:

u =def μμMG(O)

ui=def μμMG(I)

UK=def μμMG(K)

Note, that the concepts of the comparative and the superlative cannot be reduced by the reduction rule: If x is greater than y, it does not imply that x is great, since it can be small and y even smaller. If I say that a grasshopper is taller than an ant, this does not entail that a grasshopper is tall. If I say that x is the ugliest of all flowers this does not imply that it is ugly, since there aren't any ugly flowers, but only that it is the least beautiful among all flowers.

Towards the conclusion of this discussion I would like to introduce the term "weak continuity." This is only now the time to define it since it leans on comparative concepts. Above, (Chap. ###), I defined (ordinary) continuity as a state-of-affairs in which each part of the object has a partial common boundary with at least one other part of that object. There is, however, a different type of continuity, which I will call "weak continuity." In contrast to ordinary continuity, which has to do with the internal relation within a given object, weak continuity will be defined as a relation between two objects. However, this difference is merely technical, and is useful for reasons of convenience, since it would be equally possible, given the principle of free composition, to formulate each of them in the converse way.

 Two objects will be considered weakly continuous iff x is more something than y, and for any place between x and y we can find an object z that is less something than x and more something than y, and the same is true for objects that are between them. We will denote "weakly continuous with" by WEC and will define (at some length):

x1⮭WEC;x2 ≡ ∃Y x2μY;x1˄∀z1,Ez2,z3(z1⮭μY;x1˄z1⮭λY;x2) z2μY;x1˄z2⮭ λY;z1˄z3⮭Y;z1˄z3⮭λY;x2

I am fully aware that some will contend that I am embracing the naïve concept of continuum, the one abandoned thousands of years ago with the discovery of the irrational numbers, which allegedly break the continuity of the number line. Yet, even if we admit that the irrational numbers do break that line (and I personally am not sure that sizeless units can break a continuum, but let us leave that aside), this cannot be to the detriment of the above concept's right of existence. These are plainly two different concepts: Continuity in the ordinary sense does not capture the number line, while weak continuity definitely does.

 Weak continuity applies to both individua and concepts, and mainly to the latter, therefore the discussion of it in the context of the comparative, which is an operator applying to concepts, is very much in place. Some parts of the colour spectrum present a weak continuity: light yellow, darker, then even darker, and so on. Pulling the bow on the string of a violin (in contrast to strumming a guitar or a harp) presents a spectrum of sounds. When a certain odour is smelled in the air and then gradually fades away we face a spectrum of odours. And so on.

 At this stage we can turn to the issue of quantification.

### Quantification

In predicate calculus the two famous quantifiers are used: the existential (∃) and the universal (∀). As some logicians have pointed out, these are not the only possible quantifiers but simply the two most useful ones. The existential quantifier denotes "there is at least one…," i.e. if we denote the quantifying number by n, we say that n≥1; but the quantifier could just as well be n=1, n=2, n=3 or n≥1,500,000. Also, the universal quantifier, that seemingly denotes "all the …" (actually it is not that clear, as explained below), and we could suggest quantifiers such as "most of the …," the minority of the …," etc. As we build the mechanism of quantification in a way that will suit our improved logic, we have to build it in a more general way, so as to reflect the metaphysical assumptions underlying the operation of quantification.

 If we assume, as said above, that "is greater than…" and "is greater than/equal to…" are well defined (and indeed, they will be defined below) then, and only then, will we be able to define the existential quantifier as accepted in the tradition of contemporary logic:

∃x x⮭y ≡def ny≥1

Further below I will continue to use the conventional notation, only for the convenience of the reader, even though it might be misleading.

 We will now turn to the universal quantifier. In truth, I do not fully understand what it exactly is. When we pronounce it in speech we say "for all x," but what kind of quantifier is "for all"? "All" is a quantifier, of course, but what is the "for'? In contrast to the existential quantifier, that expresses an assertoric message (there is x!) the universal quantifier does not include such a message explicitly. Hence the well-known questions arise, such as whether the universal quantifier premises the existential one (i.e. "the existential premise", which, of course, does bear an assertoric message). Anyway, the very asymmetry between the two is troublesome.

 In principle, a quantifier is meant to tell us the number of items existing in a collection of the extension of a given concept (or the number of the members of the non-empty set of its extension; this doesn't matter at the moment). Therefore, when coming to create a formal quantifier, its basic form is that of number, as described above:

X⮭ND;y

where X denotes a concept and y denotes a number. When we want to say "All the Chinese are wise," predicate calculus forces us to formalize it as "For all x, if x is a Chinese then x is smart." This is sophistry. Admittedly, it is true, but does not directly reflect what the sentence tries to say, since the sentence in its plain form does not express a relation of entailment between being Chinese and being wise. (I am aware, of course, that entailment does not necessarily have to be causal; but here we do not have any entailment whatsoever, whether causal or otherwise). What the sentence in its plain form says is that there are a certain number of Chinese, and those Chinese are wise. Let us take the letters CHNS as denoting Chinese, and the letters WISE as denoting "wise." The correct formaliztion of the above sentence will therefore be:

nCHNS=n(CHNS∩WISE)

Namely, the number of the Chinese equals the number of the wise Chinese. The advantage of this notation over the one accepted in predicate logic is the fact that it returns the concept of quantifier to the realm of mathematics and enables us to subject it to arithmetic operations, thus paving the way for deducing further conclusions.

 The more we use the notation suggested above, the more we reduce the mystification and arbitrariness in logic. However, I am aware that this way of writing might be conceived as cumbersome in comparison to the existing practice, and I am similarly aware of people's conservative inclinations (often in the wrong places…), so I will continue to use the signs of universal and existential quantifiers accepted today. In this notation the universal quantifier will be defined as follows:

∀x x⮭Y = def nY=nO

Namely, the number of the items in the extension of the concept Y equals the number of all the objects in the world. Even though the above notation, based on n, is more correct logically and metaphysically, than the accepted notation, based on ∀ and ∃, now, that we have defined them by the more correct notation, we may keep using the customary signs. In case of need, especially when we need to understand the true nature of things for the purposes of the metaphysical discussion, we can always return to the more correct notation.

 I will offer a few examples for quantification possibilities open to us by the new method of notation. Thus, for example:

 The sentence "Some Chinese are wise," instead of the form ∃x(CHNSx˄WISEx) should be written:

n(CHNS∩WISE)≥1

Id=f we want to say that most of the Chinese are wise, the formalized sentence will be:

n(CHNS∩WISE)> ½nCHNS

If we want to say that the number of wise Chinese is one million, we will write:

n(CHNS∩WISE)=1,000,000

In the above example I referred to the sentence "For all x, x is Y." Yet, in most of the sentences in which we use the universal quantifier we do not state things about the entirety of objects, but, as in the example of the wise Chinese, about the items in the extension of a certain concept. We can therefore formulate a more general notation.

 Let n and m be variables of numbers. Let us define:

n//m x⮭y ≡def nx=m˄n(x∩y)=n

Let nO denote the number of the objects in the world. It is obvious that this number is infinity, but in order to avoid committing ourselves to a definite number (as well as getting into the plethora of problems related to this number in mathematics, logic and philosophy), I will denote it in the manner suggested above, which is more neutral.

From this form we can define the conventional quantifiers and forms related to them:

The existential quantifier:

∃x x⮭y ≡ (n|n≥1)//nO x⮭y

As long as x has not been defined as something specific it is a neutral "object." The property "object" does not add anything, so we can give it up and define simply:

∃x x⮭y ≡ ny≥1

The negation of the existential quantifier:

¬∃x⮭y ≡ 0//nO x⮭y

But here, too, the property "object" does not contribute anything, and so we may suffice with the simple form:

¬∃x x⮭y ≡ ny=0

I will return to the signs of negation in a moment. For now let us continue:

The universal quantifier:

∀x x⮭y ≡ n//n x⮭y

But the meaning of this is actually:

∀x x⮭y ≡ ny=nO

The negation of the universal quantifier:

¬∀x x⮭y ≡ (n|n<no)/nO x⮭y

But the meaning of this is actually:

¬∀x x⮭y ≡ ny<nO

Supposedly, by this I "legitimized" the continued use of the quantifiers of predicate logic, to which we have been so accustomed, without invalidating the conclusions of the new logic. In truth, however, the use of the traditional predicate calculus quantifiers is philosophically problematic. In the traditional logic the quantifiers bind variables of individua, thus giving expression to Russell's nominalist approach. In contrast, the concept calculus is based on Leibniz's approach, which is at odds with Russell's. In this approach quantifiers talk about concepts and tell us about the size of their extensions. Therefore we will have to create a notational system that will make room for some compromise between them, a compromise that will have a graphical linkage to the traditional quantifiers while essentially applying to concepts.

 According to the suggested notation, the existential quantifier will be denoted by ə and the universal one by ɒ. We will preserve the unhealthy asymmetry between the two quantifiers in terms of the assertoric message. The quantifiers will be attached, graphically as well as substantially, to the concepts to which they apply. This way we will express the fact that quantifiers quantify concepts, not xs.

The universal quantifier will be defined as above:

ɒX⮭Y ≡def nX=n(X∩Y)

The existential quantifier, which gives us the number of objects captured by a certain concept, we can define as:

ə(n)X ≡def nX=n

In terms of grammar I formulated the definition as if the subject of the sentence is nX, i.e. the number of the Xs; but in terms of metaphysics it is possible and more correct to see the sentence as referring to X itself, since, as we have seen, the basic concept from which n is inferred is NMBRD, which is attributed to a concept. Therefore the formulation must not mislead us, and we should treat the sentence as referring to the concept.

 Now we can discuss the existential quantifier. Here, too, the quantifier relates to a concept, and tells us about the size of its extension. The ordinary, existential, quantifier is only a particular instance of the broader existential quantifier, i.e. the determination that n is greater or smaller than 1 is just one possibility out of many to determine the number of the items captured by the given concept. For the sake of brevity we will mark this specific quantifier without any number beside it:

 əX ≡def nX≥1

Since the quantifiers always describe an extension of a concept, if we want to assert that a certain individuum exists we will have to determine the existence of a uniextensional concept that captures it. Thus, for instance, if a is an individuum, we will state its existence as follows:

ə(1)a↑

Yet, since a↑ is a uniextensional concept, i.e. a concept capturing one object, the bracketed addition (1) might be considered redundant, and the very same information mght be conveyed also in the following sentence:

əa↑

Still, there is no problem with the longer form and both of these modes of expression, that are logically tantamount, will be acceptable.

 Before we continue, we must dedicate another discussion to the concept of negation. We have already seen that we can define negation as complement, but we have also seen that it can be defined as an assertion regarding quantification. i.e. nX=0. Although we have no formal proof for it, we can state the equivalence:

x⮭CKy ≡def n(x↑∩y)=0

An argument might be raised that not all negation sentences are sentences about the negation of existence. But, as we remember, we have embraced Brentano's thesis, according to which all sentences can be reduced to existential sentences, and so we can definitely contend that every negative sentence can be reduced to a sentence on the negation of existence. Can we also reduce the negation to a positive existential sentence? And, furthermore, how does this assertion stand vis-à-vis our assertion above about negation as complement. Both questions are given one answer: Indeed, the complement can be transformed to quantification and vice versa. As we have seen above, the application of Brentano's thesis to the definition of negation as complement renders the following formulae:

¬x⮭y ≡ ¬ə(x↑∩CKy)

¬x⮭y ≡ ə((CKx)↑∩y)

These two formulae are built by the use of the ordinary existential quantifier, that now is within the framework of the larger theory of quantification which will help us in this work. In fact, the combination of both of them by disjunction renders the full meaning of negation in the light of Brentano's thesis:

 ¬x⮭y ≡ ə(x↑∩CKy)˅ə((CKx)↑∩y)

These quantifiers will serve us in building our new calculus.

### Laws and laws of logic

As I wrote above (Chapter ### the mereology of concepts), a sentence bound to the universal quantifier is called a **law**. We are thus able to denote a law by the letters LA and define it as follows:

Φ⮭LA ≡def (Φ=ɒx⮭y….)

When we want to denote a law as an object we can do it by a title or index, or just cite its content after the expression LA and a colon. Thus, for example, the law of non-contradiction will be formulated as follows:

LA: ¬(Φ˄¬Φ)

I further wrote there that a law pertaining to concepts regardless of their contents is a law of logic. We can thus denote **law of logic** by the letters LL and define it:

Φ⮭LL ≡def (Φ=ɒK⮭y….)

By "law of logic" I understand a law of basic logic alone. Of course, I do negate the right of existence of particular logics, pertaining to non-empty concepts, such as deontic logic (if not inferred from modal logic), epistemic logic and others; but these should be considered as calculi built through logic, and not as logic in the pure original sense.

When we want to denote a law of logic as an object we can do it by a title or index, or just cite its content after the expression LL and a colon, as was said above regarding laws in general.

**Non-logical law** is simply a law that is not logical. We will denote such a law by the letters NL and will define it as follows:

Φ⮭NL≡def Φ⮭LA˄Φ⮭C'KLL

And here, too: When we want to denote a non-logical law as an object we can do it by a title or index, or just cite its content after the expression NL and a colon. If, for instance, we will want to mention the law stating that water contains oxygen, we will denote water by "water," oxygen by "oxygen" and the predicate contains by CONTAIN, and will write:

NL: water⮭CONTAIN;oxygen

The system of logic is the system that includes all the laws of logic. Such a system is a collection (not a set or a sum) of all those laws. We can denote it by SLL and define it:

SLL = def CN↓\*ɒΦ(Φ⮭LL)

Some may contend that SLL ought to be conceived of as a set whose members are its laws; or as a sum in which all the laws of logic are its parts. But in truth it is a system whose laws are united by conjunction, yet still preserve their boundaries (one cannot partition a law of logic and connect it to a part of another law of logic). Therefore they constitute items thereof. (By this I take a different stand than the one I took in *Thoughts and Ways of Thinking*, where I had not yet developed the concept of collection and saw the system of the laws of logic as a set.)

 Similarly we can build the system of all the non-logical laws. It, too, constitutes a collection, and will be denoted by snl:

Snl = CN↓\*ɒΦ(Φ⮭NL)

As I wrote above, the foundational concepts enumerated in this book, and in particular the mereology of concepts, are the infrastructure for the above-mentioned logic.

### Analyticity

The mereology of concepts enables us to revive the concept of analyticity, that has long been the object of philosophical attacks, and through it gives a better infrastructure for the basic concepts of modality. This renewal, however, presents some difficulties that have to be removed first. I do not intend to enter this vast topic in the limited scope of this book, but will sketch a few lines that will continue the above discussions.

 The concept of analyticity has known many definitions. The two main competitors in this field are Kant and Frege. The two are usually taken as dissenting concerning the definition of analyticity. Kant contended that an analytic sentence is one in which "the predicate B belongs to the subject A as something which is (covertly) contained in this concept A." (Kant, 1929, p. 48). Frege, in contrast, defined an analytic sentence as one whose truth can be determined by the use of the laws of logic alone (Frege, 2007, sec. 3). In the light of our discussions above, the controversy is not grave, since the laws of logic can be reformulated as laws of mereology of concepts, and so they actually constitute laws of the containment of concepts by other concepts. Still, when we come to concrete applications, some doubts arise. I would not like to enter even into the least of them here, and certainly not involve myself in the thicket of discussions that have developed among contemporary philosophers, but cannot avoid the clarification of the way I will use these concepts in this book. And since the definition of analyticity I will present below looks rather non-intuitive, at least at first sight, I should precede it with an explanatory discussion.

 Let us begin with Kant's definition. Kant used two tests for analyticity, which he conceived – incorrectly, in my opinion – as one: The logical test and the psychological one. The logical test (which he inherited from Leibniz) says that an analytic sentence is one whose veracity can be determined through the use of the law of non-contradiction alone. The psychological test (which he inherited from Hume) says that an analytic sentence is one in which as soon as I bear in mind the subject I also bear in mind the predicate. This latter test, however, should not find a place in logic, and therefore neither in metaphysics. Furthermore, some have already pointed out the relative character of this test: Let us suppose for the moment that the definition of gold is "a yellow metal" (as Kant wrote) and therefore the sentence "Gold is a metal" is an analytic sentence. Yet, when prehistoric man first discovered that gold is a metal and enthusiastically came to tell it to his wife, it was a great innovation, and therefore a synthetic sentence. Today we may consider the sentence "Gold is the element whose atomic number is 79" as an analytic sentence, but when Mendeleev discovered this fact it was synthetic. The analytic-synthetic distinction is supposed to be logical, and therefore a-temporal, and we may not leave it to changing cultural and epistemic situations.

 In order to partly recover Kant's definition we should therefore assume that a sentence is analytic only when its components appear expressly and not "tacitly" (as Kant postulated). Therefore only a sentence asserting that "a yellow metal is a metal" will be allowed as analytic. "Gold is metal" should be considered analytic iff it is additionally given to us, and expressly, that "Gold is a yellow metal."

 However, even the definition of such a sentence as analytic is not free of problems. As we remember, a gryphon is an animal half lion, half eagle. According to the above definition, the sentence "Gryphon is an animal" is not analytic, while "an animal half lion half eagle is an animal," is. The problem with that sentence is that, according to our above statement, it premises the strong existential premise (which is related to Brentano's thesis), which assumes that an animal half lion and half eagle exists. This statement is problematic not only in terms of reality (taken that gryphons do not really exist), but also in terms of internal consistency of Kant himself: It was Kant, of all philosophers, who insisted that the predicate "exists" cannot be a part of an object's concept, and therefore cannot be inferred analytically. It seems, then, that a sentence which is Kant-wise analytic contains an existential premise. i.e. a synthetic component. And if so, there can never be an analytic sentence according to Kant. But here we should ask: Was Kant right in that?

 The resolution of this entanglement can be found only if we fully understand the meaning of the sentence "An animal half lion half eagle is an animal." At first sight, it looks like a sentence of the type ɒ(X∩Y)⮭X, but this is not so, since that sentence refers to an overlap between two extensions and not between intensions, as Quine rightly noted regarding chordates and renates (Quine, 1953). What this sentence actually comes to tell us is that the concept "animal half lion half eagle" is a part of the concept "animal." The correct formulation of the sentence will therefore be:

(X∩Y)⮭PP;X

We can therefore conclude that an analytic sentence in the amended Kantian sense is one that (explicitly) asserts that an intersection concept is a part of one of the concepts that intersect in it. But if we apply Brentano's thesis to this, the above sentence will entail the following one:

 ə((X∩Y)∩PP;X)↑

In other words, there is a concept which is the intersection of X∩Y and PP;X. Lacking specification, it is clear that this intersection concept is X∩Y. But can we say that the sentence ɘ((X∩Y)∩PP;X)↑ is indeed analytic? Apparently we return to the previous problem, i.e. that every analytic sentence contains a premise of existence, which undermines its analyticity according to Kant. Yet, there is a difference between the sentence in its present form and the same sentence in its "naïve" form. Here we do not premise the existence of objects but the existence of concepts. This is an important difference because the existence of concepts – all (consistent) concepts – we have already premised as a sweeping rule, i.e. the rule of conceptual existence. Hence we may conclude that any contention regarding the existence of a concept is "untelling" with relation to the rule of conceptual existence, and therefore should be considered analytic.

 There is another premise of existence that we have taken as a sweeping rule, which has to accompany any other proposition, and that is the rule of basic existence ("The world exists"). Therefore, a sentence that might be reduced to this rule will also be considered "untelling" and, as such, analytic.

 We can therefore summarize that an analytic sentence is a sentence that, once reduced to an existential sentence through Brentano's thesis, asserts one of the following: either (a) the existence of some (consistent) concept, or (b) the existence of the world. Indeed, this definition is far off the common definitions that prevail in contemporary philosophy, and even seems counterintuitive at first glance, but an unbiased analysis affirms it.

 Let us formalize it. The concept "analytic" will be denoted by the letters ANA. We can now define:

Φ⮭ANA =def Φ⊧(x⮭K˄əx)˅Φ⊧əu

So far I have renewed Kant's definition of analyticity to fit the metaphysic of this book. But here we must mind Frege's convincing argument, that not only the containment of the predicate concept by the subject concept creates an analytic sentence, but any sentence whose veracity can be determined through the use of the laws of logic alone should be considered as such. Thus, for instance, the sentence "Either it is raining or it is not raining" is not less analytic than "An animal half lion and half eagle is an animal."

 With some effort, we could formalize Frege's definition. In fact it contends that a sentence is analytic iff the conceptualization of all of its constituents renders a law. Since the definition does not relate only to atomic sentences but also to molecular ones (such as "Either it is raining or it is not raining"), it does not relate only to sentences of the form x⮭Y, but also to sentences bound by logical connectives. We could overcome this problem by assigning a sign for a variable of a connective (such as the currently conventional Ω). However, I think all of this is useless, since the connectives can be expressed, in a somewhat cumbersome way, in sentences reformulated according to Brentano's thesis. The relations between sentences can be reformulated as relations between objects. Thus, for instance:

¬Φ = əCK(x↑∩Z)

Φ˄Ψ = ə((x↑∩Y)↓∪(w↑∩Z)↓))↑

Φ˅Ψ = ə((x↑∩Y)∪(w↑∩Z))

And so forth. In a similar way we can continue and express all the existing connectives, since they can be expressed by any pair from the above connectives. And here comes a interesting point, maybe even surprising: Given this reformulation of the sentences and the connectives, we do not need to make any changes in the definition of analyticity, and it can remain just as it is now: analytic sentences are those that, having been reformulated according to Brentano's thesis, state the existence of a concept, or of the world. This is not the place to examine it in all the laws of logic by which (according to Frege) analyticity is determined, but it appears that such an examination will approve the above assumption.

 In view of the above we can probably define a synthetic sentence. I could presumably just define it by negation, i.e. as a sentence that is not analytic. This, of course, is a possible definition. But since we have clarified the nature of analyticity, we can easily define syntheticity in an equally positive way. A synthetic sentence will therefore be any sentence which, when reduced to an existential sentence by Brentano's thesis, asserts the existence of an individuum, excluding that of the world. We will denote "synthetic" by the letters SYN and define:

Φ⮭SYN =def Φ⊧x⮭I˄x≠u˄əx

Here, too, we see the contribution of Brentano's thesis to the simplification and the resolution of logical problems long dealt with. Moreover, by this we have opened a pathway to the definition of logical necessity as well.

### Modality

Since every logical necessity is derived from analyticity, we can reformulate the foundational concepts of modal logic, which are the foundational concepts of logical necessity, in the terms of analyticity.

 The predicate "logically necessary" will be denoted by NC. If Φ is a sentence, the sentence "Φ is necessary" will be written as follows:

Φ⮭NC

For the sake of our well-entrenched habits we will convert NC to the operator accepted in modal logic –  – and will state the identity between them:

Φ = Φ⮭NC

Accordingly, the concept "logically possible," denoted by PS, will be defined, following the convention, as the lack of logical necessity:

Φ⮭PS≡def ¬Φ⮭NC

◊Φ=¬Φ

At this stage we should ask ourselves what are the natures of necessity and possibility, and how do they relate to the concepts of analyticity and syntheticity. The answer is simple: A necessary truth is one that is expressed via an analytic sentence, while a possible truth is one that is expressed via a synthetic sentence. The reason is that when we talk about a logical necessity we talk about a necessity whose veracity is learned by the use of laws of logic alone, i.e. by formal means alone. This is Frege's definition of analyticity, which we have adopted here. (I add parenthetically that since we have seen above that in concept calculus the controversy between Kant and Frege becomes obsolete, the same is true for Kant's definition as well). We can therefore state:

Φ= Φ⮭NC=Φ⮭AN

◊Φ= Φ⮭PS=⮭ΦSN

All this is true iff Φ is an atomic sentence. In the case where it is a molecular one, i.e. containing connectives, the analyticity can be determined through the use of truth tables. These things are known and need not be repeated in this framework. Put simply: In the level of atomic sentence, any logical necessity is analyticity. Any possibility is non-analyticity. From this point on we can develop modal logic. But this job has been undertaken by others and is not required for our discussion here.

## A reminder: the basic signs of concept calculus

In this chapter I do not mean to say anything new, only to summarize from the preceding chapters the elements from which the new logical calculus I wish to introduce here is built. I do not present here a well-ordered axiomatic, but a list of the main and most useful signs, for purposes of summary and reminiscence.

Concept calculus is built from sentences, object-terms, sentential connectives and non-sentential connectives. A special type of object required for notation is numbers.

Variables of sentences will be denoted by Φ,Ψ, sometimes with indexes.

Constants of sentences will be denoted by p,q,r, sometimes with indexes.

Variables of sentences will be denoted by x,y,z,w, sometimes with indexes.

Constants of object terms will be denoted by a,b,c,d, sometimes with indexes.

Variables of numbers will be denoted by m,n,s.

The universal quantifier denotes the size of an extension, i.e. the number of the items in a collection of objects. It will be denoted by ɒ accompanied by the concept it quantifies.

The existential quantifier denotes the (holistic) part of an extension, i.e. a part of a collection. It will be denoted by ɘ accompanied by the concept it quantifies.

The sentential connectives will be:

¬ - negation

˄ - conjunction

˅ - disjunction

→ - condition

↔,≡ - bi-condition, equivalence

Non-sentential connectives will be:

⮭ - predication

⮯ - capture

\* - of

C – universal complement

CK – universal conceptual complement

C'K – partial conceptual complement

∩ - intersection

∪ - union

↑ - uniextensional concept of…

↓ - singular object of

↑↑ - plain (multiextensional) concept of…

↓↓ - plain (non-singular) object of…

Q(X) - the quality of predicate X

P(x) – the predicate of quality x

μ – more than…

λ – less than…

μμ – most…

λλ – least…

Now let us recall a few rules, focusing on those in which the present calculus introduces a change in relation to predicate calculus:

The reduction rule:

x⮭Y;z1,...znx⮭Y;zm|(m≥n)

Brentano's thesis:

x⮭y  ə(x↑∩y)

Quality and predicate:

Q(p(x))=x

P(q(X))=X

In principle, the foundational rules and premises of concept calculus require a re-alignment of logic. Some basic rules of logic, such as modus ponens, modus tolens, de Morgan's rules and more will now be deduced from the basic premises of concept calculus, including the mereology of concepts. But this re-alignment is beyond the scope and aims of this work.

## The order of any world whatsoever, the primeval concept and the first partitioner

### The order of any world whatsoever

If we were asked to define freely, in a philosophical yet informal language, what "the world" is we would say: plurality subjected to order. If we want to be more precise in this understanding, it includes two requirements: (a) there are at least two beings (in contrast to the Parmenidean world and the experiences of the mystics); and (b) general functions apply to these beings. But those functions are but concepts, in the sense adopted in this book. It turns out, then, that the order of the world means objects captured by concepts. I have already stated several times that these concepts are required not only for creating order, but for discriminating objects from one another, i.e. for their very existence as plurality. In other words, the fulfilment of the first requirement is a necessary condition for the fulfilment of the second one. At the same time, however, it is also a sufficient condition, since from the moment we have a concept – even one – we have plurality. That is because this concept captures itself (a concept of concept is a concept, and therefore captures itself), and this way we have the plurality within a single object, which at the same time functions both as capturing and captured. All this leads us to the conclusion that the order of the world is, put simply, the existence of at least one concept. Once there is such a concept, the world is ordered. This is not to be demonstrated empirically or speculatively; this is a logical (analytical) necessity.

 Moreover, according to the theorem of universal conceptualization, any collection of objects is captured by some concept. This is because any combination of objects may be considered as one object (according to the rule of free union), and then it will obviously be captured by a concept that will determine the boundaries of the unified object. Furthermore, according to the rule of universal conceptualization, even if we continue to see all the objects as separate, we will be able to find a unifying concept for them. This concept will expressed by the graph that determines the relation between the first object and the second, and then between the second and the third, and so on (no matter what is the order of the objects). This graph is itself an object, and as such it is captured by a concept, which is general by its nature. Here, too, we are speaking of a logical necessity.

 I have already equated this, above, to mathematical function, and I allow myself to repeat it: Any collection of numbers can be considered as a collection of arguments of a general function, and there are countless functions that may describe them. If we draw, between some points, a certain graph, we can continue this graph by reiterating its course again and again to infinity, and by this turn it into a general function of regularity and "order." In the very same way we could draw another graph, i.e. a graph connecting those points in a different manner, and reiterate that graph to infinity just as well. If we added more points that would take the graph to a different direction, we could reiterate that graph as well in countless possibilities. Namely, any line, even the most "arbitrary" one we can imagine, expresses a general function. This, too, is a logical necessity.

 Still more than that: Every graph is an object, and in being such, it is discriminated through its boundaries. But these boundaries are determined by concepts, and thus, it is discriminated through a general function.

 Although I am not interested in involving theological discussions here, it seems that it will be useful to address the concept of miracles for illustration. In theological literature there are various understandings of this concept. Usually a miracle is described as a one-time divergence from the order of nature and its general laws, but at times we find statements taking about miracles as divergences from any order or regularity whatsoever. Theologians often don't notice the difference between these two understandings and use them interchangeably, but it should be highlighted. The first concept of miracle is one that can be discussed in the theological framework proper or as part of the science and religion discourse. Yet the second concept is logically impossible, and a theologian who embraces it must first embrace an extreme position, viewing miracle as free not only from the laws of nature but also from the laws of logic. From a logical point of view, there is no room for a miracle which diverges from any regularity, since even the most extraordinary event has boundaries, and therefore has a concept which captures it, and therefore is a part of a general order that governs it. Furthermore, by the very fact that the concept of miracle is a concept, all the miracles are captured by it, and by this alone a general regularity is imposed on them. This regularity might be different from that of nature, and not based on a few laws like it, but still it will be a regularity by virtue of the common element which all the miracles share. This concept might be compatible with that of Judah Loew of Prague, who talks about miracles having "their own particular order"(Maharal, 1975, second introduction, pp. 22-26). But we will suffice with this brief theological intermezzo, which was not made here for its own sake but in order to sharpen the notions of order and the divergence from it.

 So now we will return to our main discussion. The discrimination of an object is the partition of the world to x and the complement of x. Once this partition exists, we have got two objects, i.e. plurality. But this plurality is necessarily subject to one concept, i.e. to a unifying order. No doubt, the world has an immense plurality of objects, which are captured by an immense plurality of concepts, but necessarily – and by this I mean a logical necessity – there is one concept that captures all of them.

### The foundational concepts of any world whatsoever

"Foundational concept" is a primitive concept, i.e. a concept that cannot be built from other concepts, but other concepts are built from it. Here we should remind ourselves once again that we distinguish between two types of foundational concepts:

1. The primary foundational concepts: The foundational concepts required for the discrimination of objects (and thus for the creation of a plurality) in any world whatsoever.
2. The concrete foundational concepts: The foundational concepts required for the discrimination of objects in our world.

The concrete foundational concepts are the subject matter of the second metaphysic, while in the present section our focus is the first one. The primary foundational concepts, i.e. those that pertain to any world that might exist as an ordered plurality, are in fact those that capture any being qua being. These concepts do not necessarily refer to concrete objects, and therefore do not belong to one or another concrete world, but to any world whatsoever. Therefore we can define a primary foundational concept as one that captures any object whatsoever. We will denote a primary foundational concept by the letters BK and will define it:

X⮭PFK ≡def ∀y X⮯y

(I did not formally express the postulate that a primary concept is irreducible, and in fact it is not needed; we may suffice with keeping it as a procedural postulate).

In light of this definition, the best way to extract those concepts is through the foundational equivalence:

∀x x⮭O≡x⮭UK≡x⮭ID≡x⮭NID≡x⮭MF≡x⮭ICP≡x⮭CL≡x⮭UCM≡x⮭ND≡x⮭MG

However, in its present form the foundational equivalence contains pairs of concepts that might be further reduced, whether because they are inverse or because one can build them from other foundational concepts. Let us examine them one by one.

1. Object (O) is cannot be reduced to any of the above concepts, but is inverse to the concept of concept (K).
2. Something (UK) actually overlaps the concept O and adds no further content to it (as I have already noted above, ###, right after its definition). We can therefore give it up.
3. Identical (ID) is irreducible. Admittedly, this concept is the negation of the concept of nonidentical (NID), but I have already stated that the concept of identity precedes any relation that might capture the object, including that of negation, let alone the fact that negation itself is a certain form of nonidentity.
4. Nonidentical (NID) is irreducible. One might have argued that it can be reduced to the concept of complement (CM), in its negating function, and the identical-part (IP), and each of these can be reduced further. But in truth each of these options already presupposes the concept of unidentity.
5. Manifold (MF) is irreducible, but is inverse to the concept "component" (CP).
6. Identical-component (IC) can be reduced to identical (ID) and component (CP), and component, in turn, is an inverse of manifold.
7. Closure is irreducible, but is inverse to "boundary."
8. Universal complement (UCM) can be reduced to boundary (BD), closure (CL) and nonidentical (NID) or to the concepts part (PP) and nonidentical (NID), while part, in turn, can be further reduced to component (CP).
9. Individually numbered (ND) is irreducible.
10. Sized (MG) is irreducible.

Each of the above inverse pairs might be put under one category, which indicates that one may choose which one among them is the foundational concept. The rest will be reduced to a minimal number as much as we can. The list will therefore be as follows:

1. Object/concept (O/K)
2. Identical (ID)
3. Nonidentical (NID)
4. Component/manifold (CP/MF)
5. Boundary/closure (BD/CL)
6. Sized (MG)
7. Individually numbered (ND)

This, then, is the full list of foundational concepts. These are the foundational concepts because they are the concepts from which all the other concepts that capture any object qua object are built. Therefore they capture themselves as well, and thus also discriminate themselves. On the importance of this aspect I will dwell below.

Before we continue we must fully understand the logical-theoretic importance of this list, i.e. its importance for the construction of formal logic on sound pillars. Until now the constituents of formal logic have been determined by robust intuitions of the logicians. Now, however, we have the most rigorous tools for the construction of such a system. Since the foundational concepts are the properties of being qua being, we may now stipulate that a system pretending to be purely formal is such that might be constructed through the use of foundational concepts alone.

I will add parenthetically that the foundational concepts, in contrast to what appears at first glance, are not of infinite size (even if they capture an infinite number of objects). Every object is finite, since it has boundaries which discriminate it from the rest of the world, and concepts are also included (needless to add that the concept of "infinite" is itself finite, since it has a boundary that discriminates it from the rest of the world, and in particular from the concept "finite"). It looks as if the foundational concepts are infinite, since every object is captured by them without any limitation; but this is not so, because foundational concepts are discriminated from other concepts, and therefore have boundaries and ends. Furthermore, by the very fact of their being concepts they are discriminated from individua, and by being foundational they are discriminated from non-foundational concepts, so that they have to have boundaries vis-à-vis those objects.

### The general concept of the first metaphysic, or the Primeval Concept

So, we have presented a nice list of foundational concepts. But can we stop here? The answer is no. The above-mentioned concepts are not disjoint. As we remember, any two concepts that may capture the same object are not disjoint (above, ###), and even more so when the many concepts capture all the objects. Since they are not disjoint, they have an intersection in common. Thus concept will be referred to as "the general concept of the first metaphysic," or, for short, "the First Concept" or "the Primeval Concept." This concept will be denoted by GKM1.We were supposed to define it simply as follows:

GKM1 =def CC(X|X⮭PFK)

However, before we build this concept properly, we have to ask ourselves how should we treat the pairs of inverse concepts in lines 1, 4 and 5 above (object/concept, component/manifold, boundary/closure). The answer is that since the Primeval Concept has to be an intersection concept of all the concepts of being qua being, we should select from the above pairs the all-capturing concept. Let us examine them one by one:

1) Object/concept (O/K): Every being is an object, but not every being is a concept, so we must opt for O as the concept that will be taken for the building of the Primeval Concept. Yet, having said that we have to remember that the Primeval Concept is a concept (denoted by GKM1), and therefore even if we do not include the concept K and one of the intersects of GKM1 it will be needed for building it as a second order concept.

4) Component/manifold (CP/MF): Every object is a whole, since every object has parts and therefore every object is also a manifold; also, every object is a component, since it is an item of the collection of all objects and a member of the set of all objects. (Note, the world is a whole but is not a part; but the world, too, is an item in the collection of all objects and a member of the set of all objects, and therefore it is a component). In view of this we can choose freely, based on considerations of convenience, which one of them will be taken for the building of the Primeval Concept. Since the matter at stake is the partition of the world, it is more convenient for us to choose the foundational concept MF out of the following consideration: In the realm of parthood, all objects are wholes while not all of them are parts; therefore we had better take the concept that includes the concept of whole rather than the concept that includes the concept of part.

1. Boundary/closure (BD/CL): Every object is a closure, since every object has a boundary; but not every object *is* a boundary. Therefore we must opt for CL to participate in the building of the Primeval Concept.

From all the above we can now define the Primeval Concept as follows:

GKM1= O∩ID∩NID∩MF∩CL∩MG∩ND

This concept is the formative element of the unified logical order of any world whatsoever. But in order to complete this section we have to identify the concept which enables that concept.

### The First Partitioner

I said that every object is discriminated by the concept that captures it. I then said that concept is also an object; hence, the concepts discriminating the objects were, in turn, discriminated by other concepts that discriminated them (this is inferred from the previous sentence), and those were discriminated by other concepts; and so on, and so forth. What, then, is the starting point? This question, in this version or another, has accompanied the history of philosophy since Plato. Like in other logical regressions, we have got five main strategies to tackle the problem (compare Chisholm 1964, p. 264):

1. Dismissing the question as based on false assumptions.
2. An infinite line of dependency.
3. A circular line of dependency.
4. A foundationalist line of dependency starting from first concepts that need not be discriminated.
5. A foundationalist line of dependency starting from first concepts that are self-discriminating.

If we examine these options one by one we can conclude as follows:

1. We have no reason to dismiss our assumptions as false.
2. An infinite line does not answer the question. In terms of our analysis, we "go back," i.e. from one concept to the one that "preceded" it and so on, and in this way we can theoretically "go back to infinity," but in reality we "move forward," i.e. from one concept to the next one that "follows" it. Therefore we need foundational concepts.
3. It is unclear how concept X can discriminate concept Y and at the same time concept Y discriminates concept X. I couldn't find even one example for such a state of affairs. A more complex circularity – such as that concept X discriminates concept Y, concept Y discriminates concept Z and concept Z discriminates concept X – is not possible, to the best of my understanding. We can, however, assume a circularity of two or more foundational concepts that "come together" do discriminate one another at once.
4. In the subjective sphere we might talk about foundational concepts that do not require discrimination, although that too is dubious. Anyway, in the objective sphere it is altogether implausible. An object that is not discriminated is an object that does not exist (above ###). Therefore, a concept that is not discriminated does not exist, either. As I said above, there is no object with arbitrary boundaries, and therefore the very existence of the object testifies that some concept has discriminated it.
5. A concept that captures itself and thereby discriminates itself is an option that I find no reason to negate.

The conclusion, by way of elimination, is that a self-capturing concept, or self-capturing concepts (as written in point 5) are the key to the problem (and in case we talk about more than one self-capturing concept these concepts may, in principle, entertain a circular dependence in the sense mentioned above, at point 3).

 There are quite a few self-capturing concepts, but here we need a self-capturing concept that is not dependent on other concepts for its discrimination. Of such concepts there are only seven – the foundational concepts. And indeed, each of the foundational concepts might serve as a starting point for the discrimination of objects in the world. Above all, the Primeval Concept can do this job.

 Yet, capturing per se is not the only requirement needed for the fulfilment of article 5. The foundational concept we need is not just one that captures itself, but also discriminates itself from its environs, i.e. one that creates plurality in the world. Without this additional condition, namely, if we suffice with a self-capturing concept which does not discriminate itself from its environs, that concept might be the only being in the world. In such a case we would be left with a world lacking parts and discriminations, i.e. in a Parmenidean world. This option is negated, being contrary to the rule of basic existence. We therefore need a foundational concept that creates a discrimination of the object. This foundational concept will hereafter be named "the first partitioner" and will be denoted by the letters PK. In the case that we find one such single concept, that concept will be the first partitioner, with no further action; in the case that we find more than one, the first partitioner will be the intersection of them all, provided that they are not disjoint.

As we have seen, all the foundational concepts discriminate themselves, and to a great extent we may say that each of them deserves to be considered as first partitioner. And indeed, from the moment that a concept captures itself it automatically creates a plurality, because it serves simultaneously as both capturing and captured, thus creating the pair concept-object. The very fact that this is a pair already makes it a basis for plurality. Yet, even given this fact, the previous problem arises again: Since all these concepts, headed by the Primeval Concept, capture being qua being, the act of capturing might apply to the world alone, not to any of its parts, and so too fail to create a **partition** of the world. This state of affairs will leave us with a unified, unpartitioned picture of the world, i.e. a Parmenidean one, and will not advance us in finding the key to a rational metaphysic, which is a metaphysic of ordered plurality (as stated in the rule of basic existence). For this purpose we need a concept that will actually create a partition of the world into objects of one type and objects of another type. We therefore talk about an exclusive concept, i.e. a concept that is not all-capturing.

From all the above we conclude that the first partitioner must include the following characteristics:

1. It is a foundational concept.
2. It is self-capturing.
3. It is exclusive, i.e. not all-capturing.

At first glance requirements (1) and (3) contradict each other, since foundational concepts are all-capturing. But here is the right place to recall the fact that not only foundational concepts comprise the foundational equivalence from which we built the Primeval Concept, but also the inverses of some of them. As I mentioned above, opting for one concept over its inverse was based on the specific need to pick all-capturing concepts for the Primeval Concept, but substantially speaking every concept that has an inverse contains its inverse within it, so that we can return to the inverse concepts and treat them as foundational concepts themselves. The three foundational concepts that had inverses were, as we remember, three: O, whose inverse is K; MF, whose inverse is CP; and CL, whose inverse is BD. Now we will turn to examine to what extent does each of them satisfy the three requirements stated above:

1. K is a foundational concept; it captures itself, since it is itself a concept; it is exclusive, since not all objects are concepts.
2. CP is a foundational concept; it captures itself, since a member is also a component and CP is a member of the set of foundational concepts; but it is not exclusive, since every object is a component, as explained above (in the previous chapter), and therefore CP (just as its inverse, MF) is all-capturing.
3. BD is a foundational concept; it does not capture itself, since BD itself is not a boundary but only a concept that determines boundaries; and it is exclusive, since not every object is a boundary.

The above analysis shows that there is only one foundational concept – the concept of concept (K) – that satisfies all the three requirements for a Primeval Concept. K is therefore the first partitioner of the world, being the first partitioner of any world whatsoever. It should certainly be emphasized that the first partitioner is not a concept added to the Primeval Concept, but one that is inferred directly from it. That is because K is the inverse of O and O is one of the concepts that comprise GKM1. When the Primeval Concept captures itself it contains K *in adjecto*, and therefore also K's possibility to capture itself. In its function as first partitioner K holds both aspects: It is both capturing and captured. In its function as a captured object it is discriminated from its environs, since not all the objects are concepts.

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We have thus clarified the foundations of the world qua world, i.e. as an ordered plurality. We have seen not only the foundational concepts of that world, but also the Primeval Concept, which is an intersection of them all, and the first partitioner, which is inferred from it. But we have remained in very high spheres, those of purely formal concepts. In order to descend from those spheres down to our world, the world of nature, we will have to make moves similar to those we have done in this section also with regard to less formal concepts that shape our lives.

## Second Section: the Foundations of the Order of Our World

## Framework concepts: Forms of being, ontological entireties, pools, and ontological spheres

### Between the first and the second metaphysics

Thus far we have discussed the conditions for the existence of a world per se, i.e. any world whatsoever, as ordered plurality. The first metaphysic, discussed in the previous section, provided us with the collection of concepts required for the first partition of the world, regardless of the concrete nature of that partition. Here, however, we have to arrive in *our* world, which is the subject matter of the second metaphysic. By this expression, our world, I mean first and foremost the real world, but as long as I haven't proven its existence it can refer to any other world in which the subject lives, such as the world of his mind or the one expressed in the language he speaks. These two – mind and language – are related to two **forms of being**, because in our world the plurality and its order exist in such forms. Our world is not one in which objects are captured by empty concepts, but by concrete properties: colours, sounds, tastes, etc. Its objects exist in time, and some also in space. Things happen to them in time, including events that bring them into existence or terminate that existence. Some of these events act as causes for other events. These things, or at least some of them, are elements of our world, and will not be provided to us by the first metaphysic. In this section we will therefore discuss the way in which our world is partitioned, i.e. the foundational concepts that enable us to have the concrete partition of the world we know. By this we will develop the second metaphysic.

 The discussion of the second metaphysic will necessitate a more concrete discourse, but also a less rigorous one, compared to what we have had thus far. Its arguments might be further tightened with the help of the sciences (even if the second metaphysic is meant to give a basis for the sciences, not vice versa). Indeed, the sciences themselves might need to make some progress until they can make such a contribution. However, we cannot escape this discussion, because the two levels of metaphysics are both required for a full understanding of the foundations of being. It is important, however, to note already at this stage: Even if the list of foundational concepts presented below in this section is not absolute, and others may wish to improve it in the future, the method of its construction must be the one I will construe here, and also soon will improve its clarification. First is the elucidation of the characters of the various forms of being, then the elucidation of the foundational concepts of the realm of thought through an analysis of the most basic types of our ideas; and then the elucidation of the foundational concepts of other forms of being according to the nature of their relation to thought.

 A few introductory words: A world is something that contains plurality, and in any world that contains plurality (i.e. a non-Parmenidean world) there are concepts that are common to all the objects, and thanks to this every world is subject to an order. In the course of our discussions we have seen all those concepts: Object/concept (O/K), identical (ID), nonidentical (NID), component/manifold (CP/MF), boundary/closure (BD/CL), sized (MG), and individually numbered (ND), the intersection concept of which renders the Primeval Concept. No world that contains plurality can exist without these concepts, and of course cannot exist without the Primeval Concept. However, we can imagine a world without minds or without matter, a world without time, a two-dimensional world, or otherwise lacking other ontological properties of our world, even if we understand that each of these worlds would be far different from ours. It is relatively easy to imagine a world without minds – we can think about the world before the emergence of life on it – and it is not too difficult to imagine a world without matter, especially after we have read Berkeley. Aristotle taught that time is "the measure of motion," and if we accept his opinion we will conclude that a motionless world would also lack time. Of course, we are not obliged to accept his opinion (and many good people have rejected it), but the very fact that we can grasp it manifests that we can imagine a world without time. A two-dimensional world is certainly easy to imagine: Just take some picture and suppose that it is the world and there is nothing beyond it.

 As I have written above (Chap. ###), and as we will see further below, I distinguish between four forms of being: The noumenal existence (or "thing in itself"); reality; thought; and language. The existence of the noumenon, i.e. an entity that transcends the ego, I will prove below; but since we don't have access to it, we cannot know anything about its nature, and therefore we will discuss mostly the other three forms of being. Each of them exists "within," so to speak, other objects, that will be called "pools." The noumenon is unknown to us, and so we cannot say anything about the pool(s) in which it exists (or perhaps doesn't); Reality exists in one pool, that of space-time; thought exists within all the thinking pools, i.e. the subjects; while language exists in the verbal pools – the various languages. Each of these pools constitutes a sort of "small world" with a plurality of ordered objects, and therefore in each of them, *mutatis mutandis*, the first metaphysic applies. All the pools of the same form of being join together into what I will call "ontological sphere." The ontological spheres are therefore: the noumenal, the real, the mental and the verbal. The first two will be called "the objective spheres" while the last two will be named "the subjective ones." In contrast to the noumenal entirety, which is beyond our reach, the real sphere will be defined by a pragmatist definition, according to which this sphere is within our reach. And so we have three accessible spheres, while the fourth is an unknown.

 Each pool contains many objects, and therefore every pool contains concepts, and consequently every pool has an order (as expounded in the first section). First and foremost it is the order required for any world whatsoever, but we usually find an order that is additional to that one, such as laws of physics and other internal relations.

 Since the definition of the real is also related to subjects (as we are about to see), it is clear that there is a strong linkage between the order of the real world and that of the subjective world. In both of them we are bound to work with primary partitions made by the subject. The subject perceives the world through its sources, and these sources do not only represent the world but, as a precondition for representation, "decide" on the foundational concepts of its partition. Yet these very sources also enable us to overcome the raw foundational concepts and build other foundational concepts, more elementary. Thus, for example, the sensual qualities of colour, smell, taste etc. are reduced by science to the motion of elementary particles (atoms in the past, quanta today) from which the perceived qualities are created. This is how a gap is opened between the two spheres: while the subject perceives the world by foundational concepts such as colours, odours and tastes, science teaches us that there are concepts that are more basic than those. However, we must remember that any scientific proof begins from the data received from the primary sources, i.e. those that are built by the raw foundational concepts. After all, even the most sophisticated scientist has to begin his research with raw colours, odours, tastes, voices and tactile qualities (and even when he develops an intricate theory he has to account for those data). In view of this complexity we will have to understand the gap between the foundational concepts of the subjective spheres and those of reality, but also the linkage that nevertheless exists between them.

### Forms of being and ontological spheres

To clarify these questions we have to delve into the four forms of being and four ontological spheres that each of them creates. Let us first introduce the system of signs that will serve us.

 The letters FB will denoted the concept '… is of form-of-being …'. This is a primitive concept.

Since every object is of some form of being, we can state this as a rule, which will be called the form of being rule:

ɒO⮭FB

The form of being itself will therefore be denoted by the letters: qFB

The indexical letters R,J,T,V will denote the real, noumenal, mental and verbal forms of being, respectively. Since we know almost nothing about the noumenal world, the J form will be used quite seldom. Still, as will see, it is not entirely redundant.

Our notation will therefore be as follows:

x⮭R means: x is a real object.

x⮭J means: x is a noumenal object. The meaning of the term will be expounded below.

x⮭T means: x is a thought object. In our context, it means that it is an idea in the mind. We will usually take a particular subject, the ego, which will be denoted by S1.

x⮭V means: x is an object in the language. In our context it is a verbal expression in English. By this I mean a word or any collection of words, small or large, that has a meaning (and we will not enter now into the question of what meaning is).

We can therefore state that:

R⮭PP;FB

J⮭PP;FB

T⮭PP;FB

V⮭PP;FB

All of these concepts denoting the forms of being will be taken as primitive. Indeed, they are all parts of FB, but we don't have good tools to describe the differentia, and therefore we will give up on the determination (let alone its formalization) of the conceptual partitioner that determines their boundaries.

 Following the above notation we will mark the letters R, J, T and V as upper indexes, and they will denote the ontological sphere to which the object belongs. Namely, xᴿ, xᴶ, xᵀ, and xᵛ will denote the object x in the real, noumenal, mental and verbal spheres, respectively, as follows:

xᴿ means a real object.

xᴶ means an object in itself (noumenal object).

xᵀ means an idea of x (usually it will be in the pool of the ego, S1).

xᵛ means the verbal expression x (in the pool of the English language).

These indexes will be called ontological indexes.

The marks delta and epsilon – δ and ε – will denote variables of forms of being, whether as predicates or ontological indexes

Accordingly,

x⮭δ will denote the statement that x is a form of being δ.

xᵟ will denote the object x in the form of being δ.

We can now define existential quantifiers which relate to the different forms of being: existing in reality, existing in thought, existing in language. We will mark them as ontological indexes at the side of the ordinary existential quantifier:

əᵟx =def əx(x⮭δ(

When we write the existential quantifier without mentioning the form of being, it will denote əᴿ, unless otherwise stated.

In the case that we want to note that a certain object exists in two or more forms of being, we will denote it by two ontological indexes separated by an upper comma. For example:

əᴿ˴ᵀx = def əx(x⮭R˄x⮭T(

Now we can define ontological entireties.

By "ontological entirety" I understand the sum of all the objects of a certain form of being. We will denote it by the letter u accompanied by an ontological index. We will formalize this definition:

uᵟ =def x (x⮭δ)

Hence we can define the four existing entireties:

The entirety of reality:

uᴿ=x (x⮭R)

The entirety in itself (or of the noumenal)

uᴶ=x (x⮭J)

The entirety of thought:

uᵀ=x (x⮭T)

The entirety of language:

uᵛ=x (x⮭V)

From this we arrive at a new definition of "the world": the world is the sum of all the ontological entireties:

 u = σx(x⮭ɒδ)

Of course, all the notations of the previous section will continue to apply, and will be able to join those of this section. Thus, for instance, if we wish to say that x is a concept in the entirety of thought we will be able to state: x⮭(K∩T), and so on.

 We have enumerated four forms of being. Are there any more, beyond these? The subjective perception of animals might be a certain form of being, and perhaps different animal minds have different forms of being. Whoever accepts David Lewis' theory regarding the reality of possible worlds (Lewis, 1986) might also see them as having some form of being, and might even contend that they are in the same "family" as the sphere of reality (against my assertion above regarding the singularity of this sphere). One mght also claim that small models of places – houses, neighborhoods, cities – are also a form of being. Constructions made of building blocks (such as Lego) might also qualify as forms of being. Paintings, photographs and other visual media may also become candidates. I do not tend to count them as forms of being, but I wouldn't like to decide about all of these cases here, because I wish to focus on the four ontological entireties I mentioned above, and from them will the reader be able to infer, *mutatis mutandis*, about other ontological entireties, insofar as they can be considered as such.

 Already at this stage we have to state the rule of the bequeathing-to-parts of the form of being. This rule says that if an object is of some form of being, any part of it is of the same form of being. There is no way to prove this rule, so we have to take it as an axiom:

x⮭δɒ(IP;x)⮭δ

And for short:

δ⮭BQ(PP)

It should be noted that this bequeathing is only to parts and not to wholes. There is no bequeathing-to-wholes since according to the principle of free union we can see any two objects as parts of a third one. Therefore we can see the idea of a concept in someone's mind and a vase in reality as parts of a single object. We can therefore state the non-bequeathing-to-wholes of the form of being:

δ⮭NBQ(WH)

Now we will examine the special objects that "contain" the ordinary objects in the various forms of being, namely the pools.

### Pools and ontological spheres

The premise of this book is that objects exist "within" some other object, and this existence make them "contained" as contents within that object. The latter objects of the latter type are called, as I said, "pools." There is no way to describe the nature of a pool and its relation to the object that is "within" it, so we will take them as primitives. We can, however, give examples: Physical objects exist "between" space-time (though we can imagine forms of being which exist in four-, five-, or six-dimensional spaces, and so on, as the imagination flies); ideas in the mind exist "within" the subject; words and sentences exist "within" language. Only with regard to noumenal objects can we not say that they exist "within" pools because we know nothing about them, and at this stage we haven't even proved their existence.

 The objects that are "within" pools will be called "contents" of those pools. In what sense can we talk about one object being "within" a pool and another "outside" it? As for the pool of reality the answer is simple – when it occupies some place within the cosmic space in a certain time – but with regard to the subjective pools we encounter a problem. In principle, the entirety of objects within a pool is supposed to be "a small world," i.e. some mirroring of the real world, but if we want to concretize this test, or actually formalize it, we will meet some problems. If, for instance, we try to say that for any object in reality there is at most one object in a given subjective pool, we will have difficulty explaining both the fact that the same object appears to us differently in dreams and while awake (in the pools of thought) and the existence of synonyms (in the pools of language). All of these examples show us that determining the boundaries of the pools is not done according to the objects that populate them. Furthermore, supposedly we could discuss the pool-content relation of as a type of manifold–component relation. Yet, even though I toiled much over the developing of such an option, I only discovered that we lack the formal tools for that. Under these constraints we will have to take the predicate of being "within" – i.e. the predicate "… is a content of …" – and its inverse, "…is a pool of …," as primitives.

 Let us denote "pool" by the letters PL. The inverse, "content," will be denoted by CT. In principle, each of them could be taken as a primitive from which its inverse would be defined. We will take PL as a primitive and will define the predicate of "content" by it:

x⮭CT;y ≡def y⮭PL;x

A pool of objects of the form of being δ will be marked by a ᵟ at the right side of the pool letter. We will define:

x⮭PLᵟ =def x⮭PL˄ɒCT;x⮭δ

When we wish to denote "object x in pool PLn" we will write the pool in square brackets after the pool letter:

x[PLn] =def x|x⮭CT;PLn

When we want to say that a certain object exists in a certain pool PLn we can write it this way:

əx[PLn] = def əx˄x⮭CT;PLn

In principle it ought to be marked by an index (accompanying the quantifier or the predicate) showing what x's form of being is, but, as I said above, when the answer is clear from the context we can omit it, and usually it will refer to reality.

It goes without saying that if x is a content of pool of the form of being δ, its own form of being is also δ:

əx[PLnᵟ↔x⮭δ

Since a pool is a "small world," terms related to the world will apply, *mutatis mutandis*, to pools. Thus, for instance, a full concept of an object x was defined above as the intersection concept of all the concepts that capture x. However, when I speak about a full concept of x within pool PLn, I will mean the intersection of all the concepts in PLn that capture x:

FK(x)[PLn]=def CC;ɒ(y[PLn],z[PLn]…)|y[PLn],z[PLn],…⮯x

Now let us turn to the description of the pools that will be at the centre of our discussions.

### The nature of pools of different forms of being

**The pool in the realm of reality** is space-time. This pool unifies space and time, i.e. it is the one and only object captured by the union of the concepts of space and time. Theoretically, we could have the three-dimensional space alone as a pool of reality, but in addition to extended objects there is another type of real objects: subjects. Indeed, the objects that are "within" the subjects – the ideas – are thoughts, but the subject itself is not thought but exists as a real object, independent of thought, as we were taught by modern philosophers. This existence is not in space, but it happens in time. That is why time is the common substratum between the subject and the objects conceived by it, as we were taught by Kant and others.

 We will take the concept of time as a primitive and will denote it by the letters TI. As for the three-dimensional space, since it comprises three dimensions we should take the concept of dimension as a primitive. This concept will be denoted by DMn, where n denotes the number of dimensions. In view of that we can define the n-dimensional space as the union of those dimensions. However, we are talking about a union of objects, not of concepts. We will denote it by SPn so that n denotes the number of dimensions in this union:

SPn↓ =def DM1↓∩DM2↓∩….∩DMn↓.

The familiar three-dimensional space will therefore be denoted by SP3↓. Time will be denoted by TI↓. When we wish to emphasize that we discuss them in the context of reality we will add the appropriate index: SP3↓ᴿ and TI↓ᴿ.

These signs denote the concrete space and time, but when we want so signify the concept of time we will denote it by SP3, and when we want to denote time we will use TI.

Let us denote the concept of "space-time" by the letters SPT and define:SPT = (SP3↓∩TI↓)↑

The concrete space-time will therefore be denoted as: SPT↓

Indeed, space-time is simply the union between space and time. The uniextensional concept of this object is the one that captures this union.

Since in reality there is only one space and one time, we will not need anything beyond that. For the completeness of the discussion, however, I will state explicitly that we assume the existence of only one cosmic space and only one line of time, and therefore only once space-time:

əᴿ(1)SP3

əᴿ(1)TI

əᴿ(1)SPT

* Which means:

SP3ᴿ⮭NMBRD;1

TIᴿ⮭NMBRD;1

SPTᴿ⮭NMBRD;1

Now we will state the fact that space-time is the pool of reality:

SPT↓⮭PLᴿ

But since we know that there is only one space-time we may state this too:

SPT↓=def (PLᴿ)↓

Below we will re-define the concepts of space and time, but that will not affect their status as primitives.

**A pool in the realm of thought** is a subject. The concept of "subject" will be denoted by a capital S, and we will define it as following:

S =def PLᵀ

Constants of subjects (i.e. known individual subjects) will be denoted by a capital S accompanied by a number: S1, S2, S3 etc. S1 will denote the ego. In contrast, variables of subjects will be denoted by writing a letter signifying a number variable instead of that of the number: Sn, Sm.

A pool in the realm of language is a language. The concept of "language" will be denoted by a capital L and will be defined as follows:

L=def PLᵛ

A variable of the pool of language will be denoted by capital L accompanied by a numeral variable, n or m: Ln or Lm. Constants (i.e. individual languages) will be denoted by capital L accompanied by numbers: L1, L2, L3.

Pools will be denoted by an indexical numeral following their letter of form of being. Thus, for instance: xᵀ¹ will denote pool no. 1 in the sphere of thought.

At this point we may ask: Why in the realm of thought is a pool a subject while in the realm of language a pool is a language? In the realm of language, too, subjects are working, and they are those that create the texts! The answer is that in this matter (as in other matters in life) there is no symmetry. Ideas which exist in John's mind exist only in his mind, and even if he successfully conveys them to Mary, Mary will be able to have similar or parallel ideas, but not the very same ideas that are borne in John's mind. In contrast, a word which exists in a text is the same both to John or to Mary, even if each one understands it in a slightly different manner. That is why John and Mary are pools in the realm of thought while the text is a pool in the realm of language.

The collection of all the pools in a given form of being will be called the ontological sphere of that form of being. I am talking about a collection and not a sum, since half a pool is not an item of an ontological sphere. An ontological sphere will be denoted by the letters OS, and will thus be defined as follows:

OSᵟ=def CN;ɒPLᵟ

Now we can define all the four ontological spheres:

The sphere of reality will be defined as follows:

OSᴿ=def CN;ɒPLᴿ

The sphere of the noumenon (the "in-itself") will be defined as follows:

OSᴶ=def CN;ɒPLᴶ

The sphere of thought will be defined as follows:

OSᵀ=def CN;ɒPLᵀ

The sphere of language will be defined as follows:

OSᵛ=def CN;ɒPLᵛ

An object that is "within" an ontological sphere will also be called a "content" of that sphere (similar to pool):

x⮭CT;y˄y⮭IT;zx⮭CT;z

Which means that the predicate CT ("… is a content of…") is bequeathing-to-items:

CT⮭BQ(CN)

Or, in the general formulation:

CT⮭BQ(MF2)

Is this predicate also bequeathing-to-items? Not in ordinary bequeathing, but only in weak ones. If object x is a content of an ontological sphere y – there is some pool (i.e. some item of y) in which x is a content.

Now let us turn to the relations between different spheres, and to some extent also to those between different pools within the same sphere.

### Parallelism and sub-parallelism

I stated that an object might have at least four forms of being: Thought (including perception, of course), language, reality and noumenon. The relation between an object in a certain form of being to that "same" object in another form of being will be called hereafter **parallelism**. This concept should be elucidated. Parallelism is not necessarily identity, and in fact identity between spheres is impossible since the very difference in forms of being makes the objects nonidentical. The relation of parallelism is therefore more complex. Let us denote the predicate "… is a parallel of…" by the letters PL and define it at first in a semi-formal manner:

Object x will be said to be parallel to object y iff:

1. x and y exist in different ontological spheres, δ and ε, respectively.
2. x is captured by some concept Z and y is captured by some concept W.
3. Between all the objects captured by z in δ and those captured by W in ε there is a permanent relation RN.

Let us discuss these parameters one by one:

1. x and y exist in different ontological spheres. The relation of parallelism is between different ontological spheres and between pools contained in different ontological spheres, not between two pools within the same ontological sphere. Some intuition tells us that a similar series of parameters might serve also with regard to a relation between pools. Indeed, that relation will be called sub-parallelism, and will be discussed very briefly below.
2. x is captured by some concept Z and y is captured by some concept W. This condition is always satisfied, since every object is captured by some concept (in virtue of the axiom of universal capturing), and I only stated it for the sake of the following.
3. Between all the objects captured by z in δ and those captured by W in ε there is a permanent relation RN. It is noteworthy that this condition does not necessitate that for any object captured by Z there will be an object captured by W or vice versa, but those that are captured by both of them maintain a fixed relation. It should be emphasized here that the parallelism between two singular objects cannot be determined unless the relation between them is a law (in the sense attached to it above).

Now we can formalize:

x⮭PR;y ≡def

1. x⮭δ
2. y⮭ε
3. δ≠ε
4. əZ⮯x
5. əW⮯y
6. əLA: əRN|ɒδ⮭RN;ɒε

The sign LA is meant to emphasize that we are talking about a law, but in fact it is rendered redundant by the very appearance of the universal quantifier at the beginning of the sentence.

RST characteristics of the relation PR: It is non-reflexive, symmetric and transitive.

¬PR⮭RFLX

PR⮭SMTR

PR⮭TRNS

In principle, parallelism might exist between any form of being and any other. If we acknowledge other forms of being, beyond the above four, this concept will apply to them just as well. However, I do not tend to assume the existence of other forms of being, and in any case I will not discuss them here.

 The nature of the noumenal world, insofar as we prove its existence, is not known to us and there is no way it can be known to us. Therefore the relation between it and any of the other forms of being will remain an unknown. But further on I will argue that there is some parallelism between it and some of the objects of the real world, and consequently some indirect relation between it and some of the objects of thought and language. This relation will not add much information; since the noumenal world is not known to us, we can say almost nothing about that indirect relation as well. The little we can say will be discussed below, but beyond that the noumenal world, even after we prove its existence, will remain out of the discussion. The main relations are thus between the other three forms of being: reality, thought and language.

 But here arises the question: How can we determine a parallelism between objects in different spheres while in some ontological spheres there is more than one pool? The simple question is that at the end of the day the relation of parallelism exists between an object from a specific pool in a given ontological sphere and a specific object in another ontological sphere, and not necessarily with another object in that sphere. However, since we are discussing the question at the level of principles only, we will have to invent for the discussion an imaginary construct named "the typical pool." In other words, for any ontological sphere that has more than one pool, we will relate to only one pool. Thus, the sphere of thought will be represented by one subject and the sphere of language by one language. The question of parallelism will then be asked with regard to those pools. Here one might ask – which subject and which language? The answer is that we may take whatever subject and whatever language we wish, for exemplification alone, and assume for the sake of discussion that they are typical and representative of other pools as well, until otherwise proven. In the case that is proven otherwise, we will apply our conclusions to those pools with the changes necessitated by their divergences from the ideal types.

 For illustration, let us take the question of parallelism between the three ontological spheres known to us, in all the possible parallelisms, from a combinatorial point of view: Between thought and reality, between language and reality and between language and thought. I will sometimes call them "arenas of parallelism." It is noteworthy that parallelism, in spite of its symmetric connotation and the symmetric characterization given to it above, is symmetric only in the logical sense, but not necessarily in the functional sense. The subjective aspires to reflecting or representing the objective (i.e. thought and language aspire to reflect reality) and not vice versa, and therefore parallelism will be examined from the ideas and verbal expressions in comparison to reality. As for the subjective spheres: Since thought and language are both subjective – I sometimes think about words and language often represents thoughts – the parallelism between them will always be examined in both directions. Let us analyze each of these in its turn:

**Parallelism between the sphere of thought and the sphere of reality**: In principle, if I have an idea of the Western Wall in my mind and in reality there is an object that is the Western Wall, there is parallelism between the two ontological spheres. However, we may not assume that the Western Wall of my mind is identical to that of reality. Thus, for instance, the Western Wall of my mind has a white-yellowish colour, while I have already mentioned that the concept of colour is not relevant without a subject that perceives the objects by senses, nervous system etc. Therefore, we must examine if there is some concept X that captures the real Western Wall, and another concept Y that captures the Western Wall of my mind, and whether between X and Y there is some constant relation RN such that in any case that X exists in reality (i.e. the circumstances that cause the sensation of white-yellowish colours, for example) Y exists in my mind (the sensation of the white-yellowish colour). If that happens, there is a parallelism.

 The Western Wall is an individuum, but the same triple test applies to concepts as well, as I exemplified above a propos of the discussion about the concept of white-yellowish. I will exemplify it with a concept in the next parallelism as well.

**Parallelism between the sphere of language and the sphere of reality**: Language is counted among the subjective ontological spheres, since there is no language without subjects. Even if after the creation of the language it can stand on its own, without a subject that operates it, its preliminary formation requires a subject, or actually multiple subjects. (This argument is almost trivial, and therefore must be accepted without necessarily committing ourselves to theories which exclude the possibility of private language or to Grice's theory of meaning). Therefore, the parallelism between the sphere of language to that of reality exists iff whenever there appears a verbal expression (a word, a sentence, etc.) in the sphere of reality – whether in a visual or auditory form – it evokes in the subject the meaning attached to it in the language. (Homonyms that evoke different ideas will be examined in their context).

**Parallelism between the sphere of language and the sphere of thought**: A parallelism between these two will be created when a verbal expression represents a thought or when a thought reflects a verbal expression. The feeling of gladness in my mind is unlikely to be fully conveyed by the words "I'm glad." It is obvious that this sentence, which has a single verbal representation, might represent tens, if not hundreds, of different subjective feelings. As I said, this incongruity does not negate the possibility of parallelism. The relation between the verbal expression and the thought will be considered parallelism iff it is done on the basis of a constant rule that links these types of objects (thoughts and verbal expressions) in the two spheres. In other words, whenever one sort of feeling wakes in me, the same type of verbal expressions (that might have many tokens) will be able to represent it. Needless to add, that even though these two spheres – thought and language – are both subjective, there is no necessary parallelism between them. There are thoughts that the language cannot express, whether because language is not rich enough or because these thoughts are essentially ineffable, and there are expressions that one say or write but not think. The best examples for the latter are expressions that contain a *contradictio in adjecto* such as a triangle with four sides. I can say it, I can write it (as I have just done!) but not create an idea of it. And I will return to this example further below.

We have found that in all the three arenas of parallelism we can determine the existence or nonexistence of parallelism according to the triple test stated at the beginning of this chapter.

Alongside the relation of parallelism, which stands between objects in different ontological spheres, there is a relation between objects that are in different pools within the same ontological sphere. The table that John sees in front of his eyes and the table that Mary sees are supposedly the same table, even if each of them is a different subject, and each of them sees the table in a slightly different way. The word "table" in English and the word "שולחן" in Hebrew is the same word, even if it has a slightly different connotation in each language. We can therefore determine the existence of a relation between objects that are "within" different pools of the same ontological sphere and are very similar. We will call this relation sub-parallelism and will say that an object x is sub-parallel to object y iff:

1. x and y exist in the same ontological sphere.
2. x and y exist in different pools.
3. x is captured by some concept Z and y is captured by some concept W.
4. Z and W are very similar.

What is the definition of 'very similar' and what is the degree of similarity required to meet this criterion? We could discuss this at length, but for the purposes of our current study it is not really needed, since in this book I will not discuss the relations between pools. In fact, as I have already written, I will not tackle the question of the plurality of pools at all, but rather discuss one typical pool in each ontological sphere. For this reason there is no need to enter into all of that here, nor to introduce a formal sign for the relation of sub-parallelism.

To summarize this chapter, let us remember that at the present stage we have got six primitive concepts: Those of form of being (FB) and pool (PL) and the four concepts of forms of being: R, J, T, and V. From this point we will develop the discussion of them and the ontological spheres related to them, each in its turn. We will begin with the sphere that is the most accessible to us and is therefore the least presumptuous to study – the sphere of thought. Here we will therefore not discuss the "external" world, but rather the way we as subjects perceive the world.

## The sphere of thought: Foundational concepts

### The thinking subject

Ever since Descartes, the starting point of any discussion of the world must be the sphere of thought. When coming to discuss the foundational concepts of the various forms of being we have to begin with those of the sphere of thought, i.e. from the most basic ways in which ideas exist. In this chapter I will dedicate the entire discussion to it, and therefore I will omit the sign of thought ᵀ, and will return to it once I will have to deal with more than one sphere.

 The sphere of thought exists because the thinking ego exists. The fact of my own existence is proven. On this issue I will follow in the footsteps of Descartes, subject to limitations I will impose on it below. This means: I perceive the ego immediately, and I perceive it as a "thinking entity." The ego is a subject, and it is the subject I named S1 above.

 There is no subject without ideas "within" it: at least one idea (i.e. a datum, in which there is at least one thought object), be its truth value whatever it be. In the case where there is only one such idea, this idea is the subjective world of S1; insofar as there are more – their union is the subjective world of S1.

 But the ideas do not really exist "within" the subject. This is a very common metaphor, but a misleading one. The subject is not a spatial tool, and ideas are not goods that are put into it. And the same is true for all the other kinds of thought, including desires, feelings, sentiments and the rest, to which we will not relate here. On the one hand, the subject is a union of thoughts, including ideas; but on the other hand we cannot assume that any free union of thoughts will be considered as a subject, since in that case we can take half of my thoughts and attribute them to one subject and the other half to another. We are talking, then, about a union of thoughts that has some boundaries, which constitute the boundaries of the subject. It is right to say, therefore, that the subject is identical with its "subjective world" (if we may call it thus), i.e. with the union of all its thoughts, including the ideas it conceives, and the boundaries of the subject determine the boundaries of that "subjective world."

 The argument that there is an ego is, therefore, also the argument that there is a subjective world of theego, and that world is identical with the ego itself. To this argument it is common to add another one, which consists of two: Beyond the boundaries of its world the ego cannot know anything, and therefore cannot prove the existence of anything that is not "within" the subject. This argument will hereinafter be called the solipsist hypothesis. However, in a somewhat paradoxical way, the solipsist hypothesis affirms the existence of one entity that is not "within" the subject. That, of course, is the thinking ego itself. We will return to this point further below.

### The data-creating functions in the sphere of thought

We have already defined the pool of the sphere of thought. That is the subject Sn. The content of a subject is an idea, or a datum. Even though data can be in either the form of object or of sentence, we will have them in the sentential form. According to Brentano's thesis, there is no problem with "translating" the sentential expression to a non-sentential one. Variables of sentences will be denoted as common by the letters Φ,Ψ. Constants will be denoted by p, q, r, also as common. The concept of 'datum' will be denoted by DT and will therefore be defined:

DT= def CTᵀ

The subject perceives itself as a pool of thoughts and emotions. Descartes talked about "thoughts" in a broad sense, as including all mental acts, emotions among them. Classical philosophy spoke about three types of mental acts: thoughts, emotions and desires. Our classification will be different: I will distinguish between acts in which the subject cognizes the world, i.e. aspires to represent it, and acts in which it tries to react to those cognitions, but with a reaction that does not belong to the cognition itself. Only the former will be called *thoughts*, while the latter, non-cognitive reactions will be called emotions. According to this classification, desires are a particular form of emotions; in other words, the concept of desire is a part of the concept of emotion, in its above definition.

 I will add, parenthetically, that all the attempts made so far to classify emotions were unsuccessful. Even analytic attempts, such as those made by Hobbes and Spinoza, look artificial, and other attempts, of a more speculative character, seem completely arbitrary. However, all this is not relevant to us, because the current book relates only to the cognitive functions of the subject. I will also note that in the scope of this work I do not intend to discuss the processes that make the self acquire the ideas, nor their preconditions. Cognitive scientists of various disciplines will doubtlessly do that much better than me. What I intend to do here is present the preconditions and processes that help us understand what it *means* to contain an idea in the epistemic level.

 The subject's cognitive acts are also divided into two: data receiving and acts of decision. Both concepts are closely related to basic concepts of Source Theory (Brown, 2017), which I will not develop here anew. I will only mention that our thinking is carried out by truth sources (or just sources, for short), which transmit data to the subject. Among those sources we often enumerate the senses, intellect, imagination etc., but these are only primary sources, while there are also secondary ones such as texts ("testimonies"). Data receiving takes place when the truth sources of the subject transmit data to it, while decisions are made when the subjects determine when and how one or another of its cognitive capacities will be operated. Decisions do not belong to the realm of emotion since most of them are unconscious and involuntary, and they all belong to the cognitive act itself. Among the decisions we can count the decision to operate a source, to adopt a source (i.e. accept its data as true), reject a source (i.e. not accept its data as true) or the decisions to determine the boundaries of objects (including concepts) in one "place" or another, to tag certain perceived properties, to follow them, and more (these terms will be expounded below). Some of the decisions are preconditions for the reception of data, others take place after their reception has been made, and sometimes the data are preconditions for the decisions.

 Decision is called that way even though it is usually involuntary, because it is always non-necessary, i.e. from a logico-metaphysical perspective the subject could make another decision.

 The subject receives data from truth sources. Human truth sources are built on four types of cognitive functions, each of which has a few sources. These constitute our *basic cognitive tools*. The concept of cognitive function – or function, for short – will be denoted by the letter F. The different functions will be denoted by added indexes: F1, F2, F3 etc. Within them there can be secondary functions, tertiary, etc. As we will see below, these functions create function concepts. The concepts of the functions and the secondary functions will all be taken as primitives.

 The sources transmit data to the subject according to the function by which they act on the mind. The predicate "transmits that…" will be taken as a primitive and will be denoted by the letters TM, while the concept "source" will be denoted by the letters SRC and will be defined by it as follows:

SRC=def x|x⮭TM;y

In the case where the source transmits a proposition, a colon will follow the letters TM:

SRC=def x|x⮭TM:Φ

According to the above, if x is a source of y, y is a datum of x:

x⮭SRC;y ≡ y⮭DT;x

A constant of a source will be denoted by the letter "a" accompanied by an index; a variable of a source – by a Greek α accompanied by an index:

α1⮭TM;x

α1⮭TM:Φ

And taken that an ordinary Φ is of the form of capture sentence (attribution of predicate to subject) we can denote it as follows:

α2⮭TM: (x⮭y)

In view of Brentano's thesis, the difference between these two is not essential, and so it is clear that we may conclude:

α2⮭TM: (x⮭y) α2⮭TM;(x∩y)↓

For the sake of brevity we will sometimes use the shorter notation of Source Theory, where the colon sign denotes transmission. However, even when we use that sign we will not forget that at the end of the day we are talking about a predicate, i.e. about a linguistic representation of a concept. So let us define:

α:Φ ≡def α⮭TM;Φ

From a combinatorial point of view one can consider four modes of operation on data: creation of being out of naught (ex nihilo); creation of being out of being (ex materia); creation of naught out of being (annihilation); and creation of naught out of naught. The creation of naught out of naught is not an operation but rather a non-operation, and therefore we may delete it from the list. The creation of being out of being, however, must be split into two: the creation of x out of y, i.e. the creation of a new being, against the creation of x out of x, i.e. the preservation of the existing being. We thus remain with four superior functions. We will examine the four of them, and then the subordinate functions that depend on them:

F1 – creation of being out of naught (ex nihilo) includes the senses: sight, hearing, touch, taste and smell, as well as "the inner sense," introspection (or reflexion). The qualities transmitted by sight are colours; by hearing – voices (including sounds); by taste – tastes; by smell – odours. As for the sense of touch, it is a little more complex since it transmits a variety of qualities such as: hardness/softness (including states), heat/coldness, smoothness/roughness, stickiness/separation. All of these qualities should be taken as foundational, and we will call them the primitive qualities. The qualities of the inner sense belong mostly to the realm of emotion. Those that belong to the realm of thought are usually the subject's awareness of its own thoughts, and thus belong, in fact, to the other categories.

F2 – creation of being out of being (ex materia). This function includes the (natural, and often unconscious) use of all the logical tools presented in the first section of this book, on data accepted originally from ex nihilo functions, sometimes after recalling or forgetting (as will be explained soon). These functions include all the capacities of *reasoning*, i.e. the ability to find parthood relations between concepts; *judgment*, i.e. the ability to find relations of capturing between concept and object; and *abstraction*, i.e. the ability to isolate some concepts from the multiple concepts that capture a given object. The two last capacities are in fact two sides of the same coin, since judgment is the determinationof a concept-object relation from the concept to the object while abstraction is the determination of that relation from the object to the concept. Actions of deduction and induction are made by this function of the mind. In addition we should add the partitionary faculty, which is the capacity by which we can see whole objects as being combined of parts, and the combinatory faculty, which is the capacity by which we can see objects as parts of other objects (wholes). These two capacities will be discussed at greater length below. For the moment I will only say the obvious thing, that they, too, are two sides of the same coin. (Lamentably, in *Thoughts and Ways of Thinking* I mistakenly mentioned them as two different functions, and was also mistaken in adding to them two more functions). These two capacities are also responsible for the cutting and uniting of objects, including concepts. The mechanism consisting of all of these capacities works in various ways. When it works by ordered rules it constitutes what we commonly call *understanding*; when it works more freely, it is what we call *imagination*, in the broader sense of the word.

F3 – Creation of being from being: preservation; i.e. memory.

F4 – Creation of naught from being: elimination, i.e. forgetting.

The functions of preservation and forgetting do not create data but make use of data that have already arrived from other sources, and therefore do not help us to study the foundational concepts of the sphere of thought.

 In Peirce's classical distinction deduction and induction are functions of creating being out being: Deduction is an act of the capacity of reasoning, induction – of abstraction. Abduction also belongs to the functions of creating being out of being, since it is an act of the imagination. An explanation that is not inferred directly out of the data requires the operation of the imagination, and only after the explanation has been created one can examine its utility by the capacity of judgment.

 The foundational concepts of the senses will be denoted by F1 accompanied by indexes, as follows:

F1.1 - Qualities of sight, i.e. colours.

F1.2 – Qualities of hearing, i.e. voices (in the broad sense of the word, denoting any auditory quality).

F1.3 – Qualities of taste, i.e. tastes.

F1.4 – Qualities of smell, i.e. odours.

F1.4 – Qualities of touch, i.e. any feeling of pressure on the skin.

Each of these qualities is in fact a spectrum of subqualities, i.e. of particular qualities, to which the foundational concepts are partitioned: colour – to the particular colours (yellow, green, blue etc.); voice – to particular voices, etc. These will be defined as secondary qualities and will be arbitrarily denoted by secondary indexical numbering. I say 'arbitrarily' because in the realm of raw thought the various concepts of colours, voices, tastes, smells and tactile perceptions do not have definitions based on partitioner concepts that divide the concepts of colours, voice and the rest. These often exist in science, which reflects the real world, but in the subjective world of the thinking agent the definitions of those perceptions are made almost exclusively through tropes. If we ask someone what is yellow, he will most likely answer: "The colour of the sun," and if you ask for a definition of "salty" he will say "the taste of the sea." For this reason the numbering of the secondary functions does not reflect a partition based on a partitioner concept, but an arbitrary partition. Thus, for instance:

F1.11, F1.12, F1.13 will denote yellow, green, blue etc (the identity of the colours does not matter, since the numbering is arbitrary, for purposes of illustration alone). Hues of colours will be denoted by additional figures after the decimal point.

F1.21, F1.22, F1.23 etc. will denote various voices, including sounds and noises.

F1.31, F1.32, F1.33 will denote salty, sour, sweet, bitter etc. Hues of taste will be denoted by additional figures after the decimal point.

The particular qualities of the other senses are more complex, so I will not dwell on them here, since all I present here is only for illustration. For this reason they are also open to changes and precision.

 In philosophy it is accepted to add a sixth sense, which will be denoted by F6, and is known as introspection. That is "the inner sense" by which we perceive ourselves. It transmits to us that we feel cold, hot, that we love or hate someone. I will return to this sense below.

Let us begin with F1. The actions of the senses reflect decisions. The first decision is to act by truth sources which transmit the sense data F1.1-F1.5. As I have said, this is not a choice, but we could imagine another state of things, in which, for instance, we would have a sense denoted by F1.7, that would transmit to us particular qualities on the spectrum of F1.7. A second decision is the partition within each spectrum. This, for example, science tells us that our ability to discern colours exists only in intervals of 5 nanometer wavelengths. We could imagine a different state of things, in which the discernment would be on the basis of larger or smaller intervals on the spectrum. (This turning to science should not mislead us: Indeed, in the realm of thoughts we deal with primitive qualities alone; this mention is meant only for illustration).

Indeed, modern science taught us that all these qualities are reducible to more basic elements: elementary particles, quarks, quantums and more. Whether it is about these units or others, there is *something* that populates space-time. This "something" might be called protomatter, since it fulfils the role that matter fulfils in the raw consciousness. However, even if from a physical point of view the entities are indeed more basic, from the point of view of the "pure" sphere of thought the raw, primitive (in the physical, not the logical sense) qualities are the more basic ones. That is because we do not perceive the protomatter but infer its existence by using our basic cognitive tools, which transmit to us the raw qualities. This is an important point, since in this chapter I have no intention of discussing the sources of knowledge as an epistemological question, but rather to expound in a quasi-phenomenological way the foundational concepts of thought. For this reason, when we talk about the concepts of the sphere of thought we will remain loyal to our approach of preferring raw concepts, and accept the concept of matter as one such. Matter is therefore that "something" which has a magnitude in space, as perceived in the sphere of thought. Yet, being thus defined, it may not be taken as a primitive, since it is defined by the concepts of magnitude and space. And indeed, as we will see below, I will not count it among the foundational concepts of the sphere of thought.

I will stress this point once again: In contrast to the space-time of the sphere of reality, which will be discussed below, here I refer to the space and the time of the sphere of thought, i.e. the subjective space and time. These are the "raw" space and time and not the space-time of the theory of relativity or any other scientific theory.

Supposedly we could, for example, imagine an a-temporal perception of colours, but that is a theoretical construct. Perception itself is a temporal event (i.e an event happening in time). However, this theoretical construct is important since it helps us to decompose the idea. Space and time cannot be reduced to a more foundational concept, and so we must take them as primitives, i.e. as foundational concepts of the sphere of thought.

We will now turn to F2. Creating being out of being is the role of the intellect. The tools of intellect are merely the foundational concepts enumerated in the first section, with their interrelations. Since they have already been displayed above, I will not discuss them here one by one, but relate to the entire system, i.e. the concept calculus. F2 is, put simply, the function that receives data from another source, mainly from memory, and applies concept calculus to them. However, I will mention some important secondary functions as well:

F2.1 is the function of capturing, i.e. the function by which the subject determines that there is a relation of capturing between a concept and an object. This function has two aspects: judgment and abstraction. Judgment is the determination that a given object is of some property or relation, i.e. that a certain concept captures it, while abstraction is the ability to conceive the concept as such. That is, as an entity that might capture, in principle, other objects as well. Maybe I should have allotted to each of them a separate secondary function, i.e. F2.11 and F2.12 respectively, but since in essence we are not talking about different actions but about different focuses of the subject's attention, I cannot see the difference between them as pertaining to epistemology, and will therefore leave them with the same notation.

F2.2 is the function of composition, i.e. the function that determines that between two objects there is a component-manifold relation. As could have been expected, this function is divided into three secondary functions:

F2.21 is the function of partition, which determines the existence of part-whole relation. This function actually serves as a "combinative faculty" as well. According to the principle of free union, every two objects might be taken as parts of a third one. However, in real life we do not relate to any pair of objects as part of a unified object. The determination of which objects will be taken as parts is therefore a result of a decision, and it is done through the use of the combinative faculty on objects on which the subject decided to operate that function. Usually it will be operated on objects which have some continuity or functional linkage between them, but this does not have much to do with our concerns. This function also serves as the "partitionary faculty," i.e. the faculty that takes parts as objects with their own existence in spite of their being parts of a whole. Here, too, there might be room for allotting to each of these functions a secondary function of its own, i.e. F2.211 and F2.212, but since in essence the actions they carry out are not different and the apparent difference between them is only a matter of point of view, I do not see the difference between them as pertaining to the realm of epistemology and will therefore will leave both of them with the same notation.

F2.22 is the function of itemhood, which determines the item-collection relations. Here, too, one might divide the function into two secondary functions: one that sees two objects as maintaining an item-collection relation, and the other that sees the items as existing on their own.

F2.23 is the function of membership, which determines the member-set relations. Here, too, one might divide the function into two secondary functions: one that sees two objects as maintaining a member-set relation, and the other that sees the members as existing on their own.

I would like to emphasize that these functions do not apply only to individua but also to concepts, and therefore function F2.21 is responsible for the determination of parthood relations among concepts, which makes it the function responsible for our deduction processes.

Imagination, too, is nothing but the operation of some of the functions introduced above, but when they are combined together, often in more complex combinations than the ones presented above. Therefore it should be counted among the functions that create being out of being. We will denote it by F2.3. I will not be able to enumerate the combinations by which imagination works, since they are numerous and often very complex, but I will mention some of them and will survey them very briefly: fantasy (denoted by F2.31), extrapolaton (denoted by F2.32) and abduction (denoted by F2.33).

F2.31 – Fantasy. What happens in our minds when we build an image of a centaur, i.e. a creature with a human head and a body of a horse? We can describe it in two ways: one that the subject extracted out of his memory an idea of a horse, operated the faculty of partition on it and isolated the body of the horse, then extracted an idea of a man, operated the faculty of partition and isolated the head of the man, then operated the combinatory faculty on the two individual ideas, operated the faculty of forgetting on the distance between these two parts as they were in his memory, and created the individual image of the centaur. At this point he operated the faculty of abstraction as well, and created the general concept of a centaur. The other, shorter, way is that the subject simply created a concept out of an intersection (i.e. combination) of the concepts "with a human head" and "with a horse's body." The difference between these two ways is in fact the difference between the two possible ways of operation open to the subject: Does he create the new idea through concrete thinking (on ideas arriving from the senses) or by abstract thinking (on concepts).

F2.32 – Extrapolation. What happens in our minds when we see a line with a certain trajectory and continue it with the same trajectory? We simply perceive it by the sense of sight (F1.1), keep it in our memory, perform an abstraction of the concept that captures that line and then connect (by the combinative faculty) the line perceived by sight with its continuation built by the concept alone. According to the rule of universal conceptualization, there are an infinite number of concepts that capture the line, and therefore, in the absence of a given function, there are an infinite number of ways to extrapolate it. We can, however, talk about more or less simple extrapolations, or about more or less economical ones.

F2.33 – Abduction. This term coined by C.S. Peirce denotes the action by which we offer an explanation or a hypothesis for a given collection of data. In contrast to the other two methods offered by Peirce, deduction and induction, in which the result arises from the data, here the path is a little less straightforward: Usually the subject takes from the memory a set of members captured by two unrelated concepts. Abduction is a function that offers a third concept (and sometimes fourth, fifth, etc.) which provides a linkage between the two concepts that is not based on a logical linkage (i.e. on relations of capturing or partiality of concepts). Often the offered linkage is a causal one (and the concept of causality will be discussed below). The new concept is called an *explanation* or a *hypothesis*. Due to the difference between abduction and the two other methods (deduction and induction), and due to the fact that deduction is based on imagination, one can raise for any such set a large number of explanations. The subject selects from them what seems to be "the best explanation," and does it through a series of rules, among them parsimony (in accordance with Ockham's razor), the scope of explained data, and more. This method does not render an evident certainty, but it is yet one of the ways by which we attain data we consider to be true. Often this does not end the process, as we now have to apply the new explanation to the data, i.e. to check if the new concept indeed captures the data to the expected degree. Since there is more than one possible explanation, the choice remains open, and so when the explanation is not the best, or ceases to be the best due to change in the data, the road is open to another explanation. All of these issues were broadly discussed in the philosophy of science, and are not of our concern here. Yet, abduction is not only a part of scientific method, but is part and parcel of the epistemology of our everyday life. In countless cases in which we are in doubt we raise – whether consciously or not – explanations that might settle the matter, and opt for one of them which we see as "the best explanation."

 The common denominator of all these actions, and to actions of the imaginations on the whole, is that they do not necessitate a single result. There are countless combinations by the way of fantasy; countless ways by which one can extend a given graph through extrapolation, in case its function is not given; and there are countless concepts that may be suggested as explanations for a given set of data. Another common denominator is that all these functions make use of materials taken from memory. It is noteworthy that these three secondary functions certainly do not exhaust all the possibilities of combination between the above-stated functions, and therefore do not exhaust all that we call acts of imagination. This is only an illustration of some of the widespread and useful actions of this faculty.

 As I said, in using these functions the subject actually takes ideas already existing in the mind by virtue of F2 and creates new ideas out of them. Thus, the operation of F2, to all its divisions, is dependent on decisions. The mind decides on which data it will act and which functions it will operate on these data from the many rules of concept calculus.

F3 is the function of preservation, i.e. the capacity of memory. This function takes ideas that were attained by the mind through F1 and F2 and helps them keep appearing, or makes them re-appear, in a fashion similar to the one in which they appeared at first. When I say "similar" I do not mean only the difference between a primary experience and a secondary one, but sometimes a deeper difference, resulting from the capacity of forgetting.

F4 is the function of elimination, i.e. the capacity of forgetting. This function is actually the decision not to operate the function of preservation on a certain idea or a part of it. However, since it produces a cognitive condition that stands in its own right, I found it right to grant it a status of an independent function.

Note: These are the basic cognitive tools because any truth system requires them. Even in a system in which the basic cognitive tools are only means for transmitting data from another source (for instance: a written text) and even when the other source mandates that in certain cases one must reject certain data transmitted by the basic cognitive tools, this very order is transmitted through the basic cognitive tools, so that it is impossible to do without them. Therefore no system can ever reject the basic cognitive tools categorically, as such rejection leads to an altered version of the liar paradox. Does this fact endow a system based exclusively on the basic cognitive tools, i.e. Western Rational System (WRS), with an epistemic advantage over other systems? There is no way to conclude that, as I have shown elsewhere (Brown, 2017). It only grants it a greater degree of universality, stemming from the fact that the basic cognitive tools are common to all the existing and ever-possible systems. This might be an advantage, but not an epistemic one, i.e. not an advantage from the perspective of the systems’ chances of attaining truth. However, that system – WRS – is the one pertinent for this book, as we discuss here the foundations of *rational* metaphysics.

The rational order of the world is determined through the basic cognitive tools, but these, in their turn, also act by an order. This order determines the "division of labour" between the sources and the relations between their data. It is called a *source model* or just a *model* for short. By such a model data are allowed into the system or rejected from it. An adoption of any other source into the model also requires that it go through the test of the basic cognitive tools, as set in the source model of the system. The order of the world described in the rational metaphysic, in both its parts, is therefore the order of the world constructed out of the source model of WRS.

All the cognitive functions mentioned above create different types of data, and therefore also create general concepts of those types. These general concepts will be named *function concepts* and will be denoted by KF, and usually followed by an index: KFm, with the Fm denoting the primary and secondary functions. Thus, for instance, KF1.1 will denote the function concept of "visual datum" (i.e. a colour or something created by colour relations); KF2.21 will denote the function concept of being in part-whole relation with something; KF2.3 – being a construct of imagination; KF3 – the function concept of being preserved in one's memory; and KF4 – being absent (from one's mind).

It is easy to see that as far as the secondary functions of F2 are concerned, we can actually re-notate all the concepts of the first section in terms of F2 and its ramifications. Admittedly, in that case we will not talk about them as mere concepts but as concepts within the realm of our thought, the latter only differing by the fact that we relate to them qua existing in a certain form of being.

The concept of function per se, concept KFm, which means "a datum of a cognitive function," is actually an overlapping concept of CTᵀ, which has already been determined as a foundational concept (i.e. as a primitive), and therefore need not be determined here again as such. As for function concepts of primary functions, it appears that each of them requires a foundational concept. Let us examine them one by one:

The function concept of F1, i.e., "sense datum," requires a new foundational concept because we cannot describe a sense datum as an intersection (or a consequence of any other operation) between two existing concepts.

The function concept of F2, i.e. "a datum of intellect or imagination," also requires a new foundational concept. Indeed all the relations determined by intellect and imagination are based on foundational concepts of the first section (as stated below), and there thay have already been declared as foundational concepts, but the fact that a certain datum is a result of the use of these tools cannot be expressed by existing concepts and therefore requires a new one.

The function F3, i.e. a "datum of memory," requires a new foundational concept, since even if the preserved datum were identical with the original one, it would not be enough to note this identity, since it would not reflect the action taken for the "retrieval" of the original datum in a later stage. Indeed, it is even more the case when the memory datum is not identical with the original datum, but only similar to it. Hence it is clear that a new foundational concept is required for the function of preservation.

The function concept of F4 is 'a datum of forgetting'. Supposedly forgetting has no data, and we are actually talking about the absence of data. Absence certainly does not require a foundational concept. However, the assertion that a particular datum is absent from the mind does not express the fact that it was there previously. For this purpose the concept of forgetting needs a foundational concept of its own.

And what about the secondary functions? Here we have to distinguish between the secondary functions of F1 and those of F2 (As remembered, functions F3 and F4 do not have secondary functions).

The function concepts of the secondary functions of F1 denote properties of sense data, which we cannot discriminate from one another by a conceptual partitioner of the primary function concept (namely, we don't have a partitioner for the concept of "sense datum" that will render the concept of "taste" for instance). Therefore we have to take each of those secondary function concepts as foundational concepts. In contrast:

The function concepts of the secondary functions of F2 are the concepts of intellect and imagination. Their action itself is covered by the foundational concept of the primary function, KF2. To this foundational concept we do not need to add new foundational concepts since, as I have said, the concepts of intellect are actually the concepts of the first section and their articulations, while the concepts of imagination are concepts that are created by the same functions as those of intellect, but used in a different way than they are by intellect. This way or the other, we are talking about functions already defined above, so that we do not need to create new foundational concepts for them. Functions determined by tropes also do not require new foundational concepts, since we can construct them by the concept of the secondary function ("the colour…") and the concept of the trope-object ("… of the sun").

The function concepts of tertiary functions of F1 can be discriminated by tropes, and therefore, as I said, do not require new foundational concepts.

We can therefore conclude that all the function concepts of the primary functions, but not solely those but also the function concepts of the senses, are foundational. However, the secondary functions of the data of intellect and imagination do not require foundational concepts and will be built from existing foundational concepts. The list of primitive function concepts will therefore read as follows:

KF1 – a datum of thought created ex nihilo (sensual).

KF1.1 – a visual datum.

KF1.2 – an auditory datum.

KF1.3 – a datum of taste.

KF1.4 – a datum of odour.

KF1.5 – a tactile datum.

KF1.6 – a datum of introspection.

KF2 – a datum of thought (intellect or imagination) created from an existing datum.

KF3 – a preserved datum of thought (memory).

KF4 – a lost (usually forgotten) datum of thought.

As noted above, All these functions exist within many subjects (almost in every subject, excluding only disabled people), and consequently the function concepts related to them exist in the minds of many people. Therefore, when we refer to different subjects, we need to mention who possesses the functions at stake. For this purpose we will use again the asterisk of possession. If we wish to talk about function Fm of a subject whose number is n we will write it this way:

Fm\*Sn

Yet, the different functions of every subject transmit the data to that subject. My sense of vision does not transmit data to anyone but me; insofar as I wish to convey the data of my sense of sight to someone else, I must do it by auxiliaries, most prominent of which is language. For this purpose we need a predicate denoting that a given subject contains a certain datum. That predicate has already been introduced above; it is the predicate of containment (of a datum in a pool), denoted above by CT. We will recognize that it refers to the sphere of thought when it relates to subject and idea.

Now we can state the axiom of transmission:

Sn⮭PL;x ↔ ə(F⮭TM;x)

This sentence may also be taken as a definition:

Sn⮭PL;x ≡def ə(F⮭TM;x)

Now I can rephrase my words more precisely: Function Fm transmits to the subject data which are all of KFm-ish character: The sense of sight transmits visual data, the sense of hearing – auditory, etc. Intellect transmits concepts; memory – data similar to the sense data from which they were taken, yet different from them, and imagination is based on memory data, and this is why the data of imagination are similar in character to those of memory. As for the data of elimination (forgetting) – they are 'empty' data. That is why we can denote any datum by the function that transmitted it, namely: xFm. We can also state this:

Sn⮭CT;xFm ↔ Fm⮭TM;xFm

Of course, one can substitute more than one source for Fm, and even a source model (in the sense attached to it in *Thoughts and Ways of Thinking*).

So far I have phrased the description with terminology based on source calculus, which is based on the process of transmission. Now, however, we can describe things from the perspective of the thinking subject in the stage that follows that process. In continuation of the above statement we can state:

Fm⮭TM;xFm Sn⮭CT;xFm

Namely, any transmission by cognitive function m makes the receiving subject contain an idea of the type transmitted by that function.

When a pool contains a datum, i.e. an idea, we can write it in two ways: either as the containment of an object (Sn⮭PL;x) or as the containment of a sentence (Sn⮭PL;Φ). According to Brentano's thesis, there is no essential difference between them. In fact, containing an object means Sn⮭PL;(x↑∩Y), while containing a sentence means

Sn⮭PL; Φ:(x⮭Y). As we remember, according to Brentano's thesis these are tantamount.

### Introspection

Whenever the subject contains an individuum, it also contains the concept which discriminates that individuum, since otherwise it would not have been perceived. However, when I see a box in the street I am not necessarily aware of all the concepts which determine its boundary lines and make me discriminate it from its environs. Should those concepts be considered as ideas contained in my thought? The answer is negative. If we say it is positive, we will have to say that every subject contains all the properties of all objects, including the laws applying to those objects. It is ridiculous, in my opinion, to say that every person holds in mind the entire quantum mechanics. A subject will be considered as containing a certain idea only if it is conscious, in some degree, of its existence. This is true not only for ideas of concepts, but also to those of individua. If I look at some landscape and am utterly unaware of some object within it, one cannot say that it is contained in my mind.

 Should we say, then, that the condition for an object to be considered as an idea is that the subject be in a state of full consciousness with regard to it? That is a far-reaching conclusion, and therefore here, too, the answer is negative. Full consciousness is not just about awareness of the existence of an object, but also a condition in which the subject thinks about itself as being in a state of thinking. This is certainly not needed. A subject will therefore be considered as containing an idea when it has the potential of attaining full consciousness with regard to that containment, if only it turns its thought in that direction. This is a condition less than full consciousness but higher than an utter lack thereof. I don't see any need to attach a new predicate to it, since it fully overlaps the predicate "…is containing a thought content in a pool of thought."

 When the subject thinks of itself, we will describe it this way: Sn⮭PL;Sn. This is the state we often call self-consciousness, or reflexion. However, we should distinguish between a basic reflexion and a concrete one. In the former the subject just thinks of itself; in the latter it thinks of itself as thinking a certain idea. We have already seen above that reflexion is a sense, i.e. a cognitive function of its own, which we called F1.6. We can therefore state the equivalence:

Sn⮭CT;xF1.6 ≡ Sn⮭PL;(Sn⮭PL;Φ)

We might see this equivalence as a definition of function F1.6, but I do not tend to see it as such, since it does not express the character of the action of self-perception. That character is undefinable, just as one cannot define a sight or a sound. For this reason I will leave F1.6 as a primitive, just as all the other foundational functions. However, for purposes of brevity we will name it a new predicate. We will denote it by the letters RFPL; Φ or RFPL;x, with Φ or x denoting, respectively, the datum which the subject is self-consciously containing.

Sn⮭RFPL;Φ=def Sn⮭PL;(Sn⮭PL;Φ)

Self-consciousness will be discussed again below, but now we will turn to an action of the subject that is not necessarily conscious.

### Decision

I have mentioned the concept 'decision' several times in previous chapters. Indeed, it is of pivotal importance in this work. In our discussion it bears a unique sense, unrelated to the one attached to it in everyday discourse or philosophical action theory. On the formal level we will accept it as a primitive, but in a free informal speech we may define decision as an action in which the subject determines something without a logical necessity, i.e. where from a logical point of view it could determine otherwise. Decisions are of normative character (in the logical sense of the word). I will emphasize: a decision is not always deliberate, often not even conscious.

 I will denote the concept "… decides that…" by DC. The first place will be a subject, denoted by Sn, with n denoting an index. The second place will denote the object of the decision. If it is an object it will be denoted by an ordinary letter, while if it is a sentence it will be denoted by a sentence, written after a colon (see below).

Now we can state the *rule of the contingency of decision*:

Sn⮭DC;x◊Sn⮭DC;y(y≠x)

where Sn denotes, as we remember, a variable of a subject.

In the sentence I wrote here the decision, i.e. the second place, appears as an object – x or y; but usually it will appear as a sentence. In that case the predicate DC will be followed by a colon: (:). This colon will follow only the predicate DC and must not be confused with the colon denoting transmission ("transmits that…"). The rule of the contingency of decision can therefore also be phrased as follows:

Sn⮭DC:Φ◊Sn⮭DC:Ψ(Φ≠Ψ)

And consequently also this way:

Sn⮭DC:Φ¬LL:Φ

Which means that a decision is always something that could have been different, and thefore is not subject to a universal law.

We should presumably negate the applicability of any law whatsoever, whether logical or non-logical, to a decision, but in such a statement we would be taking sides in the question of determinism and free will (as it is wrongly entitled in philosophical literature), a question which is out of the scope of the present work.

The concept inverse to DC will be DD. We can translate it as "… is decided by…" Since we are discussing the tools for the description of the world and not those of the description of the subject in the world, we will take DD as a primitive and will define DC by it, and not vice versa:

DD=def INV;DC

Namely:

x⮭DD;Sn ≡ Sn⮭DC;x

However, we may apply the reduction rule to x⮭DD;Sn and state: Whenever something is decided by some subject, it is decided.

x⮭DD;Snx⮭DD

The contrary of the decided is the given. What is given is what is the case that is not as a result of a decision. I could have tried to offer a positive definition of this concept, but I prefer to do it simply by the negation of the decided. We will denote the given by the letters GN and will define it as follows:

x⮭GN ≡def x⮭¬DD

Theoretically, we could take the given (GN) as a primitive and define the decided (DD) as its negation. I opted for DD for reasons of convenience.

### Time and change

Above we enumerated space-time as one of our foundational concepts. We mentioned it as the sole pool of the sphere of reality, a status that actually makes it identical to that sphere itself. Yet, space and time, as well as their union (by virtue of the principle of free union) to the object known as space-time, are also central concepts in the sphere of thought. Furthermore, the sphere of reality has been described as containing two types of objects: bodies extended in space and thinking (subjects). Even though the sphere of thought is subjective, and the ideas contained in it are not extended in space, the data which constitute the contents of those ideas definitely include space and the objects extended in it. After all, I have an idea of space and ideas of bodies extended in it. This urges us to clarify the character of the concepts of space, time and matter, as they appear to us in their pre-philosophic and pre-scientific forms in the sphere of thought. Let us start with the concept of time.

 The use of the different functions is done in cognitive actions. These actions occur in time. Sensation and introspection, the use of intellect (judgment, reasoning), preservation and elimination, are all done it time. Decisions too, whether conscious or unconscious, are made in time. However, the contents of these functions are different; the contents of sense data are always of objects extended in space and made of matter, while the contents of all the other functions can be either of extended or non-extended ones. All of these three concepts – space, time and matter, that served Newton as building bricks of his mechanics – are in fact the three building bricks of the simple, naïve picture of the world. We will begin our discussion with sensation, in which all the three coincide.

 In principle, we ought to add a time component to all of our predications. The "raw" subject conceives time as a line, which we have already marked above as TI↓. We may take some fixed magnitude – an hour, a day, a year– and also select some point of time as a starting point, be it the Creation, the birth of Jesus, the Hijrah or my drinking coffee this morning, and by that we will be able to locate events on the timeline. Our time points will be denoted by the sign t followed by a number denoting the place of that point on the timeline in relation to the starting point. For this reason, every sentence in the concept calculus should have been phrased in a form expressing the time for which it is true, such as:

x⮭y;tn

Namely, in time-point n the case was such that x was y.

When we wish to state that a certain object x existed in time-point tn we will write it as follows:

əx↑;tn

As we remember, the quantifier always binds the concept, but since x↑ denotes a uniextensional concept, the quantifier that applies to it affirms the existence of the sole individuum captured by it.

Now we can define simultaneity, preceding and following of time-points. The predicate SMPT denotes "…is simultaneous with…," the predicate PDPT denotes "…precedes …," and the predicate FLPT denotes "… follows…" Let us define:

x⮭SMPT; y =def əx↑;tm˄ əy↑;tn˄ n=m

x⮭PRPT; y =def əx↑;tm˄ əy↑;tn˄ n>m

x⮭FLPT;y =def def əx↑;tm˄ əy↑;tn˄ m<n

Lets us add two more needed definitions:

x⮭SMPRPT; y =def əx↑;tm˄ əy↑;tn˄ n≥m

x⮭SMFLPT;y =def def əx↑;tm˄ əy↑;tn˄ m≥n

Let us assume that an object begins to exist at time-point n. (As we will see below, the determination of this point should not be taken for granted, but let us assume that we can decide on it). We can therefore say that that object was "born" in that time-point. We will denote "…was born at…" by BRN and will define:

x⮭BRN;tn ≡def əx↑,tn˄¬əx↑(t<n)

The moment of the birth of any object will be called tn0x. Therefore:

ɒx⮭BRN;tn0x

Maybe for the sake of symmetry we should have also defined the concept "…corrupted," i.e. ceased to exist, but, as we will see later below, we will not need this concept in our discussions, and so I can leave its developing to other works.

We can also broaden the predicate BRN and also state the place of the object in pool PLn:

x⮭BRN;PLn,tn ≡def əx[PLn]↑,tn˄¬əx[PLn]↑(t<n)

Let the sign 🡂 denote the unit ranging in (weak) continuity "between … and…," i.e. between one numbered point (of time, space or any other continuum) and another, while it is consented, as a rule of grammar, that the sign 🡂 will always be flanked by numbers so that the number at its right side is larger than the one at its left side.

 It is agreed that:

ət(n🡂m)n⮭WEC;m

In view of that, there can be a sentence stating that a concept captured an object not only in a time-point, but during a time unit, as follows:

x⮭y;t(n🡂m)

Namely, during time-point m and time-point m the case was that x was y.

We can also state that a certain object existed in a certain time unit:

əx↑;t(n🡂m)

However, since the time constituent is a part of the predicate, we are allowed to omit it, by virtue of the reduction rule. This is the justification for the common practice of writing the predicate without mentioning time. The meaning of this form of writing is that in a certain time (not always!) the case was as described.

Now we can define a *change*. Every change is a change in time, and therefore in principle every change must have the formal constituent of time, but it, too, might be omitted by the reduction rule. Since we are talking about a complex predicate, its definition is also complex to some degree. Let us denote the predicate changes (or "changed," or "will change," etc.) by CH1, and define it as follows:

x⮭CH1;(y,z),t(n🡂m) =def x⮭y;tn˄x⮭z;tm˄n<m˄x⮭IP;CKy

Namely: If x is y at time-point n and x is z at time-point m, it has changed from y to z. The last conjunct requires that the change be in the same "arena": an object may change from yellow to green or from small to large, but cannot change from small to green. By this requirement we also express the fact that an object cannot change by first being captured by a larger concept and then by its part, or vice versa. We cannot say that a certain object changed from elephant to animal, just as we cannot say that it changed from animal to elephant.

 It should be noted parenthetically: One of the most important changes that we measure is the change of place, which we call "motion" (it will be discussed separately). We measure motion easily because we have precise units of distance. We can also measure the speed of the change by the concept of velocity, which we define as distance divided by time. Place is a property just as any other property. If we could have precise units for measuring the "distances" between other properties we could, at least theoretically, measure the changes they go through with the same degree of precision, and develop with regard to them concepts like the speed of change, the acceleration of change, etc. Today we have such units only for a very small number of properties and even they are not as useful as the concept of motion.

We say that an object changes when a concept X captures it at a certain time-point t1 and ceases to capture it at a later time-point t2. We have already declared CH1 as the predicate denoting a change in the concept capturing the object. But what if the object keeps standing unchanged? To this we should answer that not only do changes occur in time, but also every passage of time changes the object. This change will be called a change in the *age* of the object. Every existence of an object in time carries with it a change in the age of the object. In view of this, we can now state the law of change, which is: Every object existing in time changes:

əx↑;t(n🡂m)↔ x⮭CH1;(y,z),t(n🡂m)

The concepts of age and age change require formalization. If we take a certain time-point as a starting point of the existence of an object (its "birthday"), then even if all its properties remained the same, it grows "older" with every time unit that passes it. The concept of age will be denoted by AGn, where the n denotes the time unit that has passed since the "birth" of the object and the tm denotes the "present," i.e. the time in which the statement is made:

x⮭AGn;tm ≡def x⮭BRN;tn0x˄əx↑t(n0x🡂m)

Namely, x is at the age of n at time-point tm iff it was born n time units before tm and maintained a continuous existence between the moment of its birth and tm.

This statement can be said in the other direction as well: it is provable that iff x has existed for a certain time unit of time, it has changed its age to be the same as that time unit:

əx↑;t(n🡂m)≡ x⮭CH1;(n🡂m),(AG(n-n0x)🡂(m-n0x))

Now we can formulate the concept of age change. We will denote it by the letters CHAGn,m and will define it as follows:

x⮭CHAGn,m ≡def x⮭CH1(AGn,AGm),t(n🡂m)

As I said, sometimes the change is only of age, yet sometimes there are other changes *in addition* to that of age. In fact, the concept of change of age turns the concept of change to a very large concept. Even if an inanimate object stands unmoving and without any other development it will be considered as changing. We therefore need a concept that will denote a change in the narrower sense, i.e. a change that is not of age only. We will denote that concept by the letters CH2. The definition of change will thus be rather simple:

CH2 =def CH1-CHAGn,m

Hence we can define time unit. We will denote a time unit beginning in a given starting point and ending at time-point x by the letters tux. Let us define:

tux =def qAGtx

Namely, a time unit is the quality of "being at the age of x." Hence we can define time on the whole. We have already denoted (cosmic) time by the letters TI↓ and now we can define it as follows:

TI↓=def σqAGtx

Namely, time is the sum of all the time units together. Admittedly, this is a rather Humean definition of time (but differs from that of Hume by the fact that the primitive expression of time is a concept which captures objects), but let us not forget that we are currently discussing the sphere of thought, in which we examine objects from the perspective of the pre-philosophic and pre-scientific experience. In that sphere we do not experience the cosmic time as one object but rather as a union of time units, combined together and extrapolated into what we call our cosmic timeline.

### Place

A place is a part of the three-dimensional cosmic space, which we denoted above by SP3↓. In the "natural" space, the one that prevails in the sphere of thought, places are determined, so to speak, as coordinates of three axes. For this purpose we can determine their origin in whatever place we wish, and measure from it the distances to any of the three axes. Motions on these axes will help us measure changes of place.

 A change of place does not differ from any other change. Let us take the signs pn, pm etc to denote points in space. These signs denote the intersection of three-dimensional coordinates within the cosmic space. The sign LCpm will be used as a primitive sign of the predicate "is at (spatial) point pm." When there is no other indication, we will assume that they maintain (at least) weak spatial continuity, i.e.:

x⮭LC(pm🡂pn) pm⮭WEC;pn

Now we can discuss motion. A change of place will be denoted by the letters MV. We can define it as follows:

x⮭MV;(pm,pn) ≡def x⮭CH1;(LCpm,LCpn)

If we add the time constituent to this expression we can also say:

x⮭MV;(pm,pn),t(n🡂m) = def x⮭CH1;(LCpm,LCpn),t(n🡂m)

Now we can give a new definition, also Humean, to the concept of (cosmic) space. If that space was denoted above by SP3↓, we can now define it as follows:

SP3↓=def σqLCpm

Having developed the concepts of space and time, and consequently the concept of space-time that is a combination of them, we can turn to the concept of matter.

### Matter

In modern philosophy (and actually long before it) the material substance was defined as the extended substance, i.e. that which occupies place in the three-dimensional space. And I have already expressed my wonder above: Why particularly the three-dimensional space? Is a two-dimensional surface, or even a one-dimensional line less material than a three-dimensional object? In fact, even a point, which that has no magnitude, is a part of the material world, and even more so if we remember that such a point will always appear on some line. We should therefore define the material substance as one that has magnitude in at least one dimension. We will denote by XTpm,pn the predicate "… extends between point pm and point pn" and will define it as follows:

x⮭XT;pm,pn ≡def x⮭MG;(n-m)

And according to the reduction rule it is clear that an object of some extension is an object with magnitude:

x⮭XT;pm,pn ≡def x⮭MG

Hence we can define matter. Matter is nothing but qXT. Note, that matter is not a foundational concept since it is not a primitive. Its definition is built on another concept. Indeed, from a metaphysical point of view the definition of matter does not rely on its being of such or such qualities but from the fact that it occupies some dimension.

 But, as noted above, matter is also the content of the pool of space. We can therefore state the equation:

qXTᴿ = (CT;PLᴿ)↓

However, we must remember that the concept of matter exists in the sphere of thought, too. Here we will have to define the idea of matter as a parallel in the sphere of thought of the real matter; but there is no use elaborating on it here.

Mass can be defined as the magnitude of matter. If we denote the concept "… is a mass" by the letters MASS, then mass itself will ne qMASS and will be defined:

qMASS = def qMG\*qXT

Two extended objects are of *indirect spatial continuity* if there is at least one extended object that borders on both of them. Let us use the letters INDSC to denote "… has an indirect spatial continuity with…" and define:

x⮭INDSC;y =def x⮭XT˄y⮭XT˄əXT(C'K(x∩y))(CPBD;x˄CPBD;y).

From this point we can construct all the concepts of Newton's mechanics. Of course, this reaches far beyond the scope and goals of this book. If one asks why I engage here with the foundational concepts of Newtonian physics when those have already become old, and modern physics built itself on other foundational concepts, I have already replied to that, but will, nevertheless, sharpen my point here. I do not come to present here the foundations of the "true" physic, but the foundations of the sphere of thought. Newton, in contrast to Einstein and other modern physicists, took the intuitive concepts from which our sensual world is made, reduced them to three foundational ones, and constructed his scientific world from them. In a way, his method is not so different from that of Aristotle, even if its implementation is very different. Since our sensual world is the basis for the work of empirical science, the world view of modern physics is supposed to be developed, in this way or the other, from the foundational concept of that world. Even when we reach the heights of the theory of relativity and quantum mechanics, the data with which the scientist starts working are those he perceives. If his final explanations are far removed from those sense data, it is because he develops them by way of abduction ("the best explanation") which is, as we have seen above, one of the cognitive functions. (In fact, this function is very often used to explain sense data.) For this reason, when we discuss the sphere of thought, these should be our building blocks.

### The different types of change

Before we move on, we have to continue to develop our concept of change. Above, we have defined CH2 as a concept describing a change other than a change of age. Sometimes we will wish to refer precisely to a change that is neither of age nor of place. That is, so to speak, a change in the "character" of the object. We will denote it by CH3, and will define it as follows:

CH3=def CH2-MV

Or

CH3=def CH1-AG-MV

Having described the nature of changes, we can now state the rule of mereological change. According to this rule, any change in the part is a change in its whole. Let us formalize it:

x⮭CHyzɒ(PP\*x)⮭CH1;(y,z)

According to the reduction rule:

x⮭CHɒ(PP\*x)⮭CH1

With the quantifiers of traditional logic we would formalize it this way:

∀x x⮭CH ∀y (y⮭PP;x)y⮭CH1

But x itself also changes, and so we can say:

x⮭CHɒ(IP\*x)⮭CH1

And if we use the quantifiers of traditional logic:

∀x x⮭CH ∀y (y⮭IP;x)y⮭CH1

All of this might also be said of the narrower predicates of change, CH2 and CH3. Namely, predicates CH1, CH2 and CH3 bequeath-to-wholes. Formally speaking:

CH1⮭BQ(WH)

CH2⮭BQ(WH)

CH3⮭BQ(WH)

Of course, if change as a general concept bequeaths-to-wholes, this does not mean that every particular concept of change also bequeaths-to-wholes. One cannot deduce from a whole concept (change) to a partial concept (a particular type of change). Thus, for instance, if a part of someone's body became red, that does not mean that his entire body became red, and if someone ate an apple it does not mean that the entire world ate an apple.

 Predicate CH2 is the predicate "changed"; the change itself, insofar as it is time limited, will be called "event." The time limits of an event are determined by decision. Therefore, even if this concept has a connotation of a short change, in ontological terms it might also refer to changes that endure for millions of years. Since change is always a concept that has an object, event is also something that happens to an object. In fact, event is the quality of the concept "changed." We will denote an event by CHE2\*x,t (n🡂m), where x denotes the object in which the event takes place, and the constituent t(n🡂m) denotes the time unit in which the event took place, and will define as follows:

CHE2\*x;t(n🡂m) =def qCH2\*x;t(n🡂m)

But according to the reduction rule we may give up the time constituent, and then we get:

CHE2\*x=def qCH2\*x

And hence we can state:

x⮭CH2;(y,z)  əCHE2\*x

And also:

x⮭CH2;(y,z);t(n🡂m) əCHE2\*x;t(n🡂m)

Above, we defined when a time point precedes another, when it is simultaneous with another and when it follows it. As for events, we will define an event as preceding another when its starting point precedes that of the other, and the same with all the other temporal relations.

 The predicate PRCHE2 will denote "event … precedes event …," the predicate SMCHE2 will denote "event … is simultaneous with event …" and FLCHE2 will denote "event … follows event …" We will define:

x⮭PRCHE2;y =def x⮭CHE2˄y⮭CHE2˄x⮭BRNtm˄y⮭BRN;tn˄m<n

x⮭SMCHE2;y =def x⮭CHE2˄y⮭CHE2˄x⮭BRNtm˄y⮭BRN;tn˄m=n

x⮭FLCHE2;y =def x⮭CHE2˄y⮭CHE2˄x⮭BRNtm˄y⮭BRN;tn˄m<n

We will add two more definitions: simultaneous/preceding (SMPRCHE2) and simultaneous/following (SMFLCHE2):

x⮭SMPRCHE2;y = x⮭CHE2˄y⮭CHE2˄def x⮭BRNtm˄y⮭BRN;tn˄m≤n

x⮭SMFLCHE2;y =def x⮭CHE2˄y⮭CHE2˄x⮭BRNtm˄y⮭BRN;tn˄m≥n

And what about an event that does not include a "positive" change, i.e. whose only change is that of age? In ontological terms, a change of type CH1 deserves a list of concepts similar to the one we built for CH2. However, such an event is not called an event in natural language. We will therefore call it event1, will denote it by CHE1 and will define it as follows:

CHE1\*x;t(n🡂m) =def qCH1\*x;t(n🡂m)

Accordingly we will state, *mutatis mutandis*, the list of above developments:

x⮭SMCHE1;y =def x⮭CHE1˄y⮭CHE1˄x⮭BRNtm˄y⮭BRN;tn˄m=n

x⮭PRCHE1;y =def x⮭CHE1˄y⮭CHE1˄x⮭BRNtm˄y⮭BRN;tn˄m<n

x⮭FLCHE1;y =def x⮭CHE1˄y⮭CHE1˄x⮭BRNtm˄y⮭BRN;tn˄m<n

x⮭SMPRCHE1;y =def x⮭CHE1˄y⮭CHE1˄x⮭BRNtm˄y⮭BRN;tn˄m≤n

x⮭SMFLCHE1;y =def x⮭CHE1˄y⮭CHE1˄x⮭BRNtm˄y⮭BRN;tn˄m≥n

And the same is true for CH3, but an event of that type is useless (and it, too, is not called "event" in natural language), so we will refrain from presenting all its developments here. The uniformity-loving reader will be able to complete this part by himself.

### Operation

Let us return to the issue of decision. We have mentioned a few types of decisions, but did not discuss them systematically. Two types of decision deserve a particularly elaborate treatment: decision to operate (some concept or function) and decision to tag (i.e. "follow" objects on the timeline). We will discuss decisions of tagging below, and for the moment will concentrate on operation decision.

 A decision to operate is simply a decision to start using a particular cognitive function or concept. Since the very appearance of the function or the concept in the subject is actually the beginning of its activity, we may well say that their operation is the decision to start having them in the subject. Therefore we can define operation by the existing concepts of birth, content and subject.

 The concept "operates" will be denoted by OPRT and will be defined as follows: Sn⮭OPRT;x ≡def Sn⮭DC:(x⮭BRN;Sn).

 The concept "is operated" is the inverse of "operates." It will be denoted by the letters OPD and will be defined as follows:

x⮭OPD;Sn ≡def Sn⮭OPRT;x

Here, too, since we deal with creating tools for the description of the world and not the description of the subject's actions on the world, we will take OPD as preceding OPRT. We should therefore define OPRT as an inverse of OPD:

OPRT ≡ INV;OPD

As I noted above, for our purposes there are two types of operation at stake: of functions and of concepts.

**The operation of a function**: The predicate which expresses this is "function … is operated." It will be denoted by the letters OPDF, and will be defined as follows:

x⮭OPDF =def x⮭F˄x⮭OPD

When I receive from a source some data of some sort, it means that I decided (involuntarily) to receive qualities of that sort. Thus, for example, the decision to operate the sense of sight determines that I receive visual data; the decision to operate intellect determines that I receive ideas of concepts; the decision to operate memory determines that I receive ideas of the sort preserved in memory, which are similar to those from which they were originally absorbed (senses, intellect etc.), but still differ from them; and the decision to operate forgetting determines that I receive "empty ideas" instead of existing ones, or instead of parts thereof. The decision to operate imagination is probably the most complex one, and depends on the sources from which imagination builds its new ideas, and the order of the actions within that process.

**The operation of a concept**: The predicate which expresses this is "concept … is operated." It will be denoted by the letters OPDK, and will be defined as follows:

x⮭OPDK =def x⮭K˄x⮭OPD

The decision to operate functions is *ipso facto* a decision to operate concepts, i.e. the function concepts of those functions. Thus, the decision to operate the sense of sight is a decision to operate the concept of colour; the decision to operate intellect is the decision to operate the concept of concept, i.e. to think about the world in a way of applying general data to individual ones; and so on.

 Is it possible to operate a function without operating a concept? Of course not. Is it possible to operate a concept without operating a function? The answer is negative here, too, since any datum of thought, including any concept in thought, is received from a cognitive function. We are talking, therefore, about two sides of the very same action. We can even state it as a biconditional:

x⮭OPDF↔x⮭OPDK

That sentence notwithstanding, we must distinguish between these two sides of the same coin.

The concepts contained by the subject are those which constitute its *partition of the world*. Every decision to operate a function or a concept is a decision to partition the world in some way. But from that point and on further decisions of partition are made as continuations of the decisions to operate the functions and concepts and are added to those decisions. The concept of colour, for example, includes all the spectrum of colours. So here starts a secondary partition. That partition should not be taken for granted, since, as we know, in different cultures the spectrum of colours is partitioned in different ways. But even if there were a universal unanimity with regard to the way of partition, the very fact that it *could* be different testifies that it is the result of a decision. The same is true for the concept of "concept." This concept includes all the concepts in the world, and can be partitioned in numerous ways, according to the mereology of concepts. The subject can partition them according to their contents (for example: concepts of colour, sound etc.) or according to second-order characteristics (for example: the size of their extension). One way or another, in any partition according to any criterion, we could imagine a different partition. Therefore every partition is a result of a decision.

 We have already stated the notation of decision: DC. The predicate "operated" will be denoted by OPDF or OPDK, according to the issue discussed. The sentence "Sn decides that function X be operated" will be written: Sn⮭DC:X⮭OPDF, and the sentence "Sn decides that concept Y be operated" will be written: Sn⮭DC:Y⮭OPDK.

 We previously stated that the decision to operate a function means a decision to operate its function concept. Hence we can state:

Sn⮭DC:Fm⮭OPDFSn⮭DC:KFm⮭OPDK

The decision to operate a concept is, in turn, a decision to determine (partial) boundaries of the object, as expounded in the first section. Therefore, any decision to operate a concept is *ipso facto* a decision to operate its conceptual complement.

Sn⮭DC:x⮭OPDK≡Sn⮭DC:CKx⮭OPDK

Therefore we may conclude that the decision to operate a function is also a decision to operate the conceptual complement of the function concept:

Sn⮭DC:Fm⮭OPDF≡Sn⮭DC:CK(KFm)⮭OPDK

Namely, any decision to operate a function is a decision to operate both its function concept and its complement.

What bothers us here is the question of the partition of the world, and since we reached the conclusion that the primary partition is a result of a decision of the subject, we turned to the truth sources in order to learn what this decision is. I have already emphasized that the distinctions made by the subject are not according to photons, wavelengths, density of molecules etc., but according to colour, sound, taste, odour, hardness and heat. Each of these foundational concepts is of course a spectrum that continues to be partitioned into the various colours, sounds, tastes, odours, degrees of hardness and degrees of heat. If we had another sense, which we might call FF, it would transmit to us the FF-ish qualities, that would offer us another partition of the world. These qualities, too, could be explained by science as results of elementary particles, but from the perspective of the sphere of thought the relevant concepts would be the function concepts of the raw sense FF.

 All these qualities aspire to reflect qualities in the sphere of reality. But we will discuss that later.

### Crisp versus vague boundaries

The subject observes from a distance the yellow box laid on the sidewalk. It cannot see its boundaries very well. Philosophers of our time differ on the question of vague boundaries, whether they are an epistemic condition or exist in the sphere of reality.

 Above (Chap. ###) we saw that in reality, too, there can be crisp boundaries (linear functions) or vague ones (areal functions). Actually, in the sphere of reality all the possible types of boundaries exist. The Big Concept is partitioned in reality to all the possible parts, and therefore is partitioned indefinitely. It is partitioned not only into all the possible partitions through linear functions, but also into all the possible partitions through areal functions. Therefore, both of them exist in the sphere of reality. Of course, only a small minority of the countless partitions overlap the partitions of the thinking subjects. In most of the cases, the partitions made in thought are useful to the subjects (and I will dwell on it below).

 The subject seeing a box whose boundaries are vague often sees an object which is the box (or a part of it), an object which is not the box ("the rest of the world"), and a "dubious" area. Here the constituents of Fuzzy Logic related to doubt can enter. In this case, the doubt stems from the subject. It is also possible, however, that the object itself would have areal boundaries. Such, for example, is the case of the partitioned sheet (black – gradually brightening grey – white), in the above illustration. In that case, the constituents of Fuzzy Logic related to partial application of predicates will enter the scene. Indeed, in both ways it should be emphasized that the concept of capturing itself is binary and sharp. Either a concept captures an object or it does not. A partial application of a predicate is not a partial capturing of the object. Even a judgment regarding the partial application of a predicate must be determined sharply, and it is that determination which constitutes the capturing of the object by the concept. Yet, what was said here about objects, and was exemplified by individua, is true for concepts as well: Concepts, too (even in reality and certainly in thought) might be of either linear function boundaries (crisp) or areal function ones (vague). The boundaries of both types are full-fledged boundaries.

 As I have already noted, believing that vague boundaries are not boundaries is not just a metaphysical epistemological stance, but an approach with methodological and even ideological bearings. Many of the theories fashionable today, especially those of the postmodernist school, seek to present vague boundaries existing between two phenomena, x and y, declare them "blurred" and consequently deduce that there is neither x nor y, only a single continuous object. This assertion is definitely incorrect. As I wrote above (first section): If we have a paper sheet on which the colour turns from white through continuously darkening grey up to black, this does not mean that there is no black and no white. Rather, there is black, there is white and there is grey of varying hues. Where does the boundary pass? That is a typical sorites, but even in the classical sorites there is a heap and a non-heap, and the question regarding the location of the boundary does not negate these concepts.

 Boundaries are determined through concepts; concepts are determined through cognitive functions; cognitive functions are determined through a decision; therefore we may conclude that decision also determines whether the boundaries will be crisp or vague.

### Useful and useless partition

Further below I will demonstrate that the sphere of reality exists. I will also demonstrate that in that sphere all the partitions, both of linear and of spatial boundaries, exist. Then I will also demonstrate that the partitions of the sphere of reality are available to the thinking subject. If so, which of the partitions existing in the sphere of reality exist also in the sphere of thought? The answer is that the subject decides to take useful partitions, as required by his or her needs and limitations. As I noted above, the partition of the world in the sphere of thought is a result of decision, and therefore it cannot be judged in terms of right or wrong. However, it can be judged in terms of useful or useless (Compare Varzi, 2011). We can partition the world in such a way that the concept of the union between the dark side of the moon and the tip of Cleopatra's nose will be considered a foundational concept. However, since the use of this concept will be very rare, and the object it conceptualizes has no role in our lives, we can conclude that it results from a useless partition.

 The partition of the world might be different in an even more basic aspect, that is already touching on the margins of the first metaphysic. Our partition of the world, as described in the first section, is done pretty much in a pyramidal structure: At the top of the pyramid stands the primeval concept with all its components and it is partitioned, by the concept of concept (the first partitioner), into all the other foundational concepts. These concepts constitute, to a great extent, the second line (from above) of the pyramid. The foundational concepts intersect each other intricately and create various intersection concepts, from which the laws of logic are constructed. These concepts constitute the third line, then the fourth etc. At some stage this pyramid moves forward to the second metaphysic. That stage begins with the foundational concepts of that metaphysic (which we are trying to discover at this these very moment), and continues further on to their mutual intersections. At the bottom of the pyramid stand the uniextensional concepts. This picture of the world is pretty much Leibnizian. Leibniz himself even aspired to quantify thus structure by his Characteristica Universalis. However, this structure itself is not necessary. I am not talking about the primary and basic decision to go out of the unified perspective of the mystic into a world of ordered plurality. That decision is already taken as a given. Rather, I am talking about the decision to build the pyramid of such a large number of lines. We could imagine, for instance, a partition in which there is only a line consisting of the primeval concept and another consisting of uniextensional concepts. In such a partition we couldn't group objects into species, and consequently could not make any generalizations or discover any laws applying to those objects. We could also imagine more moderate partitions than that one (if we define moderate as closer to our partition of the world) and yet even they could provide us an altogether different picture of the world. Thus, for example, if the concept of colour were partitioned to only two parts – say, black and all the rest – the way we would perceive the boundaries of visual objects would be entirely different. Here, too, one must add that all of these ways of partitioning the world are such that we may judge them to be less useful than our way of world partitioning.

 Useless partition of a concept often occurs when the concept is too large or too small for the issue at stake. Thus, for instance, the union concept of "green" and "equine" is too large, because the objects captured by it will usually be either equine or green, so that the union is superfluous. On the other hand, the intersection concept of "green" and "equine" is too small, since it doesn't capture any object. However, we cannot reduce the entire question of usefulness/uselessness to the breadth of the concept's extension. Sometimes when we find that a certain concept is of zero or one-item extension this discovery is very useful. Furthermore, all the individua in the world have uniextensional concepts that capture them. We must conclude, then, that in a case that a concept with small extension is found useless, it is not its small extension which makes it such.

 I do not have a principled key for classifying a concept as useful or useless, because it depends, among other things, on the deciding subject. We know the famous (and false) stories about the Eskimos having tens of different terms for different types of snow, the French having a similar number for different types of cheese and the Bedouins for different types of camels. The Eskimos' partition would be useless for the Bedouins, and vice versa. No doubt, in this matter language plays a highly important role, but language, in turn, reflects thought. Today there is often an emphasis on the role of language as creating thought, but we must remember that the converse is also the case. Whichever way we look at it, there is a large overlap between them, and therefore we can learn from the partitions of language about the partitions of thought.

We can take another example. Suppose there is a machine A manufacturing a certain type of shoe as part of factory B which is located in country C. If we go to the factory and ask "Where is this shoe manufactured?" the workers will most likely direct us to machine A. If we ask the same question in the city or country where the shoe is manufactured, we will be directed to factory B. If we ask outside that country, we will be answered that the shoe is "made in country C." The answer that the shoe is "made in country C" is trivial, almost ridiculous, within factory A. In contrast, if we discuss it in the context of the entire economy of country C the assertion that factory A is the one that manufactures the shoe is not trivial at all. Similarly, if we live in a country outside C, to which country C exports the shoe, the assertion that the shoe is "made in country C" is a useful statement. I guess that if one day there is an inter-planetary commerce, even the statement "This shoe is manufactured by Planet Earth" will not be trivial. To sum up, the concept "made in country C" might be useful or useless depending on the changing subjective circumstances, and the partition of the world that generated that proposition might change accordingly. Some partitions might be useful for some subjects and useless for others. Moreover, even for the subject itself it might be that in some time or some set of circumstances one partition will be useful, and in another – not.

We can say that all of us can find the partition based on the senses useful (and the same is true, *mutatis mutandis*, for all the sources enumerated above). That is true, but here too the issue is not that simple. First: The senses, too, reflect usefulness, i.e. animals that had senses of some sort survived in the conditions in which they lived while other animals that did not have those cognitive tools became extinct in the same conditions. Second: As I said, there are different ways to partition the concepts of sense data themselves. Thus, for instance, some say that in certain tribes there is one word for pink and orange, and the same word for azure and green. I don't know if such tribes exist, what is important is that their alleged partition of the world *could* exist. In other words, we could imagine a state of things in which there would be an idea for the union concept of pink and orange, while there would be no idea of pink and no idea of orange. Such a partition would generate in us a picture of the world that is entirely different from the one we have today, since the number of objects we would perceive as discriminated from their environs would considerably decreas**e**. Consequently, all of our attitudes to those objects and our ability to construct laws relating to them would change altogether. And so on with the derivatives of those changes.

 The discussion above goes back to the question of operation, addressed above, as well as to the question of tagging (i.e. the decision to "follow" objects on the timeline), to be addressed below. The operation of functions and concepts can be done in multiple ways, and the same is true for the tagging of objects, but for some causes we decided on the ways of operation and tagging that we have. This decision has two layers: The first is that of the innate capacities, such as senses, intellect, etc. These implant in us very foundational concepts, of the first and second metaphysics. Above it comes the second layer, where we transcend the innate capacities and already operate some unconscious discretion, which decides on further partitions. It is in this layer where we can find, among other things, the social-cultural-political considerations, pointed at (and overstated) by the critical theories. In view of the irrelevance of the logico-metaphysical tests (the question of truth being not pertinent to decisions), these decisions are judged by the criterion of usefulness. It is very likely that some of the decisions we made with regard to the partition of the world were not the best in terms of usefulness, but not to a degree that impeded our survival. In both of the layers the decisions are for the most part involuntary, but it seems that the decisions of the second layer are easier to change than those of the first. And still, the second layer, too, is well entrenched in our minds. In any case, once we discuss decision, whether voluntary or not, we are dragged into the normative arena.

 But one thing should nevertheless be said about the metaphysical aspect of the useful partition. Partition does not create objects but only reveals objects. The partition of the world is the work of the subject, by means of decision, but the ability of the concept to capture objects does not depend on the subject but on the nature of the concept, the nature of the object, and the relation between them. When concepts capture objects it is because the latter "indulge" them. The subject cannot decide that a white object lends itself to being captured by the concept "black." It can refrain from operating the concept of white altogether, but then the object will not gain any boundaries in terms of the function of colour. (Such a partition, needless to say, will be useless with regard to that aspect). Partition is therefore of great importance, but not everything depends on it.

### Causation

Above I have defined event as a continuous change enduring over a time unit whose boundaries are determined by decision. Now we can discuss some concepts related to it, first of which is the causal linkage.

 The concept of causation is among the most central concepts in our lives: A boy kicked the ball, and therefore the ball flew; the ball hit the window, therefore the window shattered; the window shattered, therefore the shard fell down; the shards fell down, therefore a girl was wounded. And so on, and so forth. Ancient philosophy took the concept of causation for granted, and deliberated only with regard to its effect on human will. Modern philosophy, however, raised questions about it, especially following Hume's sharp criticism of it. The Scottish philosopher defined causation as a relation of three constituents: contiguity, succession and necessary connection (Hume 1739, Book 1, part 3, sec. 2). It seems that in "necessary connection" Hume understood a logical linkage, in the spirit of Hume's fork. Therefore when he could not find the linkage between two physical events which we are structured to see as cause and effect, he inferred that they do not entertain a cause-effect linkage, and doubted its very existence. It seems, however, that Hume was wrong in his definition. Not only does the common understanding of causation not require a logical linkage, but even contradicts it. Suppose someone said: "All of John's body got wet, and as a result his hand got wet, too," something would sound strange to us in that statement. The causal linking ("and as a result") is incorrect here, since the linkage between the body's wetting and the hand's wetting is not causal but rather logical. Since the hand is a part of the body, and "got wet" is a concept bequeathing-to-parts – when the whole gets wet so does the part. The linkage sought by an ordinary person who speaks about causation is a stable linkage assuring that always whenever the cause takes place so will the effect, yet that linkage is not a logical one but one that is established by a non-a-priori law of nature. Furthermore, Hume himself was entangled in a tension, if not an outright self-contradiction, in this matter: A logical linkage was supposed to be contained in the concepts of the objects involved in the causal event, while Hume contends that causation cannot be attributed to the objects themselves (ibid). It seems, therefore, that we should adopt an amended definition of causation. I won’t get into the jungle of definitions offered for causation in modern philosophy, but will only state the one that will serve us in our discussions below.

Causation is a relation (and therefore a concept) between two events, one of which we call cause and the other effect, that satisfy the following requisites (conjunctively):

1. There are two events x and y, such that x takes place in object z and y takes place in object w.
2. x is not identical with y.
3. y is not later than x.
4. There is an indirect spatial continuity between x and y, i.e. between the object(s) z taking part in x and the object(s) w taking part in y.
5. There is a general non-logical law stating that whenever x takes place in z, y will take place in w.

I will briefly expound each of these points:

1. It is crystal clear that a causal linkage can also exist between two events within one and the same object, and therefore it should be noted that z=w is possible. This point also includes the possibility of multiple causes for one event since, according to the principle of free union, we may see the many causes as parts of one unified cause.
2. I usually use different letters to denote different objects, and do not bother to stress the nonidentity each time. Here it is important to state that x and y are different since z and w do not have to be different.
3. Already Hume was wondering if the cause must precede the effect. At the end of the day, it is a question of definition. In this book I will follow the assumption that the cause might be simultaneous with the effect, but not later than it (namely, the concept of "cause" does not include final causes; by this I do not say that such causes do not exist, only that this book will not include them by the word "cause").
4. He word "indirect" is misleading here, since it is not in its plain meaning. According to the definition of indirect continuity (above, ###), it rather means "not necessarily direct." In other words, it denotes a condition in which there is no spatial detachment between z and w. Of course, there is no such detachment when z=w (an object is not detached from itself).
5. As stated above, a non-logical law is a universal sentence which does not belong to the logic expounded in the first section, nor is inferred from it.

Now let us formalize this. The letters CSE will denote the concept "…causes change in…" i.e. "… is a cause of…":

x⮭CSE;y ≡def

1. x⮭CHE2\*z;t(m1🡂m2)˄ y⮭CHE2\*w;(m3🡂m4)
2. x≠y
3. x⮭SMPRCHE2;y
4. x⮭INDSC;y
5. əNL: x↑⮭CHE2\*z;t(m1🡂m2) y↑⮭CHE2\*w;(m3🡂m4)

As I said, causation is a relation. This relation is non-reflexive and asymmetric, but transitive, since if x is the cause of a cause of y it is also a cause of y. We can therefore state the RST characteristics of CSE as follows:

CSE⮭NRFLX

CSE⮭NSMTR

CSE⮭TRNT

Namely:

x⮭CSE;y˄y⮭CSE;zx⮭CSE;z

The concept of causation stated here is appropriate for all those pairs of events in daily life to which we refer as maintaining a cause-effect relation, as in the examples presented above. Point 4 negates the possibility of attributing causal linkage to two events (especially long-standing conditions) existing in two distant places without any dependence of the one on the other, as in the sentence "Thanks to human beings’ wars the asteroids move in the heavens." Obviously, at every given moment humans are at war, and at every given moment asteroids move in the heavens, but the causal linkage between them will be determined only if there is a spatial continuity between those fighting and the moving asteroids. Point 5 answers the requirement stated by Hume of "necessary connection" between cause and effect, by presenting a quasi-necessary linkage that is not logical. With such a concept of causation we work all the time in our sphere of thought. An empirical survey of that sphere will demonstrate that causation prevails in all the arena of the material world, i.e. the entirety of objects extended in space (and in the sphere of thought this means ideas of objects extended in the idea of space). Causation is thus a part of the order of our world.

 Still, the above definition evokes many questions. First, what are the boundaries of an event? Since we decide those boundaries by decision, we should ask, more precisely, what are the boundaries that are relevant to the definition of causation? As for the continuity, taken that we can find spatio-temporal continuity between any two objects, what is the continuity required here? And if causes and effects are events, we should ask, following contemporary philosophers, what type of event qualifies as a cause and what type as an effect? Suppose a mountain climber fell and was injured, may we consider the lack of proper safety conditions as a cause of his falling ("negative cause")? If a girl holds a book and consequently it does not fall, may we consider that non-falling as a result of her holding ("negative effect")? And should the law of gravity be considered as a cause for the falling ("general cause")?

 According to the definition used above, the word "event" is used here in the narrow sense, i.e. only as a "positive" change, such that it does not include merely a change of age. We might, however, build a larger concept of cause in which, in the above conjunction, we will put CHE1 instead of CHE2. We may call such a cause "cause1." The concept of cause1 will thus be rather large, and will also include "negative" events, i.e. the non-existence of changes. According to this broad definition, negative causes are also causes and negative effects are also effects. There is much sense in recognizing such causes, since negation is only the complementary concept, and the complement is a concept just as any other concept, and therefore an object just like any other object. Such a cause does not have to be prior to the effect, and might just as well be simultaneous with it. The reason that the girl's book is not falling is that she is holding it, and the two actions take place at the same time.

 Laws of nature are also events. Indeed, laws of nature are events of very long endurance (this, per se, is not a defect, since from a pure ontological point of view the entire history of the world can be considered as one long event). However, we have seen that a part of the definition of cause is the requirement that there be "a general non-logical law stating that whenever x takes place in z, y will take place in w." The law is a part of the reason, and therefore cannot itself be a whole reason.

 The boundaries of cause and effect are not determined by logical or metaphysical considerations. When the girl rubs a match on the matchbox we can supposedly contend that this event was said to be the cause since throughout the entire action the same subject hold control over it, and he or she can choose to stop or continue it, while the effect is already out of their control. Supposedly, we could say the same thing about the boy kicking the ball, in the example given above. However, that example offers a contrary case, where the flying ball constitutes a cause and the broken window is the effect, and both of them are out of any control. Furthermore, both the cause and the effect could be partitioned to a chain of "internal" causes and effects. The postulated conclusion is that the boundaries of the cause and the effect are a matter of decision and the decision is made according to the criterion of usefulness. Causes and effects are useful events when one can find types of events (concepts) which capture both of them and serve as building blocks for the construction of a non-logical law.

### Factor and action

We have defined causation as a relation between events; and we agreed that an event is an object; but it is clear that beyond that there are objects taking part in every event. An object that takes part in the event of the cause and influences the event of the effect (i.e. without it the effect would not take place, according to the non-logical law involved), is called a **factor**.

We will denote factor by FKR and will formalize it with the help of the formula of causation stated above:

z⮭FKR\*y ≡def

z⮭FKR\*y ≡def

1. x⮭CHE2\*z;t(m1🡂m2)˄ y⮭CHE2\*w;(m3🡂m4)
2. x≠y
3. x⮭SMPRCHE2;y
4. x⮭INDSC;y
5. əNL: x⮭CHE2\*z;t(m1🡂m2)↔y⮭CHE2\*w;(m3🡂m4)

It should be noted that in the last point the sign of conditional was replaced by that of biconditional, since z will be considered as a cause of the effect only if it is a necessary condition thereof, i.e. if the effect takes place if *and only if* z takes part in the cause.

 When the factor is a subject, the cause will be called an **action.** The concept "acts" or "performs an action" will be denoted by the letters ACT and will be formulated as follows:

x⮭ACT= x⮭CSE˄FKR\*x↓⮭S

And hence it is clear that

ACT⮭PP;CSE

We may, of course, eliminate the last predicative constituents by the reduction rule. We should also remember that this entire move is built on a change type 2, CHE2, and we may build similar definitions for change type 1 or change type 3. I will not bother the readers with that, however.

Action itself will be denoted by qACT.

Here I must add that I well aware of the discussions in contemporary philosophy on the definition of action and the agent-action relation. This includes other topics such as whether that relation should be considered as causal, whether will or intention are necessary for counting an event as action, and more. These discussions are important for the philosophy of mind, for moral philosophy and maybe for our concerns as well. As for myself, I adopted the above definition, which contains a constituent of causality but does not contain a constituent of intention. This does not imply that I presume to decide the above controversies. It only means that this is the definition that best serves our needs in the present work and will therefore be used hereafter.

 But here comes the question of the identification of the factor from a mereological point of view. This question regards not only action, but any causal event. When will we attribute the influence on the effect to z, when to its part and when to its whole?

 Let us return to the example of the shoe, given above. As we remember, to the question "Who manufactured this shoe" one person will reply by saying the name of the country where it was made, another by saying the name of the factory, and another will give the name of the factory worker or point at the machine which manufactured that shoe. In view of that we came to the conclusion that the identification of the manufacturer depends on the partition of the world, and the latter is a matter of decision, determined pretty much by the question of usefulness. We have also seen that in that example the useful partition is not universal, so that different people will find different partitions useful, and even the same person might partition the world in different ways, depending on the time or circumstances. The question mentioned there, of who manufactured the shoe, is actually a question of the identification of the factor. Consequently we should say that the identification of the factor is also subject to the subject's decision on the partition of the world, and therefore to the usefulness test. With regard to identification of factors we can try to compose a list of indicators for useful partition, that should work for most of the subjects in most of the cases. I make no pretence of exhausting this list here but I will present two examples:

1. If there is a way to describe the factor as a whole or as its part, we will usually see the part as reflecting greater precision, and therefore as more useful. We often attribute a causal action to the whole as a result of ignorance of the identity of the factor or its relation to the effect, and when we learn the details of the event we attribute it to the part. This preference will be called "the selection of the part." But we should add an exception to this rule:
2. If there is a way to describe the factor as a whole or as its part, and the whole is a human, we will usually see the identification of the whole (i.e. the human) as reflecting greater precision, and therefore as more useful. Thus, for instance, we will prefer to say that "John cut the bread" rather than "John's hands cut the bread." This preference will be called "the selection of the human." This exception shows how much the entire issue is a matter of decision, since the concept "human" has no advantage in the ontological level, and the only reason we give him an additional weight is that, being humans, it is central in our minds.

Can the above example lead us to the conclusion that the concept "…causes change in…" is bequeathing-to-wholes? That is, can we attribute to any whole a status of factor in relation to an event whose factor is a part of it? A few examples can illustrate the question. According to the principle of free union we may talk about the object uniting John Lennon and the bright side of the moon as one object. Let us name it Lemoon; can we say, then, that Lemoon wrote "Imagine" and shines on the earth at night? If I am a resident of Jerusalem, and wrote a letter to a friend, can we say that Jerusalem wrote him a letter? That Planet Earth wrote it? Can we attribute to the world any action performed by any object? Our "natural" intuition says no to all of these questions. Yet I have noted above that whenever we encounter "intuitions" and "natural" we have to evoke our suspicion lest these are but prejudices and long-entrenched customs of thought. Admittedly, simple intuitions are often more correct than the complex constructs created by the professionals, including philosophers (no less than others!) but suspicion must be not be suspended.

From a purely ontological standpoint, one cannot avoid the conclusion that the concept "causes change in…" is bequeathing-to-wholes. I have already noted above ( ###) that the fact that a whole has a part with a property Y says something about it (the whole), too. It does not necessarily say that property Y captures the whole as well (bequeathing-to-wholes), but it does say that the whole "has a part that is Y." However, when the whole "has a part that is a cause of z" one can definitely see it (the whole) as a cause of z too. The selection of the particular part that causes z, i.e. bringing the predication to a higher degree of precision, does not negate the possibility of seeing the entire world as contributing to the causation. Pointing at the whole as the factor might bring some less useful statements, and therefore uninteresting and maybe even ridiculous, but they are not false. They are not false just as the statement that the shoe is made in country C is not false, even when it is said on the site of factory A.

Let us take the sentence "Vesuvius caused the destruction of Pompeii." In our sphere of thought, pointing at Vesuvius as the cause of the destruction is definitely plausible. In our customs of speech if we said that the cause was Italy, it would sound ridiculous; Europe – even more ridiculous. However, suppose we lived in another sphere of thought where, due to different cognitive functions, for instance, we could not identify as factors objects sized less than 200,000 square kilometers. Would then, too, these assertions sound ridiculous? I assume they would not. When we do not have a possibility of attaining more precise information in terms of the selection of the part, the most precise information is the one that offers the largest part attainable under the circumstances, and that is why that part is the most useful one, even if it is small. Anyway, it would certainly not be judged as false.

In view of that, let us return to the above examples, which seem to show the opposite, and try to understand them. Did Lemoom write "Imagine"? Yes, it did, but the selection of its part will show that the part named John Lennon is the more precise part for that action. Does it also shine at night? Yes it does, but the selection of its part will show that the part named the bright side of the moon is the more precise part for that action. If I wrote a letter to a friend, can we say that the city of Jerusalem wrote it? In truth, we can, and what makes this statement absurd is the normative consideration. Usually we will say that the city of Jerusalem wrote a letter only if all its residents were signatories to it or if a representative body, such as the city council or the mayor, had written it. This is because we make a link between the author of the letter and the one responsible for it. However, these are all normative conventions, whether social or legal, that ought to be "cleaned" when we discuss this issue from ontological perspectives. From an ontological point of view there is no difference between saying that the letter is written by the city of Jerusalem and saying that the shoe is made in country C.

We can now formulate it as a simple rule, which will be named **the rule of the bequeathing-of-action-to-whole**, which will read as follows:

x⮭ACT yWH\*x⮭Act;y

(By the reduction rule we may omit the constituent y in this formula).

 Can we say, by this rule, that the entire world wrote the letter? Can we actually attribute agency to the world for any action done by any object? The answer is definitely yes. Any action performed by any part of the world is an action performed by the world (and one doesn't have to be a Spinozist to admit that).

 This sentence will be called the rule of the agency of the world. Its formulation is even simpler:

x⮭ACT;yu⮭ACT;y

(By the reduction rule we may omit the constituent y in this formula, too).

Indeed, this is an altogether useless truth (and for similar reasons there is something of the useless in Spinoza's philosophy), since when we seek the factor of an event we expect to find something more precise than just "the world," but this, again, concerns our world and considerations of its usefulness. That is why we need an act of "selection of the part"; but this act, which tells us what precise part was captured by the concept of cause, does not diminish the fact that its whole, too, was captured by the same concept.

 In view of this we can formulate the rule of the change-causing's bequeathing-to-wholes:

CSE⮭BQ(WH)

But we have already seen that ACT⮭PP;CSE, and therefore it will be correct to apply this rule to actions as well:

ACT⮭BQ(WH)

Theoretically, we could refine this assertion and say that it applies only to non-reflexive actions, since in reflexive ones the predicate constituent that is bequeathed must be the part, not the whole. This is because if we say, for example, the sentence "Danny painted himself in red" and will wish to conclude about it from the rule of the bequeathing-of-action-to-whole, we will have to say that "the state of Israel painted itself in red" or even that "the world painted itself in red." This will obviously be futile. However, in concept calculus reflexivity is not expressed independently but by a repeated use of the entire predicate, which also includes the object's constituents. Thus, for instance, in the sentence x⮭PAINTRED;x the bequeathing predicate is PAINTRED;x, and therefore the bequeathing-to-wholes cannot be WH\*x⮭PAINTRED;WH\*x, but only WH\*x⮭PAINTRED;x. In other words, from the sentence "Danny painted himself in red" we may infer that "The State of Israel painted Danny in red" – weird as it may seem from an intuitive point of view it is still correct from the ontological one – but not the conclusion that the State of Israel painted itself in red.

 I am well aware that my arguments in this subchapter will be rejected by many. Although I think that those arguments are correct, I will offer here another explanation as an analysis of causes. Let us suppose that the concept of causation is not bequeathing-to-wholes. If so, Italy did not manufacture the Italian shoes, nor brought forth the destruction of Pompeii. When we attribute to Italy a status of factor in these two events, we do it only due to lack of sufficient information. At the end of the day, when our knowledge improves, we can tell which part of Italy is the manufacturer and which is the cause of the destruction. That part was the factor all the time, but we, for epistemic limitations, could not attribute the causing to it. According to this explanation, the same is true for the imaginary state of things I describe above, in which our mind cannot track objects smaller than 200,000 square kilometers. Then, too, we would say Vesuvius was the factor in the destruction of Pompeii, and the fact that we could not discriminate the mountain does not diminish the veracity of this proposition. We could say, allegedly, that we would discover this truth ex post facto, once our epistemic capacities improve.

 We may conclude, then, as follows: I personally assume that the concept of causing is bequeathing-to-wholes, and therefore we can attribute to any effect both the focused cause and any of its wholes, including the world. I admit that we are inclined to attribute an action to factors as small as possible, but this inclination stems from considerations of useful partition rather than those of truth (that are actually irrelevant to it). But even if someone disagrees with this position he/she should concede that we might attribute the cause to some whole due to ignorance of the part, until the latter is disclosed at some later stage. Another inclination, also stemming from considerations of useful partition, is the preference we give to subjects in the identification of factors.

 Concerning this I wish to add: A decision is an action of the subject, and usually the subject also carries it out. The decision and the carrying out should be seen as cause and effect, and due to the selection of a human we will be inclined to see a human subject as the main factor in the achievement of that effect. To take some examples: John decides to raise his hand and the hand is raised; Mary decides to operate the cognitive function of sight and now she sees the world in colours; Peter decides to see the tip of Cleopatra's nose and the dark side of the moon as two objects and from now on he treats them as such. In all of these examples the antecedent is a decision of a subject and the consequesnt is the effect of that decision. In all of them the effect results from a non-logical law. Hence the decision is an event of a cause. The decision to see the human beings (John, Mary, Peter) as the main factors in the achievement of the effect – i.e. the selection of the human – is usually the one that attains a more useful partition, and this is probably why we are inclined to favour it.

### Body

We have seen that the subject takes part in the causation existing within the sphere of thought (in which it constitutes a pool). In this subchapter we will continue discussing it. However, that discussion should be divided into two: (1) Can events within the subject be causes? To that we have already responded positively; (2) Can events within the subject be effects? For both of the questions we have to examine the way the subject acts on a very particular type of material object: the body. (And I will remind us again that the entire discussion is within the sphere of thought, and therefore the space mentioned here relates to the idea of space within the sphere of thought and the matter mentioned here is the idea of matter within that sphere. The real space and the real matter that exists in it will be addressed below, at Chap. ###).

 A human **body** is a material object, i.e. one that occupies a place in space, which has a special linkage with the subject. This linkage is bidirectional, and therefore we can talk about two linkages: The linkage of **control** is the one by which the subject acts on the body while the linkage of **cognition** is the one in which the body (or parts thereof) act on the subject. That takes place when the sensory organs transmit data to the subject. We can imagine a control by the subject that does not act on the body, such as in machines operated by thought. We can also imagine a cognition which is not done through the human body, such as a person's knowledge of himself. It is almost impossible, however, to imagine a single object other than a human body in which both of the linkages exist. Therefore we may define the body as the object that maintains both of these linkages with the subject.

One can imagine another body-mind relation, which is that of a decision operated by the body. Here we enter the age-old controversy regarding freedom of the will. This question, it should be noted, does not concern all the decisions, but only the conscious ones. Any decision, however, even unconscious and even instinctive, is a reaction to a cognitive act. There is no condition in which decision is operated directly on the body without the mediation, whether conscious or unconscious, of cognition. That is because a person first knows the facts and only then reacts to them, and that reaction is preceded by a decision. Yet, even if cognition works in a causal linkage, some contend that the decision following it is not a result of a causal linkage. Furthermore, there are those who believe that this question is closely linked to the question of moral responsibility (I dissent, following McTaggart, 1973, and others), but beyond this question many thinkers point at the fact that within the sphere of thought itself we feel that we choose between two alternatives, and therefore there is no causation which necessitates the specific choice we finally make.

Furthermore, there are thinkers who believe that this issue is closely related to the question of moral responsibility, and contend that only when a decision is not entirely determined by previous causes does it attach moral responsibility to its possessor. I disagree with this contention, for reasons similar to those of McTaggart (1973) and others, i.e. that a decision born in a person "ex nihilo" does not attach a greater responsibility to its possessor than a decision born in him "ex materia." For this reason I also think that the great endeavours invested in proving "free will" have been pretty much redundant. Once this moral purpose is taken off the table, the motivation to exclude human decisions from the causal order of nature is also taken off, let alone that such a motivation ought not to have influenced the philosophical discussion in the first place. Indeed, beyond the question of moral responsibility there are those who point out the fact that in the sphere of thought itself we often feel that we choose between two alternatives and could make the opposite choice, and therefore there is no causal necessity standing behind the choice we make. Yet this feeling-based argument, too, I barely legitimate as a reason within a philosophical discourse. We thus remain with the determinist assumption, which we regularly embrace as an explanation for the world as a whole.

While the issue of decision (to be precise, a part of this issue: that of the conscious choice) is controversial, the issues of cognition and control seem to enjoy a broad consensus, accepting that the body-mind relation applies to both. Admittedly, there are conditions, such as blindness or deafness, in which the body does not transmit cognitive data to the subjec and there are other conditions, such as cerebral palsy, in which the subject does not maintain control over his or her body. Yet, when one of the linkages is absent there is still the other one to keep the body-mind relation. There are conditions in which neither of them exists, for example, where the person is defined as a vegetable, and in such cases there is indeed a question of whether there still exists a subject that is linked to the body.

When an object (including a subject) is "controlled by…" a subject, this means that whenever the subject decides that the object be something, the object actually becomes it. In other words, there is a direct (without intervention of another subject) and regular (law-based) causal linkage between the decision of the subject and the being of the object. This concept will be denoted by the letters CONTROLLEDBY and will be defined as follows:

x⮭CONTROLLEDBY;Sn = def NL: (Sn⮭DC:x⮭CH2;y,zx⮭CH2;y,z)

When an object creates data in a subject this means that whenever something happens in the object the subject decides to operate some cognitive function. In other words, there is a direct and regular causal linkage between the event in which the object took part and the operation of the cognitive function in the subject. This concept will be denoted by the letters CREATESDATAIN and will be defined as follows:

x⮭CREATESDATAIN;Sn,y = def NL: (x⮭CH2Sn⮭OPDF;y)

Yet, when we talk about causal linkage the question arises of whether the conditions for causal linkage are satisfied. If we examine the constituents of causal linkage we will find that numbers (1), (2) and (3) are satisfied in both of the above linkages. Number (5) is supposedly the main issue, but before we get to that we have to solve a severe problem regarding constituent number (4). To remind us, this constituent requires that there be an indirect spatial continuity between the object(s) taking part in the cause and those taking part in the effect. In order to determine the existence of a spatial relation between the subject and other objects taking part in that causal linkage we first have to find out whether all of them are extended in space. As for the objects external to the subject, there is no problem, since all the events at stake pass through the human body, which is extended in space, i.e. is physical; but what about the subject? This brings us to the age-old body-mind question in its ontological context. Are body and mind two distinct substances, or just different tokens of the same substance? And if so, what is that substance? The contention that matter is reducible to mind is, of course, the argument of idealism, while the contention that mind is reducible to matter is that of materialism. The contention that none of them is reducible to the other is the argument of dualism. In order to accept the existence of a causal linkage, be it the linkage of cognition or that of control, we should therefore embrace the materialist model. This is indeed the most plausible path of philosophical enquiry.

 Therefore we seem to have two alternatives before us: Either deny the causal linkage in the acts of cognition and control, or affirm the existence of such a linkage on the basis of the materialist model. Since the existence of that linkage seems to be an uncontested datum, we remain with the latter alternative. There might be, however, some third way, which will be based on some definition of the causal linkage that will engage a softer version of constituent number 4 (the requirement of spatial continuity). As I noted above, a complete omission of this constituent is inconceivable, since without it we will have to determine a causal linkage between any pair of events that take place in the world on a constant basis (such as the existence of wars and the motion of asteroids in the heavens). Yet, if we ever find a substitute constituent for causal relations linking non-spatial objects, we will probably be able to maintain the causation in the linkages of cognition and control even in dualist or idealist ontological models. This issue is not yet resolved, but for the time being, as long as that is the case, the materialist model should be preferred.

### Tagging and continuous identity in time and change

I have noted above that the operation of concepts is what determines the identity of objects. For example: If we didn't operate the concept of colours we could not discriminate the box in the street. Even when we do operate the concept of colour, if we partitioned that concept to only two partial concepts – say, black and all the rest – it is doubtful whether we would discriminate the box in the street, and even if we did, we would most probably discriminate it along boundary lines that are far different from those in which we discriminate it today. Now I would like to make another point: From the moment of the operation and onwards, all the changes that an object goes through are changes within the same function concepts, and so the secondary function concepts are continuous. If an object changes its colour, it remains coloured, i.e. the function of colours is preserved. Usually, the changes in its colours occur in weak continuity (WEC), which means that between its colour at t1 and its colour at t2 there will always be a middle time-point t1.5 in which the colour will also be an intermediate one. In these states of things, even if we do not see the continuity with our eyes, we complete it through extrapolation, and if not by that, at least by abduction ("the best explanation"). This way of completing the missing part is even more the case when the change reaches some point, then transcends the range of our senses and is renewed at some other point. In such cases we complete the "missing link" by the above cognitive functions. The continuity achieved by these uses of imagination will be named "reconstructed continuity."

 Admittedly, sometimes the change seems to be abrupt, i.e. as if the object passes without a gradual process from one property to another, distant from the first one, within the same cognitive function. Yet, (a) even then when we check the event in greater detail and in higher resolution (for instance, if we descend to the level of elementary particles) we discover greater continuity; (b) even when we cannot track such a continuity by the senses, we usually surmise that it was preserved in the process, if not in the event itself at least in its factors. And, even more important: If there is a change in the object, whether continuous or abrupt, it will never refer to all of its properties, since in that case we will no longer talk about that object as the same one object but as an object that has ceased to exist and another one was born in its place.

 Within this sequence of changes there is one crucial moment for every object in the sphere of thought. That is the moment in which it is fixed in our minds as *this* particular object. From then on we begin our tracking of it, the same tracking in which we learn about the continuity of some properties and the change of others. Suppose I am walking in the street and see the yellow box. From the moment I first saw it lying on the sidewalk and took it to mind, it received a new status in my thought as an object which I discriminate by a given concept, and am ready to ascribe to it additional concepts hereafter. I call this action **tagging**, and the concept by which a certain object is tagged within a given subject – the **tagging concept** of the object in the subject. Tagging is a decision, since it is obvious that I, and even more so another subject, could tag the object at stake by a different tagging concept. Generally, tagging is a non-voluntary action, since I do not consciously choose the concepts by which I partition the world, nor the objects I encounter and are captured by those concepts, but we have already seen that a decision, in the sense attached to this term in this book, is not necessarily voluntary.

 Suppose I first saw the box, with sharp awareness, on Wednesday, July 28, 2018, at 18:24 – and this time point will be called t1 – and it seemed to me as a yellow, cuboid, measuring approximately 70 x 50 x 50 cm, and it was lying on the pavement of Derekh Hahoresh Street in Jerusalem. To make the notation simpler, we will agree that the collection of these properties will be denoted by the letters A,B,C,D.

 The tagging concept of the box will thus be A∩B∩C∩D, and we can abbreviate it to E, provided that

E=A∩B∩C∩D

This does not necessarily mean that the box is truly E, since in another time point the subject might discover that it is not yellow, or not of the form of a cuboid, or does not measure anything like 70 x 50 x 50 cm, or was not at all lying at that place, and all those characterizations were the result of some mistake or other. However, the subject will not be able to alter the fact that in the time point in which he had his "acquaintance meeting" with the box, that was the way it was perceived in his/her mind, and therefore any change in the description of the object that may follow will come as an addition to that description. Namely, even when the subject discovers that the box is, for instance, not yellow, this discovery will mean that the object I perceived at t1 as A∩B∩C∩D is not truly A. And still, the sentence will be about "the object I perceived at t1 as A∩B∩C∩D." Indeed, the tagging concept of an object is its rigid designator, and is always done by a definite description.

 I will present another example. Suppose I am at a social event and someone approaches me. At this stage he is tagged as "the one of whom my truth sources a,b,c, transmitted to me that he has properties A1,A2, and A3." Now he introduces himself as Dr. John Smith. From now on he is carved in my mind as "the one of whom my truth sources a,b,c, transmitted to me that he has properties A1,A2, and A3 and he himself transmitted to me that his name is John Smith and his title is doctor." Suppose that on a different day I see his picture in the newspaper, and the report says that he introduces himself as Dr. John Smith, but in truth his name is not John Smith and he has no title of doctor. From this point and on he is "the one of whom my truth sources a,b,c, transmitted to me that … and he himself transmitted to me that … and the newspaper transmitted to me that …," etc. Now, even if I believe the newspaper and not the person himself, his self-testimony still remains a part of his concept in my mind, and the concept by which I perceived him in our "acquaintance meeting" remains the one that forms his initial tagging. Admittedly, at some stage the initial tagging might be forgotten, or a later description will be so reliable and compelling that it will altogether reject a previous unreliable one. However, such an event will actually be the liquidation of the first tagging and the generation of a new one, which merits the title **retagging**.

 As noted before, the tagging takes place in the "acquaintance meeting" of the subject with the tagged object. This is not necessarily the first time the subject perceives the object, but the first time it perceives its discriminated existence, i.e. thinks about it in some level of consciousness as an object.

 The tagging (as an active thing, on the subject's part) will be denoted by TG, namely: Sn is tagging by concept Y the object x at time-point t0. The definition will unfortunately be long and complex, since the action at stake is complex:

Sn⮭TG;xᵀ,Y,,t0 =def xᵀ⮭BRN;Sn,t0˄ Φᵀ(x⮭Y)⮭BRN;Sn,t0˄ɒZ(Sn⮭PL(x⮭Z),(t>0)) Sn⮭PL(x⮭(Y∩Z))

Namely: x started existing within Sn at time-point t0, and at the same time Sn contained the datum that x is Y, and from that time forward any additional concept that captures x within Sn will be added to concept Y, i.e. will intersect it as a new part of the full concept capturing x within Sn.

Of course, the concept of tagging itself is a part of the full concept of x, but, as we remember, in a given pool PLn this concept does not designate the intersection of all the concepts capturing x, but the intersection of all such concepts that are thought by Sn.

 (Yᵀ,,,t1)⮭PP;FK(xᵀ)

Indeed, according to the reduction rule we may eliminate the mention of time, but, as I have noted before, this omission does not mean that the sentence is true at all times but that it is true at *some* time.

The decision to tag with Yᵀ is dependent on a decision to operate Yᵀ:

S1⮭TG;Yᵀ,,xᵀ ↔ S1⮭OPDK;Yᵀ

But the decision to operate a concept is a decision:

S1⮭OPDK;YᵀS1⮭DC;Yᵀ

And therefore another decision could be made:

S1⮭DC;Yᵀ◊ S1⮭OPDK;Zᵀ (Yᵀ≠Zᵀ)

And in our case, a decision to operate another concept could be made:

◊ S1⮭OPDK;Zᵀ (Yᵀ≠Zᵀ)

The decision to operate a concept is a decision to determine boundaries. It is not a decision to create boundaries, since the boundaries exist by virtue of the fact that concepts capture objects, and capturing, in the sphere of thought just as in the sphere of reality, does not require an act of decision or any other act on the part of the subject. The concept yellow in thought captures the box not because we decided to do so but because the box is yellow. Yet, the decision to tag an object is the decision to give some of its boundaries a stronger status in the subject. That status will be named "fixing" not because these boundaries cannot change any more in the future, but because those boundaries are the basis for any such change. They are fixed in the sense that from now on any predicate attributed to x will be attributed to the same x that was tagged in the specific boundaries in which it was tagged.

 The act of fixing can be seen both in the issue of continuity through time and the more complex issue of continuity through other changes (which includes the continuity through time, but adds more to it).

 Continuity of identity through the change of time: Suppose the box is standing in the same place without any change in its properties. In spite of the seeming stability, it goes through a change each and every moment. If at one time point it is aged t1, a second later it is aged t2, and them after another one t3, and so on. "Aged t1,""aged t2,, "aged t3" etc. – are all properties, and therefore from a strict metaphysical perspective we can say that at each moment the object is captured by a different full concept, and consequently we can talk about that object as a different object at each time point. In order to create the continuous identity of the object we must therefore operate the functions of preservation and unification: preservation in memory of the data of previous perceptions and unification of memory data with one another and with current sense data.

 Continuity of identity through the change of properties: Sometimes in addition to the passage of time, properties of the object also change, such as its place, colour, etc. Here, too, the same functions mentioned above are operated, yet here we need a test for the creation of identity through the changes. If from t1 to t2 only the age of the object changes, it is easy to identify it, since all its other properties remain the same. But what if other properties also change? If, for instance, at t1 its properties are A∩B∩C∩D and at t2 they are F∩B∩C∩D, they should supposedly be considered as two different objects, so why should the subject operate the combinative faculty on these two objects more than on two other objects that it perceives as alien to one another?

 We seem to have two answers: First, is that the subject will operate it when one or a few properties change, but a considerable part of the properties remain unchanged. How many of them? What percentage? That is a matter of decision. Second, it will operate it only if between A and F there is a weak continuity (and even more so if there is an ordinary continuity) whether the latter is actually perceived or is being reconstructed. Apparently in most cases the subject will decide to unite the two objects of the change into one when both the options are satisfied, but, since it is a matter of decision, it might suffice with one.

 The continuity of an object is guaranteed by the fact that the change takes place in a continuous time. Time is a continuum, as defined above ( ###, where we defined: x⮭SCN ≡def ∀y y⮭PP;x∃z(z≠y) z⮭PP;x˄z⮭CPBD;y) and therefore the passage from t1 to t2 is not interrupted or skipped but moves over a line of time units that constitute a single continuum.

 As for the properties, i.e. concepts, any other change in the object is additional to the change of time, not substitutive to it. In Brentano's terms we can talk about it as a secondary continuum (i.e. one that comes above another continuum), in contrast to the continuum of time, which is a primary continuum. Such, for instance, are the changes of place and colour. If, for example, the box moves at a changing speed on the pavement and at some stage even flies a little bit in the wind, or if it stands in its place and gradually changes its colour from middle yellow to dark yellow (due to the effect of the sun), then all these changes occur continuously: The places in which the box passes in space are contiguous, and the hues of yellow to which it changes are contiguous on the colour spectrum, and consequently upon the map of concepts, as defined above.

As I said above, when we do not perceive the continuity with our senses we tend to complete it by other functions, presuming that such continuity exists. Suppose I am standing in the street and watching the yellow box. If it is flying in the wind, for instance, it might hide behind some other object for a while, and I can't see all its changes of place. Even if it remained in one place and only changed its colours, it is quite likely that for a few moments I didn't watch it and didn't see all the continuum of its changes of colour. In fact, such intervals must exist in real life, since we all blink from time to time. Since we all presume the continuity of its existence and the continuity of its change (or changelessness) in accordance with the existing graph since we are capable of completing the missing links by using our imagination and thus assume they actually exist. In this action it is not only the imagination that does the work but also the intellect, since imagination can complete the missing link in any way whatsoever. It might decide, for instance, that while I was distracted from the box it jumped to the moon and then returned to Earth, or ceased to exist and then returned to existence. The intellect, in contrast, completes it in a way that makes it compatible with the laws of such motions, both the regularity of the laws of nature applying to the box and the regularity of the graph of change (or changelessness), in a more plausible way. (Admittedly, I wrote above, at ###, that from a metaphysical point of view any way of completing the graph is just as acceptable as the other, yet our imagination, like nature itself, usually opts for the more parsimonious one).

 And this brings us to an answer to the question of the cat and the tail, raised by some critics of extensional mereology. These scholars contend that if the cat is the sum of all its parts, as postulated by the principle of extensionality in mereology, then the cat whose tail was cut off is no longer the same cat, and that is absurd. Yet, from the moment we began following the cat, i.e. from the moment we tagged it, it remains the same object as long as the changes it goes through happen as parts of a continuum. Even the cutting of its tail, which appears as a sharp change, is in fact a part of a continuum, even if very quick one, which might be observed if we tracked it in higher resolution. The tailless cat is not a possessor of the same properties as the healthy one, and in some respects one might view it as an object consisting of two separate parts (by the rule of free union, remembering that the using or non-using of that rule is a matter of decision). Yet, even if we decide that there is a cat and there is a part that was taken away from it, the continuity of the change it went through leaves it in the status of a single object even after a relatively drastic change, just as we recognized it as one and the same object after it changed its place or its countenance.

 The only question that might still be asked here is: Which of the two parts of the cat is "the cat itself" and which is the one that "was taken away from it"? From a logical perspective one can see the tail as "the cat itself" and the rest of the body as "the part taken away from it" just as much as the opposite way. Much like a human group that is split and each party wants to continue to bear the name and reputation of the original body, viewing itself as its genuine continuation, so, by analogy, is the case with regards to the parts of the cat. This equality contradicts our common sense. As in Mirabeau's famous saying on Prussia ("not a state which possesses an army, but an army which possesses a state"), we tend to say that the cat's body has a tail and not that the cat's tail has a body. One might contend that we follow the quantitative majority, but that is also false. If we imagine a crocodile or a dinosaur whose tail is longer and heavier than the rest of its body, we will say, in the event that that tail is cut off, that the dinosaur (whose main part is the body) lost its tail, not that the dinosaur (whose main part is the tail) lost its body. Where an animal is concerned it is most likely that the part in which the vital organs (brain, heart) are located will be considered as it main part. One way or another, this discussion takes us back to the age-old question of essence. This question was touched upon above, with regard to the essential properties, and we will not dwell on it further.

### Retagging

Tagging is an action in which the subject makes the "acquaintance" with an object, i.e. one in which it begins to think about it as a discriminate object and begins following the changes it goes through (be they only changes of time or changes of both time and properties) from that moment and on (sometimes limitlessly, sometimes up to a certain time point). And here it should be emphasized:

1. Basically, tagging exists when a concept captures an object, but, as I stated above, an object is not necessarily meant here in the rough commonsensical sense, but rather in the logical one. "My pair of shoes," "the Smiths," "the residents of Jerusalem" and "the human race" – each of these can be taken as an object, once we treat them as wholes. The way we treat them, as multiple objects or a single one, depends on decision.
2. Since tagging takes place via a cognitive act, it always has a source, and thus the concept of tagging is always a kind of "definite description" which also includes the source. Suppose a running man enters my field of vision; I tag him not as "the man who ran at Jones Street at time-point t1" but rather as "the object I saw as a man running in Jones Street at time-point t1," which should be better phrased n the terms of Source Theory as "the object that my sense of sight transmitted to me as a man running in Jones Street at time-point t1."

Indeed, the predicates which tag an object will be, at least at the outset, predicates denoting transmission (in the sense attached to it in Source Theory; Brown, 2017): "That which my senses perceived as such and such," "That which my parents told me of as such and such," "That which the teacher described as such and such," and above all – "That which the community (in question) refers to as such and such." The latter example is true not only for language, but also for the ideas of the sphere of thought. This stands out particularly in the case of ideas created normatively. "Bachelor" is "a man who hasn't got married" not just as a linguistic convention but as a concept created by a norm (family law); when I talk about "Jerusalem" or "France" I talk about a territorial unit whose boundaries were determined by a norm, even if I don't know its historical sources but only its social, political and legal expressions in the present. These boundaries are "Fiat boundaries," in the terminology of Smith and Varzi (2000).

 Once I have tagged an object, I keep following it with my truth sources. In the above example of the man running in the street, it is likely that I will continue to follow him running and receive data on the event from my sense of sight, and in that case my concept of it will be "the object which my sense of sight transmitted to me as a man running in Smith street in that direction between t1-t8". But if someone calls in the street "That's a thief!" my concept of him will be "the object which my sense of sight transmitted to me as a man running in Smith street in that direction between t1-t8 and my sense of hearing transmitted to me that a passerby transmitted that he was a thief"; and so on and so forth.

 Often, however, the person does not remember what was his acquaintance meeting with the object, and therefore not its tagging concept either. Let us take the above incident, continued by a variation similar to the one I presented in the previous subchapter: Maybe in the day following my seeing a man running in the street I read in the newspaper that the running man was an Olympic runner. From now on my concept of him will be something like: "the object which my sense of sight transmitted to me as a man running in Smith street in that direction between t1-t8 and my sense of hearing transmitted to me that a passerby transmitted that he was a thief and my sense of sight transmitted to me that the newspaper transmitted that he was an Olympic runner." It is likely that I will believe the newspaper more than the suspicious passerby and the more I follow news reports about the runner, his Olympic achievements and the rest of his career, the more I will come close to the condition in which his tagging will be changed to "the object that a variety of (undefined) reliable sources transmitted that he is an Olympic runner that did this and that." At that stage the first tagging will be pushed aside, and most probably will be forgotten. In the course of life it might often happen, therefore, that an object is retagged.

A new and more reliable datum does not necessarily entail retagging. Let us consider the following example: None of us remembers our acquaintance meeting with our parents or the street where we grew up, but it is obvious that in the child's mind a certain object is tagged as dad because some reliable sources transmitted to him that this is his father and another object is tagged as mum because reliable sources transmitted to him that she is his mother. If, suppose, when this child grows up and discovers an adoption certificate telling that he is in fact he was adopted by those parents, or learns it by a DNA test, it is most likely that he will reject the reliable (yet indefinite) sources on which he relied for the even more reliable sources of official documentation and science, since that is how his hierarchy of sources is built. Still, it is quite plausible that the first tagging will not be wiped out, and the child will tag his parents as "the objects that reliable sources transmitted to me that they are my parents and my sense of sight transmitted to me that the official document / DNA test transmitted that they are not my biological parents." Namely, the old tagging will be preserved, and the new datum will be added to it. That is so because the new discovery, more reliable as it may be, will have difficulty pushing out an older well-rooted belief.

Even though none of the tagging, neither the first nor the second, is done by choice, both are decisions. That is so since both of them are made by human action that could be different. It should be emphasized that the action of tagging is subjective, technical, coincidental, relative and changing, and therefore must not affect the truth value of propositions.

### A subtotal: The foundational concepts of the sphere of thought

In this chapter we have discussed the sphere of thought. This chapter stands on the shoulders of the previous chapter, in which we discussed the forms of being in general. Before we proceed, let us summarize the list of foundational concepts emerging from the primitive terms of the discussion in these two chapters.

**The foundational concepts of the forms of being:**

FB – having a form of being.

R – real.

J – noumenal (in itself).

T – thought.

V – verbal.

PL/CT – pool/content.

**The foundational concepts of the sphere of thought:**

TI – time.

DMn – dimension.

TM – transmits (a datum).

KF1 – a datum of thought created ex nihilo.

KF2 - a datum of thought created out of an existing datum.

KF3 - a datum of thought that has been preserved.

KF4 - a datum of thought that has been lost (forgotten, in most cases).

KF1.1 – a visual datum.

KF1.2 – an auditory datum.

KF1.3 – a datum of taste.

KF1.4 – a datum of smell.

KF1.5 – a tactile datum.

KF1.6 – a datum of introspection.

DD/GN – decided by/given

As we will see, this list will serve us in all the other spheres. We will now turn to the sphere of language, which is closely linked to the sphere of thought.

## The sphere of Language: Basic characteristics

### Language as a dependent form of being

Language partitions the world. A Parmenidean unpartitioned world is a world that no language can describe, and therefore, as Wittgenstein recommended in the case of a similar issue, thereof one must be silent. Yet, in a world in the sense we discuss here, i.e. a world of ordered plurality, language presents an order of its own, not completely detached from, yet not fully overlapping, that of thought.

 Language, as we know, is a collection of sensory objects perceived by the senses of hearing or sight (maybe even touch, as in Braille writing). Beyond the fact that they are data of thought by virtue of their being sense data, they are created in the thought of the transmitter and activate the thought of the receiver. Since these are objects that exist in various levels of the sphere of thought, we may ask why they qualify as a separate form of being rather than being considered as a derivative form of thought.

 The answer to this is that the pools of language are indeed products of the pools of thought, but they sometimes include orders that are different from those of the pools of thought. The latter point justifies seeing language and thought as separate forms of being. On the one hand, the pools of thought (the subjects) contain thought objects (ideas) that are not expressible in words; and on the other hand the pools of language sometimes contain verbal objects (language expressions) that thought pools do not and cannot contain. Let us take for example the verbal object "four-sided triangle." This object exists in language (I have just written it here in words!), while the pools of thought do not and cannot contain an intersection of these two concepts. This point shows that language may offer a partition of the world that is different from that of thought, and that, in turn, corroborates the assumption that it constitutes a different form of being.

 Yet, facing the existence of such expressions we meet two interrelated problems, one from a metaphysical perspective, the other from a linguistic one. From the metaphysical perspective, I have already written above that any secondary order of the world comes in addition and subject to the first order, which is the first metaphysic stated in the first section. But this first metaphysic contains the law of non-contradiction, and so an expression like "four-sided triangle" cannot exist in it. From the linguistic perspective, some will claim that the expression "four-sided triangle" is meaningless, and therefore is not a part of language. The solution for this problem is simple, and I will return to it after I have expounded some required terms.

 I said there are objects which exist in the sphere of language and do not exist in the spheres of thought and reality. This assertion evokes an interesting issue. In the Jewish tradition, probably following Greek or Arab sources, it is accepted to distinguish three levels of human activity: Thought, speech and deed (see for instance: *Zohar* vol. 3, p. 83a; ibid, p. 255b). Usually, thought is considered as the farthest from reality, speech as closer, and deed as part of reality proper. Now, according to our findings, that is not the case. The sphere of language – that is, speech – is the "wildest" one, and the sphere of thought is more "restrained" than it. "Paper endures all," goes the folk saying, while thought (as Hume has already taught us) does not endure at all. "Four-sided triangle" is something one can say and write, but not think. In this respect, language is the sphere farthest from reality. Nevertheless, the intent of the traditional thesis is clear, and even justifiable in another respect: Thought remains entirely within the subject, i.e. in a form of being completely different from reality, while language is made of representative objects which exist in reality: Voices (utterances), sights (letters),etc.

 If in some instances the signs of language do not necessarily reflect a thought (due to the fact that the thought cannot contain them), do we need to restrict our previous statement, that the signs of language are created in the thought of the subject? The answer is negative. First, because the condition in which there is such an impediment is created in connecting two contradictory words, but even in such a condition each separate word does reflect a thought. Second, and mainly, because even if the signs do not reflect thoughts at all, it cannot change the fact that a thinking subject is the one that created them.

 It is, however, noteworthy that, due to the reasons I mentioned above, language is a form of being that is weaker than the two other accessible ones. Its entire existence is given to it by virtue of the combination of sense data and signification data. Language is thus entirely dependent on the sphere of thought, a dependence we do not find between other pairs of spheres. We may conclude, then, that it is a dependent and therefore weaker than other forms of being, yet a form of being that is irreducible to another, and therefore should nevertheless be counted as a form of being of its own.

### Language as a subsystem

Language is a subsystem of a cultural system. When I say subsystem I mean that the sources of the cultural system are those that "authorize" other sources, and the data transmitted by the latter are taken as distinct from the collection of the data of the greater system. In our issue, the sources of the cultural system "authorize" the community of the speakers of the language to determine the relation between the sense datum and the thought it is required to evoke. Even if a language has a recognized institution for deciding which expressions are legitimate in that language and which are not, it is obvious that it does not determine the boundaries of the language, and furthermore, it draws its authority from the consent of the community of speakers.

As said above, any word or expression in language is double sided: On the one hand it is a sensual object (graphic, auditory or tactile), and on the other it is a signification unit. We will call these two sides the sensual and the significative, respectively. The sensual is an object perceived as having a form, a sound or a texture. The significative side is the thought evoked by the sensual object in the subjects using the language (and all the attempts of modern philosophers to play down the role of thought in their theories of significance is just sophistry). This distinction between the two sides stands out when we try to apply mereology to language. Let us take the expressions "the black horse" and "the tale of the black horse." Which one is the part and which is the whole? From the perspective of the sensual side, the latter is the part of the former, but from the perspective of the significative side, the opposite is true. We will have to remember this in order to avoid confusion.

 We talk, therefore, about objects of the type of signs. These differ from other sense data due to the fact that the subject that created or used them transmits its thought by them to another subject, thus creating a similar thought within the latter. (As semioticians emphasize, a "sign" does not have to be made by a subject, since smoke is also a sign of a fire and rain is a sign of clouds; but here we will discuss only signs transmitted by subjects. Icouldrestrict the discussion to purposeful transmissions, but such an additional requirement would entangle us with undesired complexities, and I will therefore desist.) For several generations this description of language is considered outdated and naïve, but in my opinion there are none better or more correct.

 Every language has it semantics and syntax. (Today we add pragmatics as well, but it is irrelevant to our current concern.) Syntax discusses the rules by which one can connect given words in a way that will create new significantion units, while semantics deals with the ability to attach existence to the objects of these units. In the terms of Brentano's thesis, syntax postulates rules for creating an intersection concept out of given concepts (including concepts of individua, which are uniextensional intersection concepts), while semantics deals with the ability to attribute existence (extension) to them.

### Language as pool

Language as a universal phenomenon is a form of being and the entirety of the meaningful words in the world is the ontological sphere of language. But here, too, most of the action takes place in the pools. The pools of language are the particular languages: Hebrew, Greek, Latin, English, French etc. are all pools. As said above, the concept of language will be denoted by L, the particular languages will be denoted by L accompanied by indexes, and the variable of language will be denoted by Ln. The community of the speakers of Ln will be denoted by h(Ln).

 What determines the boundaries of a particular language as a pool? Can every dialect of a language be considered as a pool of its own? Uriel Weinreich argued that the difference between a language and a dialect is political: "A language is a dialect with an army and a navy." (Weinreich 1945, p. 13). Whether we agree or not, it seems that the boundaries in this issue are not sharp. In principle, it seems that the boundaries of language from the perspective of its status as a pool are determined by its semantics and syntax, in contrast to its pragmatics and phonetics. But here too there is room for sorites-type questions: A difference of how many words makes a separate language. What degree of difference in rules of syntax makes us recognize something as a new language. These are interesting questions, but we have no tools to decide on the answers. We can proceed, however, without deciding them, since the discussion below will focus on the principled level, and on that level we can take the concept of language, even that of a particular language, in some ideal sense.

 However, even if a language is "a small world" and therefore qualifies as a pool, there might be other "small worlds," even smaller than it, that deserve a similar status. I mean textual units of rich descriptive texture. We often talk about "the world of the Bible" or "the world of Shakespeare." Beyond the fact that it is a nice metaphor, this also implies that a text might serve as a subsystem, and therefore as some limited representation of what is. If I ask someone "Is it true that Macbeth killed Macduff"? – obviously the question does not refer to the real world but to Shakespeare's oeuvre. Consequently, the answer will also refer to that oeuvre. In fantasy books, for instance, not only are the characters and their adventures different from those of reality, but even the laws of nature are dissimilar thereto. Thus they come close to creating a comprehensive worldview that is an alternative to that of reality. Allegedly we could see them as independent pools. Yet, it seems that such an approach will not only create an embarrassing inflation of pools, but will also miss the independent nature of language as contrasted to the dependent character of texts. Literary, theatric, cinematic or any similar types of work are all created through a language that is spoken and written by thinking subjects (pools of the sphere of thought) and are perceived by such subjects. Beyond language they do not have anything of their own. The analogy we can give for it in the sphere of thought is a dream. Just as a dream is nothing but an event within a pool of thought (a subject) and no-one would even consider thinking of it as a pool of its own, the same is true for textual works. Texts and other similar works, and even dreams, should therefore be considered as smaller units than pools, which we may call "underpools."

 What is a text? For our concerns we will not require that a text necessarily be a "small world," since we would stumble into a problematic sorites with regard to the definition of such an object. A text will be any collection of words in a certain language (contents of a linguistic pool) which transmit something. From that perspective, the size and quality of the text do not matter, nor does the medium by which it is transmitted. The sentence my wife told me this morning is a text, and the same is true for the sum of all the English language books in the Library of Congress. The fact that among the books of that library we will find both the Bible (in English translations) and Darwin's *Origin of the Species* does not prevent us from seeing both of them, if we will, as parts of one text. And just as a one-page text can contain a contradiction and that fact does not negate its status as text, so also is a text that is combined of two books or more. The principle of free union applies here, too, even if its application seems somewhat unfitting. For some matters even a question, a blessing, a wish, an order or other non-assertory utterances are texts, but in this context they will not interest us. Indeed, it might be proper to determine the boundaries of a text, and that topic draws the attention of some of the philosophers of our day, but for our discussion it is not required, and anything we conclude on that matter will probably be arbitrary and controversial.

 Let us denote (the concept of) text by the letters TXT and define:

x⮭TXT = def x⮭CT;Ln˄əΦᵛ|x⮭TM;Φᵛ

But since CT;Ln is simply a linguistic expression, that is marked by a ᵛ above any object, we can abridge the definition, to be as follows:

x⮭TXT = def əΦᵛ|xᵛ⮭TM;Φᵛ

But a text is always a source that is mediated through the senses, the intellect and more, which means that it is always an indirect source. Therefore, the form α⮭TM;Φ will never stand alone, but the feature will always be:

α⮭TM:(β⮭TM:Φ)

Thanks to that we are able to give it an abbreviated form:

α⮭TM:Φ ≡def əβ| β⮭TM:(α⮭TM:Φ)

This is a convention of formulation, nothing more. It means that we may see the texts as transmitted by their initial source, the "author" of the text, and ignore the senses, the intellect and the other basic cognitive tools, particularly because this mediation always exists.

### The axiom of linguistic existence

Now we can set the axiom of linguistic existence:

α⮭TM:əxəxᵛ

Any sentence in a text that asserts that some object exists in the sphere of thought is necessarily and inherently true. That is because that assertion itself, in which the object is mentioned, is made by language, and therefore the very act of assertion gives it existence in some sort of performative utterance. However, any proposition, being usually built as an attribution of predicate to object (x⮭Y), might be reduced to existential proposition (by Brentano's thesis), and thus we can state the axiom of linguistic existence in the following way as well:

 (α⮭TM:x⮭Y)əxᵛ˄əYᵛ

But since our business here is only assertions made in language, namely in texts, we may actually say that an assertion of existence is in fact an assertion stating a parallel in the sphere of reality. The assertion that x exists (in reality) is actually the assertion that the word x has a parallel in the sphere of reality. The same is true for the sphere of thought.

əxᴿ ≡ əx|x⮭PR;xᵛ˄x⮭R

But according to Brentano's thesis any sentence is reducible to an existential sentence; therefore every sentence can be rephrased as a sentence stating parallelism:

 (x⮭y)ᴿ ≡ əᴿz|zᴿ⮭PR;(x∩y)ᵛ

By this all the questions on Pegasus and other imaginary creatures, on which philosophers of our time have toiled, all fall down. Pegasus exists in language, by its very appearance on paper (or on the speaker's lips). Any proposition attributing some property to Pegasus is in fact a proposition stating that Pegasus has a parallel in the sphere of reality. It is a proposition about the word Pegasus, not about the real Pegasus. Therefore the mere use of the name of an object as a subject in a sentence does not yet premise its existence in reality.

Here we should pay a debt of gratitude to Meinong, but also add a critical comment on him. Meinong was forced to build sophisticated metaphysical constructions on existence, as contrasted to subsistence, in order to express the difference between existence in reality as contrasted to existence in a sentence. He never clarified the nature of subsistence and so it remained a vague form of being, not required according to Ockham's razor. The above rephrasing leads us to another, simpler and clearer way.

Do concepts containing a contradiction, such as "four-sided triangle" or "a dog that is not an animal" exist in the sphere of language? On the one hand, the answer is obviously positive: In the sphere of language a "four-sided triangle" exists since I have just written it, i.e. given it a linguistic existence. On the other hand, the answer is negative, since in the first section we stated that the rules of logic enumerated there are to apply to any world whatsoever, i.e. in any form of being. The answer to this dilemma is not very difficult: Here, too, we must discriminate between the sensual and the significational aspects of words. The axiom of linguistic existence refers only to the sensual aspect, and at the sensual level there is no contradiction between the existence of the letters f.o.u.r s.i.d.e.d and the word t.r.i.a.n.g.l.e. These and those can dwell peacefully on the sheet. The contradiction is at the level of the signification units. The significational aspect, as noted above, is the one that attaches a word to a thought. Here, in thought, there is indeed no existence of a four-sided triangle.

This latter point leads us to a further elucidation of the role of syntax. I said above that the role of syntax is to offer rules for the combining of words into new signification units. But now the question arises, at which level is that combination made? If at the sensual level, we have seen that such rules do not exist, since letters and words function just as other objects in space; if at the significational level, a "four-sided triangle" is supposed to be meaningless, so why do we consider it as syntactically correct? The answer is that syntax works with existing significational units that have already been recognized as such separately, and sets formal (i.e. "external") conditions for combining new signification units, yet those formal conditions are necessary but not sufficient conditions. An intersection concept will be a new signification unit if we add to the formal conditions some material ("internal") conditions referring to the relations between the concepts involved. These constitute a new stage, beyond the role of syntax. (This insight may help us understand and criticize some of the arguments that have nourished the critique of language in modern philosophy from the later Wittgenstein and onward, which do not all necessarily lead to the conclusions drawn by the philosophers; this is not the place to dwell on that). Indeed the task of language is to evoke ideas in thought, but in the case described above it cannot fulfil that task, since we cannot think of a four-sided triangle, and therefore in these states of affairs the words return to function as mere sensual objects and not as signification units.

### Between different languages: differences in the partitions of the world

This book does not walk in the paths of critical theories but is based on an analytic method, and therefore I will not elaborate on matters heavily discussed in the works of the structuralists and their followers. However, some insights that came up in the realm of structuralism, in particular its classical version, may elucidate for us in what way language serves as a pool. I will only say here some brief words as a starting point for a discussion that may be continued by others.

 As I said, when we speak about words in language we do not mean only their auditory, graphic or tactile forms, but also their function as signification units. As such, they have boundaries, and furthermore, they fixate these boundaries in our minds. This applies mainly to words of concepts, which make our partition of the world, but to lesser extent they also apply to names of individua.

 Concepts: We all know the nice (even if not true) stories about exotic languages in which the partition of the world is different. According to an urban legend, the Eskimos have dozens of words to denote various types of snow, but do not have a word for snow as a genus. According to some other legend, the Bedouins have dozens of words to denote various types of camels, but do not have a word for camel as a genus. In all the languages there are numerous words for various colours, but almost no words to denote different odours, and, as I noted above, they are often characterized by the use of tropes ("the smell of rose" or "the smell of a rotten fruit").

 Individua: Are proper names parts of language? On the one hand, they function like any other word in language, i.e. transmitted by auditory or graphic signs representing objects in thought and sometimes pretending to represent objects in reality. On the other hand, they presumably do not belong to any particular language, since a person's name is his/her name in any language. In truth, however, this should not exclude them from being parts of language: First, because in fact there are names that do change from one language to another. The names of Abraham, Moses, Julius Caesar, Jesus, Charles the Great, Peter the Great, Frederick the Great and thousands of other people are not identical in all languages. The Temple Mount, The Great Wall of China and the Eiffel Tower also have different names in different languages. Furthermore, language is not just a speech medium but also a writing medium, and when different languages have different alphabets, the proper name is necessarily written in a different way in each language, and is consequently different in each language. But even if it were different, and all the names were pronounced and written exactly the same way in each language, it would not change the fact that names are parts of language, just as universal words do cease to be parts of particular languages. Languages do not claim exclusivity over words. And still: Do the names of identical objects in different languages have different boundaries for those objects? Are Abraham, Avraham and Ibrahim not the same object, as far as its boundaries are concerned? Words can be universal and the same is true for proper names, but does language actually change the denotation of a proper name? This brings us to an interesting entanglement, which I will only be able to touch upon.

 We can supposedly suggest the following argument: When an English speaker says "Abraham" he refers to the man who bound Isaac, since that is the way Abraham is described in the Bible, which constitutes a part of his or her cultural system. In contrast, when an Arabic-speaker says "Ibrahim" he refers to the man who bound Ishmael, since that is the way he is described by the commentators of the Koran, which constitutes a part of his or her cultural system. If so, they do not refer to the same person. Is it indeed so? In my opinion it is not, and not only because there are Arabic-speaking Christians and English-speaking Muslims. As I noted above, the tagging of an object is done by a source. Therefore, even for the most zealous Christian Abraham is not just "the man who bound Isaac," but "the man of whom the Bible says he bound Isaac," and even for the most zealous Muslim Ibrahim is not just "the man who bound Ishmael" but "the man of whom the commentators of the Koran say he bound Ishmael." At some stage of their lives, the Christian and the Muslim will probably learn about each other's religion, and then will also define Abraham/Ibrahim in the same way, and if they don't learn, they could in principle agree to define him in the same way: "The man of whom the Bible says he bound Isaac and the commentators of the Koran say he bound Ishmael." Even if the first tagging of a concept is made by the concepts that capture it according to some sources, its description becomes richer through further concepts that capture it according to other sources; and just as much as this is so in the sphere of thought, so it is in the sphere of language.

 We see, then, that naming objects in language does not necessarily depend on the cultural system in which that language is particularly active. Does this weaken my earlier argument, that language is a subsystem of a cultural system? Strange as it may seem the answer is no. That is because the terms "cultural system" and "subsystem" should be dealt not in their simple sense but in their accurate epistemological sense. As I have said, a subsystem is created when the sources of a given system "authorize" other sources to transmit data with regard to a given type of objects. In this accurate sense, all the languages in the world are subsystems of all the cultural systems in the world. That is because any cultural system, at least from those we can reckon to be existing in our world, authorizes the community of speakers of any language to transmit the data of that language – semantics, syntax and pragmatics alike. The Muslim cultural system, for example, does not only authorize the speakers of Arabic to determine the words of Arabic and their denotation, but also authorizes (in the formal epistemological sense) the speakers of any language Ln to determine the words of Ln and their denotations, since in that system there is no other test for speaking Ln.

 The boundaries of concepts and individua, as determined by language, are closely related to their boundaries in the sphere of thought. Since words transmit thoughts, the boundaries of the thought objects, when language does its job properly, create in the mind of the hearer (or the reader) ideas of objects with boundaries that are very similar to those boundaries. However, as the structuralists and their successors taught us, in so doing language acts not only as reflecting consciousness but also as shaping it. But it should be remembered that since boundaries of concepts are determined by the community of the speakers of the language at stake, i.e. by a large number of subjects, it is clear that when coming to express perceived objects not all of them have the very same thought objects in mind. Since the number of the perceived objects is also large, it is obvious that they cannot evoke exactly the same thought objects in all the hearers. For this matter the following test will apply (here as well as in the sphere of reality, as described below). The boundaries of object (i.e. word) x in language Ln are the boundaries common to all the subjects who are members of the community of speakers of Ln, i.e. h(Ln). It is clear, however, that not just any inarticulate member of the community of speakers will be able to alter the boundaries of its words; there must be some indeterminable percentage of the speakers of the language that can affect the boundaries of its words. A lower percentage of the speakers who have other boundaries for a word will not be able to alter its boundaries, and will be considered as speaking the language incorrectly. If this happens with many words, they might be excluded from the community of speakers. If that percentage increases, however, they might come to determine the boundaries of words.

### A subtotal: The foundational concepts of the sphere of language

Since language is a form of being that is dependent on thought, it does not have foundational concepts of its own. Its foundational concepts are those of the sphere of thought. The only concept related uniquely to the sphere of language is the concept of the linguistic itself, V. This concept, which was introduced above as part of the display of the various concepts of forms of being, was taken as a primitive due to the independent aspect of the linguistic form of being, which does not enable its full reduction to the thought form of being.

## The sphere of reality: basic characteristics

### The forms of being not dependent on the ego: The noumenal and the real

So far we have discussed the world as it is perceived by us in a "rough" manner in thought and its representation in language, which in turn is also dependent on thought. But is there a real world outside the thinking subject? In truth the question is divided into two: First, is there a world outside of me? And second, in the case that the answer is positive, does that world include only other subjects or there is some object that is not dependent on subjects ("matter")? As we know, a person who argues for the existence of such a world is a realist (there have been many of that type). A person who contends that there is no world outside his own soul is a solipsist (and there were not any such in the history of philosophy), and a person who believes that there is no world outside the many souls that exist is called an idealist (and there were numerous philosophers of this type, too, especially until the 19th century).

 The argument of the solipsist was born out of Descartes' famous challenge (it is not coincidental that we do not find the term solipsism or any equivalent term in any period prior to Descartes). Descartes believed he succeeded in answering it, but many have thought that the question was stronger than the answer. Berkeley, arguably the most brilliant and good tempered of modern philosophers, thought he succeeded in proving the existence of "other minds" (Berkeley 1710, I, 145), and his proof became a cornerstone in philosophy in spite of its patent weaknesses. His argument wanted to prove that there are beings outside the self, but those are subjects and not "matter." Kant said that philosophy's failure to prove the existence of the outside world is a "scandal to philosophy and to human reason in general" (Kant 1929, ed. B, preface p. xl, fn. a), but since he too did not succeed in that, he remained with idealism. But if Berkeley's proof of the existence of "other minds" is unconvincing, having found no better one we remain not with idealism but with solipsism. This option has always been conceived as intolerable. Schopenhauer said of solipsism ("theoretical egoism") that "it can never be refuted by proofs," but "as a serious conviction … it could be found only in a madhouse; as such it would then need not so much a refutation as a cure" (Schopenhauer, 1969, Book II, 19, p. 104).

 Yet, paradoxically, the solipsist thesis might help prove the existence of an external world. As I noted above, even the solipsist assumes the existence of an external object not dependent on the subject. That being is the thinking ego itself. But from the moment we acknowledge the existence of one such object, we have acknowledged an ontological entirety not dependent on a subject, ontological entirety being the sum of all objects of the same form of being. The disagreement between the (non-extant) solipsist and his rivals is therefore not on the question of whether there is a world not dependent on the thinking ego, but only the "population" of that world. The solipsist "populates" it with one object, while his rivals, of all schools, wish to "populate" it with greater density. In fact, even this characterization is not accurate, since according to the principle of free partition we may partition the thinking ego as well, and then will find more than one ego; but to that the solipsist will claim that only the self-conscious ego exists in reality, and there is only one such ego.

 Generally speaking, analytic philosophy gave up the formal proof of an external world and left it, following the conventional scientific hypotheses, as a plausible conjecture or as a notion of our "robust sense of reality" (In the words of Russell, 1919, pp. 169-170, and the similar argument in Moore, 1925). An exception to this rule is Frege, who provided an interesting quasi-proof (Frege 1977, pp. 18-23), which did not attract the attention it deserves. Below I will present a plausible proof that follows, to a great degree, in the footsteps of Frege but also leans on the argumentation of this book. I cannot declare for sure that it will remain unchallenged, but as far as I could follow it seemed to me correct. Either way, in the following lines I will assume the existence of an external world as a tentative working hypothesis, until I reach the full proof of that hypothesis in the next chapter. But before we examine the answer to the question of the existence of an external world we have to understand what we actually mean by such a world.

 Before anything else, we must emphasize that reality is indefinable. This is because reality is the most basic existence. The very word "exist" denotes primarily real existence and only then other forms of being. Allegedly, we could define the real as "existing outside the subject," but this is actually a negative definition, while real existence obviously has a positive one. The pseudo-definitions I will present below are not definitions, as they do not present the essence of reality but merely tests for its identification. We will thus continue to take the concept of real as a primitive, i.e. as a foundational concept.

 How, then, will we characterize reality? The method for the elucidation of this issue was provided to us by Berkeley. Several times he reiterates that before we assert that something exists we have to examine what is the way we use the word "exist" in language. He contends that any use of this word denotes something perceiving, perceived or perceivable (Berkeley 1710, I, 3). Embracing this method, the Irish bishop preceded analytical philosophy by almost two hundred years. I will also employ this method, even if to reach conclusions far different from Berkeley's. As we have seen above, the word "exist" is not limited to sense data. In any world whatsoever, even one that does not contain sense qualities, there are concepts, and in our world the concepts are not perceived by the senses (even if individua that they capture are perceivable). Furthermore, even among the individua there are some that are not perceived by the senses, and we learn about their existence mainly through their effects, such as microscopic creatures and elementary particles. Yet, even if our conclusion is different, Berkeley's basic method is correct: We have to clarify what we actually *mean* by using our various concepts of existence, and in the current context, that of being real.

 If we analyze our speech about reality properly we will find that we use the word "real" in two different meanings. When realist philosophers say of an object that it is real they mean that its existence is not dependent on the subject. When idealist philosophers say it, they usually mean that it satisfies the requirement of coherence (similar, though not identical, to the one used today in epistemological theories of truth), i.e. that it is integrated properly (without contradiction) into a collection of objects perceived by the same subject. Some will add the requirement of inter-subjectivity, postulating that the object at stake be perceived by other subjects as well. These subjects, however, are also parts of the ego's picture of the world, and therefore this requirement must be added to the former: within my coherent world, the existence of other subjects will be determined, and these will perceive the object at stake (and I will know it through the use of language). But this formulation is not sufficient, either, since the other subjects perceive many objects, both real and unreal. We must postulate, therefore, that the other subjects, too, perceive the object at stake as part of a coherent order (the spiral structure of the postulations might seem circular, but is not such).

 This becomes a little more complex since even realist philosophers such as Locke accept coherence as a test for reality, and we could imagine such philosophers who would accept inter-subjectivity as well for the same purpose. However, it should be pointed out that what constitutes for the idealists an essential property of the real is for the realists merely an epistemic test for its identification. In the latter’s view the essential property is, as I said above, that of existence independent of the subject.

 Let us try to unravel the entanglement with the help of pre-philosophical thinking, which might indeed be of special import when we employ Berkeley's method. If we ask a non-philosopher whether there is a real world outside his mind, he will surely answer in the affirmative, and so we will classify him as realist; but if we tell him that that outside world is inaccessible and we will never be able to know anything about it, he will most probably reply that if so that world is not "the real world" for him, but a meaningless and irrelevant world. If, in the world outside me, the yellow box before me is not a yellow box but, say, a green camel, which I will never access, why should I care, and what is "real" in it? For the pre-philosophical person the real world is the one to which one wakes up from a dream or to which one exits from an illusion; if the "external" world is one to which no-one ever wakes up or to which no-one ever exits, it is meaningless and therefore, to a great extent, **un**real. What, then, is the world to which one does wake up and to which one exists? At this point, if we push our pre-philosopher to the wall, he will give an answer quite similar to that of the idealists: That is the world in which I experience a collection of objects that is coherent, ordered, large, enduring and stable; a world that is not only "in his head" but is common to all "sane" people, meaning, those who are not in a state of dream or illusion. Supposedly, this is a variant of the idealist concept of reality but, once again, we must be accurate: while for the idealist these two characteristics are essential properties, for the pre-philosophical person, similar to the realist, they are merely epistemic tests for it. For this reason this concept should be taken only as quasi-idealistic.

 The pre-philosophical person thus lies in the grip of contradiction. On the one hand he conceives the real world as existing outside and independently of the thinking subject, yet on the other hand as accessible to him as a thinking subject and to other thinking subjects like him. Admittedly, there is a common denominator between the two contradictory components: Both express the understanding that the real is not dependent on the single subject, i.e. the ego. In the realistic component this is so because the real is not dependent on any subject whatsoever and in the quasi-idealistic component it is so because inter-subjectivity takes it beyond the realm of the single subject. This, however, cannot resolve the contradiction.

In truth, there is no way to resolve this contradiction, nor need to do so. That is because these are two different concepts of reality, mixed together by natural language and by pre-philosophical discourse. One concept of reality is the one characterizing the real as existing outside the subject and independently thereof; the other is the one identifying it as belonging to a coherent, ordered, large, enduring and stable entirety of objects and as being perceived by all "sane" people.

 Note, that I took the quasi-idealistic concept in an improved version: First, I did not suffice with the object's belonging to a coherent world, but assembled on it the requirements of being coherent, ordered, large, enduring and stable, or COLES for short. In addition I added the requirement of inter-subjectivity, as described above, i.e. the existence of other subjects sharing that coherent, ordered, large, enduring and stable world, who also perceive the object in question as part of that world.

 Even if we take it that reality of the first type does exist, and even if we admit that there is some fixed relation between the two types of reality (both of which I seek to prove in the next chapter), the reality which is external to the subject is irrelevant to us and therefore does not qualify to be named reality (similar ideas were raised by Mainländer, 1876). The concept called in this book "real," denoted by R, is therefore the real of the first type, the accessible one. The other concept is called here the "noumenal," denoted by J, and is the one inaccessible to us.

### The double test of reality

As I noted above, reality itself is a primitive. The two tests I mentioned, the COLES and the inter-subjectivity, certainly do not grasp the full scope of its nature. If I presented them as exhaustive I would doubtlessly sin in the understanding of reality, as well as to its living concept in our minds. What is real is real because it exists **really**, i.e. in a strong form of being, not because it belongs to this or that entirety or because there are partners to the way I perceive it. For this reason, as I said, I preferred to describe the tests as epistemic tests for reality, not as essential properties of it. An object will therefore be recognized as real iff it satisfies both of these tests conjunctively. We will now examine what they are about.

The COLES test:

The COLES test states that an object is real iff it belongs to an entirety of objects that is coherent, ordered, large, enduring and stable. Therefore it can be recognized only if the subject contains such an entirety. Namely, there cannot be a real object without a real world into which it integrates. Let us examine these properties one by one:

Coherent: This property appears first because of the respect we must show to it, given its honourable place in the tradition of modern philosophy. Today it serves mainly in the epistemological discussions of the concept of truth (in which it is often presented as contrary to the test of correspondence, and sometimes also to the pragmatic truth test). In that context the issue is the coherence of sentences, since the concept of truth is relevant to them alone, while we are talking about the coherence of objects, yet the difference is not big, especially if we recall Brentano's thesis. We can therefore say that an object x is real if the sentence "x is real" can be integrated into a whole and non-contradictory collection of sentences (that are, in turn, reducible to existence sentences, according to Brentano's thesis).

**Ordered**: This property might be understood as a part of coherence, but it includes a little more than it. An order exists when all the objects in a given collection are under an array of general laws, from which there is no deviance. The object at stake will be recognized as real only if it, too, is subject to these laws. Indeed, according to the rule of universal conceptualization any collection of objects might be considered as ordered, and if so the requirement of order does not exclude anything and is consequently redundant. That is, however, not the case: when we postulate that the object integrate into the existing order we require that the laws be determined in advance without it, and not such that will be created ex post facto in a way that will include it.

**Large**: A world will be recognized as real only if it is large. Coherence and order may exist easily when we talk about three or four objects, but these may not be considered as a world. A world is a large and rich collection of objects. How large is "large"? This is immeasurable. Certainly there is no room for stating a number of objects (not only because of sorites problems but also because of the principle of free partition), but I believe most of us will be able to agree on most of the suggestions. Where we cannot agree, i.e. in the doubtful cases, we will judge severely, and will not recognize the object in question as being large enough to qualify for the COLES test.

**Enduring**: A world will be recognized as real only if it is enduring. Coherence and order may exist easily in a dream, which often has laws of its own, and that world might even seen large to the dreamer; Yet, a dream cannot be considered as a world if one wakes out of it after several minutes. A world is a long-lasting collection of objects. How long is "long"? This is incommensurable. Certainly there is no room for stating a definite time unit, but I believe most of us will be able to agree on most of the suggestions. Where we cannot agree, i.e. in the doubtful cases, we will judge severely, and will not recognize the object in question as existing for long enough to qualify for the COLES test.

**Stable**: A world will be recognized as real only if it is stable, i.e. that its objects and laws do not change too rapidly. A world without such stability cannot be recognized as real. Once again: There are no formal or otherwise rigorous criteria for defining stability. Most of us will probably agree on most of the suggestions, and the doubtful cases will be judged severely.

Reality is an exclusive property, namely, one will always assume that there is only one real existence. Therefore, if the subject has two COLES collections she will not assume that there are two real worlds, but rather will examine which of them is *more* COLES. Indeed, since some of the properties constituting the COLES are unmeasurable, we do not have binary concepts here, and thus can unhesitatingly talk about "more COLES" or "less COLES." In that determination I believe the third and fourth constituents, size and length of time, will have considerable weight.

By this we solemnly return the concept of reality to the bosom of the "robust sense of reality" after it was deprived of it by Descartes and the modern philosophers who followed in his footsteps. When a person reads Descartes' question she is supposed to ask herself: OK, suppose I live my entire life in a dream, from which I am to wake up one day; but wake up where? What is it in the real world that will be so different from the illusory one? What will be the decisive difference from which I will learn that so far I was in a transient subjective experience? As noted, a realist philosopher will answer that reality, in contrast to dream, is not dependent on the subject. That is correct, of course. Yet, from the rank and file's point of view the problem is not the dependence on the subject but the ability to maintain a coherent, ordered, large, enduring and stable picture of the world. A man who mistakes the dream for reality does not err between two equal possibilities on his part, that differ only by their placing in relation to the subject ("inside" or "outside" it), but between a "bad" possibility and a "better" one; reality is better by virtue of its being more coherent, ordered, large, enduring and stable. If, for instance, we lived most of our lives in a dream, only seldom sallying into reality, and the world of that dream were more coherent, ordered, large, enduring and stable than that of the world "external" to it, I guess there would come in the dream some alternative Descartes who would wonder how we can be certain that we truly live in a dream and not in some "transient reality." As I said, it is for this reason that in this book I use the term reality in that sense, the allegedly non-philosophical one (and unscientific one). This is the first among the two accumulative tests for the identification of reality.

The inter-subjectivity test:

This reality test states that an object is real iff all the subjects having reality tests similar to mine perceive the object at stake in a way similar to mine. As I said, this test is built accumulatively on the former test, and the subjects mentioned in it are subjects that are conceived as parts of the real world by virtue of the COLES test. And here we may ask: If so, what is their benefit? If we accept the fact of their real existence only by virtue of the COLES test, their ontological status cannot be stronger than that of the world proved by the COLES, and therefore whatever they think or say will never be stronger than what is proved through COLES. Hence the test based on them is superfluous. Yet, this is another place where we need to fulfil the requirements of the pre-philosophical mind. First, because this is desirable according to Berkeley's method; second, and more important, because of the need to strengthen our reality test. Particularly because the real existence is ontologically weaker than the noumenal one, being still dependent on the subject, we seek to strengthen it in whatever way we can. Apparently this intuition can also be found in the pre-philosophical person. From his point of view, one of the proofs that he lives in reality and not in a dream is the fact that others apart from him see, hear, touch, taste and smell the "same" (meaning very similar) objects as he does. The ego learns what other subjects perceive through what they tell him by language. Indeed, in a dream, too, people may appear and tell the dreamer that they see objects just the way he does, thus strengthening his dream perception, but these were not proven as existing by the COLES test. In reality, in contrast, since the real existence of the other people has been proved through the COLES test, their testimony is accepted as referring to the real world.

 Let us examine the constituents of the inter-subjectivity test:

**All the subjects having reality tests similar to mine:** This constituent comes to exclude dreaming people, hallucinating people, etc; however, this issue is not that simple, and draws us into a long epistemological discussion.

 According to the premises of Source Theory, there is no such thing as mere "existence," but any existence is "existence in system," and this holds also for the most basic of all existences – the real one. Subjects believe data according to the truth sources they have adopted and different sources might transmit different things on the existence or non-existence of objects in reality. As I clarified elsewhere (Brown, 2017), a truth system – i.e. a given source model with all the data transmitted by it – can even be a personal one, and even an arbitrarily fabricated one. Thus, for instance, we may imagine a person who adopted a source model transmitting that the reality test is only that which has wings is real, while another might adopt another source model transmitting the opposite. There is no logical way to negate these possibilities. These possibilities lead us into relativism among systems which, while proper in other issues, is very problematic when it comes to the judgment of something as real. As I said before, reality requires some extra strength, that is supposed to cross through subjects and systems.

 In fact we can overcome the relativity to a limited level. In the first step we have to remember that personal and arbitrary source models such as the ones I raised above as possibilities, and others like them, are not really workable and cannot provide their activators with an outlet from the nihilistic absurdities of formal epistemology (Brown, 2017). A workable system, according to the pragmatic guidelines of Source Theory, is a multi-agent *cultural* system, or, as it is called when developed and preserved for a long time, a tradition. Belief in the existence of an object in reality, just as any other belief, cannot be determined by whimsical systems, but only by traditions.

 However, this test too does not save us from the problem of the relativity of reality, since different traditions recognize the reality of different objects. Thus, for instance, some traditions that recognize the real existence of ghosts and demons do not recognize the existence of atoms and molecules whilst the Western Rational System (WRS) recognizes the real existence of the latter while denying that of the former. It seems then, that here too, we have to embrace a certain type of relativism. Yet, here we reach the second step.

 We have to distinguish between the concept of reality in a given system, as it is determined through its tests (for the concept of reality itself cannot be defined), and the contents of the concepts forming these tests. Now, if we examine the existing cultural systems – from east and west, from north and south – I assume we will reach a surprising consensus regarding the reality tests. This is thanks to two reasons: (1) Because of the strength required for the concept of reality in the minds of people, it is likely that any cultural system will include some aspect of universality, i.e. that reality be recognized by "every sane person"; (2) All the possible systems, and of course also the existing ones, *necessarily* include the basic cognitive tools as truth sources (even if they are placed at a low level in the hierarchy of sources, compared to others); and therefore all the truth sources already have some universal constituent (and that by no means implies that the basic cognitive tools deserve universal supremacy for that reason). If that is so, all the cultural systems of the world accept similar reality tests, namely, COLES and inter-subjectivity. Whoever claims the reality of objects that are incompatible with these tests is considered a dreamer in any cultural system. But here again the question comes up, even more intensely: In the light of the proximity between the reality tests, how did it happen that people from different cultural systems recognize different things as real? For instance, how come that one tradition recognizes the real existence of ghosts and demons but does not recognize that of atoms and molecules, while another recognizes the latter and denies the former? Here we reach the third step, in which we need to succumb to relativism.

 As I insinuated above, there is a difference between the system's concept of reality, as determined by its tests (a concept that to a great extent is similar in all the existing traditions) and the contents of the concepts forming those tests. As for the COLES test, different systems have different concepts of order (some may involve supernatural forces, other do not), and the other properties as well may change from one to the other. As for the inter-subjectivity test, here the requirement is that the reality of an object be recognized by every sane person, but the criteria for sanity change from one culture to another. Thus, for example, a man claiming to see ghosts and demons will immediately lose his status as sane in cultures embracing the Western Rational System, not so in various other cultures. In the latter, by contrast, someone who said that wood and iron are both made of very similar particles, and only their varying compositions make them different would certainly be considered as imagining. At this point, it seems, we have to say goodbye to universality and acknowledge that the population of real things will not be determined in absolute terms, crossing over any possible truth system of the subjects. "Reality tests similar to mine" will thus be found only in subjects who share my cultural system. Yet, we must add a small modification to that, and here comes the fourth step.

 In spite of the above, there are a considerable number of objects that will be recognized as real in any cultural system. Will the objects of this collection be recognized as objects whose reality is more evident? I do not tend to favour this conclusion, since it attempts to build a concept of truth arching above systems, a possibility that I do not accept. Still, this fact should not be dismissed, especially when we seek a concept of reality that would be as strong as possible.

 And finally comes the fifth step: Howsoever we look at this question, one thing is clear: A real world exists in every existing cultural system. There is not even one system that affirms the premise of Descartes' challenge, i.e. solipsism.

We can summarize, therefore, that from the perspective of the inter-subjectivity test an object will be recognized as real iff it is perceived in a similar way by all the activators of the cultural system in which that object was transmitted, when operating that system properly (according to its own requirements). If we return to the yellow box lying in the street: That box will be recognized by me as existing in the real world iff all the subjects who share the system in which their senses serve as truth sources and who operate their senses properly would see it as a yellow box. The entirety of the real is the sum of all the real objects.

**The above-mentioned subjects perceive the object under discussion in a way similar to mine**: When I say that the objects perceived by different subjects (activators of the same truth system) must be "similar" to one another I fully understand that this concept is unquantifiable and therefore is vague and prone to relativist interpretations. But it should be stated: Even when the same subject perceives the same object in different situations or from different angles, it does not perceive it in an identical way in all of them. But just as much as the same subject can identify the perceived object as one, in spite of those changes, so also two or more subjects can identify an object as one in spite of the differences in perception between them.

Indeed, even if I assume that there are other subjects and that those subjects perceive the objects in a way similar to the one in which I perceive them, I have no reason to assume that they perceive it in *exactly* the same way. We can talk about the similarity relation SIM1 between the way my wife Iris perceives the yellow box and the way I perceive it; a relation SIM2 between the way Iris perceives the brown notepad and the way I perceive it; and a relation SIM3 between the way Iris perceives the round table and the way I perceive it. However, the three relations SIM1, SIM2 and SIM3 are all objects, and therefore a common concept captures them (according to the rule of universal conceptualization). Hence there is a fixed relation between the world as perceived by Iris and the world as perceived by me.

 The existence of such a similarity relation is learned through language. Language, as a means of communication between subjects, enables me to know, if only partially, how another subject perceives the same objects that I perceive. The fact that different subjects apply the same concepts, represented by the same words, to common objects, and do so consistently and along a rich experience proves that these subjects ultimately have a similar perception of those objects. We can see it as a sort of "communicational coherentism" (as opposed to epistemological coherentism).

 This world – the world of objects shared by many subjects, who perceive them similarly – is the real world in the concrete sense of the word, the one fitting Russell's demand for "a robust sense of reality." We can take its existence as a given, since the source model of our truth system transmits the fact of its existence. Furthermore, as we have seen, all the cultural systems transmit its existence, even if they differ on the "population" of that world.

 Unfortunately, we cannot formalize any of the reality tests, since neither of them offers clear-cut criteria, certainly not measurable ones. Even if we could find a logical definition of coherence, we do not have rigorous and accurate concepts for "ordered," "large," "enduring" or "stable." And even if we succeeded in formulating the inter-subjective sharing in the terms of Source Theory, we would have difficulty formulating the nature of "similarity" required between objects and the way its existence is determined. I believe such formulations will be attainable when our formalization capacities improve, or maybe never. However, this lack of formalization does not impede the wholeness of the theory, since in any case we are talking about tests (i.e. criteria for reflecting reality in our thoughts), not the essence of reality itself. Reality itself is and will remain a foundational concept.

### Space-time and the only pool of reality

The objects of reality exist "within" some object. i.e. within a pool. That pool is space-time. Since reality has only one space-time, it has only one pool. In that, reality differs from all the other forms of being. Since we have defined ontological space as the sum total of pools, space-time is therefore also the ontological space of reality. Space and time satisfy the reality tests presented above, and should therefore be recognized as real.

 However, in our discussion of the sphere of thought we mentioned another type of real objects: subjects. The ideas contained "within" the subjects are of the mental form of being, not the real one, but the subjects themselves are parts of the COLES world shared by all the subjects. Even Berkeley agrees that subjects are not ideas, and therefore are not dependent on subjects. However, subjects do not exist in space, which means that they are not material. If so, do they not exist in the sole pool of reality? The answer is, of course, negative. That is because subjects exist in time. (They might also exist in space, being only the actions of atoms and molecules, as the materialists contend, and then the challenge is easily dissolved; yet I wish to answer it without commitment to a materialist theory.) The subject is a thinking subject, and thinking is an action which takes place in time. For this reason we cannot imagine a subject outside time (Kant has dwelt on it at length in his discussions on schematism). But since, above, we have taken space-time as one object (unlike Kant), the subject is contained in that object by virtue of its existence in time.

### The realm of metaphysics and the realm of science

The concept of reality is a foundational concept standing on its own, but its tests were built from the world as perceived by the subject, i.e. from the sphere of thought. Our reality tests are in fact criteria to help us discover which data from those existing in the sphere of thought qualify as real. That reality is supposed to transcend the subjectivity of the sphere of thought, and therefore transcend the epistemological realm. For this reason the list of foundational concepts of the sphere of reality are the foundational concepts of the sphere of thought, with the epistemological concept TM subtracted. Being a concept relating to the subject as a thinking entity, its sole place is the sphere of thought.

 As far as proving the existence of an object in reality is concerned, the starting point of every subject is the ideas within it. It is not only that any object in reality to which it can relate is a parallel of an object in thought, but even more than that. Any proof of any proposition regarding the real world, even one that transcends the foundational concepts of the sphere of thought, will have to start from data received by the basic cognitive tools of the subject, and eventually will have to return to them, in order to explain the events conceived by them. The basic cognitive tools are, as we remember, cognitive functions of the sphere of thought. We find, then, that the foundational concepts of reality lean, insofar as the subject is concerned, on the foundational concepts of thought. This appears to be a support for the idealist thesis, but that is not the case.

The case is that reality is a foundational concept. The fact that the proof of the existence of objects (as well as laws) has to move from thought to reality does not necessitate the conclusion that reality is dependent on thought. (This position was the usually tacit premise of the 17th century rationalists, most of whom thought that logical dependence reflects ontological dependence, namely, that if the proof of x in thought depends on assuming the existence of y then in reality, too, x depends on y.) Once we have arrived at the realm of reality by the basic cognitive tools, we can reach conclusions regarding the existence of objects far removed from those of thought. That way, for example, modern physics deprived matter of its status as a foundational concept, crowning energy in its stead. By this, however, it cannot abolish the status of matter as a foundational concept of the sphere of thought. Furthermore, even if physics ever becomes as solid as a rock it will never be able to change the fact that the first data from which the scientist sets out are those of the sphere of thought (mainly sense data), and it is to these data that she will have to return in order to prove that her theory passes the test of experience. However, this fact will not be able to change yet another fact, namely that the sphere of reality will be explained, at the end of the day, through foundational concepts (and laws) far different from those of the sphere of thought, and certainly will not be reduced to thought.

It is the task of science, not of metaphysics, to discover the foundational concepts (and laws) of the sphere of reality. Metaphysics is supposed to discover the foundational concepts of any world whatsoever, present the foundational forms of being and elucidate the initial foundational concepts of "our world," the sphere of thought, and by them build hypotheses and present explanations concerning the sphere of reality. The building of the hypotheses themselves and the presentation of the explanations themselves are within the scope of the role of science. Since science is a rational discipline, it can rely only on the basic cognitive tools, on which the Western Rational System is based. As every picture of the world, it must rely on the foundational concepts of the first metaphysic, as expounded in the first section.

It is science that is supposed to determine, by the foundational concepts of the sphere of thought, the partition of the world of the sphere of reality. Therefore it is that, that will decide whether one can reduce the sphere of reality to extended objects (taken that space is perceived separately from time), and the actions of subjects can be explained by elementary units of such objects, or we cannot use such a reduction and consequently must conclude that subjects are separate pools within the sphere of reality. (Actually, even if its conclusion is reached by following the second option it will not undermine the status of the thought concept T as a foundational concept of the second metaphysic, since, as I said, these very conclusions were reached by science through the use of the elementary concepts of the sphere of thought.) I might also conclude that the space of reality (once again, separated from time) is three-dimensional or n-dimensional but even then, as in any other case, that space, when unified with time, will remain the pool of the sphere of reality. That is because the premise accepted in this book that any form of being characterized by multiplicity of objects requires a single object "within" which all those objects be contained and which will render them their coexistence.

Science is also what is supposed to discover to what degree causality prevails in the sphere of reality, and whether it is all-embracing or partial. It is also commissioned to find out to what level there is a causal link between different ontological spheres, especially between thought and reality and between reality and thought (which is known in philosophy as the mind-body problem). We may also expect that science would try to answer the question of whether or not cosmic space has physical boundaries and whether or not extended objects can be infinitely divided or not. But whatever are the findings of science on these questions, they will never refute the fact the *concept* of the cosmos has boundaries (just as any other concept), and even the physically indivisible atom must have parts (upper and lower, left and right, etc.) just like any other object. That is because these facts are parts of the order of any world whatsoever, as expounded in the first section of this book, and no scientific theory can negate the first metaphysic. If it ever tries to refure it, it will cut the branch on which it is sitting, for the very possibility of scientific thinking depends of the acceptance of the premises of the first section.

### Sub-total: The foundational concepts of the sphere of reality

Apart from the concept of the real, R, which we have already accepted as a primitive, the sphere of reality does not require any new foundational concepts. The foundational concepts of the sphere of reality are those of the sphere of thought, subtracting the epistemological concept of transmission (TM) and adding the foundational concepts provided by science. These foundational concepts, which will be found if and when science finishes its task, will join the foundational concepts of the sphere of thought and together they will constitute the second metaphysic. We have dwelt on the elementary postulates of that metaphysic above, in our discussion of the sphere of thought. It should be emphasized once more: The foundational concepts of the sphere of reality are not necessarily those of the sphere of thought, but the latter are the foundational concepts from which any researcher must set out, and by which he or she can study reality. That study can be carried out only by the use of the basic cognitive tools, also enumerated above, and which also constitute a part of the material for a second metaphysic. The foundational concepts of the second metaphysic discovered by science do not necessarily overlap those of the sphere of thought, but must begin from them and must return to them.

It appears that by this our work has been completed. Seemingly, here ends the role of metaphysics, and from here on is the realm of science. Yet, before we can summarize the main conclusions of this book and say goodbye, we have to try to say something about the fourth form of being – the noumenal. If anyone is reading this book at all, and if that unique person made it up to this point, I will ask of him or her a bit more patience to read the next not-too-long chapter.

## The sphere of the noumenon: its existence and some of its very basic characteristics

Thus far we have assumed the existence of the noumenal world as a tentative working hypothesis. We have also assumed that it has an independent form of being, named J. We have taken J as a primitive, assuming that J is not R and none of them can be reduced to the other. What stands at the basis of this assumption? Why shouldn't we assume that the noumenal and the real are one and the same, especially after we have also taken the concept of the real, R, as a primitive, and stated that the epistemological tests for its identification are not exhaustive regarding its essence?

 The answer is that the real, even if we concluded that its essence is not exhaustively grasped by its tests, remains closely linked to the sphere of thought. As I have noted, the foundational concepts from which the metaphysician and the scientist set out to study reality are those of the sphere of thought, and even if, during his journey, the scientist finds a new theory that inaugurates new foundational concepts, pushing the foundational concepts of thought to the second rank, he still bases his findings on a process that started its way in the sphere of thought. Furthermore, any theory must, at the end of the day, explain objects and events in the sphere of thought. Indeed, the scientist, the researcher of reality, came from the sphere of thought and to the sphere of thought shall he return.

 This is not just about things related to scientific work, but also to the essential characteristics of the two objective forms of being. The sphere of thought is by its very nature a sphere accessible to subjects, at least to some of them and at least to some degree. However, the subject is often conceived as imprisoned within its boundaries (we will see below that this is not very accurate), and therefore there is some level of reality that it cannot access. That level is not reality itself, since the latter is of a form of being that the subject can access, and so we have to define it as a distinct form of being. Being unable to characterize it, we must accept it as a primitive standing on its own. J will thus remain a foundational concept, different from that of R. But now we have to prove that it is not an empty concept and there is something over there, in the inaccessible level of reality.

 Indeed, so far we have assumed the existence of the noumenon and the existence of multiple subjects (other than the ego) as tentative working hypotheses. Now we can start discussing the body of these assumptions. Beyond the very existence of such a world we will not be able to say anything about it – by the very nature of the noumenal. But its existence must be proved, at least to a high degree of plausibility. In addition we might be able to say a few things about its relations to other spheres and hence nevertheless learn something about its nature. As I noted above, not all the proofs will be evident, some will only have high plausibility.

### The existence of the noumenal world

I have already proved my own existence (above, ###) in the spirit of Descartes' proof, which I will qualify below. As we remember, that proof reached as far as proving the existence of the ego, while we have accepted as a temporary working hypothesis the existence of other thinking subjects ("other minds"). Now we will move a step forward in this course to prove that there are other objects outside the ego, among them subjects different from the ego.

 First, let us recall what I have written: Even the solipsist thesis premises the existence of an external world, not dependent on the subject, but it "populates" it with one single object: the subject itself. That existence is not just a real existence, in its above sense, but is a noumenal one. The ego is not just a part of the COLES ontological entirety to which many subjects direct their ideas, but should be taken as an object standing on its own, beyond and above any contact with the sphere of thought. We can thus accept the existence of the object S1ᴶ as a given.

 We will now turn to the proof of the existence of the noumenal world, i.e. the one external to the thinking ego. I will present two possible proofs, and will say in advance that from my point of view the main one is the second.

**The first proof** is the simpler of the two, and arguably the most commonsensical. It suits the spirit of the 17th and 18th century philosophers (and therefore will probably vex the philosophers of our day). It is a *reductio ad absurdum* proof, and goes as following: If the solipsist thesis is true, then the entire world is a product of my mind, and if so, then any part of it is also a product of my mind. But if so, then I wrote Shakespeare's *Macbeth*, composed Mussorgsky's *Boris Godunov*, painted Van Gogh's *Starry Night*, developed Einstein's Theory of Relativity and invented the spaceship, for all of these are parts of the world contained in my mind. Now, since I know, by introspection, that I cannot do even one of these actions, let alone all of them, then they are not products of my mind, and if so, the solipsist thesis is false. QED.

There is much to discuss in this proof, but I brought it just as an aperitif. I wish to dedicate the main discussion to the proof that makes use of the tools presented in this book. So I will now introduce **the second proof**, the more important one for our matters:

First, let us remember that there are two possibilities: Either the only existing world is that which is within a given subject S1, which is the ego, or there is a noumenal world outside of it. The solipsist thesis is that there is only a world within S1; namely, that S1 is the only noumenal object. We can formulate it as follows:

S1⮭J˄¬əxᴶ|x≠S1

Or

S1⮭J˄əxᴶ→x=S1

According to this, any object is an idea in S1. An object x contained within S1 (i.e. an idea of object x) will be denoted as xᵀ¹. Having said that, we can now turn to the body of the proof:

1. Every object is discriminated through its boundaries.
2. The boundaries of an object are determined by a concept.
3. The concept discriminates the object from its environs.
4. The concept which determines the boundaries of S1 is S1↑.
5. The use of a concept in order to determine boundaries is a result of a decision.
6. A decision is an action taken by a subject.
7. Subject S1ᴶ decides on all the boundaries of all the objects contained in it.
8. S1is an idea contained in S1ᴶ (I perceive myself).
9. Hence, S1ᵀ¹ exists.
10. S1 discriminates between S1ᵀ¹ and the sum of the other ideas in it, i.e. CKS!ᵀ¹.
11. Either S1ᴶ=S1ᵀ¹ or S1ᴶ≠S1ᵀ¹.
12. If it is true that S1ᴶ=S1ᵀ¹, then an idea can discriminate itself, and that is false (by 6).
13. If it is true that S1ᴶ≠S1ᵀ¹, then S1 discriminates itself from environs external to it.
14. And if so, there are objects external to S1, i.e. there are noumenal objects, not contained by S1.

QED.

The object existing outside S1ᴶ will be named CS1ᴶ. Above we spoke about a "world" external to the ego, but it is obvious (from our discussions in the first section) that even if there is only one object external to S1ᴶ, it is, for our purposes, a noumenal external "world." The two of them together are "the world" as a whole, and each of them is the universal complement of the other.

 Allegedly one could challenge this proof. Fichte was the first philosopher to discuss this matter; he first discussed the ego that limits itself in order to make place for other beings (Fichte 1970, pp. 107-110). But this is a nice metaphor suitable for philosophers of the romantic period, but not so much for formal-ontological discourse.

 I will explain some of the points in the above argumentation that may not be understood at first glance. It should be expounded by tools borrowed from the philosophy of language and one very informal illustration.

In my opinion, following Frege, (1977) and Wittgenstein, (1969), the solipsist thesis is a statement that loses the meaning of the word "existence" or that of the word "I" (ego). The entire meaning of the phrase "existing within the ego" is given to it by virtue of the premise that something exists outside of me; once we give up this premise the phrase "existing within the ego" becomes synonymous to mere "existing" and consequently loses its meaning.

I will illustrate it through a story (and I hope no criminal investigation will be filed against me because of it…).

When my eldest son was a kid, I told him he was so cute that I would swallow him in one big bite. The child took the saying verbatim, though did not really believe it, and answered with a big jocose smile: "You can't do that!" His reaction motivated me to keep playing the game so I replied: "Of course I can!" After a short argument on whether or not I could swallow him in one bite, I insisted and said: "I definitely can, here it comes!" and here I uttered the appropriate monster-eating noises and made a gesture of swallowing him. "See?" I said, "Now you're in my stomach!"

* "You didn't swallow me at all," the little sceptic insisted.
* "Of course I did," I responded, "You're now in my stomach!"
* "So how come that we have here a table and a chair?" he challenged.
* "I swallowed them, too!"
* "And the window, and the door and the walls?"
* "I swallowed them, too! I swallowed all! Now all is in my stomach!"

Do not worry, eventually I told the kid the truth (as I said, he didn't believe me anyway, even for a minute), but more importantly, both of us received here an important elucidation of the question of the existence of an "external world." Once the entire world is in my stomach, the existence in my stomach becomes the definition of existence, and consequently there is no meaning to any other existence, including "existence within the ego." In the case of my stomach, just as in Descartes' original challenge, the point still remains, because it might be that one day all will exit from my stomach (in Descartes' case: exit from my dream, or the illusion created by the malicious demon), and then we will find that "the external world" is altogether different. In the solipsist thesis, though, the concept of existence cannot be anything other than "existing within the ego," and therefore exiting from one's self is impossible from this point of view. In such a theory the concept of "external world" is rendered meaningless, and so one cannot even affirm or negate its existence. Furthermore, even in the world that is within the ego the solipsist will clearly distinguish between the ego and other objects, and so will become entangled in a vicious circle. Hence is the refutation of the solipsist thesis.

 The object entirely external to the ego is the rest of the noumenal world. The noumenal world is the union of all the objects existing "in themselves," including the ego, and if there is at least one noumenal object in addition to the ego, then a noumenal world exists. Now we will elucidate some of its aspects.

### On the "population" of the noumenal world

 Taken that we have proven, to a high level of plausibility, that there is a world external to the ego, we should now turn to the question of its nature, as well as its relation to our internal world. As we know, Hume has already taught us that even if an external world exists, and even if I assume some relation between it and our minds, we will never be able to know anything about this relation beyond its basic existence. Kant took a position not very far from that. Both of them concluded that from the perspective of philosophy this leads to the embracing of the idealist thesis, i.e. that there is no world outside the mind. (Kant referred to the "thing in itself" as to what is "given" within the mind.) The consensus between these two giants must be respected, but in the light of the tools we have received in the previous discussions we can say a little more on the "population" of the noumenal world and its relation to our sphere of thought.

As stated above, when we talk about internal and external existences we actually talk about three types of existence that are theoretically possible: (1) existence as subject; (2) existence as idea of a subject; (3) existing as an object that is neither a subject nor an idea thereof, i.e. "external" existence. Even though the subject itself exists "externally" to the subject, as I demonstrated above, the first two types are linked with the concept of subject, and therefore we finally remain with only two possibilities:

**The idealist thesis**: Outside the subject S1 there are other subjects, and any object that exists – does so as an idea within one or more of these subjects. In other words, objects exist only in types of existence (1) or (2).

**The realist thesis**: Outside the subject S1 there is at least one object that is neither a subject nor an idea within it. In other words, at least one object exists in type of existence (3).

Given that there is an additional *subject* beyond S1ᴶ, that subject will be named S2. As we remember, the object external to S1ᴶ we call CS1ᴶ. Now we can clarify the conclusions:

1. There exists something outside of S1ᴶ, which is CS1ᴶ (proved above).
2. CS1ᴶ is either (a) S2 or (b) an idea contained in S2 or (c) an object existing outside of S1 and S2.
3. If (a) CS1ᴶ is S2, then it exists not merely as an idea, i.e. it has an external existence.
4. If (b) CS1ᴶ is an idea of S2, then necessarily S2 exists, and not as mere idea, i.e. it has a noumenal existence (as above).
5. If (c ) CS1ᴶ exists outside S1 and S2, then CS1ᴶ has a noumenal existence.
6. Hence, in each of the three possibilities there is some object with noumenal existence (i.e. such that is not dependent on the subject).

Put simply, both in the idealist and the realist theses there is something that is not an idea within S1. That is not new. Since we negated the possibility of solipsism, we remained, by way of elimination, with either idealism or realism.

But at this point we have to advance our understanding of these to include alternatives. From the perspective of the history of philosophy, we seem to talk about two possibilities of equal standing. But that is not the case. Here we have to address the consideration of the burden of proof, i.e. ask ourselves: In a state of no evidence, which is the data that we will take as a basic premise until otherwise proven, and which is the one we will take as requiring proof. The question of the burden of proof, often considered as pivotal in the realm of law, is of great importance to philosophy, perhaps no less so than to law. The fact that philosophy has not given it much attention so far, especially if we compare it to legal theory, is in my opinion an oversight. This is not the proper place to fully repair that oversight, but I will say what is needed for our purposes. When there are two propositions, one proposition p, transmitted by source 1 and asserting that some object is X and another proposition q transmitted by source 2 and asserting that the same object is IP;X – we will see the former as the basic premise and the latter as requiring proof. The reasonableness of this maxim is clear. The two parties agree that the object at stake is X, but source 2 wants to argue one more thing, and therefore the burden is on him. For example, if I argue that the object seen in the binoculars is a man and my friend says it is a Chinese man, the burden is on him. That is because a Chinese man is a man, so we both agree on this property, but my friend seeks to argue more than that.

As for our issues: Realism contends that there exists something which does not depend on mind, but does not say anything about its essential properties. Idealism, in contrast, contends that there exists something that does not depend on mind, but also contends that that thing is of only one type: subject. According to the above maxim, the realist thesis should be taken as a basic premise, while idealism remains with the burden of proof on it. But since we cannot cross the walls of the boundaries of the subject and access the noumenal world, idealism will never be able to lift that burden, and therefore realism has to be taken as a solid thesis.

We can add here another argument to somewhat support the above conclusion: Let us assume the idealist thesis. According to that thesis, there are at least two subjects in the noumenal world. (All the idealists accept some proof of the existence of other minds, and it is obvious that there are more than one such, but I take here the minimalist assumption, which can later be applied to larger numbers.) Let us call them S1ᴶ and S2ᴶ. However, if there are objects, there are concepts which discriminate them. Therefore, according to the idealist thesis, there are at least two uniextensional concepts which capture S1ᴶ and S2ᴶ, which we will name S1ᴶ↑ and S2ᴶ↑. By that we have proved that the noumenal entirety contains at least two objects (we do not forget: concepts are also objects!) that are not subjects. But if there are two subjects, all the rules regarding the relations between subjects expounded in the first section of this book apply to them. And if so, there are now much more than just two concepts. In that case, even according to the idealist thesis, the noumenal entirety is populated by many objects that are not subjects. Thus they already come very close to the realist thesis.

Indeed, the realist thesis is often linked with the argument that the noumenal entirety contains sensible individua: tables and chairs, trees and stones. Another assumption usually added to it is that there is some relation between these individua and their parallel individua in the spheres of thought and reality. We haven't yet proven that (we will come close to that below), but already from this point the idealist's burden of proof becomes even heavier than in the previous phase. Now the basic (and agreed) assumption is that there are noumenal objects and they are not necessarily subjects, and of course it is the idealist who is required to prove that they cannot be individua. Since we can say categorically that the idealist cannot withstand this burden, we categorically remain with the realist thesis.

By this we have brought to quite a simple solution the problem whose unsolved condition was considered by Kant as a "scandal to philosophy and to human reason." We have proven the existence of an external world, populated not only by subjects, as a solid datum.

### Unity and plurality in the noumenal entirety

Now we can ask what the character of the noumenal world is. Here, too, we are facing two alternatives: One is that it is a unity without distinctions, i.e. a Parmenidean world, and the other, that it contains plurality. I have already written that the premise of this book is the rule of basic existence, which negates the Parmenidean thesis and assumes that the world is of plurality and distinctions. But now we can support this rule with some evidence, which probably does not amount to a proof but still might corroborate the premise underlying it.

 We showed above that there are at least two objects: S1 and its complement. I keep restating that S1 was proven to be existing and was not just an idea in S1. Once there are two objects, there is plurality. Hence there is plurality even in the noumenal world. Hence it is not Parmenidean. This will be hereafter named the **rule of the noumenal plurality**.

 Against this one may contend that the rule of the noumenal plurality is not necessary, since the plurality was created by S1 in its decision to discriminate between itself and "the rest of the world," taken that it could decide not to set such a boundary. But this condition could be neither of two cases: Either there would be no subject or there would be no world. If there is no subject, there is no-one who could decide anything, and then everything would be null; and if there is no world, we remain with the subject and its internal world, and that world is doubtlessly a world of plurality. There is yet a third alternative, that of a subject that erases not only the boundary between itself and the world, but also those of the partitions within itself. Then what is left is indeed a Parmenidean world, without any boundaries, discriminations or partitions whatsoever. This is the world of the mystic, who experiences the absolute unity, which includes the aspect of self-negation (*annihilatio*). I am not sure a person can truly experience it. The action of erasing the boundaries of the experiencing ego is also an action of erasing the experience itself. But even if we take it that such a decision to create a Parmenidean world is possible, I insist that even then the rule of noumenal plurality still holds. That is because when the subject sets the boundary between itself and what is external to it, this placing of the boundary is later found to be useful (we run all our lives with it), in contrast to a condition in which it would include within the ego other things such as the dark side of the moon. If it is found useful, it is because it fits the objective order of the world, which is also of plurality.

 I do not intend to blur the fact that the above is an argument of corroboration, not a proof, and that the Parmenidean conception was, and remains, an alternative that cannot be easily dismissed for any conception of the world as plurality. Therefore we will remain with the assumption that the world is a world of plurality on the basis of our working hypothesis (the rule of basic existence) and not on the basis of a quasi-proof (still, I will try to further corroborate it later below). At the end of the day it should always be remembered that the external world which is relevant to our lives is not the noumenal world but the real world (as defined above), and that world is certainly a world of plurality.

 We have said a few things about the noumenal world: That it exists, that one does not have to assume that it is populated only by subjects and that it is a world of plurality. From this point on, all the principles of the first metaphysic, as elaborated in the first section, apply to it. Among them is the fact that every plurality is ordered, since it is captured by concepts (by the rule of universal conceptualization). In principle, we ought to stop here, since the noumenal world is outside our reach, and whatever we try to say about it would be speculative.

 That is correct, but not fully correct. We ourselves live in the sphere of thought, and one of the cognitive functions that build this sphere is abduction, i.e. the suggestion of the best explanation (function 5.3 in the list presented above at Chap. ###). As we remember, the cognitive functions, abduction included, build the world of thought within all the subjects, and consequently also the world of reality. Now the question is whether that function is also relevant to the relation between the spheres of noumenon and thought. If it is, we can say one more thing about the noumenal world. I am not sure about the answer to this question, but will present its different sides.

 We agree that the subject S1 is a "citizen of all the worlds": It is an object in all the ontological spheres, among them the sphere of thought and the sphere of noumenon. The relation between S1ᴶ and S1ᵀ is therefore a well-defined one: a relation of parallelism, denoted as PR. S1ᵀ is not the subject itself but the idea of the subject, while S1ᴶ, in contrast to any other noumenal object, is not altogether concealed, but is known to us through introspection. However, we cannot tell whether there is a relation that can be characterized between CS1ᴶ and CS1ᵀ, excluding the fact that they both are not S1. We cannot even tell with absolute certainty that there is a relation of nonidentity between them. Nevertheless, since CS1ᴶ and CS1ᵀ are both objects, the rule of universal conceptualization applies to them (as it applies to any pair of objects, as stated in the first section), and therefore we can say that some relation exists between them. Let us call this relation RN2. If so, we have two different concepts: PR and RN2. But these are also two objects. Hence, there is a concept that captures both of them (again, by the rule of universal conceptualization), and if so, there is a general principle of order prevailing between the spheres of thought and noumenon.

 This does not end the issue. Since the principles of the first section apply to any world whatsoever, the principle of free partition and the rule of universal conceptualization apply also to the noumenal world. The noumenal world can be partitioned to two, four, or an endless number of parts, just as can the world of thought. If so, we can contend further: Against the object x1ᵀ in the sphere of thought there must be an object x1ᴶ in the sphere of noumenon; against x2ᵀ there is x2ᴶ, and so on and so forth. For short, against any xnᵀ there is an xnᴶ. However, between each two objects of such a pair there is a relation, allegedly unique (until otherwise proven), which might be called RNn. Yet, these relations are concepts, and all these concepts have one concept that captures them (according to the rule of universal conceptualization). Let us call this concept σRN. In the light of all that, there is a unifying order prevailing on the multiple relations between the sphere of noumenon and the sphere of thought.

Now we are arriving at the question from which we set out: Can we say anything about this order? Can we say that σRN is a concept that produces a useful partition of the world? Can we determine, for example, that it includes a causal relation (CSE)? It is obvious that we cannot say anything about the noumenal that will be based on the ex nihilo functions, such as the senses. But here enters abduction. If we legitimize abduction as a cognitive function proper for this discussion we may contend that "the best explanation" for the relation between the ontological spheres includes a causal relation: The noumenal world is one of the causes of the way we perceive the world. But may we use abduction, which is a function of the sphere of thought, to explain relations external to that sphere? On the one hand, the answer is no, since those functions relate only to the cognitive tools of the sphere of thought, and through it the related spheres of language and reality, but not beyond them. On the other hand, the answer is yes, since the things we did allow ourselves to say about the noumenal entirety were reached through the use of our cognitive tools, without which we cannot say anything about anything. It's a dilemma, and I cannot decide it here.

### The noumenal subject and its boundaries

As I have emphasized several times, the subject also has boundaries. For this matter, the subject is an object like all the rest. A person discriminates his self (ego) from "the rest of the world." Indeed, these boundaries, too, are a result of a decision. Hume and Nietzsche, each in his own way, thought there is no necessity to see the subject as one, and contended that it may be dismantled into many subjects. Hume thought "there is no impression constant and invariable [of the soul]. Pain and pleasure, grief and joy, passions and sensations succeed each other, and never all exist at the same time." (Hume, 1739, Book I, Part IV, sec vi). Nietzsche thought that what is called the subject is just the arena of the struggle between various drives and passions. "'Subject' is the fiction which would fain make us believe that several similar states were the effect of one substratum" (Nietzsche, 1913, sec. 485). By this they diminished the "size" of the subject. In contrast, babies and mystics are capable of enlarging it. Some psychologists say that a baby at the stage of symbiosis sees her mother as a part of her ego, and a mystic can see himself as one with God, the world or both. A union with the world means the complete erasing of the boundaries of the ego. Any such determination of the boundaries of the ego is a result of a decision to set the boundaries according to some other concept (or, in the case of union with the world, refraining from setting any boundaries). Even if we agree with Descartes that the ego is the most solid datum, which ego are we talking about? In what boundaries? It is important to state here something of which Descartes was not aware: The determination of the boundaries of the ego precedes the cogito, and that determination is made by a concept. Hence, the concept of the ego precedes the cogito. The proof of the existence of S1 is stipulated by the existence of concept S1↑ that would capture the ego and would discriminate it. Indeed, even the knowledge of oneself depends on a decision concerning boundaries. We cannot say that one way or the other is preferable from a logical or ontological point of view.

 By this we receive a new perspective on Descartes' challenge that rightly worried so many philosophers. As we remember, Descartes asked how can one know that an external world exists, and that not everything is a dream. However, when the subject itself is the one who sets its own boundaries by a decision, then the existence of an external world (i.e. anything standing outside these boundaries) is a consequence of that decision. (By this I proved, above, the existence of the noumenal entirety). Once the ego determines its own boundaries, it determines them as contrasted to the external world. The external world is the complement, or a part of the complement, of the ego. I have already said more than once that with regard to the entire world there are two alternatives. Either a boundariless Parmenidean world, like that of the mystic, or a world of boundaries, where the ego has to have boundaries just like anything else. Once the ego has boundaries, it discriminates itself from what we call "the external world."

 But here comes the expected question: If the boundaries of the ego are determined by decision, there is a subject that makes it (by the definition of decision). Who, then, is that subject? Here there are two alternatives: Either it is the ego itself, or it is another subject (or other subjects), existing outside the noumenal ego, in the complement of that ego in the noumenal entirety. In fact, both of these alternatives look absurd, since both of them assume the existence of the ego and its complement, which means the existence of boundaries. If the ego is not yet discriminated when it determines these boundaries, how can it determine them? And how can its complement, or any of its part, determine them? On the other hand, if it is discriminated, its boundaries are already determined, what is it that it has to discriminate?

The solution is not that complex. Since we discuss the noumenal entirety, where we haven't proved the existence of any discriminated object, we are in a Parmenidean world. What exists is only the world, and none of its parts, including the subject, are discriminated. In this state of things, we can tackle the matter in a way similar to the one I used above (at ###), when I wrote about the selection of the part, but without committing ourselves to the describing of the action taken here in causal terms. Similar to what I wrote there, we often attribute an action to the whole as a result of ignorance of the identity of the factor or its relation to the action, and only when we learn the details of the event we attribute it to the part. I called this move "the selection of the part." This selection often reflects a useful partition, since the attribution to the part is conceived as a higher degree of precision.

 In our issue, in order to have a subject, that subject has to have boundaries; in order to have boundaries, there has to be a decision to determine them; in order to have a decision, there has to be a deciding subject. In a condition of no boundaries at all, the world exists in a form of unpartitioned Parmenidean world. However, since the world contains in it, albeit without discrimination, the subject, even many subjects that have, even without discrimination, all the capacities of subjects, it (the world) can partition itself, make discriminations within itself and set interior boundaries within itself. Only retrospectively we will be able to identify, given the rule of the agency of the world, that even if the entire world performed those partitions, the part that performed them was the one that was later to be discriminated as subject(s). The discrimination of that part was made through a concept that created a certain partition of the world by which all the objects known as subjects were captured. That partition proves itself as a useful partition of the world from now on as well, and actually becomes the strongest discrimination in our minds; the discrimination of the identity of our egos.

 The subject could also discriminate another object, which includes the subject. As was said, it could discriminate, for example, the object constituting a union of itself and another subject, like his mother, or with a collection of other subjects. It could also discriminate only a part of itself, for example to see itself as a collection of momentary sensations (à la Hume) or mental forces (à la Nietzsche), each of which can be considered as a subject for itself. If it did so, the concept of the ego would be diminished or enlarged accordingly. However, it seems that any other concept of the ego would be much less useful. The present partition of the world is such that the subject discriminates itself in such a manner that the boundaries of the subjects are those in which it (the subject) is tied to a given body, i.e. controls it and is affected by it. These boundaries of the subject are most probably the most useful among the possible partitions of the world between a subject (in any boundaries) and its complement. However, even if the subject discriminates a smaller or larger subject, it should be remembered that it (the subject) is the one that sets the boundaries, and therefore could split a subject that is too large or unite a subject that is too small with other subjects according to how needed or useful it would be for it.

 Once the subject discriminates itself it also tags itself. And once it tags itself it keeps following itself in all its continuous changes, thus giving itself existence along the timeline. Finally, the subject can decide to diminish or enlarge itself, but there will always be a subject to do it, since without a subject there is no decision.

 The self-discrimination of the subject creates not only the boundaries of the subject, but also those of its complement. I mean, here, not the universal complement but rather the one that complements it to the rest of the entirety of the noumenon. Since the noumenon is defined as that which is external to the subject, by this the subject determines not only its own boundaries but also those of the entirety of the noumenal.

 Since the noumenal entirety has more than one object, and nevertheless all its objects belong to the same entirety, they exist "within" some common object, at least one such. Thus we can state that the noumenal entirety has a pool, and at least one such. How many, we cannot tell. A noumenal pool will be denoted by PLᴶ, but, lacking information about the noumenal, we can know nothing about the nature of its pool(s). The collection of all the noumenal pools constitutes the ontological sphere of the noumenon, i.e. the OSᴶ.

 As it was said, the above analysis might serve as basis for discussions beyond the scope of metaphysics, in the existential as well as in the moral, cultural (esp. in its post-structuralist context), psychological and theological realms. This, however, is not the place for those discussions. We should suffice with the non-negligible step forward we have made here in the realm of metaphysics.

## The order of our world and the Second Concept

### The foundational concepts of our world

In the first section of this book I collected all the foundational concepts displayed throughout the first metaphysic, and from them created the Primeval Concept. This concept was in fact the intersection of all the foundational concepts. The same should be done in the present section in order to create general foundational concept of the second metaphysic. That concept will be denoted by GKM2. As we will see in a moment, it will not be built by a simple intersection of all the foundational concepts of the second section, for now we do not deal with "being qua being," which is supposed to be captured by all the foundational concepts, but with objects of different types. Therefore, some of the foundational concepts will be related to each other by intersection but others by union. We must first elucidate which are the concepts that have to be built by union, and combine these union concepts by way of intersection. That intersection will be the concept that captures every object in our world, and will therefore serve as the Primeval Concept which forms the unifying order of our world, in addition to the Primeval Concept which forms the order of any world whatsoever.

 There are a few differences in the ways I build the Primeval Concept in each of the sections. In the first metaphysic I didn't include, among the concepts comprising the Primeval Concept, any partial concepts when we had their wholes as foundational concepts. In the second metaphysic I will not refrain, when needed, to take as primitives even concepts that are not foundational. When I say "when needed" I mean the very specific case in which we have a partial concept and a whole concept and we take both of them as primitives because we do not have a way to reduce the partial one to the whole one with a precise partitioner. From a logical perspective, in a case where there is a partial concept and a whole concept we ought to take the whole as a foundational concept and reduce the partial one to it, yet we have encountered a few cases in the course of our discussions where we could not carry out such a reduction because we lacked the partitioners of the whole. For this reason, for example, we will have to take both the concept of "form of being" and the concepts of the particular forms of being – thought language, reality and noumenon – as primitives. That is because we do not have the means to build them properly out of the concept of form-of-being. The same is true for the concept of sense and the concepts of the particular senses: sight, hearing, touch, taste and smell. Even though the concept of sense alone is foundational and we ought to build the other concepts by partition of that concept, unfortunately we do not have partitioners that are precise enough, and so we will have to take all of them as primitives. Some will contend, and rightly, that in this way metaphysics is built from epistemic constraints. Others may add that in the future science might provide us with better tools that will enable us to be more accurate about this, and fully reduce the partial concepts to foundational ones. As I noted above, in such a case we can reformulate our second metaphysic according to the progress of science. As long as that has not been done, however, we are called, indeed, to formulate our metaphysic from the prevailing epistemic constraints while declaring that we do not present a dogmatic metaphysical truth but rather chart the way a proper metaphysic ought to be built from the data it has at hand.

 As for concepts that have inverses, in the first metaphysic I displayed them in a list from which I selected the concept that best fitted for building the Primeval Concept, and assumed that its inverse can be derived from it. In the second metaphysic, however, I won't refrain from taking both of the inverse concepts and uniting them into one.

The union concept of having a form of being

Every object in the world has a form of being. The concept "form of being" is not reducible to any more foundational concept and therefore will be taken as a primitive. It is denoted by FB.

 We know four forms of being: Thought, language, reality and noumenon. Every object in our world is of one or more of these forms of being. Since we cannot build them from more foundational concepts, each of them will be taken as a primitive. They are denoted, respectively, by T, V, R, J.

 The union concept of having a form of being will be built as the union of all these five concepts - FB and the four particular forms of being - in spite of the overlap between FB and the particular forms. This union concept will be named UFB and will be defined as follows:

 UFB =def FB∪T∪V∪R∪J

We can formalize the statement that every object has some form of being:

ɒO⮭(FB∪T∪V∪R∪J)

And for short:

ɒO⮭UFB

The union concept of being related to a pool

Every object in our world is either a pool or a content of a pool or a collection of pools of a certain form of being, i.e. an ontological sphere. Therefore, among those, only the concept of pool is foundational. But here we also have to be precise: A subject is a pool of thought, PLᵀ. A language is a pool of language: Plᵛ. But the pool of reality – the space-time – is more complex a concept, since we can diassemble it to more basic elements: dimension and time. The concepts of dimension and time, SPn and TI (respectively), are foundational. The concept of the three-dimensional space SP3 is built from the concept of dimension SPn.

 The concept of being related to a pool is a union concept of the concepts of pool, dimension, time, content, and collection of pools. That is so in site of the visible overlap between some of these concepts. This concept will be named UPL and will be defined as follows:

UPL =def PL∪SPn∪TI∪CT∪CN;ɒPL

We can formalize the statement that every object relates to a pool in either of the above ways:

ɒO⮭(PL∪SPn∪TI∪CT∪CN;ɒPL)

And for short:

ɒO⮭UPL

The union concept of being a cognitive function concept

Every object in the entireties of reality, thought and language is captured by at least one function concept of our cognitive functions. The concept of "function concept" is, as we remember, KF, but this concept does not enable us to infer all the concepts of all the cognitive functions from it. Even in the concept KF1, denoting sense perception, we cannot define each of the senses by a precise partition of KF1, and so we will have to put into the union concept of the function concepts, both the wholes and their parts. We will denote the union concept of these concepts UKF and will define it as follows:

UKF=KF1∪KF1.1∪ KF1.2∪ KF1.3∪ KF1.4∪ KF1.5∪ KF1.6∪ KF2∪ KF3∪ KF4

This time we cannot say that every object is captured by this concept, since we do not know if that is true for objects in the noumenal entirety. Indeed, the little we could say about that sphere was discovered by rational argumentation, namely by the use of our basic cognitive tools, but these do not reach to the essence of the noumenal objects. Therefore we cannot say that every object is captured by UKF, but only that every object is captured by the union concept of UKF and J. In an articulate formulation we can therefore say:

ɒO⮭(KF1∪KF1.1∪ KF1.2∪ KF1.3∪ KF1.4∪ KF1.5∪ KF1.6∪ KF2∪ KF3∪ KF4∪J)

And for short:

ɒO⮭(UKF∪J)

The union concept of being related to decision

Finally, every object in our world is either decided or given. We will name the union concept of these two possibilities UDD and will define it as follows:

UDD =def DD∪GN

We can state formally that

ɒO⮭(DD∪GN)

And for short:

ɒO⮭UDD

The only concept we took as a primitive in the sphere of thought and did not take as one of the composites of the Second Concept is TM (transmits a datum). That is because it fully belongs to the sphere of thought and therefore we cannot attribute it or its inverse to all the objects in our world. This omission does not impede the completeness of the theory since this concept is epistemological, and only epistemological, and therefore is not required for the building of a metaphysic. Admittedly, one can say that the same is true for the concepts of "decided" and "given," and that they too are subjective in essence. But that is not correct because their union indeed captures any object (and they are helpful as second order predicates for the description of concepts and boundaries).

Now let us see how we can build the general concept of the second metaphysic by all these.

### The general concept of the second metaphysic, or the Second Concept

We can now define the general concept of the second metaphysic, also known as the Second Concept. This concept is, as said above, the intersection of the primitive concepts that capture every object in our world beyond the foundational concepts of the first metaphysic, and therefore it is the constitutive concept of the order of our world. The list of concepts from which we are supposed to build the Second Concept is the list of primitive concepts declared in this section, minus TM that belongs to epistemology alone. These are in fact the concepts just enumerated in the previous subchapter. In view of our analysis above, they are all irreducible. We will only have to add the Primeval Concept to them.

 Now we can build the Second Concept, which I have already named GKM2. In accordance with the above, it will be defined as follows:

GKM2 =def GKM1∩UFB∩UPL∩(UKF∪J)∩UDD

In an articulate formulation, that displays the foundational concepts in full, the definition will be as follows:

GKM2 =def GKM1∩(FB∪T∪V∪R∪J)∩(PL∪SPn∪TI∪CT∪CN;ɒPL)∩

(KF1∪KF1.1∪ KF1.2∪ KF1.3∪ KF1.4∪ KF1.5∪ KF1.6∪ KF2∪ KF3∪ KF4∪J)∩

(DD∪GN)

Indeed, a complex formula and not the most elegant one. But our world is also complex and not always elegant, so no wonder the concept that constitutes it is similar.

 Now we can state formally:

ɒO⮭GKM2

In the first section, after we had defined the Primeval Concept, we turned to discuss the question of what discriminates that concept itself. However, when we discuss the foundational concepts of the second metaphysic we are not required to have such a discussion or anything approaching it. Once the plurality is taken as a fact, by virtue of the first metaphysic, the gateway is open for the entering of the foundational concepts of our world without infinite regress. The foundational concepts of our world are provided, as noted above, by our truth sources and, when we focus on rational metaphysics, these are the basic cognitive tools standing at the basis of Western Rational System. Indeed, as we have seen, the concepts provided to us by these sources, accompanied by the foundational concepts without which we could not have built the function concepts of these sources, are those which constitute the list of foundational concepts of the second section. Consequently, these are the foundational concepts comprising the Second Concept.

 So we have managed to determine the basic concept that combines all the foundational concepts required for the creation of the second metaphysic, that of our world. That metaphysic is the one that constitutes, on the shoulders of the first one, the basis for the entire construction of science. From this point on, the continued use of the cognitive tools of WRS is the role of scientific thought.

## Overall summary

Time to sum up. We have studied metaphysics, i.e. the theory that elucidates the foundational concepts of the world. In "the world" I mean here ordered plurality of objects. I accepted the rule of basic existence, assuming that there is more than one object, i.e. that it is not Parmenidean, and from that point I set out to elucidate its unifying order. In fact, this elucidation is of the way the partition of the world was done. I built here a metaphysic with two layers, to each of which a section of this book was dedicated: The first metaphysic, discussing the foundational concepts of any world whatsoever, that is, "being qua being" (in Aristotle's words), and the second metaphysic, discussing the foundational concepts of our world.

In each of them I also presented a general unifying concept. In the first metaphysic it was the intersection concept of all the concepts of "being qua being," which was defined as the general concept of the first metaphysic (i.e. the Primeval Concept). However, since I understood the requirement to discover a foundational concept that would also serve as a "first partitioner" of the world, I extracted out of the Primeval Concept the concept K that was crowned as that partitioner. These two constitute the Archimedean point of any world as ordered plurality.

In order to reach that point I built the concept calculus, offering an alternative to the existing predicate calculus. This calculus is based on what I called "mereology of concepts," that was later integrated into the broader manifold theory. This alternative enables a richer expression of parts of sentences, and therefore a greater variety of possibilities to make inferences therefrom. In this calculus the foundations of mathematics also found their place, not as being built on logic but as parts of it. That was achieved by the simple and well-known notion that a quantifier is built on the concept of number. Having done all of these we reached a few important conclusions. In any world whatsoever, excluding the Parmenidean one, which does not belong to this discussion, there is a plurality of objects. Those objects are discriminated through boundaries and those in turn are determined by the concepts that capture them. The concepts determine the partition of the world. All the concepts depend on the foundational concepts. Therefore, any world in which there is plurality necessitates the foundational concepts, the concepts that are the basis of the order of the world. Therefore the world as plurality is necessarily subject to order. Any two objects or more are captured, as a logical necessity, by a single concept, the intersection concept of all the foundational concepts being the Primeval Concept. The Primeval Concept determines the one and all-embracing order of the world of plurality, an order from which no object can diverge because without it, it will not be able to be discriminated as an object. From the Primeval Concept the first partitioner is inferred, that is the concept of concept (K), by whose virtue there is a possibility to partition any world whatsoever.

In the second metaphysic I turned to discuss the various forms of being and their interrelations. I presented the four forms of being and proved the existence of an object (or objects) in each of them, those being: thought (T), language (V), reality (R) and the noumenon (J). I proved that there is a mental object (by the cogito); that there is a linguistic object (by the axiom of linguistic existence); that there is something real (by proof); and that there is something noumenal (by another proof). The objects of each of these exist "within" other objects, which I named pools. The pools of thought are subjects, the pools of words and expressions are languages; the pool of reality, reducible to more basic foundational concepts, space-time. The existence of an object in various forms of being is called "parallelism," and I elucidated the conditions for its existence.

My starting point was the entirety of thought, focusing on its epistemic sides (i.e. without addressing will and emotions). This entirety is built by cognitive sources, which form cognitive functions, which, in turn, create its foundational concepts. These functions work in a "source model" which determines their "division of labour" and the relations between the data produced by them. In the present book I focused exclusively on the functions determining the Western Rational System (WRS), i.e. the basic cognitive tools. The identity of the functions and the division of labour prevailing in each model, including that of WRS, is a result of a "decision," in the sense attached to this word in the present book, i.e. a determination of the subject that, from a logical point of view, could be different (leaving aside any connotation of will or consciousness). In other stages of the process of cognition we find some more moves based on decision, while others are results of logical necessity. The latter were termed "given."

Having elucidated the nature of language as an entirety based on thought but nevertheless enjoying some independence, I turned to discuss reality. In spite of being a primitive, I characterized the real as the form of being providing us with the coherent, ordered, large, enduring and stable (COLES) world, to which all the subjects direct their thoughts as a common arena. Nevertheless, I also accepted the existence of another external world, entirely unrelated to subjects, named the noumenon. I proved its existence to a high level of plausibility, using the boundaries of the subject. By this, I believe, I brought to an end one of the main questions that bothered modern philosophers. By this I demonstrated as well that the boundaries of the ego are also decided, and therefore the ego can be smaller or larger than the way we know it by introspection. The present boundaries, based on the linkage between the ego and a specific body, are the most useful ones. I did not address the extra-metaphysical implications of this conclusion.

These are the foundations of rational metaphysics, i.e. the one created in the framework of the partition of the world through the basic cognitive tools. Other metaphysics are also possible, based on different partitions of the world. Beyond all of these, the worldview standing as alternative to all the other metaphysical worldviews is that of Parmenidean metaphysics, which lacks any partition of the world, and therefore is also devoid of any discriminations, boundaries or concepts. However, the foundations of Parmenidean metaphysics should be written, or be silent therefrom, by mystics, not by whoever seeks to write the foundations of *rational* metaphysics. The rational person, standing in the world of plurality that is subject to a unifying order from which nothing can escape, does not want to make it disappear nor to disappear within it, but rather to know it profoundly and learn one's own place within it. That is the role of the sciences, ranging from the natural sciences to the humanities. But this book is not meant to satisfy that quest, not even to begin doing it, only to lay its intellectual foundations. The rational world is no less fascinating than that of the mystic. It invites us to know it.

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