Consider the following combinatorial market problem:

A seller wishes to sell a set $M$ of $m$ items to $n$ consumers.

Each consumer $i$ has a valuation function $v\_i:2^M \rightarrow \reals^+$ that assigns a

non-negative value $v\_i(X)$ to every subset of items $X \subseteq M$.

The valuation functions can exhibit various combinations of substitutability and complementarity over items. As standard assumptions,

valuations are assumed to be monotone ($v\_i(Z) \leq v\_i(X)$ for any $Z \subseteq X$) and normalized ($v\_i(\emptyset) = 0$).

Each consumer $i$ has a {\em quasi-linear} utility function,

meaning that the consumer’s utility for a bundle $X \subseteq M$ costing $p(X)$ is $u\_i(X,p) = v\_i(X) - p(X)$.

An allocation is a vector $S=(S\_1,\ldots,S\_n)$ of disjointed bundles of items,

where $S\_i$ is the bundle allocated to consumer $i$.

The social welfare of an allocation $S$ is the sum of consumers' values for their bundles,

i.e., $SW(S)=\sum\_{i\in [n]}v\_i(S\_i)$.

An allocation that maximizes the social welfare is said to be socially efficient.

A classic market design problem is setting prices so that socially efficient outcomes arise in “equilibrium”.

Arguably, the most appealing equilibrium notion is that of a Walrasian Equilibrium (WE) \citep{walras1874}.

A WE is a pair of allocation

$S = (S\_1, \ldots S\_n)$ and item prices $p = (p\_1, \ldots, p\_m)$, where each consumer maximizes his or her utility, i.e.,

$$

v\_i(S\_i) - p(S\_i) \geq v\_i(T) - p(T)

$$

for all $T \subseteq [m]$, and the market clears. Namely, all items are allocated.\footnote {More precisely, unallocated items have a price of $0$.}

A WE is a desired outcome, as it is a simple and transparent pricing that clears the market.

Moreover, according to the “First Welfare Theorem”, every allocation that is part of a WE maximizes the social welfare\footnote {Moreover, every allocation that is part of a WE also maximizes welfare over all feasible {\em fractional} allocations \citep{nisan2006communication}.}.

Unfortunately, Walrasian equilibria exist only rarely. In particular, they are guaranteed to exist for the class of “gross substitutes” valuations \citep{kelso1982job}, which is a strict subclass of submodular valuations, and which, in some formal sense, is a maximal class for which a WE is guaranteed to exist \citep{gul1999walrasian}. Given the appealing properties of a WE, it is not surprising that a variety of approaches and relaxations have been considered in the literature in an attempt to address the non-existence problem.

\paragraph{{\bf The endowment effect.}}

The {\em endowment effect}, coined by

%Thaler

\citet{thaler1980toward}, posits that consumers tend to inflate the value of the items they own.

This phenomenon was later validated by experiments, which realized and quantified the magnitude of the effect \citep{knetsch1989endowment,kahneman1990experimental,list2011does,list2003does}.

Today, it is widely accepted that the endowment effect is evident in many markets.

However, the endowment effect has apparently been studied mainly via experiments. Recently, Babaioff, Dobzinski and Oren [\citeyear{babaioff2018combinatorial}] (henceforth, \babaioff) proposed a formal model for studying the endowment effect.

Taking a behavioral economic perspective, their work harnesses the endowment effect in order to extend market stability and efficiency.

In this work, we introduce a new framework that provides a more flexible formulation of the endowment effect, thus enabling us to generalize and extend Babaioff et al.’s work to richer settings.

\paragraph{\babaioff's formulation.}

\babaioff\ propose capturing the endowment effect in combinatorial settings

by formulating an {\em endowed valuation} function.

Given some valuation function $v$, and an endowed set $X \subseteq M$, the endowed valuation function, parameterized by $\alpha$, assigns the following real value to every set $Y \subseteq M$, referred to as the endowed valuation of $Y$ with respect to $X$:

\begin{equation}

\label{eq:endowed-babaioff}

v^{X}(Y) = \alpha \cdot v(X \cap Y) + v(Y \setminus X \mid X \cap Y),

\end{equation}

where $\alpha \geq 1$ is the {\em endowment effect parameter}, and $v(S \mid T) = v(S \cup T) - v(T)$ denotes the marginal contribution of $S$ given $T$ for any two sets $S,T$.

The idea behind this formulation is that the value of items already owned by the agent ($X \cap Y$) is multiplied by some factor $\alpha$, while the marginal value of the other items ($Y \setminus X$) remains intact.

An {\em endowment equilibrium} is then a Walrasian equilibrium with respect to the endowed valuations, i.e., a pair of $S = (S\_1, \ldots S\_n)$ and item prices, $p = (p\_1, \ldots, p\_m)$, where each consumer maximizes his or her endowed utility

$$

v\_i^{S\_i}(S\_i) - p(S\_i) \geq v\_i^{S\_i}(T) - p(T),

$$

and the market clears.

The main result of \babaioff\ is that when consumers' valuations are submodular and $\alpha \geq 2$,

there exists an endowment equilibrium that gives a 2-approximation to the optimal, even fractional, social welfare with respect to the original valuations.

Babaioff et al. also show that the existence result does not extend to the more general class of XOS valuations.

In particular, for every $\alpha>1$, there exists an instance with XOS valuations that does not admit an endowment equilibrium.

The specific function given in Equation~(\ref{eq:endowed-babaioff}) is one way to formulate the endowment effect in combinatorial settings, but is certainly not the only one. For example, supposing a consumer is endowed some set $X$, it is not clear, a priori, how to reevaluate some set $Z \subset X$, subject to the endowment effect. \babaioff\ established a non-existence result for the case where $v(Z)$ is multiplied by some parameter $\alpha$. Can a more flexible formulation of the endowment effect circumvent this impossibility result?

\subsection{A New Framework for the Endowment Effect}

\label{sec:contribution}

In this section we provide a new framework for various formulations of the endowment effect based on fundamental behavioral economic principles. Beyond circumventing impossibility results, our framework seems the optimal way to treat this problem, as there is no single formulation that fits all scenarios.

Specifically, our framework allows for reasoning about different ways of defining the value of a subset $Z$ of an endowed set.

We hope that our work will inspire further discussion regarding meaningful endowment effects in combinatorial settings, as well as experimental work that will shed more light on appropriate instantiations for different scenarios.

A crucial component of our framework is a partial order $\prec$ over endowment effects, which is {\em stability preserving}; i.e., given two endowment effects, $\Endow,\Endow'$, such that $\Endow \prec \Endow'$, a Walrasian equilibrium with respect to the endowed valuations according to $\Endow$, is also a Walrasian equilibrium with respect to the endowed valuations according to $\Endow'$ (Corollary~\ref{cor:strengthKeepEndEq}).

As in previous work, we take a “two-step” modeling approach; i.e., a consumer has a valuation function $v$ prior to being endowed a set $X$, and an {\em endowed valuation} function

 $v^X$ after being endowed a set $X$, which describes the inflation in value due to the endowment effect.

Our framework is based on two basic principles, set forth below.

\paragraph{The “loss aversion” principle.}

The loss aversion hypothesis is presented as part of prospect theory and is argued to be the source of the

endowment effect~\citep{kahneman1990experimental,kahneman1991anomalies,amos1979prospect}.

This hypothesis claims that

\begin{center}

{\em

 people tend to prefer avoiding losses to acquiring equivalent gains.

}

\end{center}

The loss aversion principle can be formulated as follows:

\begin{align} \label{eq:lossBeatsGain}

 v^{X \cup Y}(X \cup Y) - v^{X \cup Y}(Y) \geq v^{Y}(X \cup Y) - v^Y(Y) \lotsOfSpace \forall X, Y \subseteq M

\end{align}.

The left hand side term signifies the loss incurred due to losing a previously endowed set $X$, while the right hand side term signifies the benefit derived from being awarded a set $X$ that was not previously owned.

The loss aversion inequality states that the loss incurred due to losing $X$ is greater than the benefit derived from getting $X$.

\paragraph{The “separability” principle.}

This second principle, proposed by \babaioff, states that

the endowment effect with respect to set $X$ should maintain the marginal contribution of items outside of $X$ intact.

That is, given set $Y \subseteq M$, only the value of items in $X \cap Y$ may be subject to the endowment effect.

This principle is formulated as follows:

\begin{align} \label{eq:externalMarginalSame}

v^X(Y \setminus X \mid X \cap Y) = v(Y \setminus X \mid X \cap Y) \lotsOfSpace \forall Y \subseteq M

\end{align}.

In section~\ref{sec:endEffectFrame} we show that these two principles imply that

the value of set $Y$ for a consumer that is endowed a set $X$ is given by:

\begin{align\*}

 v^X(Y) = v(Y) + g^X(X \cap Y) \lotsOfSpace \forall Y \subseteq M,

\end{align\*}

for some function $g^X : 2^X \rightarrow \reals$, such that $g^X(Z)\leq g^X(X)$ for all $Z\subseteq X$.

The function $g^X$ is referred to as the {\em gain function} with respect to $X$.

It describes the added effect an endowed set $X$ has on the consumer's valuation.

An endowment effect formulation, or, in short, an endowment effect, is then given by a collection of functions

$\{ g^X\}\_{X \subseteq [m]}$ that satisfy the above condition.

An endowment {\em environment} is given by a vector of endowment effects for the consumers $\Endow = (\Endow\_1, \ldots, \Endow\_n)$, where $\Endow\_i$ is the endowment effect of consumer $i$.

We discuss each effect $\{g^X\}\_X$ using the term $g^{X}(Z \mid X\setminus Z)$, the additional loss incurred upon losing a subset $Z$ of an endowed set $X$ due to the endowment effect.

In Definition~\ref{def:dominanceRelationS} we provide a partial order over all endowment effects, based on this loss.

\paragraph{The {\em Identity} and {\em Absolute Loss} endowment effects.}

Let us consider the formulation of \babaioff\ within our framework. The endowed valuation with respect to $X$ is:

 $$

 v^X(Y) = \alpha\cdot v(X\cap Y) + v(Y \setminus X \mid X \cap Y) = (\alpha-1)v(X \cap Y) + v(Y).

 $$.

For the case of $\alpha=2$, which is the case that drives their positive results, this endowment effect can be formulated in our framework by setting $$

v^X(Y) = v(X \cap Y) + v(Y).

$$

In this case, the gain function is defined by $g^X(X \cap Y) = v(X\cap Y)$. Thus, we refer to this endowment effect as the {\em Identity} endowment effect, and denote it by $\Endow^{I} = \{g\_I^X\}\_X$, where $g^{X}\_I = v$.

Note that the additional incurred loss is $g\_{I}^{X}(Z \mid X\setminus Z)=v(Z \mid X\setminus Z)$.

We are now ready to introduce a different endowment effect, that we refer to as the {\em absolute loss} endowment effect.

In this effect, the gain function with respect to an endowed set $X$ is

$$

g\_{AL}^X(Z) = v(X) - v(X \setminus Z).

$$

I.e., $\Endow^{AL} = \{g^X\_{AL}\}\_X$.

For this effect, it holds that the additional incurred loss is $g\_{AL}^{X}(Z \mid X\setminus Z)=v(Z)$.

For subadditive consumers,

this effect demonstrates a “stronger” loss aversion bias than Identity endowment effect with respect to the relation $\prec$, defined in Definition~\ref{def:dominanceRelationS}.

\of{Consider adding the definition of $\prec$.}

\begin{proposition} \label{prop:ALdominatesI}

 For a consumer with a subadditive valuation $v$, it holds that

 $\Endow^{I} \prec \Endow^{AL}$.

\end{proposition}

Intuitively, it can be imagined that in the absolute loss effect, a consumer amplifies the loss of a subset $Z$ of an endowed set $X$ by “forgetting” the fact that $X \setminus Z$ remains in the consumer's hands.

\subsection{Existence of Equilibria and Welfare Approximation}

In this section we present our existence and approximation results.

Our approximation results hold with respect to the optimal welfare according to the {\em original} valuations, and even with respect to the optimal fractional allocation.\footnote{

Note that according to the First Welfare Theorem, an endowment equilibrium always gives the optimal welfare with respect to the endowed valuations.}

Recall that \babaioff\ prove that for the Identity endowment effect,

every market with submodular consumers admits an $\Endow^{I}$-endowment equilibrium that gives a $2$-approximation welfare guarantee.

For the larger class of XOS consumers, \babaioff\ show that an endowment equilibrium may not exist even with respect to an endowment effect $\alpha \cdot \Endow^{I} = \{\alpha \cdot g : g \in \Endow^{I} \}$ for an arbitrarily large $\alpha$.

This negative result may lead to the conclusion that while the endowment effect improves stability for submodular valuations, XOS markets may remain unstable even with respect to endowed valuations.

However, we show that this negative result is an artifact of the specific formulation chosen by the authors.

As established in the following theorem, the stronger Absolute Loss endowment effect leads to existence and approximation results for markets with XOS valuations.\footnote{

Note that “stronger” here is not in the sense of an increased value of $\alpha$. Indeed, no finite $\alpha$ suffices for such result.}

\vspace{0.1in}

\noindent

{\bf Theorem 1.} [Theorem.~\ref{thm:existenceXOSabsolute}]

There exists an algorithm such that every market with XOS consumers and every initial allocation $S' = (S'\_1, \ldots, S'\_n)$

returns an $\Endow^{AL}$-endowment equilibrium $(S, p)$, such that $SW(S) \geq SW(S')$.

\vspace{0.1in}

The algorithm is a variant of the algorithm used by \cite{fu2012conditional,christodoulou2016bayesian}.

A direct corollary of Theorem~1 is that for every market with XOS consumers, every optimal allocation $S$ can be paired with item prices $p$, so that $(S, p)$ is an $\Endow^{AL}$-endowment equilibrium.

\of{Moreover, we show that every $\Endow^{AL}$-endowment equilibrium guarantees $1/2$ of the optimal welfare.}

\vspace{0.1in}

\noindent

{\bf Theorem 2.} [Theorem.~\ref{thm:twoApx}]:

 Every $\Endow^{AL}$-endowment equilibrium gives at least $1/2$ of the optimal welfare.

\vspace{0.1in}

The theorem above shows that a stronger endowment effect enables the extension of the equilibrium existence (and approximation) result from submodular valuations to XOS valuations. Can this result be extended further?

One answer, although unsatisfactory, is yes! For example, consider an endowment effect that inflates the value of a set linearly with its size; e.g., $\Endow^{PROP} = \{ g^X(Z) = \cardinality{Z} \cdot v(X) : X \subseteq M \}$.

We show in Section~\ref{sec:subadditive} that this effect leads to a sweeping equilibrium existence guarantee for arbitrary valuations.

Moreover, every optimal allocation can be paired with item prices to form an $\Endow^{PROP}$-endowment equilibrium (Proposition~\ref{prop:EEalwaysopt}). While this sounds like a strong result, this effect inflates the value in the set's size linearly, which may be as large as $\Omega(m)$. We believe that such inflation is unreasonably high and completely misconstrues the endowment effect.

Can we attain a general positive result with a “reasonable” inflation? In Section~\ref{sec:subadditive} we show that for any endowment effect with inflation up to $O(\sqrt{m})$, an endowment equilibrium may not exist for (the strictly-larger-than XOS valuations) subadditive valuations (Proposition~\ref{prop:noEndowSubadditive}).

\subsection{The Power of Bundling}

We next study the power of bundling in settings with endowed valuations.

A bundling $B = \{B\_1, \ldots, B\_k\}$ is a partition of the set of items $M$ into $k$ disjoint bundles.

A {\em competitive bundling equilibrium} (CBE) \citep{dobzinski2015welfare} is a bundling $B$ and a Walrasian equilibrium in the market induced by $B$ (i.e., the market where $B\_1, \ldots, B\_k$ are the indivisible items).

It is easy to see that a CBE always exists. For example, bundle all items together and assign the entire bundle to the highest value consumer for a price of the second highest value.

However, while the WE notion reflects the first welfare theorem, guaranteeing that every allocation supported in a WE gives optimal welfare, no such welfare guarantee applies with respect to CBE \citep{feldman2014clearing,feldman2016combinatorial,dobzinski2015welfare}.

In this paper we introduce the notion of $\Endow$-endowment CBE, which is a CBE with respect to the endowed valuations, and provide algorithms for computing $\Endow$-endowment CBEs with good welfare for any endowment effect $\Endow$ satisfying a mild assumption.

\paragraph{{\bf Equilibrium computation.}}

\babaioff\ showed computational barriers regarding computing $\Endow^{I}$-endowment equilibria,

and raised the following question

(recall that $\alpha \cdot \Endow^{I}$ denotes the endowment effect that multiplies each gain function $g \in \Endow^{I}$ by $\alpha$):

\vspace{0.1in}

{\em

Are there allocations that can be both efficiently computed and paired with item prices that form an $\alpha \cdot \Endow^{I}$-endowment equilibrium for a small value of $\alpha$?

}

\vspace{0.1in}

The analogous question with respect to CBE and a particular endowment effect $\Endow$ would be: Are there allocations that can be both efficiently computed and paired with bundle prices that form an $\Endow$-endowment CBE? It doesn't take long to conclude that this problem is trivial for any endowment effect with non-negative gain functions. Simply, allocate all items to the highest value consumer for the entire bundle. The interesting problem here would be to compute a nearly-efficient CBE, rather than just any CBE\footnote {This is consistent with the literature on CBE, which has focused on the existence and computation of nearly-efficient CBEs \cite{dobzinski2015welfare,feldman2014clearing}.}, and can be formulated as follows:

\vspace{0.1in}

{\em

Are there approximately optimal allocations that can be both efficiently computed and paired with bundle prices that form an $\Endow$-endowment CBE for some natural endowment effect $\Endow$?

}

\vspace{0.1in}

Note that for $\alpha \cdot \Endow^{I}$-endowment equilibrium, the two problems coincide, as any $\alpha \cdot \Endow^{I}$-endowment equilibrium gives $\alpha$ approximation to the optimal social welfare.

We offer the following positive results, which essentially provide a black-box reduction from the problem of computing approximately optimal endowment CBE for significant endowment effects to the classical algorithmic problem of welfare approximation. This result applies to every {\em significant} endowment effect, where the gain functions satisfy $g^{X}(X) \geq v(X)$ for all $X \subseteq M$.

For example, one can easily verify that $\Endow^I$ and $\Endow^{AL}$ are significant with respect to all consumer valuations.

\vspace{0.1in}

\noindent

{\bf Theorem [Black-box reduction for endowment-CBE]}

\begin{enumerate}

 \item{}

 [Thm. \ref{thm:SM\_end2NoGap}]

 \quad

 {\em

 There exists a polynomial algorithm such that \underline{submodular} valuations, and every significant endowment effect $\Endow$ and initial allocation $S' = (S'\_1, \ldots, S'\_n)$,

 compute an $\Endow$-endowment CBE $(S, p)$, such that $SW(S) \geq SW(S')$.

 The algorithm runs in polynomial time using \underline{value} queries.

 }

 \item {} \label{main\_CBE\_thm\_item}

 \noindent

 [Thm. \ref{thm:GV\_end2NoGap}]

 \quad

 {\em

 There exists a polynomial algorithm such that \underline{general} valuations, and every significant endowment effect $\Endow$ and initial allocation $S' = (S'\_1, \ldots, S'\_n)$,

 compute an $\Endow$-endowment CBE $(S, p)$, such that $SW(S) \geq SW(S')$.

 The algorithm runs in polynomial time using \underline{demand} queries.

 }

 \vspace{0.1in}

\end{enumerate}

\vspace{0.1in}

\noindent

The proof of item~\ref{main\_CBE\_thm\_item} in the theorem above implies the following corollary:

\vspace{0.1in}

\noindent

{\bf Corollary}

 [Corr.~\ref{corr:OPT\_CBE}]

 For every market, and significant endowment effect $\Endow$, any optimal allocation $S$ can be paired with bundle prices $p$, so that $(S, p)$ is an $\Endow$-endowment equilibrium.

\vspace{0.1in}

We note that this result cannot be extended to all endowment effects within our framework.

In particular, for endowment effects such those for some $\beta < 1$, it holds that $g^{X}(X) \leq \beta \cdot v(X)$ for all $X \subseteq M$, there are instances that admit no endowment CBE with optimal welfare, already for XOS valuations (Proposition~\ref{prop:XOS\_SA\_noApx}). For this subclass of endowment effects, we provide approximation lower bounds as a function of the parameter $\beta$, for different classes of valuations (including XOS, subadditive, and arbitrary; see Section \ref{sec:bundles}).

\subsection{Comparison to Related Work}

Our work builds upon the recent work by \cite{babaioff2018combinatorial} that proposed the

first formulation for the endowment effect in combinatorial auctions.

They show that every market with submodular valuations admits an $\Endow^{I}$-endowment equilibrium that gives at least half of the optimal social welfare.

Other relaxations of WE have been considered in the literature in an attempt to address the non-existence problem of WE, and achieve approximate stability and efficiency for more general valuation classes than gross substitutes.

\cite{fu2012conditional} considered a relaxed notion of WE, termed {\em conditional equilibrium}. A conditional equilibrium is a pair of an allocation and item prices satisfying individual rationality, and such that no consumer wishes to expand his or her allocation, but disposal of items is not allowed. They showed that every conditional equilibrium has at least half of the optimal welfare. Moreover, every market with XOS valuations admits a conditional equilibrium, which can be reached via a “flexible ascent auction”, an algorithm proposed by

\cite{christodoulou2016bayesian}.\footnote {Our results imply that their approximation guarantee applies also with respect to the optimal fractional social welfare. (To the best of our knowledge, this was not previously known.)}

A different relaxation of WE was considered by \cite{feldman2015welfare}, where the utility maximization condition is preserved, but market clearance is relaxed (i.e., items with positive prices may be unsold). Using this notion, an equilibrium always exists (for example, price all items at some prohibitively large price), but such equilibria carry no approximation guarantees. For this notion, it is shown that even for simple markets with two submodular consumers, the social welfare approximation guarantee cannot be better than $\Omega(\sqrt{m})$.

Our results on endowment CBE (Section~\ref{sec:bundles}) should be compared with previous notions of bundling equilibria~\citep{feldman2014clearing,feldman2016combinatorial,dobzinski2015welfare}.

In these settings, the market designer first partitions the set of items into indivisible bundles $B = \{B\_1, \ldots, B\_k\}$ (these are the indivisible items in the induced market), and assigns prices to these bundles instead of to the original items,

and a CBE is a Walrasian equilibrium in the induced market.

\cite{dobzinski2015welfare} showed that every market (with arbitrary valuations) admits a CBE that gives an approximation guarantee of $\tilde{O}(\sqrt{\min\{m, n\}} )$. Moreover, given an optimal allocation, a CBE with such approximation can be computed in polynomial time. Furthermore, they provide a polynomial time algorithm that computes a CBE with a $\tilde{O}(m^{2/3})$ approximation guarantee.

This should be compared to Corollary~\ref{corr:OPT\_CBE} and Theorem~\ref{thm:GV\_end2NoGap} in this paper.

Corollary~\ref{corr:OPT\_CBE} shows that for a wide variety of endowment effects, including the one considered by \babaioff, there always exists an endowment CBE that gives the {\em optimal} welfare.

Theorem~\ref{thm:GV\_end2NoGap} shows that for a wide variety of endowment effects, including that of \babaioff, given an arbitrary allocation $S$, one can compute an endowment CBE with (weakly) higher welfare than $S$ in polynomial time.

Thus, the problem of computing nearly-efficient endowment CBEs is effectively reduced to the pure algorithmic problem of welfare approximation: a problem addressed by a vast amount of literature (e.g.,

\citep{dobzinski2005approximation,lehmann2006combinatorial,dobzinski2006improved,feige2006approximation,feige2009maximizing,feige2013welfare,chakrabarty2010approximability}).

A different notion of bundling equilibria was considered by

%Feldman {\em et al.}

\cite{feldman2016combinatorial}. This notion is a relaxed version of CBE, where some bundles (with positive prices) may remain unsold. Using this notion, for arbitrary valuations, given an arbitrary allocation $S$, one can compute an equilibrium with welfare at least half of the welfare of $S$ in polynomial time.

All the notions above consider a concise set of bundles,

a price for each bundle,

and an additive pricing over sets of bundles.

More general forms of bundle pricing, including non-linear and non-anonymous pricing, lead to welfare-maximizing results, but are highly impractical. In particular, they use an exponential number of prices \citep{bikhchandani2002package,parkes2000iterative,ausubel2002ascending,lahaie2009fair,sun2014efficient}.

\subsection{Summary}

We propose a general principle-based framework for studying the endowment effect in combinatorial markets.

We provide both existence and efficiency guarantees of endowment equilibrium (as defined by \babaioff) for a wide range of endowment effects and

consumer valuation classes.

Our main results are:

(1) There exist natural endowment effects for which an endowment equilibrium exists for XOS consumers; these equilibria guarantee $2$-approximation to the optimal welfare.

In contrast, we show that for subadditive consumers, any endowment effect that inflates at a “reasonable” rate does not suffice to guarantee endowment equilibrium existence.

%

(2) For any significant endowment effect, when allowing the seller to pre-pack items into indivisible bundles (thus turning to CBE), given any initial allocation, one can efficiently compute an endowment CBE with somewhat higher welfare. This result implies that every market admits an optimal endowment CBE. More importantly, it reduces the problem of computing an endowment CBE to the pure algorithmic problem of welfare approximation.