Salomon Maimon and the Regular Decahedron

How far was Salomon Maimon prepared to take his rationalism? His position is generally described as a return to Leibniz via Kant, a description supported by Maimon’s own summation of his system as a coalition of philosophies that included Leibniz among them as one layer. While this overarching question, which contains both a historical and a philosophical element, is not our concern here, Maimon’s writings on the decahedron raise the question of his position. Samuel Hugo Bergman already attempted to answer this question; below I will propose my own solution, which differs from Bergman’s. If this answer is accepted, it will provide us with a case in which Maimon was more radical than Leibniz in his rationalism.[[1]](#footnote-1) First, however, it is worth making a number of general notes on the principle of determinability (*Satz des Bestimmbarkeit*) in Maimon’s system, which will help us in sharpening the question.

**Background: Determinability and Creativity**

The centrality of the principle of determinability to Maimon’s system is indisputable. In order to provide a brief overview of its role, it is worth discussing the following unequivocal claim by Maimon: “[T]he *principle of determinability* laid down in this work is a principle of all *objectively real* thought, and consequently of *philosophy* as a whole too. All the propositions of philosophy can be derived from, and be determined by it.”[[2]](#footnote-2) This principle, which appears in various articulations, is reminiscent of the Aristotelian concept of genus and species and is also found in Kant’s writings; most important for our purposes, however, is its centrality to Maimon’s critique of Kant’s system. Bergman’s elaboration on the principle of determinability appears in Chapter V of his book.[[3]](#footnote-3)

For readers less familiar with Maimon’s system, it will suffice to say that determinability is the conceptualization of the relationship between a subject and a predicate. The case is one of a synthesis of concepts, one of which, the determinable, cannot be conceived of without the determinate, though the determinate can be conceived of without this particular determinable. The example Maimon tended to bring was that of the synthesis of a “straight line.” The term “straight” is inconceivable without the concept of a “line” – while a line can be conceived of without the concept of “straight,” for example in the synthesis of a “curved line.” An objective synthesis, in Maimon’s terms, requires that at least one of the component terms cannot be conceived of without the other; if it can be, then the synthesis is considered arbitrary.

Maimon’s position vis-à-vis Kant finds various forms of expression, but one is particularly interesting for our concerns. The *a priori* synthetic cognition, in his system, is not limited to a particular form of recognition – rather, it has a creative element:

Kant perceived synthetic cognition in the narrow sense. For him, cognition is synthetic if its object is not included in its subject, and it is based on construction: … In my terminology, both analytic judgments and synthetic judgments expand our cognition… analytic judgments [e.g. a triangle has three angles – M.B.] expand our cognition about a given object, whereas synthetic judgments (e.g. a triangle can be determined to be a right angle – M.B.] expand our cognition by determining a new object. (Logik, p. 29)

We are not speaking here of a mere change in terminology. Objects, according to Kant, assume existence through observation: without observation, there can be no object.[[4]](#footnote-4) In articulating his own definition, Maimon added his insight that objects are formed by syntheses acting in accordance with the principle of determinability.[[5]](#footnote-5)

This position of Maimon’s is fundamental to his system of thought and central to his critique of Kant. Maimon’s question regarding the nature of the relation between a category of the intellect and time as a form of perception in Kant’s thought can be answered only if we assume that observation is not autonomous but rather an expression of mental connections. It is worth noting that the concept of determinability can aid in the articulation of the question. The relationship between the concept of causality and the concept of time is like that between the mental concept of “seven” and the sensory concept of “sweet,” and so is unintelligible. Such categories apply to phenomena because the latter are not autonomous – at their basis stand intellectual objects, the differentials. In other words, the necessity of observation is not an absolute demand of the concept of the object. This is the point at which Maimon does return to Leibniz, as described in the book . Indeed, we are combining in Maimon’s system his statement that in the realm of mathematics we resemble God, along with his position that numbers are a product of the intellect that establishes the relationship between them, thus providing a place for the creativity of the intellect. The creation of an object according to the principle of determinability, then, is not at all surprising, but is rather a natural extension of Maimon’s fundamental positions. With this overview, we are ready to address the question of the regular decahedron.

**The Question of the Decahedron**

Regular polyhedrons are entirely symmetrical forms: all the faces are themselves regular polygons that meet at their edges; the number of sides is identical from every point, and all the sides are identical in length. The number of possible regular polyhedrons has been known for some time and was put to philosophical use in Plato’s “Dialogue of Timaeus.” As Euclid demonstrated, the number stands at five: the pyramid (tetrahedron), cube, octahedron, dodecahedron, and the icosahedron. Note that the regular decahedron is not on the list.

If this were a simple contradiction, as is the case of the round square, then a rationalist bound by the law of noncontradiction could resolve it merely by saying that there is no such thing as a decahedron, for such a form is as much of a logical impossibility as a round square. However, there is no obvious contradiction in the idea of the decahedron, and it meets the requirements of the principle of determinability – as such, its inexistence raises a question for Maimon’s concept of the synthetic. Bergman addressed this question against the background of the relationship of mental objects, which do not require observation, to objects that do require observation-construction and thus exist in relation to cognition:

We find in Maimon a certain hesitation with respect to criteria that make a distinction between thought and cognition. In one place (V., p.131) he relies only on the criteria of determinability, and in another only on that of construction. …Nevertheless, it is impossible to rely solely on the relation of determinability in creating a possible concept in the positive sense as is shown by the regular decahedron. ( )

He then raises the possibility of finding assistance in the construction of the object: “The example of the decahedron shows us then that positive possibility requires besides the relation of determinability the proof of the possibility of construction” (ibid.). However, Bergman understands the problem in Maimon’s resolution: “But what is the meaning of construction for the radical rationalist who recognizes no criterion except that of the understanding?” (p. 178). Bergman then attempts to answer his question by appealing to the distinction between consistency of an object’s definition when viewed in isolation and consistency of the same definition when we require it to be in harmony with “geometry as a whole.”[[6]](#footnote-6) He then offers an original speculation on the role of construction in Maimon's system: “Constructions rectify, as it were, the error that was made when we removed an isolated entity, such as the decahedron, from the *entire* context and analyzed its conceptual structure without considering the relations that connect this body to all other bodies in our system” (p. 178).

With this passage Bergman adds to Maimon’s explanation; thus far, there is no opposition. Part of the role of the interpreter is to offer solutions to problematic elements of the text or system of though being interpreted. Given Maimon’s ambivalence towards the role of observation, there is nothing problematic in the role assigned to it by Bergman. (See the proof of observation ???.) However, the question that arises is if this mechanism actually resolves the problem in Maimon’s system. Construction shows the concreteness of the constructed object, as well as its integration into the realm of all constructed things, as evidenced by Maimon’s own writings. The question is, however, if the opposite holds true: does the impossibility of construction prove indeterminability?

I will set aside (for now) Bergman’s question and any of its inherent problems; rather, I intend to offer another solution to the question, one based on a hint offered by Maimon in a different discussion on the decahedron. In the middle of his *Gibeat Hamoreh*, Maimon writes:

Although the concept of a decahedron, i.e., a regular polyhedron with ten faces, is possible with respect to its form, since the condition of congruent faces does not contradict their number ten, it is impossible to find such a concept in intuition…. Thus, even if the act of thought is not limited to the sensible world, our cognition is necessarily confined to what can be represented to our senses. That is why he (Maimonides) writes, “Do not criticize the words used in this chapter and in others in reference to the mind,” where he says that human understanding has its limits. This is indeed problematic, since understanding has no boundaries.[[7]](#footnote-7) (*Gibeat Hamoreh*, p. 64)

Maimon points out here that in his opinion there is no contradiction in the concept of the decahedron. The human intellect is not limited to the empirical world, though our cognition is. The continuation of the discussion hints at something deeper, leading to a conclusion that transcends the boundaries Maimon has just laid out. “Understanding has no boundaries” –

Maimon's last sentence contains the seeds of a radical interpretation of his hero Maimonides. Contrary to the interpretations of Maimonides that followed Shlomo Pines and viewed Maimonides’ secret doctrine as skeptical, Maimon offers a view according to which the idea that understanding has its limits is not the true doctrine of Maimonides. The decahedron is evoked here in order to express the view of radical rationalism that he ascribes to Maimonides – the fact that we cannot construct the decahedron does not render it absolutely impossible; it is an object of pure thought that cannot be given an intuition. What’s more, I would suggest that even this limitation is a subjective one. Maimon naturally identifies with his radical reading of Maimonides and in fact reads his own view, expressed in Kantian language, into the *Guide of the Perplexed*.

This being the case, the decahedron leads us to another passage critical to Maimon’s interpretation of Maimonides, which helps establish his philosophical position as a whole.[[8]](#footnote-8)

**3. Expansion**

Not only does this solution dovetail with Maimon’s rationalism, it also suits other theses he proposes, the most relevant in this context being his thesis on ideas; Kant’s concept of ideas of reason catches Maimon’s interest in part because Kant challenged him precisely on this point. Maimon’s response is widely taught in philosophical literature, though here we will only note that Maimon takes Kant’s notion of ideas much farther than Kant himself did, finding examples in mathematics and physics. From this we can learn that he did not see these ideas as existing outside of science but rather as playing a specifically scientific role. Maimon makes this unequivocally clear in his explication of this position: that which cannot be observed by us as a finite object can be seen as an object by the infinite intellect (p. 228). Though this is not the place to explain Maimon’s understanding of Maimonides, we can say that this position fits well with Maimon’s gloss on the *Guide of the Perplexed*. Proof by observation is subjective. Since, in his opinion, there is no contradiction in the decahedron, and since the synthesis conforms to the principle of determinability, the inability to construct the regular decahedron as an object does not rule out the possibility of its existence.

Further support for my interpretation can be found in Maimon’s writings on the infinite circle. Maimon distinguishes between ideas of the mind and ideas of reason. An idea of reason is an infinite concatenation of concepts that cannot be grasped all at once, while an idea of the mind is a finite concatenation which remains an idea due to the fact that it has no observable manifestation.

Indeed, while rejecting evidence for the claim that one infinity cannot be larger than another, Maimon refers to the form under discussion: a circle with a radius of infinity. Maimon is prepared to grant reality to such an entity and does not dismiss its existence merely because it exceeds our perceptual limits.[[9]](#footnote-9)

Luckily for Maimon, Leibniz’s writings contain a direct reference to the regular decahedron:

Essence is basically just the possibility of the thing in question. Something that

is thought possible is expressed by a deﬁnition; but if this deﬁnition doesn’t at the same time express this possibility then it is merely nominal, leaving us to wonder whether the definition expresses anything real — i.e. possible — until experience helpfully acquaints us *a posteriori* with the thing’s reality — if it does actually occur in the world. (We can settle for this way of knowing in cases where reason can’t acquaint us *a priori* with the reality of the deﬁned thing by exhibiting its cause or the possibility of its being generated.) (Third Book, Chapters 3 &3).[[10]](#footnote-10)

Leibniz does not distinguish between a decahedron and a concept such as a “square circle”; he does not hold a position like Maimon’s regarding the creativity of construction according to the principle of determinability. He later elaborates:

Leibniz requires that a synthesis be made possible by reason:

So it isn’t within our discretion to put our ideas together as we see fit, unless the combination is justified either by reason showing its possibility or by experience showing its actuality and, hence, its possibility. (Ibid.)

It is interesting to consider for a moment the difference between Maimon and Leibniz. We cannot criticize Maimon’s regular decahedron by claiming it to be an arbitrary assembly of concepts, since its assembly obeys the principle of determinability. Rather, it seems to me that their essential disagreement is regarding how to understand the evidence for the apparent fact that the regular decahedron cannot take material form in space. Leibniz understands his proof for its immateriality as necessitating the inconsistency of the decahedron. Maimon, of course, accepts this essentially mathematical claim, but only as it applies to the decahedron’s existence in three-dimensional space – a form of existence that Maimon’s reality does not require.

In this respect, Maimon was more radical than Leibniz, a statement that raises a question for the latter: why place such importance on space, when according to Leibniz himself space is not an actual substance but only the relationship between objects, and in fact other spaces may exist ( ). In order to maintain his position, Leibniz must accept some form of the principle of plenitude: that which cannot be made manifest in space contains an inherent contradiction. This is not so applicable to Leibniz, who speaks of substances; moreover, the principle of plenitude according to Leibniz is derived from the principle of sufficient cause, among others, and from the claim that the world is the best of all possible worlds (in one of its expositions). The question of whether or not it is possible to respond in Leibniz’s name, aided by an expansion of the principle of plenitude, should interest every rationalist who rejects the existence of the decahedron in any form, as we see from Leibniz’s statements.

We should not conclude without noting that this topic also leads us to the difference between Maimon’s understanding of differentials and that of Leibniz. Leibniz rejects the possibility of infinity, and along with it the possibility of differentials, which are so necessary for Maimon’s system:

The true infinite, strictly speaking, is only in the absolute, which is more basic than any composition and isn’t formed by the addition of parts… But it would be a mistake to try to suppose an absolute space that is an infinite whole made up of parts. There is no such thing: it is a notion that implies a contradiction; and these infinite wholes, and their opposites the infinitesimals, are like imaginary roots in algebra, having no place except in calculations. (Second Book, Chapters 17 &3)

For Maimon, differentials are not only needed for mathematical equations, they are the foundation of perception.[[11]](#footnote-11) The fact that there is no form of observation by which to perceive them only further legitimizes their status as a cornerstone of Maimon’s system. From another angle, it also illustrates the difference between Maimon and Leibniz.

**Summary**

This treatment cannot provide a full picture of the complex relationship between Maimon and Leibniz, but I believe it can contribute to such a comparison. Moreover, although here I have discussed the rather bizarre case of the regular decahedron, through it we have arrived at the fundamentals of rationalism.

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1. The matter is of importance even independently of its relationship to Leibniz. Husserl adopted a position similar to Leibniz’s (LI, I&15); on this point, Maimon’s position is noticeably different. [↑](#footnote-ref-1)
2. Salomon Maimon, *Gesammelte Werke*, V 368, in *Between Kant and Hegel: Texts in the Development of Post-Kantian Idealism*, George di Giovanni and H.S. Harris (trans. & ed.) (Albany: State University of New York Press, 1985), 165. [↑](#footnote-ref-2)
3. There are other roles [in addition to what?]. See for example the derivation of the distinction between subject and object, [and??] the logical relationship at the foundation of object and event [unclear]. The introductory paper at/in [??] Stanford [what is this referring to?] includes a concise articulation of this point. [↑](#footnote-ref-3)
4. Setting aside for the moment the intricate interpretation of Kant's arithmetic, this remains to a certain extent a central theme of Kant. [↑](#footnote-ref-4)
5. The Stanford paper [??] misses the point made here that not only the concept but the object itself is created. [↑](#footnote-ref-5)
6. Lachterman (“Mathematical Construction, Symbolic Cognition and the Infinite Intellect: Reflections on Maimon and Maimonides,” *Journal of the History of Philosophy* 30 [1992]: 497-522) brings Maimon’s treatment of the matter from another source .@@@. Though he compares Maimon to Maimonides in the realm of mathematics, he neither cites Maimon’s discussion from *Gibeath Hamoreh* nor even mentions the problem Bergman raises. In another note he explains the passage by suggesting a typographical error, emending decahedron to dodecahedron. He also states, erroneously, that Kant discussed the topic in his book, when in fact Kant’s discussion was of the regular hexagon. [↑](#footnote-ref-6)
7. However, an equally rich picture of matter [??], that is to say, a limited matter with ten equal surfaces, such a shape being possible, since the condition of the sides being equal does not contradict the condition of their being ten sides; nevertheless, such a shape is impossible from the point of view of the observer, that is to say, it is impossible to find a limited form of ten equal sides. Though the thought itself is unlimited, nevertheless the act of observing it is certainly limited, and such is true in every case. To this point the Master [i.e. Maimonides] said, “Do not criticize the words used in this chapter and in others,” that is to say, you should not be too critical [literal ??] in reading the Master’s words when he writes that the human intellect is certainly limited, and do not be troubled by [try to refute ??] it. For the intellect itself has no limit, as I explained… (*Gibeat Hamoreh*, I, 32, p. 64). [↑](#footnote-ref-7)
8. If this interpretation is true, it gives us another example of the connection between Maimon's philosophy and modern mathematics. Four-dimensional spheres, the Klein bottle and the polyhedral in hyperspace are legitimate objects, and the fact that we cannot construct them is to be seen as a factor of three-dimensional space, not as an objective difficulty. We should not characterize this connection with modern mathematics as an anticipation of the latter but should rather study the reasons behind Maimon's hesitations and tensions. What was the discovery that enabled the unequivocal acceptance of spatial objects that cannot be constructed in space as we know it? [↑](#footnote-ref-8)
9. Here is the relevant passage:

   This argument is fallacious, for it is possible that one infinity will be larger than another infinity, and at the same time, parts will not be equal to wholes. Let us assume a circle whose radius is an infinite line. Here the circumference is always greater than the radius even though both are infinite. Yet this does not entail that parts are equal to wholes. For it is impossible that the radius will be equal to the circumference, in such a way that a finite number of radii and their parts give the circumference, for we have proved that they are incommensurable. (*Gibeat Hamoreh*, p. 150) [↑](#footnote-ref-9)
10. In the abridged version of the translation of *New Essays*, the passage on the decahedron has been omitted. [↑](#footnote-ref-10)
11. For an updated treatment of Maimon’s theory of differentials and their relationship to observation see Simon Dufy, “Maimon’s Theory of Differentials as the Elements of Intuitions,” *International Journal of Philosophical Studies* 22, no. 2 (2014):1-20. [↑](#footnote-ref-11)