

## QUESTION 1 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

Compute the first three moments,  $m_1, m_2$ , and  $m_3$ , of the data set  $\{-1, 0, 1, 2\}$ . Report the answer to one decimal place.

$$m_1 = (-1 + 0 + 1 + 2) / 4 = 2 / 4 = 0.5 \text{ (2 points)}$$

$$m_2 = ((-1)^2 + (0)^2 + (1)^2 + (2)^2) / 4 = 6 / 4 = 1.5 \text{ (2 points)}$$

$$m_3 = ((-1)^3 + (0)^3 + (1)^3 + (2)^3) / 4 = 8 / 4 = 2 \text{ (2 points)}$$

## QUESTION 2 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

If  $X$  follows a geometric distribution,  $\text{Geometric}(p)$ , then  $E[X] = 1/p$ . Suppose that the observed data  $\{1, 2, 2, 3\}$  comes from a Geometric distribution with unknown  $p$ . Show how to compute the estimate of  $p$  using the method-of-moments. Round your answer to two decimal places.

$$1/p = (1 + 2 + 2 + 3) / 4 = 3.75 \text{ (3 points)}$$

$$p = 1 / 3.75 = 0.27 \text{ (3 points)}$$

## QUESTION 3 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

Consider an independent random sample  $\{X_1, X_2\}$  from  $\text{Binomial}(p)$ . Use the likelihood factorization criterion to show that  $U = X_1 + X_2$  is a sufficient statistic for estimating  $p$ .

$$\text{The likelihood is } L(p) = p^{X_1} (1-p)^{(1-X_1)} p^{X_2} (1-p)^{(1-X_2)} = p^{(X_1+X_2)} (1-p)^{(1-(X_1+X_2))} \text{ (3 points)}$$

$$L(p) = g(u, p) h(X_1, X_2) \text{ where } g(u, p) = p^u (1-p)^{(1-u)} \text{ and } h(X_1, X_2) = 1. \text{ (3 points)}$$

## QUESTION 4 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

Let  $\{X_1, X_2\}$  be an independent sample from a Gaussian distribution with unknown mean  $m$  and standard deviation 1.

- (a) Write down the likelihood function  $L(m)$
- (b) Write down the log-likelihood function  $LL(m)$ .

(a)  $L(m) = \frac{1}{\sqrt{2\pi}} \exp(-1/2 * (x_1 - m)^2) * \frac{1}{\sqrt{2\pi}} \exp(-1/2 * (x_2 - m)^2) = \frac{1}{(2\pi)} \exp(-1/2((x_1 - m)^2 + (x_2 - m)^2))$  (3 points)

(b)  $LL(m) = -\log(2\pi) - 1/2((x_1 - m)^2 + (x_2 - m)^2)$  (3 points)

## QUESTION 5 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

Let  $\{1, 2\}$  be an independent observed sample and exponential distribution with unknown rate  $r$ ,  $\text{Exp}(r)$ .

- (a) Write down the likelihood function  $L(r)$ .
- (b) Write down the negative log-likelihood function  $NLL(r)$ .
- (c) Find the MLE estimate of  $r$  by minimizing  $NLL(r)$ .

(a)  $L(r) = r \exp(-r * 1) * r \exp(-r * 2) = r^2 \exp(-3r)$  (2 points)

(b)  $NLL(r) = -2 \log(r) + 3r$  (2 points)

(c) The derivative of  $NLL$  is  $NLL'(r) = -2/r + 3$ . The zero of this derivative is given by  $-2/r + 3 = 0$  or  $r/2 = 1/3$  so zero is  $r = 2/3$ . The second derivative of  $NLL$  is  $NLL''(r) = 2/r^2 > 0$  so  $r = 2/3$  is indeed a minimizer. The MLE is  $r = 2/3$ . (2 points)

## QUESTION 6 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

Consider the independent observed sample  $\{(1,1),(2,2),(3,5)\}$ . Assuming the model which relates  $(x,y)$  is  $f(x)=2x+a$ .

- What is the sum-of-square residuals in the OLS problem in terms of  $a$ ?
- Find the OLS estimate of  $a$ .

(a) We have  $f(1)=2+a$ ,  $f(2)=4+a$  and  $f(3)=6+a$ . The squared residuals are  $(2-(2+a))^2$ ,  $(4-(4+a))^2+(5-(6+a))^2$ , which simplify to  $a^2$ ,  $a^2$ , and  $(a^2-2a+1)$ . The sum of squared residuals is  $3a^2-2a+1$ . (3 points)

(b) To get the OLS estimate we need to minimize this quantity. Set  $C(a)=3a^2-2a+1$ , its derivative is  $C'(a)=6a-2$  and the zero of this function is  $a=1/3$ . The second derivative is  $C''(a)=6>0$  so  $a=1/3$  is indeed the minimizer of this function. Therefore,  $a=1/3$  is the OLS estimate. (3 points)

## QUESTION 7 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 01

A certain sample admits three delete-1 jackknife samples. The values of a certain test statistic of these jackknife samples are  $m_1=4$ ,  $m_2=5$ ,  $m_3=6$

- What is the jackknife estimate,  $m$ -jack, of this statistic?
- What is the standard error of the jackknife estimate?

(a) The jackknife estimate is  $m$ -jack =  $(4+5+6)/3=5$ . (3 points)

(b) The standard error, SE, is  $SE=\sqrt{2/3 * ((4-5)^2+(5-5)^2+(6-5)^2)} = \sqrt{2/3 * 2} = \sqrt{4/3} = 2/\sqrt{3}$  (3 points)

## QUESTION 8 OF 312

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The observed value of a certain statistic is  $u = 5$ . Four bootstrap samples give estimate for this statistic as  $u_1=4$ ,  $u_2=5$ ,  $u_3=5$ , and  $u_4=6$ .

- Is it possible for the bootstrap samples to contain duplicates if the observed data doesn't have any duplicates? Explain.
- What is the standard error of the bootstrap estimate of this statistic?

(a) Yes, the bootstrap samples can contain duplicates because these the bootstrap method samples from the given sample **with replacement**. (3 points)

(b)  $SE=\sqrt{1/4 * ((4-5)^2+(5-5)^2+(5-5)^2+(6-5)^2)} = \sqrt{1/4 * 2} = \sqrt{1/2}$  (3 points)

## QUESTION 9 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

If  $X$  follows a uniform distribution,  $\text{Uniform}(0,b)$  then its  $k$ th moment is given by  $\mu_k = b^k / (k+1)$ .

Given the observed sample of  $\{0.5, 1, 1, 2, 3\}$ , Find the method-of-moments estimate for  $b$  using

- (a) The first sample moment
- (b) The second sample moment
- (c) The third sample moment.

Round your answers to two decimal places.

(a) The first sample moment is  $m_1 = (0.5 + 1 + 1 + 2 + 3) / 5 = 1.5$ . We set  $1.5 = b^{1/2} = b/2$  and solve for  $b$  to get  $b = 3$  as our estimate. (3 points)

(b) The second sample moment is  $m_2 = (0.5^2 + 1^2 + 1^2 + 2^2 + 3^2) / 5 = 15.25 / 5 = 3.05$ . We set  $3.05 = b^2/3$  and solve for  $b$ :  $b^2 = 9.15$  so  $b = \sqrt{9.15} = 3.02$  (3 points)

(c) The third sample moment is  $m_3 = (0.5^3 + 1^3 + 1^3 + 2^3 + 3^3) / 5 = 7.425$ . We set  $7.425 = b^3/4$  and solve for  $b$ :  $b^3 = 29.7$  so  $b = \sqrt[3]{29.7} = 3.10$ . (2 points)

## QUESTION 10 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

If  $X$  follows an exponential distribution with unknown rate  $r$ ,  $\text{Exp}(r)$ , then its  $k$ th moment is given by  $\mu_k = r^{-k} \cdot k!$

Given the observed sample of  $\{0.5, 1, 1, 2, 3\}$ , Find the method-of-moments estimate for  $b$  using

- (a) The first sample moment
- (b) The second sample moment
- (c) The third sample moment.

Round your answers to two decimal places.

(a) The first sample moment is  $m_1 = (0.5 + 1 + 1 + 2 + 3) / 5 = 1.5$ . We set  $1.5 = 1/r$  and solve for  $r$  to get  $b = 0.67$  as our estimate. (3 points)

(b) The second sample moment is  $m_2 = (0.5^2 + 1^2 + 1^2 + 2^2 + 3^2) / 5 = 15.25 / 5 = 3.05$ . We set  $3.05 = 2/r^2$  and solve for  $r$ :  $r^2 = 2/3.05$  so  $r = \sqrt{2/3.05} = 0.81$  (3 points)

(c) The third sample moment is  $m_3 = (0.5^3 + 1^3 + 1^3 + 2^3 + 3^3) / 5 = 7.425$ . We set  $7.425 = 6/r^3$  and solve for  $r$ :  $r^3 = 6/7.425$  so  $b = \sqrt[3]{6/7.425} = 0.93$ . (2 points)

## QUESTION 11 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

Consider an random sample  $\{x_1, x_2, x_3\}$  from  $\text{Gamma}(a, b)$ . The likelihood function is given by  $L(a, b) = G(a)^{-3} b^{-a} \exp(-x_1/b - x_2/b - x_3/b) \cdot x_1^{(1-a)} x_2^{(1-a)} x_3^{(1-a)}$  where  $G(a)$  only depends on  $a$ .

Use the likelihood factorization criterion

- (a) to find a sufficient statistic for  $a$  if  $b$  is known.
- (b) to find a sufficient statistic for  $b$  if  $a$  is known
- (c) to find a sufficient statistic for  $a$  and  $b$  if they are both  $a$  and  $b$  are unknown.

(a)  $L(a, b) = G(a) \exp(-1/b \cdot (x_1 + x_2 + x_3)) \cdot (x_1 \cdot x_2 \cdot x_3)^{(1-a)} = g(u, a) \cdot h(x_1, x_2, x_3, b)$  (1 points) where

$g(u, a) = G(a) \cdot u^{(1-a)}$  where  $u = x_1 + x_2 + x_3$  and  $h(x_1, x_2, x_3, b) = \exp(-1/b \cdot (x_1 + x_2 + x_3))$  (2 points)

(b)  $L(a, b) = g(v, b) \cdot h(x_1, x_2, x_3, a)$  (1 points) where  $g(v, b) = \exp(-v/b)$  and  $h(x_1, x_2, x_3, a) = G(a) \cdot (x_1 \cdot x_2 \cdot x_3)^{(1-a)}$  where  $v = x_1 + x_2 + x_3$  (2 points)

(c)  $L(a, b) = g(u, v, a, b) \cdot h(x_1, x_2, x_3)$  (1 point) where  $g(u, v, a, b) = G(a) \cdot u^{(1-a)} \exp(-v/b)$  where  $u = x_1 \cdot x_2 \cdot x_3$  and  $v = x_1 + x_2 + x_3$  and  $h(x_1, x_2, x_3) = 1$  (1 point)

## QUESTION 12 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

Suppose that the likelihood function for estimating the unknown positive parameter  $t$  is given by  $L(t) = \exp(-4t) t^7 / 12$ .

- (a) Write the negative log-likelihood function  $\text{NLL}(t)$ .
- (b) Find the MLE estimate by minimizing  $\text{NLL}(t)$  for  $t > 0$ .
- (c) Estimate the standard deviation of the estimate found in b.

(a)  $\text{NLL}(t) = 4t - 7 \log(t) + \log(1/12)$  (2 points)

(b) The derivative of  $\text{NLL}$  is  $\text{NLL}'(t) = 4 - 7/t$  which has the zero at  $t = 7/4$ . The second derivative is  $\text{NLL}''(t) = 7/t^2 > 0$  so indeed  $t = 7/4$  is the minimizer of  $\text{NLL}(t)$  and so is the MLE estimate for  $t$ . (3 points)

(c) The Variance of MLE is  $1/\text{NLL}''(7/4) = 1/(7/(7/4)^2) = 16/7$ . (1 point).

The standard deviation is  $\sqrt{16/7} = 4/\sqrt{7}$  (2 points)

## QUESTION 13 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 01

An observed data set of three pairs have one missing value:  $\{(1,1), (2,2), (3,z)\}$ . Your classmate has used the model  $f(x)=c*x$  and the OLS estimate for  $c$  was (correctly) computed to be  $c=3/2$  and the corresponding sum of squared residuals was  $35/18$ . What are the two possible values of  $z$ ?

The model values are  $f(1)=3/2$ ,  $f(2)=2*3/2=3$ ,  $f(3)=3*3/2=9/2$ . (3 points)

The sum of squared residuals is:  $(1-3/2)^2+(2-3)^2+(z-9/2)^2=35/18$ . (2 points)

Simplifying this equation gives  $(z-9/2)^2=25/36$  so  $z-9/2 = \pm (5/6)$  so the two possible values for  $z$  are  $9/2+5/6=16/3$  and  $9/2-5/6=11/3$  (3 points)

## QUESTION 14 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 01

Consider the observed sample  $\{0.86, 1.00, 0.59, 0.04, 0.79\}$ .

- Find the delete-1 jackknife estimates of the maximum.
- Find the jackknife estimate of the maximum.
- Find the standard error of the jackknife estimate. Round the answer to two decimal places.

a) The delete-1 jackknife samples and their maximums are:

$\{0.86, 1.00, 0.59, 0.04\} \rightarrow m_1=1.00$

$\{0.86, 1.00, 0.59, 0.79\} \rightarrow m_2=1.00$

$\{0.86, 1.00, 0.04, 0.79\} \rightarrow m_3=1.00$

$\{0.86, 0.59, 0.04, 0.79\} \rightarrow m_4=0.86$

$\{1.00, 0.59, 0.04, 0.79\} \rightarrow m_5=1.00$

(3 points)

(b) The jackknife estimate is the mean of the delete-1 jackknife maximums:

$(1.00+1.00+1.00+0.86+1.00)/5=0.97$  (2 points)

(c) The standard error is  $\sqrt{4/5 * ((1.00-0.972)^2+(1.00-0.972)^2+(1.00-0.972)^2+(0.86-0.972)^2+(1.00-0.972)^2)}=\sqrt{4/5 * (0.01568)}=0.11$  (3 points).

## QUESTION 15 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

Consider a distribution with PMF  $f(0)=p$ ,  $f(1)=\sqrt{p}$ ,  $f(2)=1-p-\sqrt{p}$  and zero otherwise and  $p$  is unknown. An independent observed sample is  $\{0,1,1,2\}$ .

- Write down the likelihood function of  $p$  for this sample.
- Write down the negative log-likelihood function.
- Find the MLE by minimizing the negative log-likelihood function.

a) The likelihood function is  $L(p)=f(0)*f(1)*f(1)*f(2)=p*\sqrt{p}*\sqrt{p}*(1-p-\sqrt{p}) = p^2(1-p-\sqrt{p})$  (3 points)

b) The negative log-likelihood function is  $NLL(p)=-2\log(p)-\log(1-p-\sqrt{p})$  (3 points)

c) The derivative of  $NLL(p)$  is  $NLL'(p)=-2/p + (1+1/(2\sqrt{p})) / (1-p-\sqrt{p}) = (1/2-\sqrt{p}) * (4+3\sqrt{p}) * p$ . Since  $p$  must be between 0 and 1, the only factor that can be zero is  $1/2-\sqrt{p}$  and its zero is  $p=1/4$ . (2 points).

## QUESTION 16 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 01

Let  $X$  and  $Y$  be two random variables with conditional distribution  $Y|X \sim N(X,s)$ . In other words, the conditional distribution of  $Y$  given  $X$  is Gaussian with mean  $X$  and unknown standard deviation. Three pairs of numbers is observed  $((1,2),(2,3),(3,2))$ .

- Write down the likelihood function,  $L(s)$  for this data.
- Write down the negative log-likelihood function,  $NLL(s)$ .
- Write down the derivative of the negative log-likelihood function,  $NLL'(s)$ .
- What is the MLE estimate for  $s$ ?

a) The likelihood function is  $L(s)=s^{-3}*(2\pi)^{-3/2}*\exp(-1/(2s^2) * ((2-1)^2+(3-2)^2 + (2-3)^2))=s^{-3}*(2\pi)^{-3/2}*\exp(-3/(2s^2))$  (2 points)

b) The negative log-likelihood function is  $NLL(s)=3\log(s)+(3/2)\log(2\pi)+3/(2s^2)$  (2 points)

c) The derivative of  $NLL$  is  $NLL'(s)=3/s-3/s^3=3*(s^2-1)/s^3$ . (3 points)

d) The only positive zero of  $NLL'(s)$  is  $s=1$ . This is the MLE estimate of  $s$ . (1 points).

## QUESTION 17 OF 312

DLBDSIS01\_Offen\_schwer\_F1/Lektion 01

An observed data set of three pairs is given  $\{(1,2),(2,4),(3,5)\}$ . There are two models to choose from: (i)  $f(x)=cx$  and (ii)  $g(x)=2x+a$ .

- Compute the OLS value of  $c$  of model (i).
- Compute the OLS value of  $a$  of model (ii).
- Determine the sum of squared residuals of the OLS model (i)
- Determine the sum of squared residuals of the OLS model (ii)
- Which model is better based on the sum of squared residuals?

a) The model values are  $f(1)=c$ ,  $f(2)=2c$ ,  $f(3)=3c$ .

The sum of squared residuals is  $(2-c)^2+(4-2c)^2+(5-3c)^2=45-50c+14c^2$ . The derivative is  $-50c+28c$ .

The zero of this expression is  $25/14$ . So the OLS value of  $c$  is  $c=25/14$ . (2 points)

b) The model values are  $g(1)=2+a$ ,  $g(2)=4+a$ ,  $g(3)=6+a$ .

The sum of squared residuals is  $(2-(2+a))^2+(4-(4+a))^2+(5-(6+a))^2=1+2a+3a^2$ . The derivative is  $2+6a$ .

The zero of this expression is  $-1/3$ . So the OLS value of  $a=-1/3$ . (2 points)

c) The minimum sum of squared residuals is  $45-50 \cdot 25/14+14 \cdot (25/14)^2=5/14$  approx. 0.357. (2 points)

d) The minimum sum of squared residuals is  $1+2 \cdot (-1/3)+3 \cdot (-1/3)^2=2/3$  approx. 0.667. (2 points)

e) Model (i) is better because its minimum sum of squared residuals is smaller than the minimum sum of squared residuals of model (ii). (2 points)

## QUESTION 18 OF 312

DLBDSIS01\_Offen\_schwer\_F1/Lektion 01

Let  $X$  and  $Y$  be two random variables with  $Y|X \sim N(cX, 1)$ . That is, the conditional distribution of  $Y$  given  $X$  is Gaussian with mean  $cX$  and unit standard deviation. We observed three independent pairs  $((1,2),(2,4),(3,5))$ .

- Write down the likelihood function,  $L(c)$  of this data based on the conditional distribution.
- Write down the negative log-likelihood of this data based on the conditional distribution,  $NLL(c)$ .
- Find the MLE estimate of  $c$  by minimizing the negative log-likelihood.
- Find the OLS estimate of  $c$  of the model  $f(x)=c \cdot x$ .

a) The likelihood function is  $L(c) = (2\pi)^{-3/2} \exp(-1/2 \cdot ((2-c)^2+(4-2c)^2+(5-3c)^2))$ . (3 points)

b) The negative log-likelihood is  $NLL(c) = 3/2 \cdot \log(2\pi) + 1/2 \cdot ((2-c)^2+(4-2c)^2+(5-3c)^2)$  (3 points)

c) Differentiating the negative log-likelihood function gives  $NLL'(c) = -(2-c)-2(4-2c)-3(5-3c) = 14c-25$ . The root of this function is  $c=25/14$ . This is the MLE estimate of  $c$ . (3 points)

d) The sum of squared residuals is  $(2-c)^2+(4-2c)^2+(5-3c)^2$ . The minimizer of this expression is the same as that of the NLL. Therefore the OLS estimate of  $c$  is the same as the MLE estimate:  $c=25/14$  (1 point).



## QUESTION 19 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 01

An observed independent sample of  $n$  numbers from a Gaussian distribution with unknown mean,  $m$ , and unit standard deviation have the following statistics: The sum of the numbers is  $a$  and the sum of the squares of the numbers is  $b$ .

- Write the negative log-likelihood function of  $m$ ,  $NLL(m)$  in terms of  $a$  and  $b$ .
- Find the MLE estimate of  $m$  by minimizing  $NLL(m)$ .
- Using the  $NLL$ , estimate the standard deviation of the estimate found in b.

(a) The likelihood function is  $L(m) = (1/\sqrt{2\pi})^n \exp(-1/2 \sum((x_i - m)^2)) = (2\pi)^{-n/2} \exp(-1/2 (b - 2a*m + n*m^2))$  (2 points). The negative log-likelihood function is  $NLL(m) = n/2 * \log(2\pi) + 1/2 (b - 2a*m + n*m^2)$  (2 points)

(b) The first derivative of  $NLL(m)$  is  $NLL'(m) = 1/2 * (-2a + 2n*m)$ . The zero of this function is  $m = a/n$ . The second derivative is  $NLL''(m) = n > 0$ , therefore  $m = a/n$  is indeed the minimizer and therefore the MLE estimate for the mean,  $m$ . (3 points)

(c) The variance of the MLE estimate is  $V = 1/NLL''(a/n) = 1/n$ . Therefore the standard deviation is  $1/\sqrt{n}$ . (3 points)

## QUESTION 20 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 01

An observed data set of three pairs have one missing value:  $\{(1, 1), (2, 5/2), (3, z)\}$ . Your classmate has used the model  $f(x) = c*x$  and the minimum sum of squares was calculated to be  $5/72$ . Your classmate also tells you that her OLS value of  $c$  is more than 1.2. Using this information, find the value of  $z$  and the OLS value of  $c$ .

The model values are  $f(1) = c$ ,  $f(2) = 2c$ ,  $f(3) = 3c$ . The sum of squared residuals gives as an equation:  $(1 - c)^2 + (5/2 - 2c)^2 + (z - 3c)^2 = 5/72$  (3 points)

Furthermore,  $c$  is the minimizer of this sum of squares so its derivative with respect to  $c$  equals zero. This gives as a second equation:

$-2(1 - c) - 4(5/2 - 2c) - 6(z - 3c) = 0$  (3 points).

Solving for  $z$  in the second equation gives  $z = 14c/3 - 2$  (1 point). Substituting in the first equation gives  $(1 - c)^2 + (5/2 - 2c)^2 + (5c/3 - 2)^2 = 5/72$ . Solving for  $c$  gives  $23/4 = 1.15$  and  $c = 1.25$ . Since we know that the OLS value of  $c$  was more than 1.2, we have  $c = 1.25$  (2 points). Finally,  $z = (14 * 5/4) / 3 - 2 = 23/6$  (1 point)

## QUESTION 21 OF 312

DLBDSIS01\_Offen\_schwer\_F1/Lektion 01

Let  $X$  and  $Y$  be two random variables with conditional distribution  $Y|X \sim N(b \cdot X, s)$ . In other words, the conditional distribution of  $Y$  given  $X$  is Gaussian with mean  $b \cdot X$  and unknown standard deviation. Four pairs of numbers is observed  $((1,2), (2,2), (3,5), (4,2.25))$

- Write down the likelihood function,  $L(b,s)$  for this data.
- Write down the negative log-likelihood function,  $NLL(b,s)$ .
- Minimize  $NLL$  with respect to  $b$ . This is the MLE estimate for  $b$ .
- Using the MLE estimate for  $b$  from part c), find the MLE estimate for  $s$ .

- The likelihood function is  $L(b,s) = s^{-4} \cdot (2\pi)^{-2} \cdot \exp(-1/(2s^2) \cdot ((2-b)^2 + (2-2b)^2 + (5-3b)^2 + (2.25-4b)^2)) = s^{-4} \cdot (2\pi)^{-2} \cdot \exp(-1/(2s^2) \cdot (38.0625 - 60b + 30b^2))$  (2.5 points)
- The negative log-likelihood function is  $NLL(b,s) = 4\log(s) + 2\log(2\pi) + 1/(2s^2) \cdot (38.0625 - 60b + 30b^2)$  (2.5 points)
- The derivative of the negative log-likelihood with respect to  $b$  is  $NLL_b(b,s) = -60 + 60b$  and its only root is  $b=1$ . This is the MLE estimate for  $b$ . (2.5 points)
- The negative log-likelihood with the MLE estimate for  $b$  is  $NLL(s) = NLL(1,s) = 4\log(s) + 2\log(2\pi) + 1/(2s^2) \cdot 8.0625$ . The derivative with respect to  $s$  is  $NLL'(s) = 4/s - 8.0625/s^3 = (4s^2 - 8.0625)/s^3$  and its only positive zero is  $s=1.42$ . This is the MLE estimate for  $s$ . (2.5 points)

## QUESTION 22 OF 312

DLBDSIS01\_Offen\_schwer\_F1/Lektion 01

Let  $X$  and  $Y$  be two random variables with conditional distribution  $Y|X \sim \text{Exponential}(b \cdot X)$ . In other words, the conditional distribution of  $Y$  given  $X$  is exponential with rate  $b \cdot X$  and  $b$  is unknown. Four pairs of numbers is observed  $((x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4))$ . We know that the  $x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 + x_4 \cdot y_4 = 1.2$ .

- Write down the likelihood function  $L(b)$ .
- Write down the negative log-likelihood function  $NLL(b)$ .
- Write down the derivative of the negative log-likelihood  $NLL'(b)$ .
- Find the MLE estimate for  $b$  by minimizing the negative log-likelihood function.

- The likelihood function is  $L(b) = b \cdot x_1 \cdot \exp(-b \cdot x_1 \cdot y_1) \cdot b \cdot x_2 \cdot \exp(-b \cdot x_2 \cdot y_2) \cdot b \cdot x_3 \cdot \exp(-b \cdot x_3 \cdot y_3) \cdot b \cdot x_4 \cdot \exp(-b \cdot x_4 \cdot y_4) = b^4 \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot \exp(-b(x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 + x_4 \cdot y_4)) = b^4 \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot \exp(-1.2b)$  (2.5 points)
- The negative log-likelihood function is  $NLL(b) = -4\log(b) + 1.2b + \log(x_1 \cdot x_2 \cdot x_3 \cdot x_4)$  (2.5 points)
- $NLL'(b) = -4/b + 1.2$  (2.5 points)
- The zero of  $NLL'(b)$  is 3.33. This is the MLE estimate for  $b$ . (2.5 points)

## QUESTION 23 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 01

Let  $X$  and  $Y$  be two random variables with conditional distribution  $Y|X \sim \text{Poisson}(b \cdot X)$ . In other words, the conditional distribution of  $Y$  given  $X$  is Poisson with mean  $b \cdot X$  and  $b$  is unknown.

Two pairs of numbers is observed  $((1,0), (2,4))$ .

- Write down the likelihood function  $L(b)$ .
- Write down the negative log-likelihood function  $NLL(b)$ .
- Write down the derivative of the negative log-likelihood  $NLL'(b)$ .
- Find the MLE estimate for  $b$  by minimizing the negative log-likelihood function.

a) The likelihood function is  $L(b) = \exp(-b \cdot 1) \cdot (b \cdot 1)^0 / 0! \cdot \exp(-b \cdot 2) \cdot (b \cdot 2)^4 / 4! = 32 b^4 \exp(-3b) / 3$  (2.5 points)

b) The negative log-likelihood function is  $NLL(b) = -\log(32/3) - 4 \log(b) + 3b$  (2.5 points)

c) The derivative is  $NLL'(b) = -4/b + 3$  (2.5 points)

d) The zero of the derivative is  $4/3$ , therefore  $b = 4/3$  is the MLE estimate (2.5 points)

## QUESTION 24 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 01

Let  $X_1, \dots, X_n$  be an independent random sample from an exponential distribution with unknown rate  $r$ . The observed sample is  $x_1, \dots, x_n$ .

- Write down the likelihood function of this sample in terms of  $r$ ,  $L(r)$ .
- Write down the negative log-likelihood function of this sample in terms of  $r$ ,  $NLL(r)$ .
- Compute the first and second derivatives of the negative log-likelihood function,  $NLL'(r)$  and  $NLL''(r)$ .
- Find the zero of  $NLL'(r)$ .
- Use the second derivative to show that the value of  $r$  found in part d) is a minimizer of  $NLL(r)$ .
- Write down the MLE estimate for  $r$ .

a)  $L(r) = \text{product}(r \cdot \exp(-r \cdot x_i)) = r^n \exp(-r \cdot (x_1 + x_2 + \dots + x_n))$  (2 points)

b)  $NLL(r) = r \cdot (x_1 + x_2 + \dots + x_n) - n \cdot \log(r)$  (2 points)

c)  $NLL'(r) = x_1 + x_2 + \dots + x_n - n/r$  (1 point)

$NLL''(r) = n/r^2$  (1 point)

d)  $NLL'(r) = 0$  if  $r = n / (x_1 + x_2 + \dots + x_n)$  (2 point)

e)  $NLL''(n / (x_1 + x_2 + \dots + x_n)) = (x_1 + x_2 + \dots + x_n) / n > 0$  (1 point) therefore  $r = n / (x_1 + x_2 + \dots + x_n)$  is indeed a minimizer of  $NLL'(r)$ . (1 point)

f) MLE estimate for  $r$  is  $n / (x_1 + x_2 + \dots + x_n)$  (1 point)

## QUESTION 25 OF 312

DLBDSIS01\_Offen\_leicht\_F1/Lektion 02

Compute the sample variance and sample standard deviation of the observed data:  $\{1,4,10\}$ .

The sample mean is  $(1+4+10)/3=5$  (3 points). The sample variance is  $1/2 * ((1-5)^2+(4-5)^2+(10-5)^2) = 1/2 * 42 = 21$  (2 points). The same standard deviation is  $\sqrt{21}$  (1 point).

## QUESTION 26 OF 312

DLBDSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X$  and  $Y$  be two independent random variables with variances,  $V[X]=2$  and  $V[Y]=3$  respectively.

- Compute the variance,  $V[5X]$ .
- Compute the variance,  $V[X+Y]$ .
- Compute the variance,  $V[5X-2Y]$ .

- $V[5X]=5^2 * V[X]=25 * 2 = 50$  (2 points)
- $V[X+Y]=V[X]+V[Y]=2+3=5$  (2 points)
- $V[5X-2Y]=V[5X]+V[2Y]=25*V[X]+4*V[Y]=25*2+4*3=62$  (2 points)

## QUESTION 27 OF 312

DLBDSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X$  and  $Y$  be two random variables with variances,  $V[X]=2$  and  $V[Y]=3$  respectively. Calculate the covariance of  $X$  and  $Y$ ,  $\text{Cov}(X,Y)$  if

- $V[X+Y]=7$
- $V[X-Y]=7$

- $V[X+Y]=V[X]+V[Y]+2\text{Cov}(X,Y)$ . So,  $7=2+3+2*\text{Cov}(X,Y)$  and  $\text{Cov}(X,Y)=1$ . (3 points)
- $V[X-Y]=V[X]+V[Y]-2\text{Cov}(X,Y)$ . So  $7=2+3-2\text{Cov}(X,Y)$  and  $\text{Cov}(X,Y)=-1$ . (3 points)

## QUESTION 28 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two random variables with variances  $V[X_1]=2$  and  $V[X_2]=1$  respectively and covariance  $\text{Cov}(X_1, X_2)=1$ . Let  $Y_1=2X_1+3X_2$  and  $Y_2=3X_1+2X_2$ .

- What is  $\text{Var}[Y_1]$ ?
- What is  $\text{Var}[Y_2]$ ?
- What is  $\text{Cov}(Y_1, Y_2)$ ?

The variance covariance matrix of  $X_1, X_2$  is  $S=((2,1),(1,1))$ .  $Y=AX$  with  $A=((2,3),(3,2))$ . The variance covariance matrix of  $Y_1, Y_2$  is  $A^*S^*A^T=((29,31),(31,34))$ .

- $V[Y_1]=29$  (2 points)
- $V[Y_2]=34$  (2 points)
- $\text{Cov}(Y_1, Y_2)=31$  (2 points)

## QUESTION 29 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two independent random variables with variances  $V[X_1]=2$  and  $V[X_2]=1$  respectively. Let  $Y_1=X_1+X_2$  and  $Y_2=2X_1+X_2$ .

- What is  $\text{Var}[Y_1]$ ?
- What is  $\text{Var}[Y_2]$ ?
- What is  $\text{Cov}(Y_1, Y_2)$ ?

The variance covariance matrix of  $X=(X_1, X_2)$  is  $S=((2,0),(0,1))$ .  $Y=(Y_1, Y_2, Y_3)=AX$  with  $A=((1,1),(2,1))$ . The variance covariance matrix of  $Y_1, Y_2$  is  $A^*S^*A^T=((3,5),(5,9))$ .

- $V[Y_1]=3$  (2 points)
- $V[Y_2]=9$  (2 points)
- $\text{Cov}(Y_1, Y_2)=5$  (2 points)

## QUESTION 30 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two independent random variables with variances  $V[X_1]=2$  and  $V[X_2]=1$  respectively. Let  $Y=X_1-X_2$  and  $Y_2=2X_1-X_2$ .

- Write the variance-covariance matrix of  $Y=(Y_1, Y_2)$
- What is  $\text{Var}[Y_2]$ ?
- What is  $\text{Cov}(Y_1, Y_2)$ ?

a) The variance covariance matrix of  $X=(X_1, X_2)$  is  $S=((2,0),(0,1))$ .  $Y=(Y_1, Y_2, Y_3)=AX$  with  $A=((1,-1),(2,-1))$ . The variance covariance matrix of  $Y_1, Y_2$  is  $A^*S^*A^T =((3,5),(5,9))$ . (3 points)

- $V[Y_2]=9$  (2 points)
- $\text{Cov}(Y_1, Y_2)=5$  (1 points)

## QUESTION 31 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two positive independent random variables with means  $E[X_1]=10$  and  $E[X_2]=20$  respectively and variances  $V[X_1]=V[X_2]=1$ . Use linearization to approximate the variance of  $Y=\log(X_1+X_2)$ .

$\text{Var}[Y]$  approx. equals  $A^*S^*A^T$  where  $S=((1,0),(0,1))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The partial derivatives of  $Y$  are  $1/(X_1+X_2)$  for  $dY/dX_1$  and  $dY/dX_2$ . (2 points)

Therefore,  $A=(1/30, 1/30)$  (2 points) and so  $\text{Var}[Y]$  approx. equals  $(1/30, 1/30)^*S^*((1/30), (1/30))=2/900$  (2 points).

## QUESTION 32 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two positive independent random variables with means  $E[X_1]=10$  and  $E[X_2]=20$  respectively and variances  $V[X_1]=V[X_2]=1$ . Use linearization to approximate the variance of  $Y=X_1/X_2$ .

$\text{Var}[Y]$  approx. equals  $A^*S^*A^T$  where  $S=((1,0),(0,1))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The partial derivatives of  $Y$  are  $dY/dX_1=1/(X_2)$  and  $dY/dX_2=-X_1/X_2^2$ . (2 points)

Therefore,  $A=(1/20, -1/40)$  (2 points) and so  $\text{Var}[Y]$  approx. equals  $(1/20, -1/40)^*S^*((1/20), (-1/40))= 1/320$  (2 points).

## QUESTION 33 OF 312

DLBDSSIS01\_Offen\_mittel\_F1/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$  be three mutually independent random variables with unit variances:  $V[X_1]=V[X_2]=V[X_3]=1$ . Let  $Y_1=2X_1+X_2+X_3$

$$Y_2=X_1+X_2+X_3$$

$$Y_3=X_1+X_2+2X_3$$

- Find  $\text{Var}[Y_1]$
- Find  $\text{Cov}(Y_1, Y_2)$
- Find  $\text{Cov}(Y_2, Y_3)$
- Find  $\text{Cov}(Y_1, Y_3)$

The variance covariance matrix of  $X=(X_1, X_2, X_3)$  is  $S=((1,0,0),(0,1,0),(0,0,1))$ .  $Y=(Y_1, Y_2, Y_3)=AX$  with  $A=((2,1,1),(1,1,1),(1,1,2))$ . The variance covariance matrix of  $Y$  is  $A^*S^*A^T=((6,4,5),(4,3,4),(5,4,6))$ .

- $V[Y_1]=6$  (2 points)
- $\text{Cov}(Y_1, Y_2)=4$  (2 points)
- $\text{Cov}(Y_2, Y_3)=4$  (2 points)
- $\text{Cov}(Y_1, Y_3)=5$  (2 points)

## QUESTION 34 OF 312

DLBDSSIS01\_Offen\_mittel\_F1/Lektion 02

$X_1$  and  $X_2$  are two random variables with variance  $V[X_1]=1$ ,  $V[X_2]=2$  and covariance  $\text{Cov}(X_1, X_2)=-1$ .  $t$  is a number between 0 and 1 exclusive.

- Write the variance of  $Y$  in terms of  $t$ .
- Find the value of  $t$  that minimizes the variance of  $Y=t^*X_1^*(1-t)^*X_2$
- What is the minimum variance? Round the answers to one decimal place.

- $V[Y]=t^2*V[X_1]+(1-t)^2*V[X_2]+2t*(1-t)\text{Cov}(X_1, X_2)=t^2*1+(1-t)^2*2-2*(1-t)=5t^2-6t+2$  (3 points)
- The minimizer of  $V[Y]$  comes from the zero of its derivative:  $10t-6$  whose zero is  $t=6/10=0.6$  (3 points)
- The minimum value of  $V[Y]$  is  $5*0.6^2-6*0.6+2=0.2$  (2 points)

## QUESTION 35 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two positive independent random variables with means  $E[X_1]=10$  and  $E[X_2]$  respectively and variances  $V[X_1]=V[X_2]=1$ .

Use linearization to approximate the variance of  $Y=\log(X_1)+\log(X_2)$ .

$\text{Var}[Y]$  approx. equals  $A^*S^*A^T$  where  $S=((1,0),(0,1))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The partial derivatives of  $Y$  are  $dY/dX_1=1/X_1$  and  $dY/dX_2=1/X_2$ . (2 points) Therefore,  $A=(1/10,1/20)$  (3 points) and so  $\text{Var}[Y]$  approx. equals  $(1/10,1/20)^*S^*((1/10),(1/20))=1/100+1/400=1/80$  (3 points).

## QUESTION 36 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two positive random variables with means  $E[X_1]=10$  and  $E[X_2]$  respectively and variances  $V[X_1]=V[X_2]=2$ . The covariance of  $X_1$ , and  $X_2$  is  $\text{Cov}(X_1,X_2)=-1$ .

Use linearization to approximate the variance of  $Y=\log(X_1+X_2)$ .

$\text{Var}[Y]$  approx. equals  $A^*S^*A^T$  where  $S=((2,-1),(-1,2))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The partial derivatives of  $Y$  are  $1/(X_1+X_2)$  for  $dY/dX_1$  and  $dY/dX_2$ . (2 points) Therefore,  $A=(1/30,1/30)$  (3 points) and so  $\text{Var}[Y]$  approx. equals  $(1/30,1/30)^*S^*((1/30),(1/30))=1/450$  (3 points).

## QUESTION 37 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 02

Let  $X_1$  and  $X_2$  be two positive random variables with means  $E[X_1]=10$  and  $E[X_2]=20$  respectively and variances  $V[X_1]=V[X_2]=2$ . The covariance of  $X_1$ , and  $X_2$  is  $\text{Cov}(X_1,X_2)=-1$ .

Use linearization to approximate the variance of  $Y=\log(X_1)+\log(X_2)$ .

$\text{Var}[Y]$  approx. equals  $A^*S^*A^T$  where  $S=((1,0),(0,1))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The partial derivatives of  $Y$  are  $dY/dX_1=1/X_1$  and  $dY/dX_2=1/X_2$ . (2 points) Therefore,  $A=(1/10,1/20)$  (3 points) and so  $\text{Var}[Y]$  approx. equals  $(1/10,1/20)^*S^*((1/10),(1/20))=3/200$  (3 points)



## QUESTION 38 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$  be three mutually independent random variables with variance  $V[X_1]=1$ ,  $V[X_2]=2$  and  $V[X_3]=3$  respectively. Their means are  $E[X_1]=10$ ,  $E[X_2]=20$ , and  $E[X_3]=30$ . Use linearization to approximate the variance of  $Y=X_1 \cdot X_2 \cdot X_3$ .

$\text{Var}[Y]$  approx. equals  $A \cdot S \cdot A^T$  where  $S=((1,0,0), (0,2,0), (0,0,3))$  and  $A$  contains the partial derivatives of  $Y$  evaluated at the expected values  $X_1 \rightarrow E[X_1]$ ,  $X_2 \rightarrow E[X_2]$ ,  $X_3 \rightarrow E[X_3]$ . The partial derivatives of  $Y$  are:  $dY/dX_1=X_2 \cdot X_3$ ,  $dY/dX_2=X_1 \cdot X_3$  and  $dY/dX_3=X_1 \cdot X_2$ . (2 points)

Therefore,  $A=(600,300,200)$  (3 points) and so  $\text{Var}[Y]$  approx. equals  $(600,300,200) \cdot S \cdot ((600),(300), (200))=660,000$  (3 points).

## QUESTION 39 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 02

$X$  is a random variable with values between 0 and 1 exclusively. We know its mean is  $E[X]=1/2$  and its variance is  $V[X]=1/8$ .

Use linearization to approximate the variance of  $Y=X \cdot \log(X)$ . Use  $\log(1/2)=-0.69$

$\text{Var}[Y]$  approx. equals  $a^2 \cdot \text{Var}[X]$  where  $a$  is the derivative of  $Y$  evaluated at  $X \rightarrow E[X]$ . The derivative is  $dY/dX=\log(X)+1$  (2 points). Therefore,  $a=\log(1/2)+1=-0.69+1=0.31$  (3 points) and so  $\text{Var}[Y]$  approx. equals  $(0.31)^2 \cdot 1/8 = 0.32$  (3 points)

## QUESTION 40 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$  be random variables with variances  $V[X_1]=2$ ,  $V[X_2]=4$ , and  $V[X_3]=6$  respectively. Their covariances are  $\text{Cov}(X_1, X_2)=-1$ ,  $\text{Cov}(X_1, X_3)=1$ , and  $\text{Cov}(X_2, X_3)=2$ . Let  $Y_1=X_1$ ,  $Y_2=X_1+X_2$ , and  $Y_3=X_1+X_2+X_3$ .

- Write down the variance-covariance matrix of  $Y=(Y_1, Y_2, Y_3)$ .
- Find  $\text{Cov}(Y_1, Y_2)$
- Find  $\text{Cov}(Y_2, Y_3)$
- Find  $\text{Cov}(Y_1, Y_3)$
- Find  $\text{Var}[Y_2]$
- Find  $\text{Var}[Y_3]$

- Let  $S$  be the variance covariance matrix of  $X=(X_1, X_2, X_3)$ :  $S= \begin{pmatrix} 2 & -1 & 1 \\ -1 & 4 & 2 \\ 1 & 2 & 6 \end{pmatrix}$ . Let  $Y=(Y_1, Y_2, Y_3)$ , then  $Y=AX$  with  $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ . The variance covariance matrix for  $Y$  is  $A^*S^*A^T= \begin{pmatrix} 2 & 1 & 2 \\ 1 & 4 & 7 \\ 2 & 7 & 16 \end{pmatrix}$  (3 points)
- $\text{Cov}(Y_1, Y_2)=1$  (1 point)
- $\text{Cov}(Y_2, Y_3)=7$  (1 point)
- $\text{Cov}(Y_1, Y_3)=2$  (1 point)
- $\text{Var}[Y_2]= 4$  (1 point)
- $\text{Var}[Y_3]=16$  (1 point)

## QUESTION 41 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X_1$  and  $X_2$  are independent random variables with means  $E[X_1]=10$ ,  $E[X_2]=20$  and variances  $V[X_1]=1$  and  $V[X_2]=2$ . Let  $Y_1=X_1 \cdot X_2$  and  $Y_2=X_1$ .

Linearization provides a formula to approximate the variance-covariance matrix of  $Y=(Y_1, Y_2)$  as  $A^*S^*A^T$ .

- Write down  $S$
- Write down  $A$
- Find the approximate variance-covariance matrix of  $Y$ .
- Use the result of c to write down the approximate Covariance of  $Y_1, Y_2$ .

- $S=\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$  (2 points)
- The matrix  $A$  contains the partial derivatives of  $Y$  evaluated at the means  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The matrix of partial derivatives is  $\begin{pmatrix} X_2 & X_1 \\ 1 & 0 \end{pmatrix}$  so  $A=\begin{pmatrix} 20 & 10 \\ 1 & 0 \end{pmatrix}$ . (3 points)
- $A^*S^*A^T = \begin{pmatrix} 600 & 20 \\ 20 & 1 \end{pmatrix}$ . (3 points)
- 20 (2 points)

## QUESTION 42 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X_1$  and  $X_2$  are random variables with means  $E[X_1]=10$ ,  $E[X_2]=20$  and variances  $V[X_1]=1$  and  $V[X_2]=2$  and covariance  $\text{Cov}(X_1, X_2)=-1$ . Let  $Y_1=X_1 \cdot X_2$  and  $Y_2=X_1$ . Linearization provides a formula to approximate the variance-covariance matrix of  $Y=(Y_1, Y_2)$  as  $A \cdot S \cdot A^T$ .

- Write down  $S$
- Write down  $A$
- Find the approximate variance-covariance matrix of  $Y$ .
- Use the result of c to write down the approximate Covariance of  $Y_1, Y_2$ .

- $S = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$  (2 points)
- The matrix  $A$  contains the partial derivatives of  $Y$  evaluated at the means  $X_1 \rightarrow E[X_1]$  and  $X_2 \rightarrow E[X_2]$ . The matrix of partial derivatives is  $\begin{pmatrix} X_2 & X_1 \\ 1 & 0 \end{pmatrix}$  so  $A = \begin{pmatrix} 20 & 10 \\ 1 & 0 \end{pmatrix}$ . (3 points)
- $A \cdot S \cdot A^T = \begin{pmatrix} 200 & 10 \\ 10 & 1 \end{pmatrix}$ . (3 points)
- 10 (2 points)

## QUESTION 43 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X_1, X_2$  and  $X_3$  are independent random variables with means  $E[X_1]=10$ ,  $E[X_2]=20$ ,  $E[X_3]=30$  and variances  $V[X_1]=1$ ,  $V[X_2]=2$ ,  $\text{Var}[X_3]=3$ . Let  $Y_1=X_1 \cdot X_2 \cdot X_3$  and  $Y_2=X_1 \cdot X_2$  and  $Y_3=X_3$ . Linearization provides a formula to approximate the variance-covariance matrix of  $Y=(Y_1, Y_2, Y_3)$  as  $A \cdot S \cdot A^T$ .

- Write down  $S$
- Write down  $A$
- Find the approximate variance-covariance matrix of  $Y$ .
- Use the result of c to write down the approximate Covariance of  $Y_1, Y_3$ .

- $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$  (2 points)
- The matrix  $A$  contains the partial derivatives of  $Y$  evaluated at the means  $X_1 \rightarrow E[X_1]$ ,  $X_2 \rightarrow E[X_2]$  and  $X_3 \rightarrow E[X_3]$ . The matrix of partial derivatives is  $\begin{pmatrix} X_2 \cdot X_3 & X_1 \cdot X_3 & X_1 \cdot X_2 \\ X_2 & X_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  so  $A = \begin{pmatrix} 600 & 300 & 200 \\ 20 & 10 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . (3 points)
- $A \cdot S \cdot A^T = \begin{pmatrix} 660000 & 18000 & 600 \\ 18000 & 600 & 0 \\ 600 & 0 & 3 \end{pmatrix}$ . (3 points)
- 600 (2 points)

## QUESTION 44 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 02

Let  $X$  be a random variable such that  $E[X]=10$ ,  $E[X^2]=104$ , and  $E[X^3]=1120$ ,  $E[X^4]=12448$

- Compute the exact covariance between  $X$  and  $X^2$ :  $\text{Cov}(X, X^2)$
- Use linearization to approximate  $\text{Cov}(X, X^2)$ . Hint: Put  $Y_1=X$  and  $Y_2=X^2$  and use the matrix formula to approximate the variance-covariance matrix of  $Y=(Y_1, Y_2)$ . What is the relative error of using linearization?
- Compute the exact covariance between  $X$  and  $X^3$ :  $\text{Cov}(X, X^3)$ .
- Use linearization to approximate  $\text{Cov}(X, X^3)$ . What is the relative error of using linearization?

a)  $\text{Cov}(X, X^2) = E[X \cdot X^2] - E[X] \cdot E[X^2] = E[X^3] - E[X] \cdot E[X^2] = 1120 - 10 \cdot 104 = 1120 - 1040 = 80$  (2 points)

b) Following the hint we will use the formula  $A \cdot S \cdot A^T$  to approximate the variance-covariance matrix.

$S$  contains just one number, it is the variance of  $X$ :  $\text{Var}[X] = E[X^2] - E[X]^2 = 104 - 100 = 4$  (1 point) and  $A$

contains the partial derivatives evaluate at the means:  $X \rightarrow E[X]$ : The partial derivatives are  $dY_1/dX=1$ ,

$dY_2/dX=2X$  so  $A$  is a column matrix:  $A = \begin{pmatrix} 1 \\ 20 \end{pmatrix}$ . The variance-covariance matrix is approximated by

$A \cdot S \cdot A^T = \begin{pmatrix} 4 & 80 \\ 80 & 1600 \end{pmatrix}$ . So the approximate covariance is 80 (1 point), which is the same as the exact covariance. Relative error is 0%. (1 point)

c)  $\text{Cov}(X, X^3) = E[X \cdot X^3] - E[X] \cdot E[X^3] = E[X^4] - E[X] \cdot E[X^3] = 12448 - 10 \cdot 1120 = 12448 - 11200 = 1248$  (2 points)

d) Similar to part b), set  $Y_1=X$  and  $Y_2=X^3$ , the partial derivatives are  $dY_1/dX=1$  and  $dY_2/dX=3X^2$ .  $A$  is

the column matrix  $A = \begin{pmatrix} 1 \\ 300 \end{pmatrix}$ . Therefore the approximate variance-covariance matrix is  $A \cdot S \cdot A^T =$

$\begin{pmatrix} 4 & 1200 \\ 1200 & 360000 \end{pmatrix}$ . So the approximate covariance is 1200 (2 points). The relative error of the

approximation is  $(1248 - 1200) / 1248 = 48 / 1248$  which is about 4% (1 point).

## QUESTION 45 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X_1$ ,  $X_2$ , and  $X_3$  are positive random variables with variances  $V[X_1]=2$ ,  $V[X_2]=V[X_3]=3$ , means  $E[X_1]=50$ ,  $E[X_2]=E[X_3]=20$ , and covariances  $\text{Cov}(X_1, X_2)=\text{Cov}(X_1, X_3)=\text{Cov}(X_2, X_3)=-1$ . Let  $Y_1 = X_1/X_2$ ,  $Y_2 = X_2/X_3$ , and  $Y_3 = X_1/X_3$ .

- Write down the variance co-variance matrix,  $S$ , of  $X=(X_1, X_2, X_3)$ .
- Compute the approximate variance-covariance matrix of  $Y=(Y_1, Y_2, Y_3)$  using linearization.
- Write the covariances  $\text{Cov}(Y_1, Y_2)$ ,  $\text{Cov}(Y_2, Y_3)$ ,  $\text{Cov}(Y_1, Y_3)$ .

a)  $S = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$  (2 points)

b) The linearization approximation is given by  $A^*S^*A^T$ .  $A$  contains the partial derivatives evaluated at the expected values  $X_1 \rightarrow E[X_1]=50$ ,  $X_2 \rightarrow E[X_2]=20$ ,  $X_3 \rightarrow E[X_3]=20$ . The partial derivatives are

$dY_1/dX_1 = 1/X_2$ ,  $dY_1/dX_2 = -X_1/X_2^2$ ,  $dY_1/dX_3 = 0$  (0.5 points)

$dY_2/dX_1 = 0$ ,  $dY_2/dX_2 = 1/X_3$ ,  $dY_2/dX_3 = -X_2/X_3^2$  (0.5 points)

$dY_3/dX_1 = 1/X_3$ ,  $dY_3/dX_2 = 0$ ,  $dY_3/dX_3 = -X_1/X_3^2$  (0.5 points)

Evaluating at the expected values yields the matrix  $A = \begin{pmatrix} 1/20 & -1/8 & 0 \\ 0 & 1/20 & -1/20 \\ 1/20 & 0 & -1/8 \end{pmatrix}$  (3 points)

The approximate variance-covariance matrix of  $Y=(Y_1, Y_2, Y_3)$  is  $1/1600 * \begin{pmatrix} 103 & -40 & 3 \\ -40 & 32 & 40 \\ 3 & 40 & 103 \end{pmatrix}$ . (2 points)

c)  $\text{Cov}(Y_1, Y_2) = -40/1600 = -1/40$  (0.5 points)  $\text{Cov}(Y_2, Y_3) = 40/1600 = 1/40$  (0.5 points),  $\text{Cov}(Y_1, Y_3) = 3/1600$  (0.5 points)

## QUESTION 46 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X$  is a random variable following an exponential distribution with rate 3. Its mean and variance are  $1/3$  and  $1/9$  respectively.

- Use linearization to approximate the covariance  $\text{Cov}(X, \text{Log } X)$ .
- Use linearization to approximate the variance  $V[\text{Log } X]$
- It is known that the expected value of  $\text{Log } X$  is  $E[\text{Log } X] = -1.676$ . Use this fact together with your answer from part a) to approximate the expected value  $E[X \text{ Log } X]$ .

a) Set  $Y_1 = X$  and  $Y_2 = \text{Log } X$ . The variance of  $X$  is  $S = \begin{pmatrix} 1/9 \end{pmatrix}$ . We can find the variance-covariance matrix of  $Y=(Y_1, Y_2)$  using the formula  $A^*S^*A^T$  where  $A$  contains the partial derivatives of  $Y$  with respect to  $X$  evaluated at the mean  $X \rightarrow E[X] = 1/3$ :

$dY_1/dX = 1$

$dY_2/dX = 1/X$

So the matrix  $A$  is a column matrix given by  $A = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ . So the variance-covariance matrix of  $Y$  is approximately  $A^*S^*A^T = \begin{pmatrix} 1/9 & 1/3 \\ 1/3 & 1 \end{pmatrix}$ . Therefore the approximate covariance  $\text{Cov}(X, \text{Log } X)$  approx.  $1/3$ . (3 points)

b) From the result of part A, the variance  $V[\text{Log } X]$  is approximately 1. (3 points)

c)  $\text{Cov}(X, \text{Log } X) = E[X \text{ Log } X] - E[X]E[\text{Log } X]$  (1 point). Substituting the values we already know:  $1/3 = E[X \text{ Log } X] - 1/3 * (-1.676)$  and solving for  $E[X * \text{Log } X]$  gives  $-0.2253$  (3 points)

## QUESTION 47 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

$X_1$ ,  $X_2$ , and  $X_3$  are positive random variables with variances  $V[X_1]=2$ ,  $V[X_2]=3$  and  $\text{Cov}(X_1, X_2) = 1$ , compute the following variances:

- a)  $V[2X_1+3]$
- b)  $V[3X_1 + 5X_2]$
- c)  $V[3X_2]$

a) If  $X$  is a random variable, then according to properties of variance,  $V(2X_1 + 3) = 2^2 * V(X_1)$  (2 points). This comes out to be  $4*2 = 8$  (2 points).

b)  $V(3X_1 + 5X_2) = V(X_1) + V(X_2) + 2*COV(X_1, X_2)$  (2 points). This comes out to be  $2 + 3 + 2*1 = 7$  (2 points).

c) If  $X$  is a random variable, then according to properties of variance,  $V(3X_2) = 3^2 * V(X_2) = 9 * 3 = 27$  (2 points).

## QUESTION 48 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 02

If  $X$  and  $Y$  are two random variables, and  $\text{Cov}(X, Y)$  represents the covariance between the two variables. Given that  $\text{Var}(X) = 1$ ,  $\text{Var}(Y) = 3$ , and  $\text{Cov}(X, Y) = 2$ , compute the following covariances:

- a)  $\text{Cov}(3X, 4Y)$
- b)  $\text{Cov}(X+3, Y+4)$
- c)  $\text{Cov}(2X + 3Y, 4X+5Y)$

a) As per the properties of Covariance between two variables  $X$  and  $Y$ ,  $\text{Cov}(3X, 4Y) = 3*4*\text{Cov}(X, Y)$  (2 points). This comes out to be  $12 * 2 = 24$  (1 point)

b)  $\text{Cov}(X+3, Y+4) = \text{Cov}(X, Y) = 2$  (2 points)

c)  $\text{Cov}(2X + 3Y, 4X + 5Y) = 2*4*\text{Var}(X) + 3 * 5 * \text{Var}(Y) + (2*5 + 3*4)*(\text{Cov}(X, Y))$  (2 points). Substituting the values =  $2*4*1 + 3*5*3 + (10+12)*2 = 8 + 45 + 44 = 97$  (3 points)

## QUESTION 49 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 03

Two dice are rolled.

What's the conditional probability that both dice are 4's if it's known that the sum of points is divisible by 4?

1.  $A \cap B$  represents the event that both dice are 4's. Then there is only one element in sample space  $A \cap B = \{4,4\}$ . (2 points). 2. If B represents the event that the sum of points is divisible by 4, then the event space a corresponding to B is  $\{(1,3),(3,1),(2,2),(4,4),(6,6)\}$ . ( 2 points). 3. Conditional probability  $P(A/B) = 1/5$  ( 2 points)

## QUESTION 50 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 03

If A and B are two events such that  $P(A) = 0.4$  and  $P(B) = 0.8$ , and  $P(B|A) = 0.6$ , then what is the conditional probability  $P(A|B)$ ?

Given  $P(B|A) = P(A \cap B) / P(A)$ . Substituting the values, we get  $P(A \cap B) = 0.6 * 0.4 = 0.24$  (3 points). We know that  $P(A \cap B) = P(B) * P(A|B)$ . This means  $P(A|B) = P(A \cap B) / P(B) = 0.24 / 0.80 = 0.30$  (3 points)

## QUESTION 51 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 03

According to the survey, the probability that a family owns two houses if its annual income is greater than \$60000 is 0.7. Of the households surveyed, 60% had income over \$60000 and 50% had two houses. What is the probability that the family has two houses and an annual income over \$60000?

Round the answer to two decimal places.

1. let I represent event that family annual income is greater than \$60000. and H represents the event it has two houses. Then  $P(I) = 0.60$  and  $P(H|I) = 0.70$  (3 points). 2. We have to find the probability that  $P(I \cap H)$  which is given as  $P(I) * P(H|I) = 0.6 * 0.7 = 0.42$  (3 points)

## QUESTION 52 OF 312

DLBSSIS01\_Offen\_leicht\_F1/Lektion 03

The  $P(A)$  represent the probability that an employee of the university has a PHD degree is 60%. Let  $B$  represent the event that employee is in sales. The probability that given the employee has a PHD degree, the employee is in sales is 10%. Of the employees without the PHD degree, 80% are in sales.

What is the probability that the employee selected at random is in sales?

(Note  $A^c$  represents the complimentary event of  $A$ ) Round the answer to two decimal places.

From the details given, we infer that  $P(A) = 0.60$ .  $P(B|A) = 0.10$  and  $P(B|A^c) = 0.80$ . ( 2 points). The probability that a randomly selected employee is in sales is given by  $P(B) = P(A \cap B) + P(A^c \cap B) = P(A) * P(B|A) + P(A^c) * P(B|A^c) = 0.6 * 0.1 + (1-0.6)*0.80 = 0.38$  (4 points)

## QUESTION 53 OF 312

DLBSSIS01\_Offen\_leicht\_F1/Lektion 03

Name the three categories to classify priors? Also, considering a likelihood Bernoulli( $\pi$ ), where does the value of the parameter most likely fall in each of these priors?

The three categories are:

- 1) Objective Priors: The parameter value lies between 0 and 1. (2 points)
- 2) Weakly informative priors: The parameter value lies between 0 and 1 (2 points)
- 3) Subjective (highly informative) prior: The parameter value mostly lies near 1/2. (2 points)

## QUESTION 54 OF 312

DLBSSIS01\_Offen\_leicht\_F1/Lektion 03

For the Poisson, Geometric and Exponential likelihood, write the target parameter and the conjugate priors?

1. Poisson: Target Parameter:  $\lambda$ (rate), Conjugate Prior: Gamma (2 points).
2. Geometric: Target Parameter:  $\pi$ (probability), Conjugate Prior: Beta (2 points).
3. Exponential: Target Parameter:  $\lambda$ (rate), Conjugate Prior: Gamma (2 points)



## QUESTION 55 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 03

Briefly explain how the window size 'h' affects the variability of the estimated density function in Parzen window?

When the window size is small, the density function has more variability (3 points). When the window size is large, the density function is smoother and has less variability. (3 points)

## QUESTION 56 OF 312

DLBDSSIS01\_Offen\_leicht\_F1/Lektion 03

In kernel density estimation, given  $x$  belongs to random sample  $\{x_1, x_2, \dots, x_n\}$ , write the function for exponential, linear and cosine kernels?

Function for Exponential kernel:  $\exp(-x)$ ,  $x \geq 0$  (2 points). Function for Linear kernel:  $1 - |x|$ , where  $-1 \leq x \leq 1$  (2 points). Function for Cosine kernel:  $(\pi/4) * \cos(\pi * x/2)$ ,  $-1 \leq x \leq 1$  (2 points).

## QUESTION 57 OF 312

DLBDSSIS01\_Offen\_mittel\_F1/Lektion 03

A fair dice is rolled twice. What is the probability of getting 2 on the first roll and 5 in the second roll?  
Also state if the events are independent or not?

1. The roll of the two dices are independent of each other. (2 points). 2. Probability of getting 2 on the first roll =  $1/6$ . (2 points) . 3. Probability of getting 5 on the second roll =  $1/6$ . (2 points). 4. Since both the events are independent, Probability of getting 2 on the first roll and 5 on the second roll =  $1/6 * 1/6 = 1/36$  (2 points)

## QUESTION 58 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

A bag contains 20 tickets numbered from 1 to 20. Two tickets are drawn one after another without replacement.

Find the probability that both tickets will show even numbers?

There are 10 even numbers from 1 to 20. Let A represent event that first draw has an even number, and B represent event that the second draw has even number. Then  $P(A) = 10/20$  (2.5 points) and  $P(B) = 9/19$  (2.5 points). Required probability is given by  $P(A \cap B) = P(A) * P(B|A) = 10/20 * 9/19 = 1/2 * 9/19 = 1/38$  (3 points)

## QUESTION 59 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

The bank has analyzed the gender of loan defaulters and also whether the defaulter was first time or repeated offender. The table is shown in image below. Assuming that an apprehended defaulter is chosen at random, find the probability that a) the defaulter is the first time defaulter given it is a male, and b) the defaulter is female given that it is a repeat offender. Round the answer to two decimal places.

Image 2

		Votes for		
		First time offender	Repeat offender	Total
Sex	Male	60	70	130
	Female	44	76	120
	Total	104	146	250

Source: Vikash Singh 2021

- a) Probability (First time offender / Male) = Probability (First time offender  $\cap$  Male) / Probability (Male) (1 point) =  $(60/250)/(130/250) = 0.46$ . (3 points)
- b) Probability (Female / Repeat offender) = Probability (Female  $\cap$  Repeat offender) / Probability (Repeat offender) (1 point) =  $(76/250)/(146/250) = 0.52$ . (3 points)

## QUESTION 60 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

The  $P(A)$  represent the probability that an employee of the university has a PHD degree is 60%. Let  $B$  represent the event that employee is in sales. The probability that given the employee has a PHD degree, the employee is in sales is 10%. Of the employees without the PHD degree, 80% are in sales. What is the probability that an employee selected at random is neither in sales nor has a PHD degree? (Note  $A^c$  represents the complimentary event of  $A$ ) Round the answer to two decimal places.

The required probability is given by  $P(A^c \cap B^c) = 1 - P(A \cup B)$  (2 points)  $= 1 - (P(A) + P(B) - P(A \cap B))$  (3 points). Substituting the values in the above formula, we get  $P(A^c \cap B^c) = 1 - (0.60 + 0.38 - 0.60 \cdot 0.10) = 0.08$  (3 points)

## QUESTION 61 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

Give the kernel density estimate of the sample data  $\{1,3,5,7,5\}$  using the Gaussian kernel with window size  $h=1$ .

The formula for kernel density estimate is:  $(1/nh) * \sum (1/\sqrt{2\pi}) * \exp(-((x-x_i)^2/2h^2))$  (2 points), where  $n$  represents the number of samples,  $h$  represents the bandwidth or window-size. (3 points). Substituting the values in the formula, we get,  $(1/5 * 1) * (1/\sqrt{2\pi}) [( \exp(-((x-1)^2)/2) + \exp(-((x-3)^2)/3) + \exp(-((x-5)^2)/5) + \exp(-((x-7)^2)/7) + \exp(-((x-5)^2)/5) ]$  (3 points)

## QUESTION 62 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

The image below shows ten data points and their respective classes. A new point  $x = -1$  is given. Classify the point to any of the three classes A, B or C?

Image 3

Points	-2.7	-1.7	-0.6	0	1	2	2.6	2.9	3.5	5
Class	A	A	B	A	B	C	B	C	A	C

Source: Vikash Singh 2021

The distance between the new point  $x = -1$  and the ten given points is calculated as  $x - x_i$ , where  $x_i$  is the points given (1 point). These distances comes out to be : (1.7, 0.7, -0.4, -1, -2, -3, -3.6, -3.9, -4.5, -6) (3 points). Arranging the distances in ascending order, the smallest distance is -0.4, which attributes to the point -0.6 in the data. (2 points). This point belongs to class B, so the new point will be classified as Class B. (2 points)

## QUESTION 63 OF 312

DLBSSIS01\_Offен\_mittel\_F1/Lektion 03

The image below shows ten data points and their respective classes. A new point  $x = 1.4$  is given. Classify the point to any of the three classes A, B or C?

Image 3

Points	-2.7	-1.7	-0.6	0	1	2	2.6	2.9	3.5	5
Class	A	A	B	A	B	C	B	C	A	C

Source: Vikash Singh 2021

The distance between the new point  $x = 1.4$  and the ten given points is calculated as  $x - x_i$ , where  $x$  is the points given. (1 point) These distances comes out to be : (4.1, 3.1, 2, 1.4, 0.4, -0.6, -1.2, -1.5, -2.1, -3.6) (3 points). Arranging the distances in ascending order, the smallest distance is -0.6, which attributes to the point 0 in the data. (2 points). This point belongs to class A, so the new point will be classified as Class A. (2 points).

## QUESTION 64 OF 312

DLBDSSIS01\_Offен\_mittel\_F1/Lektion 03

The image below shows ten data points and their respective classes. A new point  $x=0.7$  is given. Classify the point to any of the three classes A, B or C?

Image 3

Points	-2.7	-1.7	-0.6	0	1	2	2.6	2.9	3.5	5
Class	A	A	B	A	B	C	B	C	A	C

Source: Vikash Singh 2021

The distance between the new point  $x = 0.7$  and the ten given points is calculated as  $x - x_i$ , where  $x_i$  is the points given. (1 point) These distances comes out to be : (3.4, 2.4, 1.3, 0.7, -0.3, -1.3, -1.9, -2.2, -2.8, -4.3) (3 points). Arranging the distances in ascending order, the smallest distance is -0.3, which attributes to the point 1 in the data. (2 points) . This point belongs to class B, so the new point will be classified as Class B. (2 points)

## QUESTION 65 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 03

A fair dice is rolled twice. What is the probability of getting 2 on the first roll and not getting 5 in the second roll? Also state if the events are independent or not?

1. The roll of the two dices are independent of each other. (2 points) . 2 Probability of getting 2 on the first roll =  $1/6$ . (2 points) . 3. Probability of getting 5 on the second roll =  $1/6$ . (2 points). 4. 3. Probability of not getting 5 on the second roll =  $1 - 1/6 = 5/6$ . (2 points). 5. Since both the events are independent, Probability of getting 2 on the first roll and not getting 5 on the second roll =  $1/6 * 5/6 = 5/36$  ( 2points)

## QUESTION 66 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

The probability that the engineer will use good quality construction material for a bridge is 0.6. The probability that the bridge will collapse even after engineer uses quality construction material is 0.4. Also, the probability of bridge collapse by use of poor quality construction material is 0.7. one of the bridge made by this engineer collapsed. Using Baye's theorem, find the probability that the engineer used good quality construction material?

1. Let  $E_1$  = event that engineer used good quality construction material. Let  $E_2$  = event that engineer does not use good quality construction material.  $E$  = the bridge collapsed. Then we are given  $P(E_1) = 0.6$ . (1 point) .  $P(E/E_1) = 0.4$ . (1 point) .  $P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$ . (1 point)  $P(E/E_2) = 0.7$ . (1 point).
2.  $P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) = 0.6 \cdot 0.4 + 0.4 \cdot 0.7 = 0.52$ . (3 points).
3. Using the Baye's theorem, the required probability is given as  $P(E_1/E) = P(E_1) \cdot P(E/E_1) / P(E) = 0.6 \cdot 0.4 / 0.52 = 6/13$  (3 points)

## QUESTION 67 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

A bag contains 21 tickets numbered from 1 to 21. A ticket is drawn with replacement, and then a second drawn is made.

What is the probability that a) first ticket drawn is even and the second is odd, b) the first ticket is odd and the second is even. Also for each of a) and b) above, calculate how will your results be effected if the first ticket drawn is not replaced?

Let  $A$  denote event of getting even numbered ticket on the first draw and  $B$  denote event of getting odd numbered ticket on the second draw. Since the ticket drawn is replaced, the events  $A$  and  $B$  are independent.  $P(A) = 10/21$  and  $P(B) = 11/21$ . (2 points). Using the above information a)  $P(A \cap B) = P(A) \cdot P(B) = 10/21 \cdot 11/21 = 110/441$ . (2 points). If the first ticket drawn is not replaced, then the events are not independent. In that case  $P(A \cap B) = P(A) \cdot P(B/A) = 10/21 \cdot 11/20 = 110/420 = 11/42$ . (2 points).  
b)  $P(B \cap A) = P(B) \cdot P(A) = 11/21 \cdot 10/21 = 110/441$ . (2 points). If the first ticket drawn is not replaced, then the events are not independent. In that case  $P(B \cap A) = P(B) \cdot P(A/B) = 11/21 \cdot 10/20 = 110/420 = 11/42$ . (2 points).

## QUESTION 68 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

A speaks truth three out of five times. A die is tossed and A reports that it is a six. What is the probability that it was actually a six. (Note: events are defined as E1: A speaks truth, E2: A tells a lie, E = A reports a six.)

From the given information,  $P(E1) = 3/5$ , (1 point).  $P(E2) = 2/5$  (1 point).  $P(E/E1) = 1/6$  (1 point).  $P(E/E2) = 5/6$  (1 point). The required probability that actually it was a six, using Baye's theorem, is given by =  $P(E1/E) = (P(E1) * P(E/E1)) / ((P(E1) * P(E/E1) + P(E2) * P(E/E2))$  (3 points). Substituting the values, we get  $P(E1/E) = (3/5 * 1/6) / ((3/5 * 1/6) + (2/5 * 5/6)) = 0.23$  (3 points)

## QUESTION 69 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

In answering a question in multiple choice question, the probability that a student knows the answer is 0.6 and the remaining time he guesses. Assume that a student who guesses the answer will be correct 1 out of every five time, where 5 is the number of choices in multiple choice questions. What is the conditional probability that a student knows the answer to a question given that he answered the question correctly?

(Note: events are defined as E1: student knows the right answer, E2: student guesses the right answer, A= student gets the right answer.)

Round the answer to two decimal places.

From the data given,  $P(E1) = 0.6$  (1 point),  $P(E2) = 1-0.6=0.4$  (1 point) and  $P(A/E2) = 1/5$  (1 point).  $P(A/E1) =$  probability that student gets the right answer given that he knows the right answer = 1 (2 points). We want to calculate  $P(E1/A)$ . Using Baye's rule  $P(E1/A) = (P(E1) * P(A/E1)) / ((P(E1) * P(A/E1) + P(E2) * P(A/E2))$  (2 points). Substituting the values, we get  $P(E1/A) = (0.6 * 1) / ((0.6 * 1) + (0.4 * 1/5)) = 0.88$  (3 points)

## QUESTION 70 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

It is estimated that 30% of emails are spam emails. A certain brand of software claims that it can detect 99% of spam emails, and probability for a false positive (a non spam email detected as spam is 5%). Now if an email is detected as spam, then what is the probability that it is in fact a non spam email? (Note: events are defined as A = event that an email is detected as spam, B = event that an email is spam, Bc = event that an email is not spam.) Round the answer to two decimal places.

From the data given, we know that  $P(B) = 0.3$  (1 point),. So  $P(Bc) = 1 - 0.3 = 0.7$  (1 point),.  $P(A/B) = 0.99$  (1 point),  $P(A/Bc) = 0.05$  (1 point). Using Baye's theorem, we get  $P(Bc/A) = (P(Bc) * (P(A/Bc))) / ((P(Bc) * (P(A/Bc))) + (P(B) * (P(A/B))))$  (3 points). Substituting the values we get,  $P(Bc/A) = (0.05 * 0.7) / (0.05 * 0.7 + 0.99 * 0.3) = 0.11$  (3 points)

## QUESTION 71 OF 312

DLBDSSIS01\_Offen\_schwer\_F1/Lektion 03

The image below shows ten data points and their respective classes. A new point  $x = -1$  is given.

Classify the point to any of the three classes A, B or C using 3 nearest neighbors?

Image 3

Points	-2.7	-1.7	-0.6	0	1	2	2.6	2.9	3.5	5
Class	A	A	B	A	B	C	B	C	A	C

Source: Vikash Singh 2021

The distance between the new point  $x = -1$  and the ten given points is calculated as  $x - x_i$ , where  $x_i$  is the points given (1 point). These distances comes out to be : (1.7, 0.7, -0.4, -1, -2, -3, -3.6, -3.9, -4.5, -6) (3 points). Arranging the distances in ascending order, we find that the three closest points comes out to be {-1.7, -0.6, 0} (2 points). Two of these points belong to Class A and one point belong to Class B (2 points). Considering the majority vote, we conclude that the data point  $x = -1$  will correspond to Class A (2 points)



## QUESTION 72 OF 312

DLBDSSIS01\_Offен\_schwer\_F1/Lektion 03

The image below shows ten data points and their respective classes. A new point  $x=1.4$  is given. Classify the point to any of the three classes A, B or C using 3 nearest neighbors?

Image 3

Points	-2.7	-1.7	-0.6	0	1	2	2.6	2.9	3.5	5
Class	A	A	B	A	B	C	B	C	A	C

Source: Vikash Singh 2021

The distance between the new point  $x = 1.4$  and the ten given points is calculated as  $x - x_i$ , where  $x_i$  are the points given (1 point). These distances comes out to be : (4.1, 3.1, 2, 1.4, 0.4, -0.6, -1.2, -1.5, -2.1, -3.6) (3 points). Arranging the distances in ascending order, we find that the three closest points comes out to be {1, 2, 2.6}. (2 point). Two of these points belong to Class B and one point belong to Class C (2 points). Considering the majority vote, we conclude that the data point  $x = 1.4$  will correspond to Class B (2 points)

## QUESTION 73 OF 312

DLBDSSIS01\_Offен\_leicht\_F2/Lektion 04

A random sample of 25 observations produced a sample mean of 95 and sample standard deviation of 30.

- What is the formula to calculate the standard error of sample mean?
- Also, compute the standard error?

- Standard error (SE) is given by the formula,  $SE = \text{Standard deviation} / \sqrt{\text{sample size}}$ . (3 points)
- Substituting the values given in the above formula we get,  $SE = 30 / \sqrt{25}$ , which comes out to be 6. (3 points)

## QUESTION 74 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

What is the formula for one sample t-test test statistic? Assuming all conditions are satisfied, what will be the test statistic value for a sample size of 16, sample standard deviation of 2, sample mean of 28 and hypothesized mean of 25?

The test statistic value is given by the formula, test statistic = (Sample mean - hypothesized population mean) / (Standard deviation / sqrt(sample size)). (3 points)

Substituting the values given in the above formula we get, test statistic =  $(28 - 25) / (2 / \sqrt{16})$ , which comes out to be 6. (3 points)

## QUESTION 75 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

State and explain the components of the test statistic for chi-square goodness of fit test?

The test statistic is defined as  $\sum ((O_i - E_i)^2 / E_i)$  for all  $i=1, 2, 3 \dots$  up to  $n$ . (2 points). Here,  $O_i$  and  $E_i$  represents the observed and expected count, respectively, for the  $i$ th record. (3 points) The number of records is represented by 'n'. (1 point)

## QUESTION 76 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

The distribution for the number of customers that came to a retail store from Monday through Saturday is given below. There are two columns, one representing day of the week, while the other represents number of customers. The data is as below: Monday: 1200; Tuesday: 1230; Wednesday: 1180; Thursday: 1220; Friday: 1290; Saturday: 1300.

You have been asked to perform Chi-square goodness of fit test to check if the distribution is uniform.

Compute the degree of freedom for this case?

Degree of freedom for a chi-square test is given by the formula, (number of columns - 1) \* (number of rows - 1). (2 points). In this data there are two columns and six rows. (1 point). So the degree of freedom will be  $(2-1)*(6-1)$ , which is equal to 5. (3 points)

## QUESTION 77 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

You have been asked to test the hypothesis that the same proportion of teens and adults are getting affected by Covid. You have obtained a random sample of people from each group to test if there is a significant difference between the proportion of teens ( $p_1$ ) and adults ( $p_2$ ) that are getting Covid infected.

Write and explain an appropriate set of null and alternative hypotheses for your significance test?

1. Null Hypothesis  $H_0$ : The null hypothesis has a statement of equality, so it will be that there is no difference between the two proportions. So  $H_0: p_1 - p_2 = 0$ . (3 points)
2. Alternative Hypothesis  $H_1$  will be that the proportion is significantly different between the two groups. So  $H_1: p_1 - p_2 \neq 0$ . (3 points)

## QUESTION 78 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

You have been asked to test the hypothesis that the website design A leads to better conversion for an ecommerce company compared to design B. You want to test your hypothesis by randomly assigning each user one of the two designs over the course of a month and test if there is a significant difference between the proportion of users of design A ( $P_a$ ) and design B ( $P_b$ ).

Write and explain an appropriate set of null and alternative hypotheses for your significance test?

1. Null Hypothesis  $H_0$ : The null hypothesis has a statement of equality, so it will be that there is no difference between the two designs A and Bs. So  $H_0: p_1 - p_2 = 0$ . (3 points)
2. Alternative Hypothesis, 'H1', will be that the proportion is significantly higher for design A. So  $H_1: p_1 - p_2 > 0$ . (3 points)

## QUESTION 79 OF 312

DLBSSIS01\_Offen\_leicht\_F2/Lektion 04

A company X wants to test the hypothesis that there is no significant difference in the mean customer ratings of the two products A and B. It collects 20 random samples of ratings for product A and 24 for product B, and the sample mean comes out to be 4.5 and 4.3 for A and B respectively. The standard deviation is 0.3 and 0.6 for A and B respectively. You want to conduct a two-sample Z-test to determine if the mean ratings are significantly different for the two managers.

Calculate the test statistic for this test?

From the information provided, we get 1) Sample Mean of Product A = 4.5, 2) Sample Mean of Product B = 4.3, 3) Sample size of Product A = 20, 4) Sample size of Product B = 24, 5) Variance of Product A = 0.3, 6) Variance of Product B = 0.6. (3 points).

Putting the values in the test statistic, we get  $(4.5-4.3)/\sqrt{0.3/20 + 0.6/24}$ . This comes out to be  $0.2/\sqrt{0.04}$  which is equal to 1. (3 points).

## QUESTION 80 OF 312

DLBSSIS01\_Offen\_leicht\_F2/Lektion 04

A sample of 12 players in UEFA champions league give a mean goal scored per game of 2.3 goals and a standard deviation of 1. Construct a 99 percent confidence interval for the true mean of the players. (Given the critical value for t for 11 degrees of freedom is 3.11). Round the answer to one decimal place.

The confidence interval for true mean is given by:  $(2.3 - 3.11*(1/\sqrt{12})) < \text{True population Mean} < (2.3 + 3.11*(1/\sqrt{12}))$ . (4 points). This comes out to be  $1.4 < \text{True population Mean} < 3.7$  (2 points).

## QUESTION 81 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

The weight (in kgs) of Indian females is assumed to follow normal distribution. Random sample of 25 women is reported to have mean weight ( $\mu$ ) of 56 kgs, with standard deviation of 3.6.

What will be the margin of error at 90% confidence interval?

Round the answer to two decimal places.

Number of degree of freedom =  $25 - 1 = 24$ . The critical value for t-statistic at 24 degrees of freedom at 90% Confidence interval is 1.71. (3 points). The Margin of error is given by the formula =  $ME = (\text{critical value}) * (\text{standard deviation}) / (\text{sqrt}(\text{sample size}))$ . Substituting the values in the formula, we get,  $ME = (1.711) * (3.6) / \text{sqrt}(25) = 1.23$  (3 points)

## QUESTION 82 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 04

An economist measured the annual salary levels (in US\$) from sample of data scientists and data engineers. The details of the two samples is given in the image below. The economist wants to use these results to test the hypothesis  $\mu(\text{data scientist}) - \mu(\text{data engineers}) = 0$  versus  $\mu(\text{data scientist}) - \mu(\text{data engineers}) > 0$ .

Assuming all inference conditions are met, what would be an appropriate test statistic for the economist's test?

Round the answer to two decimal places.

Image 4	Mean Annual Salary (\$)	
	Data Scientists	Data Engineers
Sample Mean	80000	72000
Sample Standard Deviation	100	400
Sample Size	50	60

Source: Vikash Singh 2021

Formula for the test statistic is given as = test statistic =  $(\text{sample difference} - \text{hypothesized difference}) / (\text{standard error of the difference})$ . Using the information given, sample difference =  $80000 - 72000 = 8000$  (1 point). Hypothesized difference = 0 (1 point). Standard error of the difference =  $\text{sqrt}(100^2/50 + 400^2/60) = \text{sqrt}(200 + 2667) = \text{sqrt}(2867) = 53.54$ . (2 points). Substituting the values in the above equation, we get the test statistic value as =  $8000/53.54 = 149.42$  (2 points)

## QUESTION 83 OF 312

DLBSSIS01\_Offen\_leicht\_F2/Lektion 04

An analyst wants to analyze and compare the proportion of customer churn for a company for its two brands A and B. He observes 300 customer churn for a sample of 600 for brand A, and 200 churn for a sample of 1000 for brand B. The analyst wants to use these results to test the hypothesis  $\text{proportion}(\text{design A}) - \text{proportion}(\text{design B}) = 0$  versus  $\text{proportion}(\text{design A}) - \text{proportion}(\text{design B}) > 0$ .

Assuming all inference conditions are met, what would be an appropriate test statistic for the analyst's hypothesis test?

Round the answer to one decimal place.

Formula for the test statistic is given as = test statistic = (sample proportion difference) / (standard error of the difference in proportion). Using the information given,  $\text{proportion}(\text{design A}) = 300/600 = 0.5$  (1 point); and  $\text{proportion}(\text{design B}) = 200/1000 = 0.2$  (1 point). So sample difference in proportion =  $0.5 - 0.2 = 0.3$  (1 point). Hypothesized difference = 0. Standard error of the difference =  $\sqrt{0.5 * 0.5/600 + 0.2 * 0.2/1000} = 0.024$ . (2 points). Substituting the values in the above equation, we get the test statistic value as =  $0.3/0.024 = 12.5$  (1 point)

## QUESTION 84 OF 312

DLBSSIS01\_Offen\_leicht\_F2/Lektion 04

An economist measured the annual salary levels (in US\$) from sample of data scientists and data engineers. The details of the two samples is given in the image below. The economist wants to use these results to test the hypothesis  $\mu(\text{data scientist}) - \mu(\text{data engineers}) = 0$  versus  $\mu(\text{data scientist}) - \mu(\text{data engineers}) > 0$ . The variances of the populations are unknown but assumed to be equal.

Assuming all inference conditions are met, compute the pooled variance for this data?

Image 5	Mean Annual Salary (\$)	
	Data Scientists	Data Engineers
Sample Mean	80000	72000
Sample Standard Deviation	100	121
Sample Size	16	16

Source: Vikash Singh 2021

The pooled variance is given by the equation = variance pooled =  $((\text{sample size for data scientist} - 1) * \text{variance}(\text{data scientist}) + (\text{sample size for data engineer} - 1) * \text{variance}(\text{data engineer})) / (\text{sample size for data scientist} + \text{sample size for data engineer} - 2)$ . From the data given, we get the following information, sample size for data scientist = 16, sample size for data engineer = 16,  $\text{variance}(\text{data scientist}) = 100^2 = 10000$ , and  $\text{variance}(\text{data engineer}) = 121^2 = 14641$ . (3 points). Substituting these values in the formula, we get pooled variance =  $(15 * 10000 + 15 * 14641) / (16 + 16 - 2) = 12320.5$  (3 points)

## QUESTION 85 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

You want to test the hypothesis the average salary of employees in a function is \$50000. You select a random sample of 10 employees. At 99% level of significance what is the rejection region corresponding to alternative hypothesis that  $H_1: \mu > 50000$ . (Note: the null hypothesis is  $H_0: \mu = 50000$ .)

Round the answer to two decimal places.

The cutoff value 'Uc' must satisfy  $P(U > U_c) = \alpha = 1 - 99\% = 0.01$ . (2 points). The degree of freedom =  $n - 1 = 10 - 1 = 9$ . (2 points). The t statistic value for 9 degrees of freedom at 99% level of significance comes out to be 2.82. (2 points). So the rejection region is  $(2.82, \infty)$  (2 points)

## QUESTION 86 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

Name and describe the four paradigms of hypothesis testing used to evaluate the statistical significance of observed data with respect to pair of competing hypothesis?

1. The first part is to establish the hypothesis. (2 points)
2. Set the significance level and the test statistic. (2 points)
3. Compute the observed value of the test statistic from the sample data. (2 points)
4. Evaluate the hypothesis by comparing the observed value of the test statistic with the rejection region. (2 points)

## QUESTION 87 OF 312

DLBSSIS01\_Offen\_mittel\_F2/Lektion 04

The distribution for the number of customers that came to a retail store from Monday through Saturday varies from day to day. There are two columns, one representing day of the week, while the other represents number of customers. The data is as below: Monday: 1200; Tuesday: 1230; Wednesday: 1180; Thursday: 1220; Friday: 1290; Saturday: 1320. You have been asked to perform Chi-square goodness of fit test to check if the distribution is uniform. Formulate the a) null hypothesis, b) alternate hypothesis, c) Degree of freedom, and d) the expected frequencies of the customer visits on each of the six days?

- a) Null Hypothesis: Distribution of customer visits is uniform across days of the week. (2 points)
- b) Alternative Hypothesis: Distribution of customer visits is different across days. (2 points)
- c) Degrees of freedom =  $(2-1)*(6-1) = 5$  (1 point), and
- d) Expected frequencies on each day is given as  $(1200+1230+1180+1220+1290+1300)/6$ . This comes out to be  $7440/6$ , which is equal to 1240 (3 points)

## QUESTION 88 OF 312

DLBSSIS01\_Offen\_mittel\_F2/Lektion 04

In a school, the percentage of students with favorite subjects History, Mathematics, English, and Hindi are 10, 20, 30, and 40 percent, respectively. 300 randomly selected students reported their favorite subject type. The data is below: History: 40; Mathematics: 50; English: 100, and Hindi: 110. You have been asked to perform Chi-square goodness of fit test to check if the distribution is uniform.

Formulate the a) null hypothesis, b) alternate hypothesis, c) Degree of freedom, and d) the expected count of each of the subjects for these students?

- a) Null Hypothesis: Distribution of the subject preference is same across students. (2 points)
- b) Alternative Hypothesis: Distribution of the subject preference is different across students. (2 points)
- c) Degrees of freedom =  $(2-1)*(4-1) = 3$  (1 point), and
- d) Expected frequency for History:  $10\% * 300 = 30$ ; Expected frequency for Mathematics:  $20\% * 300 = 60$ ; Expected frequency for English  $30\% * 300 = 90$ ; and Expected frequency for Hindi:  $40\% * 300 = 120$  (3 points)



## QUESTION 89 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

Two sample polls of votes for two candidates A and B for a college post was taken, one each from two streams - Science and Commerce. The results are given in the image below. Calculate the expected frequencies of each cell.

Image 1

		Votes for		
		Candidate A	Candidate B	Total
Stream	Science	600	400	1000
	Commerce	300	700	1000
	Total	900	1100	2000

Source: Vikash Singh 2021

1. Expected Frequency for cell (Science, Candidate A) =  $(900 * 1000/2000) = 450$  (2 points) = .
2. Expected Frequency for cell (Science, Candidate B) =  $(1100 * 1000/2000) = 550$  (2 points).
3. Expected Frequency for cell (Commerce, Candidate A) =  $(900 * 1000/2000) = 450$  (2 points).
4. Expected Frequency for cell (Commerce, Candidate B) =  $(1100 * 1000/2000) = 550$ . (2 points).

## QUESTION 90 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

Two sample polls of votes for two candidates A and B for a college post was taken, one each from two streams - Science and Commerce. The results are given in the image below. You want to apply the chi-square test for independence.

Formulate the a) Null Hypothesis b) Alternative Hypothesis and c) degree of freedom

Image 1

		Votes for		
		Candidate A	Candidate B	Total
Stream	Science	600	400	1000
	Commerce	300	700	1000
	Total	900	1100	2000

Source: Vikash Singh 2021

- a) Null Hypothesis: There is no association between the frequency of each stream and the attribution to two candidates. (2.5 points).
- b) Alternative Hypothesis: There is statistical dependence between the frequency of each stream and the attribution to two candidates. (2.5 points).
- c) the degree of freedom for chi-square test of independence is given as  $(\text{number of columns} - 1) * (\text{number of rows} - 1) = (2-1)*(2-1) = 1$  degree of freedom. (3 points).

## QUESTION 91 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

A company X wants to test the hypothesis that there is no significant difference in the mean customer ratings of the two products A and B. It collects 20 random samples of ratings for product A and 24 for product B, and the sample mean comes out to be 4.5 and 4.3 for A and B respectively. The sample variance comes out to be 4 and 9 for A and B respectively. Assuming the population variance is unknown but assumed to be equal, calculate the pooled variance for t-test?

From the information provided, we get 1) Sample Mean of Product A = 4.5 (1 point), 2) Sample Mean of Product B = 4.3 (1 point), 3) Sample size of Product A = 20 (1 point), 4) Sample size of Product B = 24 (1 point), 5) Variance of Product A = 4 (1 point), 6) Variance of Product B = 9 (1 point).

Using the above information, we calculate the i) Pooled Variance =  $((20-1)*4+(24-1)*9)/(20+24-2) = 283/6.7381 = 42$ . (2 points)

## QUESTION 92 OF 312

DLBDSSIS01\_Offen\_mittel\_F2/Lektion 04

A company wants to test the hypothesis that there is no significant difference in the mean customer ratings of the two products A and B. It collects 20 random samples of ratings for product A and 24 for product B, and the sample mean comes out to be 4.5 and 4.3 for A and B respectively. The sample variance comes out to be 4 and 9 for A and B respectively. The pooled variance is 42.

Assuming the population variance is unknown but assumed to be equal, calculate the test statistic for t-test?

From the information provided, we get 1) Sample Mean of Product A = 4.50 (0.5 points), 2) Sample Mean of Product B = 4.30 (0.5 points), 3) Sample size of Product A = 20 (0.5 points), 4) Sample size of Product B = 24 (0.5 points), 5) Variance of Product A = 4 (0.5 points), 6) Variance of Product B = 9 (0.5 points). 7) The pooled variance comes out to be 42. (2 points).

Using the above information, we calculate the t test statistic = (

Putting the values in the test statistic, we get  $(4.5-4.3)/(\sqrt{42}*\sqrt{1/20 + 1/24})$ . This comes out to be 0.2/ 0.5940 which is equal to 0.34. (3 points).

## QUESTION 93 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

An economist measured the annual salary levels (in US\$) from sample of data scientists and data engineers. The details of the two samples is given in image below. The economist wants to use these results to test the hypothesis  $\mu(\text{data scientist}) - \mu(\text{data engineers}) = 0$  versus  $\mu(\text{data scientist}) - \mu(\text{data engineers}) > 0$ . The variances of the populations are unknown but assumed to be equal. Assuming all inference conditions are met, compute the margin of error for 99 percent confidence interval? (Given: critical value for 30 degree of freedom is 2.75). Round the answer to two decimal places.

Image 5	Mean Annual Salary (\$)	
	Data Scientists	Data Engineers
Sample Mean	80000	72000
Sample Standard Deviation	100	121
Sample Size	16	16

Source: Vikash Singh 2021

The pooled variance is given by the equation = variance pooled =  $((\text{sample size for data scientist} - 1) * \text{variance}(\text{data scientist}) + (\text{sample size for data engineer} - 1) * \text{variance}(\text{data engineer})) / (\text{sample size for data scientist} + \text{sample size for data engineer} - 2)$ . From the data given, we get the following information, sample size for data scientist = 16, sample size for data engineer = 16, variance(data scientist) =  $100^2 = 10000$ , and variance(data engineer) =  $121^2 = 14641$ . (2 points). Substituting these values in the formula, we get pooled variance =  $(15 * 10000 + 15 * 14641) / (16 + 16 - 2) = 12320.5$  (2 points) Pooled Standard deviation = 110.99. Margin of error is given by the formula = (critical value) \* Pooled Standard deviation \*  $\sqrt{1/(\text{sample size for data scientist}) + 1/(\text{sample size for data engineer})}$ . Substituting the values, we get ME =  $2.75 * 110.99 * \sqrt{1/16 + 1/16} = 38.15$  (2 points)

## QUESTION 94 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 04

Perform a one sample t-test using the following information. Sample size =  $n = 5$ . Sample mean =  $\bar{x} = 4$ , and sample standard deviation =  $s = 0.67$ . Construct a 95% Confidence Interval and test the null hypothesis that  $\mu = \text{population mean} = 5$ . (Critical value for 95% CI = 2.776, and for 99% CI = 4.596)

The Margin of Error (ME) is given by the formula =  $\text{ME} = (\text{critical value}) * (\text{sample standard deviation}) / \sqrt{(\text{sample size})}$ . ME at 95% CI is  $2.776 * 0.67 / \sqrt{5} = 0.83$ . (3 points). 95% Confidence Interval comes out to be  $(4 - 0.83, 4 + 0.83)$ , which is  $(3.17, 4.83)$ . (3 points) Since the CI does not contain the hypothesized value  $\mu = 5$ , the null hypothesis is rejected at 95% confidence interval (2 points)

## QUESTION 95 OF 312

DLBDSSIS01\_Offen\_mittel\_F2/Lektion 04

Perform a one sample t-test using the following information. Sample size =  $n = 5$ . Sample mean =  $\bar{x} = 4$ , and sample standard deviation =  $s = 0.67$ . Construct a 99% Confidence Interval and test the null hypothesis that  $\mu =$  population mean = 5. (Critical value for 99% CI = 4.596)

The Margin of Error (ME) is given by the formula =  $ME = (\text{critical value}) * (\text{sample standard deviation}) / (\text{sqrt}(\text{sample size}))$ . ME at 99% CI is  $4.596 * 0.67 / \text{sqrt}(5) = 1.38$ . (3 points). 99% Confidence Interval comes out to be  $(4 - 1.38, 4 + 1.38)$ , which is  $(2.62, 5.38)$ . (3 points) Since the CI contains the hypothesized value  $\mu = 5$ , we fail to reject the null hypothesis at 99% confidence interval (2 points)

## QUESTION 96 OF 312

DLBDSSIS01\_Offen\_mittel\_F2/Lektion 04

A company wants to know what percentage of its customers support its decision for strategy A. The company claims that in 19 of 20 cases, its sampling results should be no more than four percentage points off in either direction.

What's the confidence level the company is working with, and what's the sample size they should obtain? (Given: Critical value = 1.96 and standard deviation is 1 )

The proportion of customers who approve of the company's strategy is  $= 19/20 = 95\%$ . So the company is working with 95% confidence level. (3 points). The sample size can be obtained with the formula  $= 1.96 * (1/\text{sqrt}(n)) \leq 4\%$ . Solving the same, we get  $\text{sqrt}(n) \geq 49$ . (2 points) . This means  $n \geq 49^2 \geq 2401$ . So they should have obtained a sample size of at least 2401. (3 points)

## QUESTION 97 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

The distribution for the number of customers that came to a retail store from Monday through Saturday varies from day to day. There are two columns, one representing day of the week, while the other represents number of customers. The data is as below: Monday: 1200; Tuesday: 1230; Wednesday: 1180; Thursday: 1220; Friday: 1290; Saturday: 1320. Test the hypothesis that the number of customer visits doesn't depend on the day of the week at the 5% level of significance? (Note: chi-square significance value is 11.07)

1. Expected Value =  $(1200+1230+1180+1220+1290+1320)/6 = 1240$  (2 points)
2. Number of degree of freedom =  $6-1=5$  (1 point).
3. Chi-square calculated =  $((1200-1240)^2/1240) + ((1230-1240)^2/1240) + ((1180-1240)^2/1240) + ((1220-1240)^2/1240) + ((1290-1240)^2/1240) + ((1320-1240)^2/1240)$ . This comes out to be 11.78. (3 points).
4. The tabulated value of Chi-square at 5% level of significance for 5 degrees of freedom is 11.07. (1 points).
5. Conclusion: Since the calculated value 11.78 is more than the tabulated value 11.07, it is significant and the null hypothesis may be rejected. Hence we conclude that the number of customer visits depend on the day of the week at 5% level of significance (3 points)

## QUESTION 98 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

In a school, the percentage of students with favorite subjects History, Mathematics, English, and Hindi are 10, 20, 30, and 40 percent, respectively. 300 randomly selected students reported their favorite subject type. The data is below: History: 40; Mathematics: 50; English: 100, and Hindi: 110.

You have been asked to perform Chi-square goodness of fit test to check if the distribution is uniform at the 5% level of significance? (Note: chi-square significance value is 7.815)

1. Expected frequency for History:  $10\% * 300 = 30$ ; Expected frequency for Mathematics:  $20\% * 300 = 60$ ; Expected frequency for English  $30\% * 300 = 90$ ; and Expected frequency for Hindi:  $40\% * 300 = 120$  (3 points)
2. Number of degree of freedom =  $4-1=3$  (1 point).
3. Chi-square calculated =  $((40-30)^2/30) + ((50-60)^2/60) + ((100-90)^2/90) + ((110-120)^2/120)$ . This comes out to be 6.99. (3 points).
4. The tabulated value of Chi-square at 5% level of significance for 3 degrees of freedom is 7.82. (1 points).
5. Conclusion: Since the calculated value 6.99 is less than the tabulated value 7.82, it is not significant and the null hypothesis is accepted. Hence we conclude that the distribution of subject choice amongst school students is uniform. (2 points)

## QUESTION 99 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

Two sample polls of votes for two candidates A and B for a college post was taken, one each from two streams - Science and Commerce. The results are given in the image below. You want to apply the chi-square test for independence to estimate whether the nature of stream is related to the voting preference in this polls. (Note: tabulated value of chi-square statistic for 5% alpha is 3.841)

Image 1

		Votes for		
		Candidate A	Candidate B	Total
Stream	Science	600	400	1000
	Commerce	300	700	1000
	Total	900	1100	2000

Source: Vikash Singh 2021

1. Expected Frequency for cell (Science, Candidate A) =  $(900 * 1000/2000) = 450$  (1 point).
2. Expected Frequency for cell (Science, Candidate B) =  $(1100 * 1000/2000) = 550$  (1 point).
3. Expected Frequency for cell (Commerce, Candidate A) =  $(900 * 1000/2000) = 450$  (1 point).
4. Expected Frequency for cell (Commerce, Candidate B) =  $(1100 * 1000/2000) = 550$  (1 point).
5. The calculated value of chi-square statistic comes out to be =  $((600-450)^2/450) + ((400-550)^2/550) + ((300-450)^2/450) + ((700-550)^2/550)$  (2 points). Solving this gives the value = 181.82. (2 points). Since calculated value is much greater than the tabulated value, it is highly significant and the null hypothesis is rejected at 5% level of significance. (2 points)

## QUESTION 100 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

A company X wants to test the hypothesis that there is no significant difference in the mean customer ratings of the two products A and B. It collects 18 random samples of ratings for product A and 14 for product B, and the sample mean comes out to be 4.5 and 4.3 for A and B respectively. The sample variance comes out to be 4 and 9 for A and B respectively. Assuming the population variance is unknown but assumed to be equal, test the hypothesis that the mean rating for Product A is higher than that of Product B?

From the information provided, we get 1) Sample Mean of Product A = 4.5, 2) Sample Mean of Product B = 4.3, 3) Sample size of Product A = 20, 4) Sample size of Product B = 24, 5) Variance of Product A = 4, 6) Variance of Product B = 9. (2 points). Using the above information, we calculate the i) Pooled Variance =  $((18-1)*4+(14-1)*9)/(18+14-2) = 185/30 = 6.16$ . (3 points)

The test statistic is given as  $= (4.5-4.3)/(\sqrt{30}*\sqrt{1/18 + 1/14})$ . This comes out to be 0.2/1.95 which is equal to 0.10.

Degrees of freedom =  $18 + 14 - 2 = 30$ . (3 points)

The cutoff value for 30 degree of freedom at 1% level of significance is 2.46.

Conclusion: since the observed value is less than the tabulated value, the null hypothesis holds. (2 points)

## QUESTION 101 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

You want to test if the scores of students, per hundred marks, in two schools A and B are different at 1% level of significance. The following data is given: 16 Samples each of school A and B were selected randomly. The mean sample score is 78 for A and 71 for B. Sample variance for A and B are 16 and 9. (Note: the cutoff value is 2.5)  
Round the answer to one decimal place.

From the information provided, we get 1) Sample Mean of School A = 78, 2) Sample Mean of School B = 71, 3) Sample size of School A = 16, 4) Sample size of School B = 16, 5) Variance of School A = 16, 6) Variance of School B = 9. (2 points).

Using the above information, we calculate the Pooled Variance =  $((16-1)*16+(16-1)*9)/(16+16-2) = 12.5$  (3 points)

The test statistic is given as  $= (78-71)/(\sqrt{12.5}*\sqrt{1/16 + 1/16})$ . This comes out to be 5.6.

Degrees of freedom =  $16 + 16 - 2 = 30$ . (3 points)

The cutoff value for 30 degree of freedom at 1% level of significance is 2.5.

Conclusion: since the observed value is significantly more than the tabulated value, we reject the null hypothesis at 1% level of significance. (2 points)



## QUESTION 102 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

A telecom company wanted to understand if the customer churn was significantly different between men and women. The company surveyed a random sample of voters. Here are the results: Out of 80 women customers, 20 have churned. Out of 100 men customers, 30 have churned. The company wants to test if the results suggest a significant difference in churn between man and women. If  $P_w$  represents proportion of women customers who churn, and  $P_m$  represents proportion of men customers who churn. They will test  $H_0: P_w = P_m$  versus  $H_1: P_w \neq P_m$ . Assume all test conditions are met. What's the P-value associated with these sample results? (The standard normal table and z-scores value is 0.22965)

Pooled proportion of churn =  $(30 + 20) / (100 + 80) = 50/180 = 0.28$ .  $P_w = 20/80 = 0.25$ .  $P_m = 30/100 = 0.30$  (3 points). The test statistic is calculated as = test statistic =  $(0.25 - 0.3) / \sqrt{((0.278*(1-0.278)/80) + (0.278*(1-0.278)/100))} = -0.74$ . (3 points). The standard normal table and z-scores value for the observed t-statistic value of 0.74 is 0.22965 (1 point). Since this is a two tailed test, the corresponding p-value is  $2*0.22965=0.45$ . (3 points)

## QUESTION 103 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

A university offers a certain course that students can take on-campus or online. the university wanted to test if there was a difference in the passing rate between the two settings. Sample data showed that 80% percent of students passed the on-campus setting, and 75% passed the online setting.

The teachers used those results to make a 95% percent confidence interval to estimate the difference between the proportion of students who pass in each setting of the course ( $P_{\text{campus}} - P_{\text{online}}$ ). The resulting interval was approximately  $(-0.04, 0.14)$ . They want to use this interval to test  $H_0: P_{\text{campus}} = P_{\text{online}}$  versus  $H_1: P_{\text{campus}} \neq P_{\text{online}}$ . Assume that all conditions for inference have been met.

Based on the interval, what is the corresponding P-value and also explain briefly the conclusion at 5% level of significance?

In this problem, we're asked to use a 95%, percent confidence interval in a two-sided test with a significance level of  $\alpha=0.05$ , which is fine. We look to see if the interval that estimates the difference contains 0 or not. If the interval excludes 0, we reject the null hypothesis, and vice versa (3 points). The interval estimating  $P_{\text{campus}} - P_{\text{online}}$  was  $(-0.04, 0.14)$  (1 point). Since this interval contains zero, it's possible that that there's no difference between the proportion of students who pass in-person setting versus the online setting. So we can't say that there is a difference in the proportion of students who pass in each setting ( 3 points). Based on this interval, we know the P-value is greater than  $\alpha=0.05$ , and hence we can not conclude that there is a difference between the proportions. (3 points)

## QUESTION 104 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

The weight (in kgs) of Indian men is assumed to follow normal distribution. Random sample of 16 men is reported to have mean weight ( $\mu$ ) of 70 kgs, with standard deviation of 4. Calculate the margin of error and construct the 90% confidence interval for  $\mu$  ? (critical value for 15 degree of freedom= 1.753, critical value for 16 degree of freedom= 1.746, critical value for 17 degree of freedom= 1.74)  
Round the answer to two decimal places.

Number of degree of freedom =  $16-1 = 15$  (1 point). The critical value for t-statistic at 15 degrees of freedom at 90% Confidence interval is 1.753. (2 points). The Margin of error is given by the formula =  $ME = (\text{critical value}) * (\text{standard deviation}) / (\text{sqrt}(\text{sample size}))$  (1 point). Substituting the values in the formula, we get,  $ME = (1.753) * (4) / \text{sqrt}(16) = 1.75$  (3 points). The 90% Confidence Interval is given by the formula =  $(70 - 1.753, 70 + 1.753)$ . This comes out to be (68.25, 71.75). (3 points)

## QUESTION 105 OF 312

DLBDSSIS01\_Offen\_schwer\_F2/Lektion 04

An economist measured the annual salary levels (in US\$) from sample of data scientists and data engineers. The details of the two samples is given in image below. The economist wants to use these results to test the hypothesis  $\mu(\text{data scientist}) - \mu(\text{data engineers}) = 0$  versus  $\mu(\text{data scientist}) - \mu(\text{data engineers}) > 0$ . The variances of the populations are unknown but assumed to be equal. Assuming all inference conditions are met, find the 99 percent confidence interval for difference in means? (Given: critical value for 30 degree of freedom is 2.75)

Image 5	Mean Annual Salary (\$)	
	Data Scientists	Data Engineers
Sample Mean	80000	72000
Sample Standard Deviation	100	121
Sample Size	16	16

Source: Vikash Singh 2021

The pooled variance is given by the equation = variance pooled =  $((\text{sample size for data scientist} - 1) \times \text{variance}(\text{data scientist}) + (\text{sample size for data engineer} - 1) \times \text{variance}(\text{data engineer})) / (\text{sample size for data scientist} + \text{sample size for data engineer} - 2)$ . From the data given, we get the following information, sample size for data scientist = 16, sample size for data engineer = 16, variance(data scientist) =  $100^2 = 10000$ , and variance(data engineer) =  $121^2 = 14641$ . Sample mean for data scientists = 80000, sample mean for data engineers = 72000. Difference in mean =  $80000 - 72000 = 8000$  (4 points). Substituting these values in the formula, we get pooled variance =  $(15 \times 10000 + 15 \times 14641) / (16 + 16 - 2) = 12320.5$  (2 points) Pooled Standard deviation = 110.99. Margin of error is given by the formula = (critical value) \* Pooled Standard deviation \*  $\sqrt{1/(\text{sample size for data scientist}) + 1/(\text{sample size for data engineer})}$ . Substituting the values, we get ME =  $2.75 \times 110.99 \times \sqrt{1/16 + 1/16} = 38.15$  (2 points). Confidence Interval is given by the range  $(8000 - 38.15, 8000 + 38.15)$ . This comes out to be  $(7961.85, 8038.15)$  (2 points)

## QUESTION 106 OF 312

DLBDSSIS01\_Offен\_schwer\_F2/Lektion 04

The national average annual salary in Country A is \$50000. The sample of 12 professionals was conducted which gave a mean annual salary of \$ 48000 with standard deviation of 500. Construct a 99 percent confidence interval for the true mean of income levels. (t statistic value for 12 degrees of freedom 3.055, for 11 degrees of freedom 3.106).

Test the hypothesis that the sample mean is less than the national average income levels using the confidence interval.

The Margin of Error (ME) is given by the formula =  $ME = (\text{critical value}) * (\text{sample standard deviation}) / (\text{sqrt}(\text{sample size}))$ . Degrees of freedom =  $12-1 = 11$ . So critical value to be considered = 3.106 (2 points). Substituting the values in the formula, we get,  $ME = (3.106) * (500) / \text{sqrt}(12) = 448.31$  (2 points). The 99% Confidence Interval is =  $(48000 - 448.31, 48000 + 448.31)$ . This comes out to be  $(47551.70, 48448.31)$ . (3 points)

We are testing the hypothesis that sample mean  $< 50000$ . Since this is a left tailed test, we reject the null hypothesis if  $\$50000 > \text{upper bound of the confidence interval}$ , i.e., 48448.31. So we reject the null hypothesis that the sample mean income is equal to the national average income levels. (3 points)

## QUESTION 107 OF 312

DLBDSSIS01\_Offен\_leicht\_F2/Lektion 05

A telecom company build a machine learning application to predict customer churn. The loss matrix is given in the image below. If  $y$  is the true state that takes the value 1 for customer churn and 0 for non churn; and  $\delta$  is the decision of machine learning application which takes the value 0 when it predicts not churn, and 1 when it predicts churn.

Construct the loss function  $L(y, \delta)$  and using the loss matrix, report the possible  $L(y, \delta)$  loss values?

Image 6		True State Customer Churn = 0	True State Customer Churn = 1
Decision Customer Churn = 0		0	1
Decision Customer Churn = 1		1	0

Source: Vikash Singh 2021

Loss function takes the pair of true state ' $y$ ' and decision ' $\delta$ '. Given the loss matrix, we can construct the loss function as  $L(y, \delta) = 0$ , when  $y = \delta$  (2 points), and  $L(y, \delta) = 1$ , when  $y \neq \delta$  (2 points). Using this loss function and the loss matrix given, the loss values are  $L(0,0) = L(1,1) = 0$ ; and  $L(0,1) = L(1,0) = 1$ . (2 points).

## QUESTION 108 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 05

A telecom company build a machine learning application to predict customer churn. The loss matrix is given in the image below. If  $y$  is the true state that takes the value 1 for customer churn and 0 for non churn; and  $\delta$  is the decision function which takes the value 0 when it predicts not churn, and 1 when it predicts churn. Construct the loss function  $L(y, \delta)$  and report the possible  $L(y, \delta)$  loss values?

Image 7		True State	True State
		Customer Churn	Customer Churn = 1
Decision	Customer Churn = 0	0	5
Decision	Customer Churn = 1	1	0

Source: Vikash Singh 2021

Loss function takes the pair of true state ' $y$ ' and decision ' $\delta$ '. Given the loss matrix, we can construct the loss function as a)  $L(y, \delta) = 0$ , when  $y = \delta$  (1 point), b)  $L(y, \delta) = 1$ , when  $0 = y \neq \delta = 1$  (1 point), and c)  $L(y, \delta) = 5$ , when  $1 = y \neq \delta = 0$  (2 points). Using this loss function and the loss matrix given, the loss values are  $L(0,0) = L(1,1) = 0$ ; and  $L(0,1) = 1$ , and  $L(1,0) = 5$ . (2 points).

## QUESTION 109 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 05

The loss function under a decision problem is given in the image below. Here  $d1$  and  $d2$  are possible decisions and  $s1$  and  $s2$  are possible scenarios. Determine the Baye's criteria solution to the problem given  $P(s1) = 0.6$  and  $P(s2) = 0.4$ ?

Image 8	s1	s2
d1	20	10
d2	10	30

Source: Vikash Singh 2021

Under Baye's Risk criteria, the expected loss for decision  $d1 = E(L(d1)) = 0.6 * 20 + 0.4 * 10 = 16$ . (2 points). Similarly, the expected loss for decision  $d2 = E(L(d2)) = 0.6 * 10 + 0.4 * 30 = 18$ . (2 points). Comparing the two expected losses from  $d1$  and  $d2$ , we see that the risk is lower for decision  $d1$ , hence as per Baye's rule,  $d1$  is the preferred decision. (2 points)

## QUESTION 110 OF 312

DLBDSSIS01\_Offen\_leicht\_F2/Lektion 05

The regret table in terms of loss is given in the image below. Here  $d_1$  and  $d_2$  are possible decisions and  $s_1$  and  $s_2$  are possible scenarios. Explain briefly which decision is preferred using the minimax criterion?

Image 9	$s_1$	$s_2$
$d_1$	7	10
$d_2$	12	2

Source: Vikash Singh 2021

For decision  $d_1$ , the maximum regret from the values of  $s_1$  and  $s_2$  is 10. Similarly for decision  $d_2$ , the maximum regret from the values of  $s_1$  and  $s_2$  is 12. (3 points). Under the minimax regret approach, we select the minimum from the maximum regrets. In this case, the minimum from maximum regrets for  $d_1$  and  $d_2$  is 10. So decision  $d_1$  is preferable as per minimax criterion. (3 points)

## QUESTION 111 OF 312

DLBDSSIS01\_Offen\_mittel\_F2/Lektion 05

A telecom company build a machine learning application to predict customer churn. The loss matrix is given in the image below. If  $y$  is the true state that takes the value 1 for customer churn and 0 for non churn; and  $\delta$  is the decision of machine learning application which takes the value 0 when it predicts not churn, and 1 when it predicts churn. Randomly 100 customers were selected and there were 10 customers who were incorrectly identified as not churn, while there were 20 customers who were incorrectly identified as churn. Remaining 70 customers were correctly classified. Construct the loss function  $L(y, \delta)$  and report the total loss of incorrect classification for the 100 sample?

Image 6		True State Customer Churn = 0	True State Customer Churn = 1
Decision Customer Churn = 0		0	1
Decision Customer Churn = 1		1	0

Source: Vikash Singh 2021

Loss function takes the pair of true state ' $y$ ' and decision ' $\delta$ '. Given the loss matrix, we can construct the loss function as  $L(y, \delta) = 0$ , when  $y = \delta$ , and  $L(y, \delta) = 1$ , when  $y \neq \delta$  (3 points). Using this loss function and the loss matrix given, the loss values are  $L(0,0) = L(1,1) = 0$ ; and  $L(0,1) = L(1,0) = 1$ . (2 points). Since there were 30 customers who were incorrectly identified, the total loss for these 30 customers will be  $L(1,0) * 10 + L(0,1) * 20 = 1 * 10 + 1 * 20 = 30$ . (3 points).

## QUESTION 112 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 05

A telecom company build a machine learning application to predict customer churn. The loss matrix is given in the image below. If  $y$  is the true state that takes the value 1 for customer churn and 0 for non churn; and  $\delta$  is the decision function which takes the value 0 when it predicts not churn, and 1 when it predicts churn. Randomly 100 customers were selected and there were 10 customers who were incorrectly identified as not churn, while there were 20 customers who were incorrectly identified as churn. Remaining 70 customers were correctly classified. Construct the loss function  $L(y, \delta)$  and report the total loss of incorrect classification for the 100 sample?

Image 7		True State Customer Churn	True State Customer Churn = 1
Decision Customer Churn = 0		0	5
Decision Customer Churn = 1		1	0

Source: Vikash Singh 2021

Loss function takes the pair of true state ' $y$ ' and decision ' $\delta$ '. Given the loss matrix, we can construct the loss function as a)  $L(y, \delta) = 0$ , when  $y = \delta$ , b)  $L(y, \delta) = 1$ , when  $0 = y \neq \delta = 1$ , and c)  $L(y, \delta) = 5$ , when  $1 = y \neq \delta = 0$  (3 points). Using this loss function and the loss matrix given, the loss values are  $L(0,0) = L(1,1) = 0$ ; and  $L(0,1) = 1$ , and  $L(1,0) = 5$ . (2 points). Since there were 30 customers who were incorrectly identified, the total loss for these 30 customers will be  $L(1,0) * 10 + L(0,1) * 20 = 5 * 10 + 1 * 20 = 70$ . (3 points).

## QUESTION 113 OF 312

DLBDSSIS01\_Offен\_mittel\_F2/Lektion 05

When the decision  $\delta$  does not match the true mean  $\mu$ , it can either be because of underestimation and overestimation. Report the inequality for underestimation and overestimation of  $\mu$ . The following loss function is given for underestimation =  $L(\mu, \delta) = 100 * (\mu - \delta)^2$ . In case of overestimation, the loss function is  $(\mu - \delta)^2$ . Five observations are taken which has values 25, 30, 35, 40, 45.

Calculate the estimate of risk value if the true mean value is 33?

The inequality for overestimation is  $(\mu - \delta)^2$ , where  $\mu \leq \delta$ . The inequality for underestimation is  $100 * (\mu - \delta)^2$ , where  $\mu \geq \delta$ . (2 points). Out of the five observations, there are two records 25 and 30 which are underestimation, while the remaining three records 35, 40, 45 are over estimation. Using the above loss function we get the risk estimate value as =  $100 * (33 - 25)^2 + 100 * (33 - 30)^2 + (33 - 35)^2 + (33 - 40)^2 + (33 - 45)^2$ . (3 points). The loss value comes out to be 7497. (3 points)

## QUESTION 114 OF 312

DLBDSIS01\_Offен\_mittel\_F2/Lektion 05

When the decision  $\delta$  does not match the true mean  $\mu$ , it can either be because of underestimation and overestimation. Report the inequality for underestimation and overestimation of  $\mu$ . The following loss function is given for underestimation =  $L(\mu, \delta) = 100 * (\mu - \delta)^2 / (\mu^2 + 1)$ . In case of overestimation, the loss function is  $(\mu - \delta)^2 / (\mu^2 + 1)$ . Five observations are taken which has values 25, 30, 35, 40, 45. Calculate the estimate of risk value if the true mean value is 33?

The inequality for overestimation is  $(\mu - \delta)^2$ , where  $\mu \leq \delta$ . The inequality for underestimation is  $100 * (\mu - \delta)^2$ , where  $\mu \geq \delta$ . (3 points). Out of the five observations, there are two records 25 and 30 which are underestimation, while the remaining three records 35,40,45 are over estimation. Using the above loss function we get the risk estimate value as =  $(100 * (33-25)^2 / (33^2 + 1)) + (100 * (33-30)^2 / (33^2 + 1)) + ((33-35)^2 / (33^2 + 1)) + ((33-40)^2 / (33^2 + 1)) + ((33-45)^2 / (33^2 + 1))$ . (3 points). The loss value comes out to be 6.88 (2 points)

## QUESTION 115 OF 312

DLBDSIS01\_Offен\_schwer\_F2/Lektion 05

The loss function under a decision problem is given in the image below. Here d1, d2, d3 and d4 are possible decisions and s1, s2, s3 and s4 are possible scenarios. The values in the table represent cost.

Explain briefly which decisions can be immediately removed from consideration?

Image 10	s1	s2	s3
d1	10	16	6
d2	9	19	14
d3	13	17	13
d4	5	23	9

Source: Vikash Singh 2021

Comparing the cost for decisions d1 and d3 across scenarios, we see that for s1, decision d3 has cost of 13 which is greater than cost of d1 i.e., 10. (2 points). Similarly, for s2, decision d3 has cost of 17 which is greater than cost of d1 i.e., 16. (2 points). Finally, for scenario s3, decision d3 has cost of 13 which is greater than cost of d1 i.e., 6. (2 points). We see that for every scenario, cost for decision d3 is greater than decision d1 (2 points). This means decision d3 is an inferior choice compared to decision d1. So decision d3 can be removed from consideration. (2 points)



## QUESTION 116 OF 312

DLBDSSIS01\_Offен\_schwer\_F2/Lektion 05

The loss function under a decision problem is given in the image below. Here d1, d2, and d3 are possible decisions and s1, s2, and s3 are possible scenarios. The values in the table represent cost.

Determine the minimax solution to the problem?

Image 11	s1	s2	s3
d1	10	16	6
d2	9	19	14
d3	5	23	9

Source: Vikash Singh 2021

For the minimax risk approach, the first step is to find the maximum regret (cost) for every decision across the possible scenarios. For decision d1, the maximum cost is  $\max(10, 16, 6)$  which is 16. (2 points). For decision d2, the maximum cost is  $\max(9, 19, 14)$  which is 19. (2 points). For decision d3, the maximum cost is  $\max(5, 23, 9)$  which is 23. (2 points). The minimax approach solution deals with selecting the minimum cost from amongst the maximum of decisions d1, d2 and d3. (2 points) In this case the minimum(16, 19, 23) is 16. So the minimax approach solution is to choose the decision d1. (2 points)

## QUESTION 117 OF 312

DLBDSSIS01\_Offен\_schwer\_F2/Lektion 05

The loss function under a decision problem is given in the image below. Here d1, d2, and d3 are possible decisions and s1, s2, and s3 are possible scenarios. The values in the table represent cost.

Determine the Bayes criterion solution to the problem given that  $P(s1) = 0.5$ ,  $P(s2) = 0.3$  and  $P(s3) = 0.2$ ?

Image 11	s1	s2	s3
d1	10	16	6
d2	9	19	14
d3	5	23	9

Source: Vikash Singh 2021

Under Baye's Risk criteria, the expected loss for decision d1 =  $E(L(d1)) = 0.5 * 10 + 0.3 * 16 + 0.2 * 6 = 11$ . (2.5 points). Similarly, the expected loss for decision d2 =  $E(L(d2)) = 0.5 * 9 + 0.3 * 19 + 0.2 * 14 = 13$ . (2.5 points). Similarly, the expected loss for decision d3 =  $E(L(d3)) = 0.5 * 5 + 0.3 * 23 + 0.2 * 9 = 11.2$ . (2.5 points). Comparing the Bayes' expected losses from d1, d2 and d3, we see that the risk is lower for decision d1, hence as per Baye's rule, d1 is the preferred decision. (2.5 points)

## QUESTION 118 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Calculate the first and second sample moments,  $m_1$ , and  $m_2$ , respectively, of the data set  $\{1,3,3,4\}$ . Round your answer to one decimal place.

Select one:

$m_1=2.8$   
 $m_2=8.8$

$m_1=8.8$   
 $m_2=3$

$m_1=3$   
 $m_2=8.8$

$m_1=3$   
 $m_2=9$

The correct answer is:  $m_1=2.8$   
 $m_2=8.8$

## QUESTION 119 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

If  $X$  follows an exponential distribution with rate  $r$ , its mean is  $1/r$ . Suppose that we observe  $\{3.4,5.5,3.4,0.1\}$  for this distribution.

What is the method-of-moments estimate for  $r$ ?

Round your answer to one decimal place.

Select one:

0,2

0,1

0,4

0,3

The correct answer is: 0,3

## QUESTION 120 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

If  $X$  follows a beta distribution with parameters  $a=2$ , and unknown  $b$ , its mean is  $a/(a+b)$ . Suppose that we observe  $\{0.5, 0.3, 0.2, 0.3\}$  from this distribution. What is the method-of-moments estimate for  $b$ ? Round your answer to the nearest tenth.

Select one:

- 0,4
- 4,2
- 0,3
- 3,1

The correct answer is: 4,2

## QUESTION 121 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

If  $X$  follows a T distribution with  $v$  degrees of freedom, then its mean is zero and its variance is  $v/(v-2)$  for  $v > 2$ . Suppose that we observe  $\{-2.1, 0.8, 0.7, -0.1\}$  from this distribution. What is the method-of-moments estimate for  $v$ ? Round your answer to the nearest tenth.

Select one:

- 0,7
- 8,1
- 7,2
- 5,7

The correct answer is: 7,2

## QUESTION 122 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $X_1, X_2, X_3$  be a random sample (iid) from a Bernoulli( $p$ ) distribution with unknown  $p$ . Which of the following statistics are sufficient for estimating  $p$ ?

1.  $U=(X_1+X_2)/2$
2.  $U=(X_1+X_2+X_3)/3$

Select one:

- 2. only
- 1. only
- Neither 1. nor 2.
- Both 1. and 2.

The correct answer is: 2. only

## QUESTION 123 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{3,4\}$  be an observed sample from  $\text{Exp}(r)$ , an exponential distribution with unknown rate  $r$ . What is the likelihood function,  $L(r)$ ?

Select one:

- $12\exp(-7r)$
- $-3\log(3)-4\log(4)$
- $12\exp(-12r)$
- $3\exp(-3)+4\exp(-4)$

The correct answer is:  $12\exp(-7r)$

## QUESTION 124 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{-1, 1\}$  be an observed sample from  $N(m, 1)$ , a Gaussian distribution with unknown mean  $m$  and standard deviation 1.

What is the likelihood function  $L(m)$ ?

Select one:

- $1/(2\pi) * \exp(-0.5m^2)$
- $1/\sqrt{2\pi} * \exp(-0.5m^2)$
- $1/(2\pi) * \exp(-0.5*(m-1)^2-0.5(m+1)^2)$
- $1/\sqrt{2\pi} * \exp(-0.5(x-1)^2)+1/\sqrt{2\pi} * \exp(-0.5(x+1)^2)$

The correct answer is:  $1/(2\pi) * \exp(-0.5*(m-1)^2-0.5(m+1)^2)$

## QUESTION 125 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{-1, 1\}$  be an observed sample from  $N(m, 1)$ , a Gaussian distribution with unknown mean  $m$  and standard deviation 1.

What is the log-likelihood function  $L(m)$ ?

Select one:

- $\log(1/\sqrt{2\pi})*(-0.5m^2)$
- $\log(1/(2\pi))*(-0.5(m-1)^2-0.5(m+1)^2)$
- $\log(1/\sqrt{2\pi})*(-0.5(m-1)^2)+\log(1/\sqrt{2\pi})*(-0.5(m+1)^2)$
- $\log(1/(2\pi))*(-0.5m^2)$

The correct answer is:  $\log(1/(2\pi))*(-0.5(m-1)^2-0.5(m+1)^2)$

## QUESTION 126 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{0,1,1\}$  be an observed sample from Bernoulli( $p$ ), a Bernoulli distribution with unknown probability  $p$ .

What is the log-likelihood function  $L(p)$ ?

Select one:

- $2\log(p)+\log(1-p)$
- $\log(2p)+\log(1-p)$
- $\log(p)$
- $\log(1+p)$

The correct answer is:  $2\log(p)+\log(1-p)$

## QUESTION 127 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{(1,2.5),(2,3),(2,3.5)\}$  be an observed sample of  $(x,y)$ .

Assuming we estimate the relationship as  $f(x)=2x$ , what is the sum of squared residuals under  $f$ ?

Select one:

- 1,25
- 1,5
- 2
- 1

The correct answer is: 1,5

## QUESTION 128 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{(1,2.5),(2,3),(3,3.5)\}$  be an observed sample of  $(x,y)$ . Assuming we estimate the relationship as  $f(x)=c$ , where  $c$  is a constant.

What is the sum of squared residuals under  $f$ ?

Select one:

- $(2.5+3+3.5)^2-3c^2$
- $2.5^2+3^2+3.5^2-c^2$
- $(2.5^2+3^2+3.5^2)-3c^2$
- $(2.5-c)^2+(3-c)^2+(3.5-c)^2$

The correct answer is:  $(2.5-c)^2+(3-c)^2+(3.5-c)^2$

## QUESTION 129 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{(1,2.5),(2,3),(3,3.5)\}$  be an observed sample of  $(x,y)$ .

Suppose we have a model that predicts  $\{(1,2.5),(2,4.5),(3,3)\}$ , what is the sum of squared residuals?

Select one:

- 2.3
- 2.0
- 2,5
- 1.8

The correct answer is: 2,5

## QUESTION 130 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{(1,2.5),(2,3),(3,3.5)\}$  be an observed sample of  $(x,y)$ . Suppose we have a model that predicts  $\{(1,2.5),(2,4.5),(3,c)\}$ , find the OLS estimate of  $c$ .

Select one:

- 3
- 3.3
- 3,5
- 3.8

The correct answer is: 3,5

## QUESTION 131 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{(1,2.5),(2,3),(3,3.5)\}$  be an observed sample of  $(x,y)$ . Assuming we estimate the relationship as  $f(x)=c$ , where  $c$  can be one of the values  $\{1,2,3,4\}$ . Find the OLS estimate of  $c$ .

Select one:

- 1
- 4
- 3
- 2

The correct answer is: 3



## QUESTION 132 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{1,2,4\}$  be an observed sample.  
Find the delete-1 jackknife mean.

Select one:

- 3
- 2.3
- 2
- 2,5

The correct answer is: 2.3

## QUESTION 133 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{x_1, x_2, \dots, x_n\}$  be an observed sample of size  $n$ .  
How many delete-1 jackknife replicates are there?

Select one:

- $n$
- $n(n-1)/2$
- $n-1$
- $n(n-1)(n-3)/3$

The correct answer is:  $n$

## QUESTION 134 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 01

Let  $\{1,2,4\}$  be an observed sample.  
Find the delete-1 jackknife estimate of the max.

Select one:

- 4
- 2
- 14/3
- 3

The correct answer is: 14/3

## QUESTION 135 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

An observed sample  $\{0.2,0.4,0.6\}$  is drawn from  $\text{Beta}(1,b)$  distribution.  
What is the method-of-moments estimate for  $b$ ?  
Note: The mean of  $\text{Beta}(a,b)$  is  $a/(a+b)$ .

Select one:

- 1,3
- 1,4
- 1,5
- 1,2

The correct answer is: 1,5

## QUESTION 136 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

An observed sample  $\{1,2,3\}$  is drawn from  $\text{Exp}(r)$  distribution, an exponential distribution with rate  $r$ .

What is the method-of-moment estimate for  $r$ ?

Note: The mean of  $\text{Exp}(r)$  is  $1/r$ .

Select one:

- 1
- 2
- $1/3$
- $1/2$

The correct answer is:  $1/2$

## QUESTION 137 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Suppose that the sample  $\{0.2,0.4,0.6\}$  is drawn from a distribution with PDF  $f(x)=2x$  if  $0 \leq x \leq 1$  and zero otherwise.

What is the value of the likelihood function for this sample?

Select one:

- 0,3
- 0,32
- 0,33
- 0,31

The correct answer is: 0,32

## QUESTION 138 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Suppose that the observed sample  $\{1, 1, 2\}$  is from  $\text{Exp}(r)$ , an exponential distribution with rate  $r$ .

What is the log-likelihood function for this sample?

Select one:

- $-4r+3\log(r)$
- $-3\log(r)$
- $-3r+4\log(r)$
- $4r$

The correct answer is:  $-4r+3\log(r)$

## QUESTION 139 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

A sample of ten numbers is observed. The sum of the values in this sample is 5.

Assuming the sample comes from  $\text{Exp}(r)$ , an exponential distribution with rate  $r$ , what is the log-likelihood of this sample?

Select one:

- $5\log(r)$
- $-10r+5\log(r)$
- $-5r$
- $-5r+10\log(r)$

The correct answer is:  $-5r+10\log(r)$

## QUESTION 140 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Given a sample of size 10, four bootstrap samples were created having means 2,3,3,1. The sample mean of the original sample is 3.

What is the bootstrap standard error of the mean?

Select one:

- 0.4
- 0.6
- 0.5
- 0.3

The correct answer is: 0.6

## QUESTION 141 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Suppose that 3.1,3,3.2 are jackknife estimates for a particular parameter.

What is the jackknife standard error? (round to four decimal places)

Select one:

- 0.0101
- 0,0067
- 0.0301
- 0.0133

The correct answer is: 0.0133

## QUESTION 142 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Given the observed data  $\{(1,2.5),(1,3),(2,4)\}$ , and assumed model  $f(x)=cx$ , what is the OLS estimate of  $c$ ?

Select one:

- 2.6
- 1.9
- 2.3
- 1.8

The correct answer is: 2.3

## QUESTION 143 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

Given the observed data  $\{(1,2.3),(2,4),(3,6)\}$ , and assumed model  $f(x)=2x+a$ , what is the OLS estimate of  $a$ ?

Select one:

- 0,2
- 0,1
- 0,2
- 0,1

The correct answer is: 0,1

## QUESTION 144 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

A sample of 10 numbers are observed. The sum of the numbers is 31, and the sum of squares is 108.

Find the method of moments estimate of the variance,  $s^2$ .

Select one:

- $m=1.1$
- $s^2=1.2$
- $s^2=2.4$
- $s^2=1.6$

The correct answer is:  $s^2=1.2$

## QUESTION 145 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 01

The sum of a sample of 10 numbers is 16. Assuming that they come from a Poisson distribution with unknown mean  $r$ , which of the following forms is the likelihood function **proportional** to?

Select one:

- $\exp(-10r) r^{16}$
- $\exp(-16t)r^{10}$
- $\exp(-10r)r^{-16}$
- $\exp(-16r)r^{16}$

The correct answer is:  $\exp(-10r) r^{16}$

## QUESTION 146 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

The likelihood function of a Poisson( $r$ ) sample is given by  $L(r) = (1/12)^r \exp(-3r) r^6$ . What the MLE estimate of  $r$ ?

Select one:

- 6
- 3
- 12
- 2

The correct answer is: 2

## QUESTION 147 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

A sample of 10 numbers is drawn from a Beta( $a, b$ ) distribution with unknown  $a$  and  $b$ . The individual numbers in the sample are not given, but we know that the sum of the numbers is 4 and the sum of squares of the numbers is 2.

Find the method of moments estimates for  $a$  and  $b$ .

Note: if  $X \sim \text{Beta}(a, b)$ , then its variance is  $\text{Var}[X] = ab / ((a+b)^2 (1+a+b))$

Select one:

- $a=36$   
 $b=54$
- $a=4$   
 $b=6$
- $a=4$   
 $b=10$
- $a=2$   
 $b=3$

The correct answer is:  $a=2$   
 $b=3$



## QUESTION 148 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Given the observed data  $\{(1,1.5),(1,3),(2,4)\}$ , we consider two models:  $f(x)=cx$  and  $g(x)=2x+a$ . Find the OLS estimates of  $c$  and  $a$  and determine which of the two,  $f$  or  $g$ , is a better model.

Select one:

$c=2.6$   
 $a=0.4$   
 $g$  is better

$c=2.1$   
 $a=0.3$   
 $f$  is better

$c=2.1$   
 $a=0.2$   
 $g$  is better

$c=2.9$   
 $a=0.6$   
 $f$  is better

The correct answer is:  $c=2.1$   
 $a=0.2$   
 $g$  is better

## QUESTION 149 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Given the observed data  $\{(1,1.5),(1,3),(2,4)\}$ , we consider two models:  $f(x)=cx+0.5$  and  $g(x)=dx-0.5$ .

Find the OLS estimates of  $c$  and  $d$  and determine which of the two,  $f$  or  $g$ , is a better model.

Select one:

- $c=1.8$   
 $d=2.4$   
 $f$  is better
- $c=2.4$   
 $d=2.7$   
 $f$  is better
- $c=2.1$   
 $d=2.9$   
 $g$  is better
- $c=2.4$   
 $d=1.3$   
 $g$  is better

The correct answer is:  $c=1.8$   
 $d=2.4$   
 $f$  is better

## QUESTION 150 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

An observed sample of 100 numbers  $\{x_1, x_2, \dots, x_{100}\}$  comes from Gaussian distribution with zero mean and unknown precision  $t$  (precision is the reciprocal of variance). The sum of the squares of the numbers in the sample is  $ss$ .

Which of the following is the negative log-likelihood function of this sample in terms of  $t$ ?  
Note that  $K$  is a constant which doesn't depend on  $t$ .

Select one:

- $ss \cdot t^2 + K$
- $ss \cdot t/2 + 100/t + K$
- $-ss \cdot t/2 - 50t + K$
- $ss \cdot t/2 - 50t + K$

The correct answer is:  $ss \cdot t/2 - 50t + K$

## QUESTION 151 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

An observed data set of three pairs is given  $\{(1,2),(2,4),(3,7)\}$ .

There are two models to choose from: (i)  $f(x)=cx$  and (ii)  $g(x)=2x+a$ .

Compute the OLS value for  $c$  from model (i) and the OLS value for  $a$  from model (ii). Round your answers to two decimal places.

Select one:

$c=2.21$   
 $a=0.45$

$c=1.89$   
 $a=0.45$

$c=2.21$   
 $a=0.33$

$c=1.89$   
 $a=0.33$

The correct answer is:  $c=2.21$   
 $a=0.33$

## QUESTION 152 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Let  $X$  and  $Y$  be two random variables with conditional distribution  $Y|X \sim \text{Poisson}(b \cdot X)$ . In other words, the conditional distribution of  $Y$  given  $X$  is Poisson with mean  $b \cdot X$  and  $b$  is unknown.

Two pairs of numbers is observed  $((1,1),(2,4))$ .

What is the MLE estimate for  $b$ ?

Select one:

$5/3$

$1$

$2/3$

$4/3$

The correct answer is:  $5/3$

## QUESTION 153 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

An observed data set of three pairs have one missing value:  $\{(1,1), (2,5/2), (3,z)\}$ . Your classmate has used the model  $f(x)=c*x$  and the minimum sum of squares was calculated to be  $5/72$ . Your classmate also tells you that her OLS value of  $c$  is less than 1.2.

Using this information, find the value of  $z$  and the OLS value of  $c$ .

Round your answer to two decimal places.

Select one:

- 1,17
- 1,15
- 1,16
- 1,14

The correct answer is: 1,15

## QUESTION 154 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Consider a distribution with PMF  $f(0)=p$ ,  $f(1)=f(2)=\sqrt{p}$ ,  $f(3)=1-p-2\sqrt{p}$  and zero otherwise and  $p$  is unknown. An independent observed sample is  $\{0,1,2,3\}$ .

What is the MLE estimate for  $p$ ?

Select one:

- $1/3$
- $4/9$
- $2/9$
- $1/9$

The correct answer is:  $1/9$

## QUESTION 155 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Consider the observed sample  $\{1.20, 1.00, 1.59, 1.04, 1.79\}$ .  
What is the standard error of the jackknife estimate of the maximum?  
Round your answer to two decimal places.

Select one:

- 0.18
- 0.16
- 0.11
- 0.20

The correct answer is: 0.16

## QUESTION 156 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 01

Consider the observed sample  $\{1.20, 1.00, 1.59, 1.04, 1.79\}$ .  
What is the standard error of the jackknife estimate of the minimum?  
Round your answer to two decimal places.

Select one:

- 0.06
- 0.01
- 0.03
- 0.05

The correct answer is: 0.03

## QUESTION 157 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Which type of uncertainty is decreased by collecting more data?

1. Systematic
2. Statistical

Select one:

- Both 1. and 2.
- 1. only
- 2. only
- Neither 1. nor 2.

The correct answer is: 1. only

## QUESTION 158 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

How is the dependence between two random variables, X and Y, measured?

Select one:

- $\text{Cov}(X,Y)$
- $\text{Var}[X+Y]$
- $\text{SD}[X+Y]$
- $\text{Var}[X]+\text{Var}[Y]$

The correct answer is:  $\text{Cov}(X,Y)$

## QUESTION 159 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

A device, calibrated to be used between 20 and 30 degrees Celsius, is used in a room where the temperature is between 0 and 10 degrees Celsius. The researcher ignores the effect of the temperature on the device.

The resulting measurements will suffer from which of the following?

1. Systematic Uncertainty
2. Systematic Error

Select one:

- 1. only
- 2. only
- Neither 1. nor 2.
- Both 1. and 2.

The correct answer is: 2. only

## QUESTION 160 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Which of the following are correct formulas for the variance of a random variable X?

1.  $V[X]=E[(X-E[X])^2]$
2.  $V[X]=E[X^2]-E[X]^2$

Select one:

- 2. only
- 1. only
- Both 1. and 2.
- Neither 1. nor 2.

The correct answer is: Both 1. and 2.

## QUESTION 161 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Which of the following are correct formulas for the standard deviation of a random variable X?

1.  $SD[X]=E[(X-E[X])^2]$
2.  $SD[X]=\text{SQRT}(E[X^2]-E[X]^2)$

Select one:

- Both 1. and 2.
- 2. only
- Neither 1. nor 2.
- 1. only

The correct answer is: Both 1. and 2.

## QUESTION 162 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Let X and Y be two random variables.

Which of the following are true statements about the covariance,  $\text{Cov}(X,Y)$ ?

1.  $\text{Cov}(X,Y)=E[(X-E[X])(Y-E[Y])]$
2.  $\text{Cov}(X,Y)=E[XY]-E[X]E[Y]$

Select one:

- 1. only
- Neither 1. nor 2.
- 2. only
- Both 1. and 2.

The correct answer is: Both 1. and 2.



## QUESTION 163 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  and  $Y$  are independent random variables.

Which of the following are true?  $V[.]$  denotes variance and  $SD[.]$  denotes standard deviation.

1.  $V[X+Y]=V[X]+V[Y]$
2.  $SD[X+Y]=SD[X]+SD[Y]$

Select one:

- 1. only
- Neither 1. nor 2.
- Both 1. and 2.
- 2. only

The correct answer is: 1. only

## QUESTION 164 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  and  $Y$  are independent random variables.

Which of the following are true?  $V[.]$  denotes variance and  $SD[.]$  denotes standard deviation.

1.  $V[X-Y]=V[X]+V[Y]$
2.  $SD[X-Y]=SD[X]-SD[Y]$

Select one:

- 1. only
- Both 1. and 2.
- 2. only
- Neither 1. nor 2.

The correct answer is: 1. only

## QUESTION 165 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  and  $Y$  are random variables.

Which of the following are true?  $V[.]$  denotes variance,  $\text{Cov}(..)$  denotes covariance.

1.  $V[X+Y]=V[X]+V[Y]+2\text{Cov}(X,Y)$

2.  $V[X-Y]=V[X]-V[Y]-2\text{Cov}(X,Y)$

Select one:

- Both 1. and 2.
- Neither 1. nor 2.
- 1. only
- 2. only

The correct answer is: 1. only

## QUESTION 166 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  is a random variable with variance  $V[X]=5$ .

What is the Variance of  $V[3X]$ ?

Select one:

- 5
- 9
- 15
- 45

The correct answer is: 45

## QUESTION 167 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  and  $Y$  are independent random variables with  $V[X]=V[Y]=1$ , what is  $V[X+2Y]$ ?

Select one:

- 5
- 2
- 4
- 3

The correct answer is: 5

## QUESTION 168 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X$  and  $Y$  are random variables with  $V[X]=1$ ,  $V[Y]=2$ ,  $\text{Cov}(X,Y)=-1$ .  
What is  $V[2X-Y]$ ?

Select one:

- 4
- 1
- 2
- 8

The correct answer is: 8

## QUESTION 169 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that  $X_1$ ,  $X_2$ , and  $X_3$  are mutually independent random variables with variances  $V[X_1]=1$ ,  $V[X_2]=2$ ,  $V[X_3]=3$ .

What is the variance of the sample mean  $Y=(X_1+X_2+X_3)/3$ ?

Select one:

- 2
- $14/3$
- $1/3$
- $2/3$

The correct answer is:  $2/3$

## QUESTION 170 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Let  $X_1$  and  $X_2$  be independent random variables with  $E[X_1]=E[X_2]=10$ ,  $V[X_1]=1$ , and  $V[X_2]=2$ . Using the linearization formula, what is the approximate variance  $V[Y]$  for  $Y=X_1 \cdot X_2$ ?

Select one:

- 400
- 100
- 300
- 200

The correct answer is: 300

## QUESTION 171 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Suppose that the random variable  $X$  has mean  $E[X]=100$  and variance  $V[X]=225$ . Using linearization, what is the approximate variance of  $Y=\log X$ ?

Hint: The derivative,  $d/du(\log u) = 1/u$ .

Round your answer to three decimal places.

Select one:

- 0,225
- 2,225
- 0,023
- 22,225

The correct answer is: 0,023

## QUESTION 172 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Let  $X, Y, Z$  be independent random variables with variances  $V[X]=1$ ,  $V[Y]=2$ , and  $V[Z]=3$ . What is the variance  $V[3X+2Y+Z]$ ?

Select one:

- 10
- 6
- 20
- 35

The correct answer is: 20

## QUESTION 173 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 02

Let  $X$  and  $Y$  be two random variables with  $\text{Cov}(X, Y) = 2$ .  
What is  $\text{Cov}(2X, -3Y)$ ?

Select one:

- 12
- 12
- 72
- 72

The correct answer is: -12

## QUESTION 174 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X$  and  $Y$  be random variables with  $V[X] = 1$ ,  $V[Y] = 2$ , and  $V[X+Y] = 4$ .  
What is  $\text{Cov}(X, Y)$ ?

Select one:

- 1
- 0,5
- 1
- 0,5

The correct answer is: 0,5

## QUESTION 175 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X$  and  $Y$  be random variables with  $E[X]=1$ ,  $E[Y]=2$ , and  $\text{Cov}(X,Y)=-2$ .  
What is  $E[XY]$ ?

Select one:

- 1
- 1
- 2
- 0

The correct answer is: 0

## QUESTION 176 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X$  and  $Y$  be two independent random variables with  $V[X+Y]=2$  and  $V[X-2Y]=1$ .  
What is  $V[X]$ ?

Select one:

- $3/7$
- 3
- $1/3$
- $7/3$

The correct answer is:  $7/3$

## QUESTION 177 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$ , be three random variables. The variance-covariance matrix of  $X = (X_1, X_2, X_3)$  is denoted by  $S$ .

Where in  $S$  will you find  $\text{Cov}(X_2, X_3)$ ?

Select one:

- Second row third column only
- Third row second column only
- Second row third column and third row second column
- Second row third column and second row second column

The correct answer is: Second row third column and third row second column

## QUESTION 178 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$ , be three random variables. The variance-covariance matrix of  $X = (X_1, X_2, X_3)$  is denoted by  $S$ .

Where in  $S$  will you find  $V[X_2]$ , the variance of  $X_2$ ?

Select one:

- Second row second column only
- Second row first column only
- First row second column only
- Second row second column and second row first column

The correct answer is: Second row second column only



## QUESTION 179 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X_1$ ,  $X_2$ , and  $X_3$  be independent random variables with variances  $V[X_1]=2$ ,  $\text{Var}[X_2]=2$ , and  $\text{Var}[X_3]=4$ .

Which of the following are equal to  $\text{Var}[2X_1]$ ?

1.  $\text{Var}[X_1+X_2]$
2.  $\text{Var}[2X_2]$

Select one:

- 2. only
- 1. and 2. only
- 1. only
- Neither 1. nor 2.

The correct answer is: 1. and 2. only

## QUESTION 180 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

Let  $X_1$  and  $X_2$  be two random variables with  $V[X_1+X_2]=V[X_1]+V[X_2]$ .

Which of the following are necessarily true?

1.  $\text{Cov}(X_1, X_2)=0$
2.  $X_1$  and  $X_2$  are independent.

Select one:

- Neither 1. nor 2.
- Both 1. and 2.
- 2. only
- 1. only

The correct answer is: 1. only

## QUESTION 181 OF 312

DLBDDSSIS01\_MC\_mittel/Lektion 02

X and Y are two random variables with  $\text{Var}[X]=1$ ,  $\text{Var}[Y]=2$  and  $\text{Cov}(X,Y)=-1$ .  
What is the correct ordering of  $V[X+Y]$ ,  $V[X-Y]$ ,  $V[2X+Y]$  from least to greatest?

Select one:

- $V[X+Y]$ ,  $V[2X+Y]$ ,  $V[X-Y]$
- $V[X-Y]$ ,  $V[X+Y]$ ,  $V[2X+Y]$
- $V[X+Y]$ ,  $V[X-Y]$ ,  $V[2X+Y]$
- $V[2X+Y]$ ,  $V[X+Y]$ ,  $V[X-Y]$

The correct answer is:  $V[X+Y]$ ,  $V[2X+Y]$ ,  $V[X-Y]$

## QUESTION 182 OF 312

DLBDDSSIS01\_MC\_mittel/Lektion 02

X and Y are two random variables with  $\text{Var}[X]=1$ ,  $\text{Var}[Y]=2$  and  $\text{Cov}(X,Y)=-1$ .  
What is the correct ordering of  $V[X+Y]$ ,  $V[4X+Y]$ ,  $V[X+3Y]$  from least to greatest?

Select one:

- $V[X+Y]$ ,  $V[4X+Y]$ ,  $V[X+3Y]$
- $V[4X+Y]$ ,  $V[X+3Y]$ ,  $V[X+Y]$
- $V[X+Y]$ ,  $V[X+3Y]$ ,  $V[4X+Y]$
- $V[4X+Y]$ ,  $V[X+Y]$ ,  $V[X+3Y]$

The correct answer is:  $V[X+Y]$ ,  $V[X+3Y]$ ,  $V[4X+Y]$

## QUESTION 183 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

X and Y are two random variables with  $\text{Var}[X]=1$ ,  $\text{Var}[Y]=2$  and  $\text{Cov}(X,Y)=-1$ .  
What is the correct ordering of  $V[(X+Y)/2]$ ,  $V[(2X+Y)/3]$ ,  $V[(3X+Y)/4]$  from least to greatest?

Select one:

- $V[(X+Y)/2]$ ,  $V[(3X+Y)/4]$ ,  $V[(2X+Y)/3]$
- $V[(X+Y)/2]$ ,  $V[(2X+Y)/3]$ ,  $V[(3X+Y)/4]$ ,
- $V[(2X+Y)/3]$ ,  $V[(X+Y)/2]$ ,  $V[(3X+Y)/4]$
- $V[(2X+Y)/3]$ ,  $V[(3X+Y)/4]$ ,  $V[(X+Y)/2]$

The correct answer is:  $V[(2X+Y)/3]$ ,  $V[(3X+Y)/4]$ ,  $V[(X+Y)/2]$

## QUESTION 184 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 02

X and Y are two random variables with  $\text{Var}[X]=\text{Var}[Y]=1$  and  $\text{Cov}(X,Y)=-0.5$ .  
Suppose that t is a number between 0 and 1.  
For which value of t will the variance of  $tX+(1-t)Y$  be minimized?

Select one:

- 0
- 1
- 0,3
- 0,5

The correct answer is: 0,5

## QUESTION 185 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$  and  $X_2$  are two random variables with  $E[X_1]=E[X_2]=10$ ,  $\text{Var}[X]=\text{Var}[Y]=1$  and  $\text{Cov}(X_1, X_2)=0.5$ . Let  $Y_1=X_1 \cdot X_2$  and  $Y_2=X_1$ .

Use the linearization method to approximate  $\text{Cov}(Y_1, Y_2)$ .

Select one:

- 1
- 5
- 15
- 0

The correct answer is: 15

## QUESTION 186 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$  and  $X_2$  are two random variables with  $E[X_1]=16$ ,  $E[X_2]=4$ ,  $\text{Var}[X]=\text{Var}[Y]=1$  and  $\text{Cov}(X_1, X_2)=0.5$ . Let  $Y_1=X_1/X_2$  and  $Y_2=X_1$ .

Use the linearization method to approximate  $\text{Cov}(Y_1, Y_2)$ .

Round your answer to two decimal places.

Select one:

- 0,25
- 0,25
- 0,75
- 0,75

The correct answer is: -0,25

## QUESTION 187 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$  and  $X_2$  are non-negative random variables with  $E[X_1]=E[X_2]=10$ ,  $\text{Var}[X]=\text{Var}[Y]=1$  and  $\text{Cov}(X_1, X_2)=-0.5$ . Let  $Y=\log(X_1)+\log(X_2)$ . Use the linearization method to approximate  $\text{Var}(Y)$ . Round your answer to two decimal places.

Select one:

- 0,01
- 0,11
- 0,12
- 0,08

The correct answer is: 0,01

## QUESTION 188 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$  and  $X_2$  are non-negative random variables with  $E[X_1]=E[X_2]=10$ ,  $\text{Var}[X]=\text{Var}[Y]=1$  and  $\text{Cov}(X_1, X_2)=-0.5$ . Let  $Y=\log(X_1+X_2)$ . Use the linearization method to approximate  $\text{Var}(Y)$ . Round your answer to three decimal places.

Select one:

- 2,512
- 0,003
- 0,245
- 0,025

The correct answer is: 0,003

## QUESTION 189 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

X is a random variable with  $E[X]=2$  and  $V[X]=0.1$ . Use linearization to approximate the variance of  $Y=1/(1+\exp(-x))$ .

Round your answer to three decimal places.

Select one:

- 0,005
- 0,004
- 0,006
- 0,003

The correct answer is: 0,004

## QUESTION 190 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

If X and Y are two random variables, and  $\text{Cov}(X,Y)$  represents the covariance between the two variables.

Given that  $\text{Cov}(X,Y) = 2$ , what is the value of  $\text{Cov}(2X + 3, 3Y+4)$ ?

Select one:

- 6
- 2
- 12
- 7

The correct answer is: 12

## QUESTION 191 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

Let  $X_1$  and  $X_2$  be two positive random variables with means  $E[X_1]=10$  and  $E[X_2]=20$  respectively and variances  $V[X_1]=V[X_2]=2$ . The covariance of  $X_1$ , and  $X_2$  is  $\text{Cov}(X_1, X_2)=1$ . Use linearization to approximate the variance of  $Y=\log(X_1)+\log(X_2)$ . Round your answer to three decimal places.

Select one:

- 0,015
- 0,045
- 0,035
- 0,025

The correct answer is: 0,035

## QUESTION 192 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

Let  $X_1$  and  $X_2$  be two positive independent random variables with means  $E[X_1]=5$  and  $E[X_2]=10$  respectively and variances  $V[X_1]=V[X_2]=2$ . Let  $Y_1=\log(X_1)+\log(X_2)$  and  $Y_2=\log(X_1)-\log(X_2)$ . Use linearization to approximate the covariance of  $Y_1$  and  $Y_2$ ,  $\text{Cov}(Y_1, Y_2)$ . Round your answer to two decimal places.

Select one:

- 0,05
- 0,06
- 0,07
- 0,08

The correct answer is: 0,06

## QUESTION 193 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$ ,  $X_2$ , and  $X_3$  are independent positive random variables with means  $E[X_1]=10$ ,  $E[X_2]=20$ ,  $E[X_3]=30$  and variances  $V[X_1]=1$ ,  $V[X_2]=2$ ,  $V[X_3]=3$ . Use linearization to approximate the variance of  $Y=X_1 \cdot X_2 / X_3$ .

Round your answer to two decimal places.

Select one:

- 0,95
- 0,75
- 0,81
- 0,91

The correct answer is: 0,81

## QUESTION 194 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

If  $X$  and  $Y$  are two random variables, and  $\text{Cov}(X, Y)$  represents the covariance between the two variables.

Given that  $\text{Var}(X) = 2$ ,  $\text{Var}(Y) = 6$ , and  $\text{Cov}(X, Y) = 3$ , what is the value of  $\text{Cov}(2X + 4Y, 6X + 8Y)$ ?

Select one:

- 296
- 312
- 280
- 200

The correct answer is: 296



## QUESTION 195 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 02

$X_1$  and  $X_2$  are random variables with unit variances  $V[X_1]=1$  and  $V[X_2]=2$  respectively. Their covariance is  $\text{Cov}(X_1, X_2)=-1$ . Their means are  $E[X_1]=40$  and  $E[X_2]=10$ . Use linearization to approximate the variance of  $Y=X_1/X_2$ .

Round your answer to two decimal places.

Select one:

- 0,41
- 0,49
- 0,39
- 0,42

The correct answer is: 0,41

## QUESTION 196 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

Which of the following is true about frequentist statistics?

Select one:

- In frequentist statistics, it is assumed that the parameter of interest is given by prior probability.
- In frequentist statistics, it is assumed that the parameter of interest is unknown but deterministic.
- In frequentist statistics, it is assumed that the parameter of interest is known by deterministic.
- In frequentist statistics, it is assumed that the parameter of interest is a random variable.

The correct answer is: In frequentist statistics, it is assumed that the parameter of interest is unknown but deterministic.

## QUESTION 197 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

If A and B are two events, then the conditional probability  $P(A | B)$  is read as ...

Select one:

- probability of event B given that event A has occurred.
- probability of event A given that event B has occurred.
- probability of event A given that B will never occur.
- probability of event A and B to occur together.

The correct answer is: probability of event A given that event B has occurred.

## QUESTION 198 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

If A and B are two events, then the joint probability  $P(A \cap B)$  can be represented as ...

Select one:

- $P(A | B) + P(B)$
- $P(A | B) * P(B)$
- $(P(A | B) + P(B))/2$
- $P(A | B) / P(B)$

The correct answer is:  $P(A | B) * P(B)$

## QUESTION 199 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

If  $A^c$  denotes the event that is complementary to the event  $A$ , then which of the following holds true?

Select one:

- $P(A) + P(A^c) = 1$
- $P(A) + P(A^c) < 1$
- $P(A) + P(A^c) > 1$
- $P(A) + P(A^c) = 0$

The correct answer is:  $P(A) + P(A^c) = 1$

## QUESTION 200 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

If  $A$  and  $B$  are two events, and  $A^c$  is the event complimentary to the event  $A$ , which of the following expressions is correct?

Select one:

- $P(B) = P(B \cap A) + P(B \cap A^c)$
- $P(B) = P(B \cap A) - P(B \cap A^c)$
- $P(B) = P(B \cap A) + P(B \cap AC)$
- $P(B) = P(B \cap A) + P(B \cap AC)$

The correct answer is:  $P(B) = P(B \cap A) + P(B \cap A^c)$

## QUESTION 201 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

For two events A and B, the Baye's formula for the probability  $P(A/B)$  can be written in which of the following ways? (Note:  $A^c$  is the compliment event to event A)

Select one:

- $(P(B | A^c) * P(A^c))/(P(B | A) * P(A) + P(B | A^c) * P(A^c))$
- $(P(A | B) * P(B))/(P(B | A) * P(A) + P(B | A^c) * P(A^c))$
- $(P(B | A) * P(A))/(P(B | A) * P(A) + P(B | A^c) * P(A^c))$
- $(P(B | A) * P(A | B))/(P(B | A) * P(A) + P(B | A^c) * P(A^c))$

The correct answer is:  $(P(B | A) * P(A))/(P(B | A) * P(A) + P(B | A^c) * P(A^c))$

## QUESTION 202 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

If A and B are two events in a sample space S, which of the following represents marginal probability of event B?

Select one:

- $1 - P(B)$
- $P(B/A)$
- $P(A/B)$
- $P(B)$

The correct answer is:  $P(B)$

## QUESTION 203 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

Which of the following statements is true about prior distribution?

1. This distribution is the target parameter of interest set after observing any data.
2. It encodes our belief about the parameter of interest.

Select one:

- Both 1. and 2.
- Only 2.
- Only 1.
- Neither 1. nor 2.

The correct answer is: Only 2.

## QUESTION 204 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

The probability quantity occurring in the denominator of Baye's formula is called the...

Select one:

- evidence.
- target.
- statistic.
- prior.

The correct answer is: evidence.

## QUESTION 205 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

The two schools of thoughts, in Bayesian method of estimation, for choice of prior are...

Select one:

- Subjective and Objective Bayesians.
- Prior and Posterior Bayesians.
- Simple and Composite Bayesians.
- Composite and Conjugate Bayesians.

The correct answer is: Subjective and Objective Bayesians.

## QUESTION 206 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

What is the conjugate prior to a Bernoulli( $\pi$ ) likelihood?

Select one:

- Beta
- Gaussian
- Poisson
- Gamma

The correct answer is: Beta

## QUESTION 207 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

What is the target parameter for Geometric( $\Pi$ ) likelihood?

Select one:

- $\Pi$  (probability)
- $\alpha$  (probability)
- $\lambda$  (rate)
- $\beta$  (rate)

The correct answer is:  $\Pi$  (probability)

## QUESTION 208 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

What is parzen windows method?

Select one:

- It is a method to estimate the probability distribution function of a distribution from a finite sample provided you have knowledge of its parameters.
- It is a method to estimate the probability distribution function of a distribution from a finite sample.
- It is a method to estimate the probability distribution function of a distribution from an infinite sample.
- It is a method to estimate the probability distribution function of a distribution from a finite sample provided you have knowledge of the distribution.

The correct answer is: It is a method to estimate the probability distribution function of a distribution from a finite sample.

## QUESTION 209 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

One of the simplest visual tools we can use to shape of a distribution from a finite sample is...

Select one:

- Line plot
- Scatterplot
- Heatmap
- Histogram

The correct answer is: Histogram

## QUESTION 210 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

In the parzen window method for approximating the density of the distribution, the kernel function  $K$ ...

Select one:

- Lies between 0 and 1
- Is non-negative
- Lies between -1 and 1
- Is negative

The correct answer is: Is non-negative



## QUESTION 211 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

Which of the following is an application of K-Nearest-Neighbors?

1. Classification
2. Approximate the probability density directly from the data

Select one:

- Both 1. and 2.
- Only 2.
- Neither 1. nor 2.
- Only 1.

The correct answer is: Both 1. and 2.

## QUESTION 212 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 03

Which of the following is a correct statement about K-Nearest-Neighbors?

Select one:

- It is a parametric technique.
- It can be used only for classification problems.
- It has a fixed window size.
- It is a non-parametric technique.

The correct answer is: It is a non-parametric technique.

## QUESTION 213 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

Which of the following is true about Bayesian statistics?

Select one:

- In Bayesian statistics, the maximum likelihood approach is selected to estimate parameter of interest.
- In Bayesian statistics, the distribution of parameter of interest is chosen that depends on the current observed data. .
- In Bayesian statistics, it is assumed that the parameter of interest is a random variable.
- In Bayesian statistics, it is assumed that the parameter of interest is unknown but deterministic.

The correct answer is: In Bayesian statistics, it is assumed that the parameter of interest is a random variable.

## QUESTION 214 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

If A and B are two events associated with the same sample space of a random experiment, then the conditional probability  $P(A|B)$  is given by...

Select one:

- $P(A \cap B) / P(B)$ , provided  $P(B) \neq 0$
- $P(A \cap B) * P(B)$
- $P(A \cap B) / P(B)$ , provided  $P(B) = 0$
- $P(A \cap B) / P(A)$

The correct answer is:  $P(A \cap B) / P(B)$ , provided  $P(B) \neq 0$

## QUESTION 215 OF 312

DLBSSIS01\_MC\_mittel/Lektion 03

Which of the following statements is true?

1. Conditional Probability is the joint probability divided by the probability of the conditional event.
2. Joint probability is the product of the conditional probability times the marginal probability of the conditioned event.

Select one:

- Only 2.
- Only 1.
- Neither 1. nor 2.
- Both 1. and 2.

The correct answer is: Both 1. and 2.

## QUESTION 216 OF 312

DLBSSIS01\_MC\_mittel/Lektion 03

If the probability of event that it will be a bright day tomorrow is 0.6, what is the complimentary probability that it will not be a bright day?  
Round the answer to one decimal place.

Select one:

- 0,3
- 0,6
- 0,2
- 0,4

The correct answer is: 0,4

## QUESTION 217 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

Which of the following is true about conjugate prior?

Select one:

- When a prior distribution results in a posterior distribution of the different functional form with same parameters, it's called a conjugate prior.
- Conjugate priors are essential because they allow us to investigate the posterior analytically, even without formulas.
- When a prior distribution results in a posterior distribution of the same functional form with same parameters, it's called a conjugate prior.
- Conjugate priors are desirable because they allow us to investigate the posterior analytically, with formulas.

The correct answer is: Conjugate priors are desirable because they allow us to investigate the posterior analytically, with formulas.

## QUESTION 218 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

Previous probabilities in Bayes theorem that gets updated with new information is called ...

Select one:

- prior probabilities.
- independent probabilities.
- posterior probabilities.
- dependent probabilities.

The correct answer is: posterior probabilities.

## QUESTION 219 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

Which of the following statements is correct about the window-size, 'h', in Parzen windows?

Select one:

- $h=0$
- $h<0$
- $h>0$
- $-1<h<1$

The correct answer is:  $h>0$

## QUESTION 220 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

The choice of the kernel and the window size in Parzen window depends on which of the following?

Select one:

- Sample mean
- Data set
- Distribution of the data
- Sample size

The correct answer is: Data set

## QUESTION 221 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

You have parzen window method to estimate the approximate probability density function. The quality of the resulting probability density is dependent on which of the following?

1. Choice of the kernel
2. Choice of the window size

Select one:

- Both 1. and 2.
- Only 2.
- Only 1.
- Neither 1. nor 2.

The correct answer is: Both 1. and 2.

## QUESTION 222 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

Choosing a extreme value of K, such as 1-KNN will result in which of the following?

Select one:

- It is a good choice as it will reduce the distance between two data points.
- It will be ideal choice as t will be easier and faster to compute.
- It is not ideal since it will use only a small part of the neighborhood of the data points.
- It will diminish the idea of localized information.

The correct answer is: It is not ideal since it will use only a small part of the neighborhood of the data points.

## QUESTION 223 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 03

One of the most challenging steps in K-Nearest Neighbor classifier is ...

Select one:

- deciding on the evaluation metric.
- to subset the data for classification.
- selecting between 1-KK and n-KNN classifiers, where n represents the number of records.
- to find the best possible value of K.

The correct answer is: to find the best possible value of K.

## QUESTION 224 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

If A and B are two events such that  $P(A) = 1/9$ , and  $P(B) = 0$ , then what will be the value of  $P(A|B)$ ?

Select one:

- 1/9
- 1
- Not defined
- 0

The correct answer is: Not defined

## QUESTION 225 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

The earlier probabilities in Bayes Theorem that are updated with the help of new available information is defined as ...

Select one:

- posterior probabilities.
- independent probabilities.
- conjugate probabilities.
- prior probabilities.

The correct answer is: posterior probabilities.

## QUESTION 226 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

You want to use Bayes' Theorem to calculate the probability  $P(A/B)$ . Which of the following represent the necessary information you'll need to complete this task? (Note:  $A^c$  represents the event complimentary to event A)

Select one:

- $P(A)$ ,  $P(A^c)$ , and  $P(B/A)$
- $P(A)$ ,  $P(B)$ , and  $P(B/A)$
- $P(B)$  and  $P(B/A)$
- $P(A)$ ,  $P(B)$ , and  $P(B/A^c)$

The correct answer is:  $P(A)$ ,  $P(B)$ , and  $P(B/A)$



## QUESTION 227 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

Given that A and B are events such that  $P(A) = 0.3$ ,  $P(B) = 0.6$  and  $P(A \cap B) = 0.2$ , then  $P(A|B)$  ?

Select one:

- 2/3
- 1/2
- 1/3
- 3/6

The correct answer is: 1/3

## QUESTION 228 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

Given that A and B are two events, which of the following represents the joint probability  $P(A \cap B)$ ?

1.  $P(B) * P(A|B)$
2.  $P(A) * P(B|A)$

Select one:

- Only 2.
- Both 1. and 2.
- Only 1.
- Neither 1. nor 2.

The correct answer is: Both 1. and 2.

## QUESTION 229 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

Which of the following sequences represents the probability revision sequence using Baye's theorem?

Select one:

- New Information → Prior Probabilities → Application of Baye's theorem → Posterior Probabilities
- Prior Probabilities → New Information → Conjugate Probabilities → Posterior Probabilities
- Prior Probabilities → New Information → Application of Baye's theorem → Posterior Probabilities
- Conjugate Probabilities → New Information → Application of Baye's theorem → Posterior Probabilities

The correct answer is: Prior Probabilities → New Information → Application of Baye's theorem → Posterior Probabilities

## QUESTION 230 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

The prior probabilities in Bayesian estimation can be classified into ...

Select one:

- four categories.
- one category.
- three categories.
- two categories.

The correct answer is: three categories.

## QUESTION 231 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

Which of the following represents the formula used in the Parzen window method to approximate the density of the distribution using a finite sample  $\{x_1, x_2, \dots, x_n\}$ ? (Note:  $h$  is the bandwidth and  $K$  is the kernel)

Select one:

- $(1/nh) * (\sum K((x-x_i)/h))$ , where  $l$  ranges from 1 to  $n-1$
- $(1/h) * (\sum K((x-x_i)/h))$ , where  $l$  ranges from 1 to  $n-1$
- $(1/nh) * (\sum K((x-x_i)/h))$ , where  $l$  ranges from 1 to  $n$
- $(1/h) * (\sum K((x-x_i)/n))$ , where  $l$  ranges from 1 to  $n$

The correct answer is:  $(1/nh) * (\sum K((x-x_i)/h))$ , where  $l$  ranges from 1 to  $n$

## QUESTION 232 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

Which of the following statements is correct about K-nearest neighbor?

1. Increase the  $K$  will lead to improvement in classification.
2.  $K$ - stands for number of hyperparameters to be tuned for classification problems?

Select one:

- Only 2.
- Both 1. and 2.
- Only 1.
- Neither 1. nor 2.

The correct answer is: Neither 1. nor 2.

## QUESTION 233 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

While solving classification problem using K-nearest neighbor, you have 15 data points and respective classes.

Which of the following is an example of extreme classifier?

Select one:

- 1 KNN
- 3 KNN
- 4 KNN
- 5 KNN

The correct answer is: 1 KNN

## QUESTION 234 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 03

When applying K-nearest neighbors across series of data points, which of the following is true for the distance between two points p and q?

Select one:

- The distance is always smaller than or equal to zero.
- The distance is always greater than or equal to zero.
- The distance always lies between -1 and 1.
- The distance between p and q is greater than the distance between q and p.

The correct answer is: The distance is always greater than or equal to zero.

## QUESTION 235 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is considered to be at the core of statistical inference?

Select one:

- Visualization
- Descriptive Statistics
- Standard Deviation
- Hypothesis Testing

The correct answer is: Hypothesis Testing

## QUESTION 236 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

What is type I error?

Select one:

- The decision to reject the null hypothesis when it is true.
- The decision to accept the null hypothesis when it is true.
- The decision to accept the null hypothesis when it is false.
- The decision to not reject the null hypothesis when it is false.

The correct answer is: The decision to reject the null hypothesis when it is true.

## QUESTION 237 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The probability of committing a type I error is represented by...

Select one:

$\beta$ .

$\alpha$ .

$1-\beta$ .

$\mu$ .

The correct answer is:  $\alpha$ .

## QUESTION 238 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The relationship between  $\alpha$  and  $\beta$  is...

Select one:

exponential.

inverse.

linear.

quadratic.

The correct answer is: inverse.

## QUESTION 239 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is accepted if the null hypothesis is false?

Select one:

- Composite Hypothesis
- Simple Hypothesis
- Statistical Hypothesis
- Alternative Hypothesis

The correct answer is: Alternative Hypothesis

## QUESTION 240 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is true about test statistic?

Select one:

- Test statistic is a parameter that is used for calculating probability of rejecting the null hypothesis.
- Test statistic is the random variable that standardizes the quantity measuring departure from the null hypothesis.
- Test statistic is the constant value that standardizes the quantity measuring departure from the null hypothesis.
- The variable chosen for test statistic, usually, does not have a known distribution.

The correct answer is: Test statistic is the random variable that standardizes the quantity measuring departure from the null hypothesis.

## QUESTION 241 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is an example of parametric test?

Select one:

- One sample Wilcoxon Test
- Chi-square Test of Independence
- Mann-Whitney Test
- One sample t-test

The correct answer is: One sample t-test

## QUESTION 242 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

What is a non parametric test?

Select one:

- These are tests in which the hypothesis does not involve statements about sample parameter.
- These are tests in which the hypothesis does involve statements about sample parameter.
- These are tests in which the hypothesis does not involve statements about population parameter.
- These are tests in which the hypothesis does involve statements about population parameter.

The correct answer is: These are tests in which the hypothesis does not involve statements about population parameter.



## QUESTION 243 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

How many parameters does the chi-square distribution have?

Select one:

- Three
- Zero
- One
- Two

The correct answer is: One

## QUESTION 244 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The Mean of Chi Squared distribution with n-degree of freedom is...

Select one:

- $2n$ .
- $n$ .
- $n/2$ .
- $n^2$ .

The correct answer is:  $n$ .

## QUESTION 245 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The variance of Chi Squared distribution with n-degree of freedom is...

Select one:

- $\exp(n)$ .
- $2n$ .
- $n^2$ .
- $n$ .

The correct answer is:  $2n$ .

## QUESTION 246 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is true about two-sample tests?

Select one:

- These are tests that require two samples, one from each population.
- These are tests that assume that two populations are dependent of one another.
- These are tests that require two samples from the same population.
- These are tests that assume that two samples are dependent of one another.

The correct answer is: These are tests that require two samples, one from each population.

## QUESTION 247 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following statements is correct about A/B testing?

1. It is essential that we acquire data from a randomized experiment.
2. The two samples are dependent of one another.

Select one:

- Only 1.
- Neither 1. nor 2.
- Only 2.
- Both 1. and 2.

The correct answer is: Only 1.

## QUESTION 248 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

While comparing means from independent populations using a t-test, which of the following assumptions is needed?

Select one:

- The sample sizes are equal for both the populations.
- The mean of the two population is same.
- The underlying distribution from which both samples are drawn are skewed.
- The underlying distribution from which both samples are drawn are Gaussian.

The correct answer is: The underlying distribution from which both samples are drawn are Gaussian.

## QUESTION 249 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The probability of committing a type I error where we reject a true null hypothesis is represented by...

Select one:

- $\alpha$ .
- $\beta$ .
- $1-\alpha$ .
- $1-\beta$ .

The correct answer is:  $\alpha$ .

## QUESTION 250 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

The power of a hypothesis test is represented as ...

Select one:

- $1-\beta$ ;
- $1-\alpha$ ;
- $\mu$ .
- $\alpha$ .

The correct answer is:  $1-\beta$ ;

## QUESTION 251 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Which of the following is a correct statement about multiple testing?

Select one:

- When we want to test each of the many null hypothesis separately, the probability of type I error is reduced.
- When we want to test each of the many null hypothesis separately, the probability of type II error is amplified.
- When we want to test each of the many null hypothesis separately, the probability of type I error is amplified.
- When we want to test each of the many null hypothesis separately, the probability of type I and type II error is equal.

The correct answer is: When we want to test each of the many null hypothesis separately, the probability of type I error is amplified.

## QUESTION 252 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

Multiple testing is about testing...

Select one:

- two or more hypothesis.
- two hypothesis.
- confidence level of hypothesis test.
- power of a single hypothesis test.

The correct answer is: two or more hypothesis.

## QUESTION 253 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

What is the relationship between type I error and type II error?

Select one:

- Direct
- Both are equal
- Inverse
- Type I error = 1 - type II error

The correct answer is: Inverse

## QUESTION 254 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

When type I error (alpha) is very small, what is the impact on type II error (beta)?

Select one:

- Beta will be equal to alpha.
- Beta will be small.
- Beta will be large.
- Beta will be double of alpha.

The correct answer is: Beta will be large.

## QUESTION 255 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

How does the type II error (beta) affect the power of the test?

Select one:

- When beta is small, power of test is also small.
- When beta is large, power of test is small.
- Beta does not affect the power of the test.
- When beta is large, power of test is also large.

The correct answer is: When beta is large, power of test is small.

## QUESTION 256 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

In multiple testing, if FP and TP represents False and True Positives, respectively; and TN and FN represents True and False Negatives, respectively; which of the following represents total number of rejected null hypothesis.

Select one:

- TP + TN
- FP + TN
- FP + TP
- FP + FN

The correct answer is: FP + TP

## QUESTION 257 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

In multiple testing, if FP and TP represents False and True Positives, respectively; and TN and FN represents True and False Negatives, respectively; which of the following represents total number of true null hypothesis.

Select one:

- FP + TN
- FP + FN
- TN + TP
- FP + TP

The correct answer is: FP + TN

## QUESTION 258 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 04

In multiple testing, if FP and TP represents False and True Positives, respectively; and TN and FN represents True and False Negatives, respectively; which of the following represents total number of false null hypothesis.

Select one:

- FP + TN
- TP + TN
- TP + FN
- FP + FN

The correct answer is: TP + FN



## QUESTION 259 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following is an application of hypothesis testing?

1. Marketing and Psychology
2. Finance and Medicine

Select one:

- Only 1.
- Both 1. and 2.
- Only 2.
- Neither 1. nor 2.

The correct answer is: Both 1. and 2.

## QUESTION 260 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following is true about two sided hypothesis test?

Select one:

- In this test, the null hypothesis indicates a directional difference.
- In this test, the alternative hypothesis indicates that the true parameter is different than what is claimed.
- In this test, the alternative hypothesis indicates a directional difference.
- In this test, the null hypothesis indicates that the true parameter is different than what is claimed.

The correct answer is: In this test, the null hypothesis indicates that the true parameter is different than what is claimed.

## QUESTION 261 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following distributions is used to accept or not accept the null hypothesis?

Select one:

- Poisson Distribution
- Normal Distribution
- Chi-Squared Distribution
- Gaussian Distribution

The correct answer is: Chi-Squared Distribution

## QUESTION 262 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following command in R will compute the critical value of a chi-square distribution with 'df' degrees of freedom and alpha significance level.

Select one:

- `qchisq(1-alpha,1-df)`
- `qchisq(1-alpha,df)`
- `qchisq(alpha,df)`
- `qchisq(1-alpha,df-1)`

The correct answer is: `qchisq(1-alpha,df)`

## QUESTION 263 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following command in excel will conduct the t-test for two independent samples A and B, assuming the variances are unknown and unequal.

Select one:

- =T.TEST(A,B,2)
- =T.TEST(A,B,3)
- =T.TEST(A,B,2:3)
- =T.TEST(A,B,2,3)

The correct answer is: =T.TEST(A,B,2,3)

## QUESTION 264 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

For testing equality of means when the variances of population are unknown but assumed to be equal, the degree of freedom for  $n_1$  and  $n_2$  sample sizes is given by...

Select one:

- $n_1+n_2-2$
- $2*(n_1+n_2)$
- $n_1+n_2+2$
- $(n_1+n_2)/2$

The correct answer is:  $n_1+n_2-2$

## QUESTION 265 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

You are testing the difference between the means of two related populations, represented by  $m_1$  and  $m_2$ , respectively.

Which of the following would be a correct null hypothesis?

Select one:

- $(m_1 - m_2) > 0$
- $m_1 - m_2 = 1$
- $-1 < (M_1 - M_2) > 1$
- $m_1 - m_2 = 0$

The correct answer is:  $m_1 - m_2 = 0$

## QUESTION 266 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

In a hypothesis test, the p-value comes out to be less than  $\alpha$ .

What conclusion can be drawn from this?

Select one:

- Reject the Alternate Hypothesis
- Reject the Null Hypothesis
- The value of alpha should be examined
- Fail to Reject the Null Hypothesis

The correct answer is: Reject the Null Hypothesis

## QUESTION 267 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following statements is true about the power of test?

Select one:

- We want the power of a hypothesis test to be as large as possible.
- Power of the test is independent of the sample size in all cases.
- Power of test is compliment of the type I error.
- We want the power of a hypothesis test to be as small as possible.

The correct answer is: We want the power of a hypothesis test to be as large as possible.

## QUESTION 268 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

You want to construct a confidence interval estimate.  
This will be constructed around...

Select one:

- the degree of freedom.
- the population parameter.
- the point estimate.
- the power of test.

The correct answer is: the point estimate.

## QUESTION 269 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

The probability that a confidence interval contains the true value of the target parameter is called the...

Select one:

- p-value.
- power of test.
- confidence level.
- significance level.

The correct answer is: confidence level.

## QUESTION 270 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

All other things remaining constant, how is the confidence interval size affected by doubling the sample size?

Select one:

- This will multiply the interval size by two.
- This will divide the interval size by  $\sqrt{2}$ .
- This will divide the interval size by two.
- This will multiply the interval size by  $\sqrt{2}$ .

The correct answer is: This will divide the interval size by  $\sqrt{2}$ .

## QUESTION 271 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following statements is true about the margin of error?

1. The margin of error varies directly with the standard deviation of the sample.
2. The margin of error varies directly with the square root of the sample size.

Select one:

- Only 2.
- Only 1.
- Neither 1. nor 2.
- Both 1. and 2.

The correct answer is: Only 1.

## QUESTION 272 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

Which of the following statements is true about the margin of error?

1. The margin of error varies directly with the population size.
2. The margin of error varies is not affected by the value of the sample mean.

Select one:

- Only 1.
- Only 2.
- Neither 1. nor 2.
- Both 1. and 2.

The correct answer is: Only 2.

## QUESTION 273 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

The confidence interval for the true population at 99% level of confidence was reported to be (0.6,0.8).

Which of the following is a possible 95% confidence interval from the same sample?  
Round the answer to two decimal places.

Select one:

- (0.51,0.89)
- (0.56,0.84)
- (0.59,0.82)
- (0.67, 0.78)

The correct answer is: (0.67, 0.78)

## QUESTION 274 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 04

The confidence interval for the true population at 90% level of confidence was reported to be (0.6,0.8).

Which of the following is a possible 95% confidence interval from the same sample?  
Round the answer to one decimal place.

Select one:

- (0.7,0.8)
- (0.4,0.8)
- (0.5,0.9)
- (0.6,0.8)

The correct answer is: (0.5,0.9)



## QUESTION 275 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

Which of the following is true about the significance level of a hypothesis test?

Select one:

- The significance level of a hypothesis test is the highest probability of correctly rejecting the null hypothesis.
- The significance level of a hypothesis test is the mean probability of incorrectly rejecting the null hypothesis.
- The significance level of a hypothesis test is the highest probability of incorrectly accepting the null hypothesis.
- The significance level of a hypothesis test is the highest probability of incorrectly rejecting the null hypothesis.

The correct answer is: The significance level of a hypothesis test is the highest probability of incorrectly rejecting the null hypothesis.

## QUESTION 276 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

'It is easier to accumulate wealth if one starts investing before the age of 20'.  
This statement is an example of...

Select one:

- an alternative hypothesis.
- two-tailed hypothesis.
- null hypothesis.
- one-tailed hypothesis.

The correct answer is: one-tailed hypothesis.

## QUESTION 277 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

For the normal distribution, the null hypothesis, 'Ho' states that  $\mu = 25$ . against the alternative hypothesis, 'H1', which states that  $\mu < 25$ .

This hypothesis test is an example of?

Select one:

- Two sided test
- Center tailed test
- Left tailed test
- Right tailed test

The correct answer is: Left tailed test

## QUESTION 278 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

The least number of cases that need to appear in each category for chi-square test is...

Select one:

- 1
- 6
- 5
- 3

The correct answer is: 5

## QUESTION 279 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

If 'n' and 'c' are the number of rows and columns, respectively, in the contingency table, than which of the following formula can be used to calculate degree of freedom in a chi-square test of independence?

Select one:

- $(n-1)*(c-1)$
- $n+c-2$
- $(n+c)/2$
- $n*c$

The correct answer is:  $(n-1)*(c-1)$

## QUESTION 280 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

Which of the following statements is true about Chi-square test for independence?

Select one:

- It is used to test whether there is relationship between the sample estimate and population estimate.
- It is used to test whether there is relationship between two numerical variables.
- It is used to test whether there is relationship between the population mean and variance.
- It is used to test whether there is relationship between two categorical variables.

The correct answer is: It is used to test whether there is relationship between two categorical variables.

## QUESTION 281 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

The enrollment in preschool franchise for the week is given as Monday:23, Tuesday:27, Wednesday: 12, Thursday: 18, Friday: 10, Saturday: 34, and Sunday: 37.  
For chisquare test, what will be the expected number of enrollments on Monday and Sunday?

Select one:

- Monday: 23 and Sunday: 23
- Monday: 27 and Sunday: 27
- Monday: 23 and Sunday: 27
- Monday: 34 and Sunday: 24

The correct answer is: Monday: 23 and Sunday: 23

## QUESTION 282 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

When comparing two population proportions, which of the following will be an acceptable null hypothesis if the alternate hypothesis is  $p_1 < p_2$ ?

Select one:

- Null Hypothesis:  $p_1 > p_2$
- Null Hypothesis:  $p_1 < p_2$
- Null Hypothesis:  $p_1 = p_2$
- Null Hypothesis:  $p_1 \neq p_2$

The correct answer is: Null Hypothesis:  $p_1 = p_2$

## QUESTION 283 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

You are conducting a t-test for a difference between two population means. In this case, when should a pooled variance be calculated?

Select one:

- When the population variances are known but not equal
- When the population variances are unknown but assumed to be equal
- When the population variances are known and equal
- When the sample variances are easy to calculate

The correct answer is: When the population variances are unknown but assumed to be equal

## QUESTION 284 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

Which of the following influences the width of the confidence interval?

Select one:

- Small confidence level
- The estimator of the parameter of interest
- Large confidence level
- Decrease in uncertainty

The correct answer is: Large confidence level

## QUESTION 285 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

Which of the following is correct about the p-value in hypothesis testing?

Select one:

- The p-value can be used to assert that the alternative hypothesis is true.
- The large p-value indicates that the observed effect is due to random chance.
- It's the smallest value of type I error for which the data suggests that null hypothesis can be rejected.
- The probability of failing to reject the null hypothesis, given the observed results.

The correct answer is: It's the smallest value of type I error for which the data suggests that null hypothesis can be rejected.

## QUESTION 286 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

In multiple testing, following numbers are recorded. 1) The number of true null hypothesis that is rejected is 40. 2) The number of false null hypothesis that is rejected is 30. 3) The number of true null hypothesis that is not rejected is 20. 4) The number of false null hypothesis that is not rejected is 10.

Given the above information, which of the following values represent total number of null hypothesis?

Select one:

- 20
- 90
- 70
- 100

The correct answer is: 100

## QUESTION 287 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

In multiple testing, following numbers are recorded. 1) The number of true null hypothesis that is rejected is 40. 2) The number of false null hypothesis that is rejected is 30. 3) The number of true null hypothesis that is not rejected is 20. 4) The number of false null hypothesis that is not rejected is 10.

Given the above information, which of the following values represent total number of null hypothesis that is not rejected?

Select one:

- 60
- 100
- 30
- 40

The correct answer is: 30

## QUESTION 288 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

In multiple testing, following numbers are recorded. 1) The number of true null hypothesis that is rejected is 40. 2) The number of false null hypothesis that is rejected is 30. 3) The number of true null hypothesis that is not rejected is 20. 4) The number of false null hypothesis that is not rejected is 10.

Given the above information, which of the following values represent total number of false null hypothesis?

Select one:

- 60
- 40
- 70
- 50

The correct answer is: 40

## QUESTION 289 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

In multiple testing, following numbers are recorded. 1) The number of true null hypothesis that is rejected is 40. 2) The number of false null hypothesis that is rejected is 30. 3) The number of true null hypothesis that is not rejected is 20. 4) The number of false null hypothesis that is not rejected is 10.

Given the above information, which of the following values represent total number of rejected null hypothesis?

Select one:

- 70
- 50
- 40
- 60

The correct answer is: 70

## QUESTION 290 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 04

In multiple testing, following numbers are recorded. 1) The number of true null hypothesis that is rejected is 40. 2) The number of false null hypothesis that is rejected is 30. 3) The number of true null hypothesis that is not rejected is 20. 4) The number of false null hypothesis that is not rejected is 10.

Given the above information, which of the following values represent total number of true null hypothesis?

Select one:

- 60
- 30
- 40
- 70

The correct answer is: 60



## QUESTION 291 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

You want to classify customer churn for an ecommerce company. You will do this by making a decision  $D$  based on an observation  $x$ . ' $x$ ' contains information basis which the true classification  $y$  is done. You choose  $y=1$  to indicate churn and  $y=0$  to indicate non-churn. Which of the following represents a correct decision match?

Select one:

- The customer is churn, and the decision function  $D$  predicts  $y=0$ .
- The customer is not churn, and the decision function  $D$  predicts  $y=1$ .
- The customer is churn, and the decision function  $D$  predicts  $y<1$ .
- The customer is churn, and the decision function  $D$  predicts  $y=1$ .

The correct answer is: The customer is churn, and the decision function  $D$  predicts  $y=1$ .

## QUESTION 292 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

You want to classify customer churn for an ecommerce company. You will do this by making a decision  $D$  based on an observation  $x$ . ' $x$ ' contains information basis which the true classification  $y$  is done. You choose  $y=1$  to indicate churn and  $y=0$  to indicate non-churn. Which of the following represents a correct decision match?

Select one:

- The customer is churn, and the decision function  $D$  predicts  $y=0$ .
- The customer is non-churn, and the decision function  $D$  predicts  $y=0$ .
- The customer is non-churn, and the decision function  $D$  predicts  $y=1$ .
- The customer is churn, and the decision function  $D$  predicts  $y<1$ .

The correct answer is: The customer is non-churn, and the decision function  $D$  predicts  $y=0$ .

## QUESTION 293 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

You want to classify customer churn for an ecommerce company. You will do this by making a decision  $D$  based on an observation  $x$ . ' $x$ ' contains information basis which the true classification  $y$  is done. You choose  $y=1$  to indicate churn and  $y=0$  to indicate non-churn. Which of the following statements is correct about an incorrect decision?

1. The customer is churn, but the decision function  $D$  predicts it as non churn.
2. The customer is non churn, but the decision function  $D$  predicts it as churn.

Select one:

- Only 2.
- Only 1.
- Both 1. and 2.
- Neither 1. nor 2.

The correct answer is: Both 1. and 2.

## QUESTION 294 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

Which of the following statements is correct about the loss function?

Select one:

- This function measures the quality of a decision against the false state.
- This function is a measure of volatility in the decision.
- This function measures the quality of a decision against the true state.
- This function measures the median amount of risk in a decision.

The correct answer is: This function measures the quality of a decision against the true state.

## QUESTION 295 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

Which of the following statements is correct about the loss function?

Select one:

- It is a negative function.
- It is expressed in the complex number form, i.e.,  $a + bi$ .
- It is a non-negative function.
- It only takes binary values, 0 and 1.

The correct answer is: It is a non-negative function.

## QUESTION 296 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

The loss matrix is the analogous object of a ...

Select one:

- Classification Tree.
- Dendogram.
- ROC Curve.
- Confusion matrix.

The correct answer is: Confusion matrix.

## QUESTION 297 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

How can we quantify the goodness of a decision function?

Select one:

- By analyzing the fixed value of the loss per decision
- By visualizing the loss histogram
- By simulating the loss function and calculating the probabilities
- By analyzing the expected value of the loss function

The correct answer is: By analyzing the expected value of the loss function

## QUESTION 298 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

Which of the following is true about a risk function?

Select one:

- It is the expected loss for a given loss function and a given decision function.
- It is the probability of loss for a given loss function irrespective of the decision function.
- It is the probability of loss for a given loss function and a given decision function.
- It is the expected loss for a given loss function and is fixed across any decision function.

The correct answer is: It is the expected loss for a given loss function and a given decision function.

## QUESTION 299 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

Which of the following is true about Baye's risk?

Select one:

- In Baye's risk, the false state is treated as a random variable with a given prior distribution.
- In Baye's risk, the true state is treated as a constant with a given posterior distribution.
- In Baye's risk, the true state is treated as a random variable with a given posterior distribution.
- In Baye's risk, the true state is treated as a random variable with a given prior distribution.

The correct answer is: In Baye's risk, the true state is treated as a random variable with a given prior distribution.

## QUESTION 300 OF 312

DLBDSSIS01\_MC\_leicht/Lektion 05

The decision function that minimizes the Baye's risk is called...

Select one:

- Bayes decision function.
- Minimimax risk function.
- Admissible decision function.
- Maximum risk function.

The correct answer is: Bayes decision function.

## QUESTION 301 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

If  $y$  is the true state and  $D$  is the decision, then for a binary churn problem, where 1 represents churn and 0 represents non-churn, what will be the value of loss function  $L(0,0)$ ?

Select one:

- 1
- $> 0$
- 0
- $< 0$

The correct answer is: 0

## QUESTION 302 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

If  $y$  is the true state and  $D$  is the decision, then for a binary churn problem, where 1 represents churn and 0 represents non-churn, what will be the value of loss function  $L(1,1)$ ?

Select one:

- 1
- 0
- $> 0$
- $< 0$

The correct answer is: 0

## QUESTION 303 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

If  $y$  is the true state and  $D$  is the decision, then for a binary churn problem, where 1 represents churn and 0 represents non-churn, what will be the value of loss function  $L(0,1)$ ?

Select one:

- 0
- $> 1$
- 1
- 1

The correct answer is: 1

## QUESTION 304 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

If  $y$  is the true state and  $D$  is the decision, then for a binary churn problem, where 1 represents churn and 0 represents non-churn, what will be the value of loss function  $L(1,0)$ ?

Select one:

- $> 1$
- 1
- 0
- 1

The correct answer is: 1

## QUESTION 305 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

Given a loss function  $L$ , decision function  $D$ , and a random variable  $X$  whose values we observe, the risk function is given by which of the following expressions?

Select one:

- $E[L(X, L(Y))]$
- $E[L(y, D(X))]$
- $E[D(y, L(X))]$
- $E[D(y)] * L(X)$

The correct answer is:  $E[L(y, D(X))]$

## QUESTION 306 OF 312

DLBDSSIS01\_MC\_mittel/Lektion 05

Which of the following statements is true?

1. Maximum risk is the maximum of the risk function over all possible values of the true state.
2. Minimax risk function is used when for each decision function, we compute the average risk across all values of the true state.

Select one:

- Neither 1. nor 2.
- Both 1. and 2.
- Only 2.
- Only 1.

The correct answer is: Only 1.



## QUESTION 307 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

In a loss matrix, each cell in the matrix has the value of the loss incurred by the value of the A against the B. Which of the following represents the correct match for A and B?

Select one:

- A: standard deviation,  
B: false state
- A: standard deviation,  
B: true state
- A: chosen decision function,  
B: true state
- A: chosen decision function,  
B: false state

The correct answer is: A: chosen decision function,  
B: true state

## QUESTION 308 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

You want to classify customer churn for an ecommerce company. You will do this by making a decision D based on an observation x. 'x' contains information basis which the true classification y is done. You choose y=1 to indicate churn and y=0 to indicate non-churn. Which of the following statements could be a correct loss function for this case?

Select one:

- $L(y,D) = 0$ , if  $y=D$ , and  $L(y,D) = 1$ , if  $y \neq D$
- $L(y,D) = 0$ , if  $y=D$ , and  $L(y,D) < 0$ , if  $y \neq D$
- $L(y,D) = 1$ , if  $y=D$ , and  $L(y,D) = 0$ , if  $y \neq D$
- $L(y,D) = 1$ , if  $y=D$ , and  $L(y,D) > 0$ , if  $y \neq D$

The correct answer is:  $L(y,D) = 0$ , if  $y=D$ , and  $L(y,D) = 1$ , if  $y \neq D$

## QUESTION 309 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

Given a loss function  $L$ , decision function  $D$ , and a random variable  $X$  that follows a discrete distribution, the risk function is given by which of the following expressions?

Select one:

- $(\int L(y, D(X))) / P(X=x)$ , for all  $x$
- $\int L(y, D(X)) * P(X=x)$ , for all  $x$
- $(\sum L(y, D(X))) / P(X=x)$ , for all  $x$
- $\sum L(y, D(X)) * P(X=x)$ , for all  $x$

The correct answer is:  $\sum L(y, D(X)) * P(X=x)$ , for all  $x$

## QUESTION 310 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

Minimax is a decision rule used in decision theory to...

Select one:

- minimize the possible gain for a best case (maximum gain) scenario.
- reduce the variability of possible loss for a best case (maximum gain) scenario.
- maximize the possible loss for a worst case (maximum loss) scenario.
- minimize the possible loss for a worst case (maximum loss) scenario.

The correct answer is: minimize the possible loss for a worst case (maximum loss) scenario.

## QUESTION 311 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

The maximum risk for two decision functions  $d_1$  and  $d_2$  is  $x$  and  $y$ , respectively. Under which condition will  $d_1$  be the better decision function?

Select one:

- When  $x < y$
- When  $x$  and  $y$  are not comparable
- When  $x > y$
- When  $x = y$

The correct answer is: When  $x < y$

## QUESTION 312 OF 312

DLBDSSIS01\_MC\_schwer/Lektion 05

If  $\Theta$  represents the parameter to be estimated, and  $D$  represents the decision function, then which of the following represents the risk for a fixed loss function?

Select one:

- $R(\Theta, D)$
- $R(\Theta^2, D)$
- $R(1, \Theta * D)$
- $R(\Theta * D, 1)$

The correct answer is:  $R(\Theta, D)$