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Scientific abstract – *Non-degenerate Potentials on the Quiver $X_7$ and Related Problems*


The theory of quivers with potentials has various connections to representation theory and theoretical physics. For its application to cluster algebras, it is important that the potential possesses a property of being *non-degenerate*. Over an uncountable base field, any quiver admits a non-degenerate potential. However, the proof of this fact is not constructive and thus two fundamental problems still remain: the problem of explicit description of a non-degenerate potential on a given quiver and the question of its uniqueness. So far these problems have been addressed and solved for various classes of quivers, including many of finite mutation type.

The last quiver of finite mutation type for which neither of these problems was settled was the quiver $X_7$. Recently we have been able to solve them by explicitly writing two non-degenerate potentials whose Jacobian algebras are not isomorphic when the characteristic of the ground field equals 2. An interesting feature of these constructions is that for the first time, fundamental properties of the Jacobian algebra of a non-degenerate potential depend on the characteristic of the ground field.

Motivated by these findings, we plan to continue in three directions:

1. **Study the Jacobian algebras of the non-degenerate potentials found so far.**
2. **Find additional non-degenerate potentials on the quiver $X_7$.**
   The Jacobian algebra of one of the potentials we found shows features similar to those attached to quivers arising from triangulations of closed oriented surfaces of positive genus with one puncture. Motivated by this resemblance, we aim to consider certain deformations of the potentials found so far, investigate their non-degeneracy and extract numerical invariants of their Jacobian algebras with the ultimate goal of classifying all the non-degenerate potentials.
3. **Develop a theory of dimer models on closed non-orientable surfaces**
   We propose a geometric model for $X_7$ by a dimer model on the real projective plane. To this end we shall develop a general theory of dimer models on closed, non-orientable surfaces, i.e. a method to construct quivers with potentials from certain graphs on such surfaces. Contrary to the classical setup, the surface is no longer orientable and the graph is not necessarily bipartite.
   We also propose certain families of graphs on the real projective plane which are expected to yield quivers with potentials whose Jacobian algebras share properties similar to those observed so far on the quiver $X_7$.