Financial calculations

## Fundamentals of financial calculations

If you are working in the banking industry it is important to understand financial calculations, from both a technical and a practical perspective. Financial calculations are at the heart of every financial product; understanding the cash flows of financial calculations is a vital part of grasping all aspects of financial markets.

This topic will aid your understanding of the calculations and concepts discussed throughout the rest of the topics, so ensure you complete all of the sections and exercises before continuing with your learning. You can refer back to this topic to refresh your knowledge at any point.

**Financial product**

A product that helps you manage (eg savings and investment products) and use your money (eg deposit accounts and credit cards).

Some statistical calculations are used frequently in banking, so we will explain those calculations and the differences between them.

We will then move on to the concept of interest and the different ways that can be applied.

## Averages – mean, median and mode

When people talk about averages, they are generally talking about the **mean**. However, they could be referring to the **median** or **mode**, which are calculated differently and will produce different answers. Understanding the differences between these three calculations is vital in order to interpret data correctly.

### Mean

The mean of a series of numbers is the **sum of the values divided by the count of the values** (ie the number of values included). It is the most commonly used average and considers all of the values in the series with equal weighting. The disadvantage of it is that it can be skewed by outlying values (ie any extreme values one way or the other).

### Median

The median values of a series of numbers are the **values in the middle of the distribution**. It therefore ignores outliers so can be more representative of the population than the mean. The median may be used to identify the most representative value in a dataset where there is skew, such as income data where there is a floor of zero but no top limit.

### Mode

The mode of a series of numbers is the one that **occurs the most often** but may not be in the middle of the distribution.

Given a large dataset that follows a normal distribution, the mean, median and mode will be similar values.

The differences are shown best by examples. Let’s calculate the mean, median and mode for the following series of 25 numbers.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 22 | 5 | 36 | 10 | 41 |
| 150 | 3 | 97 | 84 | 23 |
| 29 | 14 | 32 | 6 | 2 |
| 30 | 15 | 1 | 22 | 14 |
| 14 | 8 | 30 | 27 | 19 |

**Mean** – total of values / count of values

 = 734 / 25

**Mean = 29.36**

**Median** – sort the values smallest to largest and select the middle value

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 5 | 6 |
| 8 | 10 | 14 | 14 | 14 |
| 15 | 19 | 22 | 22 | 23 |
| 27 | 29 | 30 | 30 | 32 |
| 36 | 41 | 84 | 97 | 150 |

**Median = 22**

**Mode** – sort the values smallest to largest and select the one that occurs the most often

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 5 | 6 |
| 8 | 10 | 14 | 14 | 14 |
| 15 | 19 | 22 | 22 | 23 |
| 27 | 29 | 30 | 30 | 32 |
| 36 | 41 | 84 | 97 | 150 |

**Mode = 14**

These simple functions are accessible in excel as AVERAGE, MEDIAN and MODE.SNGL, all written as =function(data array).

**Activity 1**

From the following dataset, calculate the mean, median and mode values. Type your answers below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 22 | 39 | 80 | 40 | 9 |  |
|  | 83 | 78 | 12 | 3 | 31 |  |
|  | 41 | 7 | 10 | 24 | 90 |  |
|  | 1 | 33 | 11 | 17 | 21 |  |
|  | 14 | 23 | 19 | 2 | 12 |  |

Mean \_\_\_\_\_

Median \_\_\_\_\_

Mode \_\_\_\_\_

## Probability

Probability can be defined as the chance of a specific event occurring. It is useful for activities like pricing risk, where you have specific outcomes, such as a borrower repays a loan or does not repay a loan.

Probability is commonly expressed as a fraction or percentage. For example, when you toss a coin there is a or 50% (or as a decimal: 0.5) chance of it landing on heads.

**FACTFIND**

Take a refresher course in how to write percentages as fractions and decimals before continuing.

BBC Bitesize – [Converting between fractions, decimals and percentages](https://www.bbc.co.uk/bitesize/guides/zc8jtv4/revision/1)

### Calculating probability

We can calculate probability using the following equation:

P(desired outcome) = P(number of possible desired outcomes) / P(all outcomes)

As an example, the probability of throwing a dice and getting a six has **one** desired outcome (ie getting a six) and **six** possible outcomes. So the calculation is:

P(6) = or 16.66667% or 0.1666667

To calculate the probability of multiple events happening, you must first determine whether the outcome of the second event is **independent** of or **dependent** on the outcome of the first event.

### Independent probabilities

This occurs when the outcome of the second event is not impacted by the first event. An example of this would be a coin toss – the outcome of the first coin toss does not impact the outcome of the second coin toss.

For example, let’s calculate the probability of two coin tosses that both produce heads. We can visualise the potential outcomes using the following diagram.

|  |  |  |
| --- | --- | --- |
| **Throw 1** | **Throw 2** | **Outcomes** |

Only one of the four possible outcomes of tossing the coin twice is the desired outcome, so there is a or 25% (0.25) probability of getting that outcome.

Another way to calculate the probability of throwing heads twice is to multiply the chance of throwing heads once by the chance of throwing heads once.

P(heads and heads) = P(heads) × P(heads)

 as a fraction

 as a percentage

 as a decimal

### Dependent probabilities

In dependent scenarios, the second outcome will depend on the first outcome.

For example, a jar contains five sweets, one of each colour: red, blue, yellow, green and orange.

You take two sweets out of the jar.

What is the probability that both the sweets you remove are any of the yellow, green or orange sweets?

For the first selection, you have 3 desired sweets in the jar, so you have a (or 60% or 0.6) chance of selecting one of the desired sweets.

For the second selection, the probability of your second selection resulting in one of the other desired colours is (which reduces to ; or 50% or 0.5) because there are 2 remaining desired sweets in the jar and there are 4 sweets in the jar in total.

The calculation of the overall probability is the same as with the coin toss:

**Activity 2**

Two dice are thrown together. What is the probability that the total number of spots facing up on the two dice when landing together will be greater than seven?

Number of desired outcomes \_\_\_\_

Total number of outcomes \_\_\_\_

P(>7) = \_\_\_\_ / \_\_\_\_ = or \_\_\_\_%

## Interest

Interest can be defined as the charge to a borrower for the use of an asset. It is fundamentally important in banking as a bank’s **net interest margin (NIM)** is critical to making money.

**NIM**

The difference between the total interest received and the total interest paid by the bank, divided by average interest-earning assets.

The bank will want to lend money at a higher rate of interest than it pays for its borrowings (eg customer deposits). The difference between what the bank earns in interest and what it pays out in interest divided by average interest-earning assets, the NIM, is a key measure of profitability for banks.

When setting an interest rate for a borrower, the bank will look at:

* the market rate that it would need to pay to borrow the money it is about to lend;
* the length of time that the borrower wants to borrow the money – a longer term will generally be more expensive;
* the riskiness of the lending, taking into consideration the creditworthiness of the borrower and any collateral that is part of the loan package (eg mortgaged assets); and
* covering a certain portion of the costs.

The amount that a bank can charge may be limited by competition; customers are likely to borrow from the bank with the lowest interest rates.

### Simple *v* compound interest charges

Simple interest charges are purely based on the:

* **amount**;
* **interest rate**; and
* **term** (ie the amount of time it has been lent for).

Whereas compound interest accrues each time interest is charged and is **rolled up**, meaning it’s added to the previous interest as well as the principal loan amount. The new total is then used for the next interest period. So the borrower is paying interest on the principal loan amount as well as the interest accrued during the term.

The following equations will calculate the total interest payable over the term.

The calculation of **simple interest** is:

The calculation of **compound interest** is:

As an example, let’s compare simple interest *v* compound interest on a four-year loan for $ 1,000 where the annual interest rate is 10%.

|  |  |  |
| --- | --- | --- |
| **Simple interest** |  | **Compound interest** |
| Year 1 | 1,000 × 10% | = | 100 |  | Year 1 | 1,000 × 10% | = | 100.00 |
| Year 2 | 1,000 × 10% | = | 100 |  | Year 2 | 1,000 + 100 × 10% | = | 110.00 |
| Year 3 | 1,000 × 10% | = | 100 |  | Year 3 | 1,100 + 110 × 10% | = | 121.00 |
| Year 4 | 1,000 × 10% | = | 100 |  | Year 4 | 1,210 + 121 × 10% | = | 133.10 |
| Total interest charged | = | 400 |  | Total interest charged | = | 464.10 |

If we use the equations we’ve learned, we will get the same results.

**Simple interest**

**Compound interest**

The difference between the two is the impact of compounding. Rather than the borrower paying interest annually, the bank has funded the accrued interest over the four-year term. Therefore, the borrower is charged more overall.

## Market conventions for interest rates

Different currencies, financial instruments and markets use differing day-count conventions to calculate interest; always read the documentation to be sure what has been applied.

The most common conventions are as follows.

* **30/360** – calculates the daily interest using a 360-day year and then multiplies that by 30 (standardised month).
* **30/365** – calculates the daily interest using a 365-day year and then multiplies that by 30 (standardised month).
* **actual/360** – calculates the daily interest using a 360-day year and then multiplies that by the actual number of days in each time period (ie taking account of months with more or less than 30 days).
* **actual/365** – calculates the daily interest using a 365-day year and then multiplies that by the actual number of days in each time period.
* **actual/actual** – calculates the daily interest using the actual number of days in the year to account for leap years, and then multiplies that by the actual number of days in each time period.

Less common is the **365/360** method, where the annual interest rate is divided by 360 but then applied to all 365 days of the year (366 days during leap year).

The differences arising due to the day-count basis are calculated as shown in the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Day-count basis** | **30/360** | **30/365** | **actual/360** | **actual/365** | **actual/actual** | **actual/actual (leap year\*)** |
| **Notional amount** |  100,000.00  |  100,000.00  |  100,000.00  |  100,000.00  |  100,000.00  |  100,000.00  |
| **Interest rate** | 5% | 5% | 5% | 5% | 5% | 5% |
| **Start date** | 02/01/2023 | 02/01/2023 | 02/01/2023 | 02/01/2023 | 02/01/2023 | 02/01/2024 |
| **End date** | 12/05/2023 | 12/05/2023 | 12/05/2023 | 12/05/2023 | 12/05/2023 | 12/05/2024 |
| **Days outstanding** | 130 | 130 | 131 | 131 | 131 | 131 |
| **Days in period** | 360 | 365 | 360 | 365 | 365 | 366 |
| **Interest charge** | 1,805.56  | 1,780.82  | 1,819.44  | 1,794.52  |  1,794.52  | 1,789.62 |

\*Actual/actual would differ from actual/365 in a leap year where there are 366 days.

**Leap year**

Leap years occur because the earth’s orbit around the sun (which happens approximately once per year) and rotation on its axis (which happens approximately once per day) are not perfectly aligned. The earth’s orbit around the sun takes 365.25 days, meaning that 1 day is added to the calendar every 4 years to make up for the missing partial day each year (Nasa, 2021).

The next three leap years occur in 2024, 2028, and 2032.

**Activity 3**

Swift Bank borrows $20m at an interest rate of 6% for a loan period of 6 months (182 days). The market convention for the loan is actual/360. How much interest is payable on the loan?

$\_\_\_\_ × \_\_\_\_% × \_\_\_\_ / \_\_\_\_ = $\_\_\_\_

## Discount instruments

Some instruments do not pay interest but are issued at a discount to the **notional amount**, with the difference between **issue price** and **maturity price** representing the interest element of the instrument. These instruments are usually known as bills.

* **Treasury bills** are issued by governments;
* **bank bills** or **bills of exchange** are issued by banks; and
* **commercial paper** is issued by corporates.

**Notional amount**

Also called the nominal value, book value, face value or par value, this is the full price of the instrument and the price that will be paid at maturity or redemption.

**Accrual instruments *v* discount instruments**

Accrual instruments are issued at nominal value and mature at nominal value plus interest, whereas discount instruments are issued at a discount on their nominal value and then mature at their nominal value.

Treasury bills and US commercial paper typically use the straight **discount rate method** (see the example that follows) so the calculation is similar to the formula for **accrual instruments**.

As we have seen already, with an accrual instrument the equation calculates the total price to be paid at the end of the agreed term, inclusive of the interest charged. With a discount instrument, the formula is used to calculate the price to be paid at **inception** (ie at time of purchase) because the price at maturity has already been determined. The interest payment is therefore the difference between the price paid at inception and the notional amount.

**Day count fraction (DCF)**

In many instances, the borrowing period is less than one year and an adjustment has to be made for the actual length of the borrowing period. This adjustment uses the DCF. For short-term borrowing, the DCF consists of two elements:

* the actual number of days in the borrowing period; and
* the interest days per year for the specific currency.

**Example: calculating the interest payment**

A US treasury bill with a nominal value of $10m has an investment period of 92 days at 2.5% interest. What is the initial purchase price?

**Activity 4**

An investor is looking to purchase $25m (notional amount) of 91-day US treasury bills at a discount rate of 2.19%. Calculate the price the investor will have to pay to invest.

$\_\_\_\_ × (1 − \_\_\_\_% × \_\_\_\_ / \_\_\_\_) = $\_\_\_\_

## Time value of money

Let’s start with a simple question: would you prefer someone to give you $1,000 today or $1,000 in 5 years’ time?

Hopefully you recognised that having money today (**present value**) is more valuable than having the same amount of money in the future (**future value**). This is due both to:

* the effects of **inflation**, which would reduce the purchasing power of the money over time; and
* the ability to **invest** money today in order to (hopefully) increase its value by a future date.

This illustrates the time value of money.

**Inflation**

A sustained increase in the general level of prices of goods and services over a period of time; a situation where the rate of growth of the money supply is greater than the rate of growth of real goods and services.

If you have money today, but do not wish to spend it today, you can invest it until such time as you do wish to spend it. This means that, during the investment period, you will attract a return based on interest rates for the period and the economic conditions prevailing at that time. This concept is known as **future value**.

The reverse is **present value**, where an amount of money due to be paid in the future is discounted to establish the amount of money needed now to meet the future demand.

**Example: inflation and the time value of money**

Let’s say government bonds or gilts (debt instruments in the form of fixed-income securities or bonds) with a maturity of one year pay a return of 0.75%. However, the annual inflation rate is currently at 2.5%.

While investing in government bonds might be regarded as a safe investment, the investor is actually decreasing the value of their money in real terms (adjusted for inflation) at a rate of 1.75% per annum.

## Future value

As we saw with accrual instruments and discount instruments, the same calculation can be used to produce two different results: either interest payable (eg on a loan) or interest earned (eg on a treasury bill).

The same applies with the compound interest calculation. In investment, a sum is invested and begins accruing interest. The interest is then added to the total, creating a new total. For the next interest period, the new total accrues interest, which is then added to create another new total and so the cycle continues for the term of the investment.

The formula for calculating future value is as follows.

In this formula, is the future value of the investment for a specified number of years (*n*). *PV* is the present value or nominal value of the investment, which is multiplied by one plus the interest rate (*i*) raised to the power of *n* (number of years).

**Example: calculating the future value**

What is the future value of 10m invested at first for 1 year, and then for a period of 5 years, at an interest rate of 4%?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Year** | ***PV*** | **×** | ***(1 + i) n*** | ***FV*** |  |
|  | **1** | 10,000,000 | × | (1 + 0.04)1 | 10,400,000 |  |
|  | **2** | 10,400,000 | × | (1 + 0.04)1 | 10,816,000 |  |
|  | **3** | 10,816,000 | × | (1 + 0.04)1 | 11,248,640 |  |
|  | **4** | 11,248,640 | × | (1 + 0.04)1 | 11,698,586 |  |
|  | **5** | 11,698,586 | × | (1 + 0.04)1 | 12,166,529 |  |

Using Excel, the formula is =FV. In the formula bar or input window, enter:

=FV(0.04,1,0,(-10000000))

* **rate**:the interest rate (in our example 4.0%, written as 0.04);
* **nper**: the number of periods (in our example 1);
* **pmt**: any payments to be made during the 5-year period (in our example 0); and
* **[pv]**: the present value (in our example 10,000,000, written as -10000000 – note this number should be entered as a negative number as it represents an outward cash flow).

The screen will then calculate the future value of 10,400,000. To solve for 5 years, simply enter 5 as the nper instead of 1.

**Activity 5**

We will now look at two investment scenarios based on an initial investment of $50,000.

1. If you invest $50,000 today, over a 20-year period, at an annual interest rate of 2%, what is the future value of your investment at the end of the term?
2. Calculate the value of your investment if interest were paid monthly, rather than annually.

**Hint**: To do this, two adjustments need to be made.

## Present value

Present valuing is, in essence, the reverse of future valuing.

In this case, we know the future value and we are going to discount the future value back to what it would be today. We need to establish an appropriate discount rate for the period concerned.

There is an alternative formula for calculating PV, which is , but for the purposes of this course we will use the formula above.

There is also a PV formula in Excel, which works similarly to the FV formula.

**Example: calculating the present value**

If the future value of an investment is calculated to be 10m in 5 years’ time, what is the present value at an interest rate of 2.5%?

This can be calculated back from the future value year by year (where *n* is 1 and the *FV* decreases each time), as follows.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Year** | ***FV*** | **÷** | ***(1 + i) n*** | ***PV***  |  |
|  | **5** | 10,000,000.00 | ÷ | (1 + 0.025)1 | 9,756,097.56 |  |
|  | **4** | 9,756,097.56 | ÷ | (1 + 0.025)1 | 9,518,143.96 |  |
|  | **3** | 9,518,143.96 | ÷ | (1 + 0.025)1 | 9,285,994.11 |  |
|  | **2** | 9,285,994.11 | ÷ | (1 + 0.025)1 | 9,059,506.45 |  |
|  | **1** | 9,059,506.45 | ÷ | (1 + 0.025)1 | 8,838,542.88 |  |

Or you can find the 5-year present value by calculating *n* as 5.

## Uses of the present value process – mortgage calculations

We will now look at an example of a typical banking service that uses the present value process: retail mortgages. For most people the largest personal transaction they will ever complete is the purchase of a home, and most people are also likely to borrow the money to purchase their home in the form of a mortgage. Mortgages are discussed in more detail in section 5.5.1.

**Example: calculating the monthly payments on a mortgage**

If we know how much we can borrow, we need to work out the monthly payment for that amount based on a given mortgage term and interest rate.

Let’s assume the amount of the mortgage is $325,000. The mortgage term is 25 years at an annual interest rate of 3.75%.

 = monthly payments

 = principal amount of loan

 = monthly interest rate

*n* = number of months the loan is outstanding

So the monthly payment for a mortgage of $325,000 over 25 years at an annual interest rate of 3.75% would be $1,670.93.

**Activity 6**

You decide to buy a house for $600,000 using a 25-year mortgage. Current 25-year mortgage rates are 3.30% per year. What is the monthly mortgage payment?

 = monthly payments

 = principal amount of loan

 = monthly interest rate

*n* = number of months the loan is outstanding

## References

Nasa (2021) *What is a leap year?*  [online]. Available at: <https://spaceplace.nasa.gov/leap-year/en/>.