**Implementing Critical Thinking Skills in Mathematics Classroom by Teaching Probability and Statistics in Daily Life**

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**Abstract**

The acquisition of critical thinking skills has been acknowledged amongst education researchers as an important educational goal. This paper presents a teaching experiment concerning a learning unit for 10th graders that combined the explicit teaching of critical thinking with the content of an existing mathematics unit called "Probability and Statistics in Daily Life". The original unit was designed to teach probability and statistics through real-life scenarios at the high-school level. To create the new learning unit described here, we took the original mathematical content and "infused" it with a progression of critical thinking skills, so that both the mathematics and the critical thinking competencies developed hierarchically – growing more and more complex as the unit progressed. Using a series of examples from the classroom implementation of the resulting unit (3 out of 15 lessons), this paper illustrates how the topics were melded together and shows the mutual benefits of this melding to both the teaching of statistics and of critical thinking. Then the paper discusses the problem of transfer of critical thinking skills and shows some promising elements in that sense, deriving from the analysis. Finally, it also discusses some educational implications of the work done, the limitations of the unit's first run and the improvements that may yet be made.

**Introduction**

The term "critical thinking" (CT) has been used in academic circles for less than a century, but evidence of the relevance of this concept in education is far older – spanning several forms of human endeavor and 2500 years of human history since the Greeks. The universal applicability of these "ancient" skills is perhaps more relevant than ever in today's complex and ceaselessly changing reality, which requires independent decision-making on a daily basis. Fostering and developing students' ability to think critically, to be capable of engaging in inquiry and evaluation based on rational considerations regarding the various messages they are exposed to in different areas of life, is therefore a particularly important part of their education (Bryan,1987; Glaser; 1972; Perkins, 1992; Regev, 1997; Swartz, 1992). While this need to focus on the promotion of CT skills has long been widely recognized by educators (Ku, 2009; Willingham, 2008), we have not yet reached a similar consensus regarding *how best this should be done*. This on-going debate raises the question of whether critical thinking should be taught as a topic in its own right, or integrated into a topic already present in the school curriculum. This question in turn raises additional questions - for instance, if critical thinking *is* integrated, which subjects should it be integrated with? Furthermore, when teaching CT skills in conjunction with another topic, should these skills be taught *implicitly*, as a hidden component 'immersed' in the primary material, or should they be an *explicit* part of the learning experience, in which the students' attention is drawn to their presence as an additional component 'infused' into the material through which it is being taught? Amongst those who believe that critical thinking should be taught in conjunction with other subjects, one of the topics suggested for this purpose is mathematics. In the field of education, mathematics has traditionally been considered a branch of knowledge particularly suited to the teaching and learning of higher-order thinking skills such as critical thinking. Mathematics curricula all over the world, including Israel, identify the acquisition of these skills as one of their goals. The idea that mathematics is a discipline suited to teaching critical thinking also appears in the research literature in a more or less explicit way (Elder and Paul, 1994; Paul, 1999; Paul & Elder, 2001; Paul, Elder & Bartell, 1997). However, in spite of this assumption, very few empirical studies to date have engaged with the question of whether the study of school mathematics indeed develops or even requires this mode of thinking. The answer to this question is far from being clear. The study upon which this paper is based attempts an approach to teaching critical thinking through mathematics that *explicitly* integrates the topic into a learning unit designed to teach high-school level probability and statistics. This topic is, I believe, particularly well-suited for the acquisition and practical application of critical thinking skills. Our model is based on the combination of two theoretical elements: the hierarchical model of critical thinking skills presented in Ennis' taxonomy (1985, 1987a, 1987b, 1989); and the "infusion approach" to teaching posited by Swartz (1992). To create the new learning unit, whose classroom implementation is presented in this paper, I took the mathematical content of an existing learning unit called "Probability and Statistics in Daily Life” (Lieberman & Tversky, 2001) and (in collaboration with one of this unit's co-creators) I "infused" it with a hierarchical progression of critical thinking skills according to Ennis' taxonomy (Ennis, 1989). In this paper, I present, in order of increasing complexity, a series of three samples from the classroom implementation of the unit. These illustrate a) how the two topics develop hierarchically together, and b) how each lesson combines them anew – calling on the students to draw on both their mathematical and their critical thinking skills to solve problems based on daily life.

**Theoretical Background**

*Critical Thinking and Ennis' Taxonomy*

Modern literature has provided a range of more or less specific definitions in an attempt to describe and characterize what "critical thinking" is (Dewey, 1933; Ennis, 1987a, 2002; Glaser, 1941; Lipman, 1991; McPeck,1981; Norris,1985; Kennedy, Fisher, & Ennis, 1991; Siegel, 1988; Willingham, 2008) To cite just a few examples, McPeck (1981,1990,1994) defines it as the *“*skills and dispositions" necessary to "appropriately use reflective skepticism”, while Lipman (1991) offers a more extensive description as “thinking that enables judgment, is based on criteria, corrects itself, and is context-sensitive.” Willingham (p. 21) goes into still more detail, defining critical thinking "in layman's terms" as "seeing both sides of an issue, being open to new evidence that disconfirms your ideas, reasoning dispassionately, demanding that claims be backed by evidence, deducing and inferring conclusions from available facts, solving problems, and so forth". This paper draws upon the definition suggested by Ennis (1962, 1963, 1985, 2002), according to which critical thinking is "reasonable, reflective thinking that is focused on deciding what to believe and do" (Ennis, 1985, p. 2) and on the hierarchical model of CT skills that goes with it. Ennis' approach to critical thinking was deemed most appropriate to my purposes for several reasons. First, unlike the other two definitions cited above, his explicitly marks critical thinking as an asset that influences how the person who employs it interacts*with the outside world*. This makes it particularly suited to the learning unit "Probability and Statistics in Daily Life," which also emphasizes the relevance of the cognitive mathematical content it teaches to understanding and making decisions about real-life situations. Secondly, Ennis (1996a, 1996b, 2002) distinguishes between two elements involved in critical thinking, which he calls "abilities" and "dispositions". "Abilities" refers to the cognitive aspects of critical thinking, to knowing *how* to think critically and *what* critical questions to pose (Ennis' list of abilities includes identifying the focus, defining terms, identifying unstated assumptions, judging source credibility, etc.). "Dispositions" refers to the socio-cultural aspect - to acquiring a "critical spirit" – a notion of *when* or *whether* something ought to be engaged with critically (dispositions include caring that beliefs be true and that decisions be justified, and taking care to present a position accurately and clearly). Finally, Ennis positions the 28 skills he associates with critical thinking upon a hierarchical structure (Ennis,1987b). In order to frame my work I chose 9 of these and correlated their hierarchical progression to that of the mathematical content in the "*Probability and Statistics in Daily Life*" unit.

*The teaching method: infusion approach*

Research into the teaching of critical thinking has shown that it is not something that students engage in naturally or develop spontaneously (Kuhn, 1999; Van Gelder, 2004). Critical behaviors like asking the right questions are *learned* behaviors, and the processes and judgments that critical thinking involves require a considerable – and deliberate - mental effort (Browne & Keeley, 2004; Onosko,1992; Tamir, 1993). Innabi and Sheikh (2006) reiterate this point, claiming that students will not think critically unless they are taught to think critically, but their position also emphasizes the importance of the *teacher*'s CT skills, because teachers’ knowledge and understanding of critical thinking will inﬂuence how they teach critical thinking to their students (Innabi & Sheikh, 2006). Given these general premises on the necessity of teaching CT skills and on the teachers who want to teach them, the most important question is: how to teach CT? As I have already stated above, though education researchers agree that teaching critical thinking is important, opinions vary as the best method of doing so. One major question that has arisen out of this debate is whether critical thinking and other higher order thinking skills are general (i.e. they can and should be taught as an independent course of study or content-dependent (and therefore must be taught in conjunction with something else) (Perkins & Salomon 1989 ; Kuhn & Dean, 2004).

**Methodology** *The "Probability and Statistics in Daily Life" learning unit as a basis for teaching CT*

"Probability and Statistics in Daily Life" is a preexisting learning unit developed by Lieberman and Tversky (2001), which was expanded and modified for the purposes of this research. I selected this unit as a basis upon which to build the teaching experiment reported in this paper because it was designed to teach mathematical content using problems and stories from daily life, and because its rationale already alluded to "elements of critical thinking", citing these among the "issues relevant to daily life" that it hoped to teach (Lieberman & Tversky 1996,2001; Introduction p.3).

The original unit was based on Tversky and Kahneman’s well-known work on making decisions in conditions of uncertainty (Tversky & Kahneman, 1974; Kahneman, Slovic & Tversky, 1982) It covers topics in statistics and probability in hierarchical order, connecting each to daily-life scenarios and decisions. The purpose of the original unit was to turn students into 'intelligent' consumers of information by introducing them to modes of thinking that went beyond the mechanics of mathematical calculation. The use of problems from daily life exposed the students to various additional fields such as medicine, economics, and law, illustrating the practical applications of probability and statistics to these fields and showing the students the ways that statistical considerations are inextricably woven into our lives. Furthermore, the unit's composition required them to analyze problems, raise questions and think critically about the numerical data and the information placed before them. Faced with problems that did not necessarily have one correct, clear-cut answer, the students learned not to be satisfied with arriving at a numerical solution, but to assess the validity of data, and to assess the problems before them in a qualitative – and not just a calculative – manner.

The revised unit is divided into three chapters:

Chapter 1: Concepts in Statistics and Probability - Statistical ConnectionThis chapter is an introduction to the basics of statistics and probability. It opens with a historical account of probability as the theory of gambling (e.g. the stories of Girolamo Cardano and Pierre de Fermat). This historical context leads into the law of large numbers and the empirical approach to probability, and to relative frequency. The chapter also includes such basic statistical tools as measuring distribution and central tendency. Finally, students are taught how to organize data into a two dimensional matrix, and how to determine the existence of a statistical connection.

Chapter 2: Statistical connection to causal connectionThis chapter evolves from the first by introducing the relationship between statistical connections and causal connections. Therefore, in addition to emphasizing the importance of understanding the nature of the statistical connection between two variables, it also focuses on the question: why is it important to know if a connection is only statistical, or is also causal? Having established that the statistical connection between variables must first be found before the causal connection can be understood, this chapter asks, "How do we test causal connections?" In this context, the chapter includes discussion of the difference between controlled, uncontrolled and observational experiments. These include explanations of methods that are employed to establish the validity of causal connections in controlled experiments (for example, the way drug trials employ tools such as the placebo effect, and blind and double blind testing to ascertain connections between cause and effect).

Chapter 3: Making decisions based on the heuristics of availability, anchoring and representativenessIn this chapter, after the students have been thoroughly versed in the mathematical, 'objective' approach to statistics and probability, introduces them to the subjective approach posited by Tversky and Kahneman (1974) and examines the differences between the two approaches. As part of this third section, the students were introduced to the three heuristic principles (availability, anchoring and representativeness) proposed by Tversky and Kahneman to account for subjective judgments made under conditions of uncertainty. After these were explained, the students were asked to apply them to a series of problems, indicating the heuristic they used to make their decision in each case. The students' experience with applying critical thinking rose to prominence here, because they now had two different methods for arriving at conclusions (the numerical method they had learned before, and a subjective one, requiring consideration of the validity of data), and were required to intelligently decide between them.

Like the original "Probability and Statistics" unit, our revised version was also designed to provide students with the statistical tools and the probabilistic models they require to assess statistics and probability in their day-to-day lives. Like the original unit, the revised unit was structured around the analysis of cases from daily life, and sought to draw teacher and students into a forum of discussion, investigation and analysis. Each lesson in the new unit, however, carried the *added* component of an *explicitly taught* set of critical thinking skills, based on the hierarchy set out in Ennis' taxonomy. The concepts in critical thinking were introduced gradually as the unit progressed, with each new concept being added cumulatively to the old, so that older skills were continually reused and recapped even as new ones were acquired (see Table 1). In some lessons I took the cases and questions from the original unit and merely added questions that explicitly draw upon critical thinking skills. In others, I took the *idea* of the original story – its mathematical purpose - and replaced the story/article suggested there with one that relied on the same mathematical principles, but was more recent/relevant and more conducive to learning critical thinking. A typical lesson was thus structured as follows: (1) Students were presented with a text describing a claim or a scenario from daily life, and asked a question about it (2) The students discussed the question as a group and tried to answer it intuitively (3) We worked as a class on the mathematical content relevant to answering the question (e.g. Bayes' Theorem, statistical connection) The original question was then re-discussed by the students in light of the mathematics, with no clear answer being provided them by the teacher (4) Finally, the elements of critical thinking were explicitly introduced and applied to the problem. One important expected element for the development of CT deriving from this four-step method was the conflicts that could arise between the students' original intuitive answers, the revised answers suggested by their later mathematical calculations, and the questions that could came into play regarding the validity of these answers once critical thinking was applied (Aizikovitsh-Udi, 2012, Aizikovitsh-Udi & Amit, 2008)

*Setting, population and data collection*

Results presented here are from a subgroup of one class taken out of a larger population of six classes [147 students in all - three experiment groups (70) and three control groups (77)]. The larger population was used for deriving quantitative results regarding the experiment's efficacy, which are not presented here. The experiment consisted of 15 sessions (90 minutes each) during the course of the academic year, and served as the "probability and statistics" section of the students' mathematics curriculum for that year. The group whose sessions are described here was the one taught by me, in my capacity as these students' regular mathematics teacher. Data collection was conducted by way of triangulation between the following sources: (i) The students' written products, including exams, in-class papers, and homework were collected (ii) Sessions were recorded, transcribed and analysed (paying special attention to their relation to CT skills). The teacher kept a log on every session. In general, data were processed by means of qualitative methods, which enabled me to follow the students' patterns of thinking and (iii) Personal interviews: 27 students were randomly chosen (four from each of the seven experimental classes) and interviewed at the end of the first and second semesters, in the middle and at the end of the unit. Personal interviews were conducted in order to reveal changes in the students' attitudes towards critical thinking throughout the academic year. The interviews were of two kinds: closed/structured interviews, based on questions chosen in advance, and open/semi-structured interviews, where only one part of questions chosen in advance was asked (and possibly modified) according to the interviewees’ answers. The interview questions were: (a) What do you think about the importance of critical thinking ability? In which fields/activities is it important? (b) Can you give an example of a situation (from school, everyday life, etc.) where critical thinking is necessary? Have you used it? Did it help you? (c) In your opinion, is it possible to develop/improve critical thinking ability? How? Do you have any suggestions for improving it? (d) In your opinion, is it possible to change dispositions for critical thinking? How? What influences the dispositions? Do you have any suggestions for improving them? (e) Did your studies in the other disciplines improve your ability for critical thinking? If yes, in what course and in what way? If not at all, why not? (f) How would you evaluate yourself in the area of critical thinking? The intended aims of the six questions were: (1)To ascertain the degree of students' awareness of the nature of critical thinking (mainly questions a, b, e, f, through the pertinence of the examples and the answers) (2) To identify students' ideas about possible fields of application for critical thinking (a, b), and in particular to identify suggested fields far from those proposed during the course (as a sign of interiorization and a premise for a possible transfer) (3) To get feedback about students' perception of which aspects/moments of the course had an impact on their CT skills and dispositions (c, d), as a way to assess students' awareness of the aims of the course.

**Results**

In what follows, I present three lessons, taken from three progressive points within the learning unit. For each, I present the daily-life topic upon which the lesson was based and show how first the mathematical content and then the thinking skills were integrated into it. In addition to exemplifying how the two topics were tied together, the succession of the lessons shows how both the mathematical content and the critical thinking skills built upon themselves hierarchically as the unit progressed, involving students in meaningful activities related to the aims of the learning unit (development of both CT and mathematical competencies).Each of the three selected samples also highlights a different element of the program. The first lesson, in which the students are sent outside of the classroom to gather data for themselves, highlights the centrality of the practical "daily life" element in the study unit. As this example shows, each daily life story or problem places the students in a position where they must draw on both the mathematics and the critical thinking skills to gather the information they need to make their decision. The second lesson is the most elaborated and detailed of the three examples, and it shows (through a long excerpt and the related analysis), how the melding working hypothesis was implemented (thus showing its feasibility). It also shows how by that point in the learning unit specific features of CT had already been interiorized by students, and how specific CT expressions entered students' language and were used by them in an appropriate way (see Discussion). The third example stresses the four step format of each lesson, and emphasizes the function served by each: it shows how the daily life scenario triggered the students' intuitive response, which was then informed and modified by the mathematics before being rethought and adjusted a second time by further application of CT. This problem shows the unit's lessons at their final and most advanced stage, where the students see that CT can be used not only to support conclusions based in mathematics, but also to look beyond them.

Example: Bayes’ Theorem – The Aspirin CaseThis final example is taken from the third chapter of the learning unit, which factors in the topic of subjective probability. The students were given the following problem:

*Your brother woke up in the middle of the night, crying and complaining he has a stomachache. Your parents are not at home and you don’t know what to do. You give your brother aspirin, but an hour later he wakes up again, suffering from bad nausea and vomiting. The doctor that regularly takes care of your brother is out of town and you consider whether to take your brother to the hospital, which is far from your home. You read from a book about children’s diseases and find out that there are children who suffer from a deficiency in a certain type of enzyme and as a result, 80% of them develop a bad reaction to aspirin, which could lead to paralysis or even death. Thus, giving aspirin to these children is forbidden. On the other hand, the general percentage of cases in which some bad reactions occur after taking aspirin is 10%. We know that 0.5% of children lack this enzyme.* (*Probability Thinking*, p. 30, with slight revisions made by the researchers)

*Should you take your brother to the emergency room? What should you do? Can aspirin consumption be lethal?*

This lesson was close to the end of the learning unit. It contains all of the mathematical elements covered by the unit, as well as nearly all of the critical thinking skills. It requires a high level of variable identification, and continues the students' work on conditional probability through Bayes' theorem. In terms of critical thinking, the students have now added g) a willingness to investigate questions beyond the basic requirements of the problem. Though this lesson does not directly address any of the heuristics posited by Tversky and Kahneman (1974), it nevertheless belongs in the subjective section of the unit because it offers the students an opportunity to make a decision based on intuition – and, having understood what the mathematics in the question entail, to *overrule* or *look beyond them*. The lesson was divided into four parts (a) First impressions: The students received the text and were asked to decide – yes or no? (b) Group discussion: A class discussion was held to consolidate all of the students' 'intuitive' answers, which they had based on their impression of the numerical data provided in the text, but had not verified through any 'real' mathematical calculations. Nearly all of the students chose to take their brother to the hospital. (c) Applying the math: We 'solved' the problem mathematically – isolating the variables, defining the conditional probability, figuring out what needed to be found (i.e. are we looking for *P(A/B)* or *P(B/A)*?) by means of Bayes' theorem, and building two two-dimensional matrixes. We concluded that there was only a 1% chance that the brother would have the enzyme. (d) Thinking critically:In light of our mathematical findings, we *suspended our initial judgment* (i.e. the decision to take the brother to the hospital) and resumed discussion of the question. The mathematical calculations proved that some of the students' initial *assumptions* had been incorrect (they had assumed that the chances would be higher, they had assumed that they could not understand the variables fully and had better 'play it safe'). Interestingly, even after the calculations had shown the chances of danger to be extremely small, most of the students *still* decided to go to the hospital. As a result, our attempt to resolve the conflict between their initial intuitions and the mathematical data led us to discuss *alternatives* – not to the students' original answer, but to their means of *supporting* that answer. Instead of basing their answer on false assumptions and ignorance, students were able to question the absolute finality and power of the mathematical result, and to consider whether mathematical solutions were always a *sufficient* basis for making daily-life decisions. As one student said, "if it's my brother, then that 1% may as well be 100%," suggesting that 'hard' mathematical facts do not always remove the discussion from the influence of other pertinent, deciding factors.

**Discussion & Conclusion**

As the examples above show, the mathematical content of statistics and probability is well suited to being 'infused' with instruction in critical thinking skills. The hierarchical patterns in which both the content and the skills can be taught complement one another structurally, with more elements of critical thinking coming into play as the mathematics becomes progressively more complex. The two instructional goals (statistics and critical thinking) are also mutually beneficial in terms of content: the mathematics provides the students with a means to engage in critical thought and pursue independent challenges and confirmations to the information they are given. The “Probability in Daily Life” unit provides the mathematics with a practical 'real world' context that can help the students comprehend the material more concretely as more than the abstract manipulation of numbers on a page. The various critical thinking skills could then be taught explicitly in the context of using mathematics to solve problems in the real world situations. The addition of critical thinking provides an added reflective dimension to the mathematical treatment rendering both the approach and the results more meaningful to students.

*Educational implications*

One of the benefits of the curricular adjustment presented here is that it did not require the complete overhaul of the entire school curriculum, but was first implemented at the discretion of the individual teacher; however, as Innabi and Sheikh (2006) stress, the teachers who implement such changes must *themselves* be able to understand and use the tools they impart to their students. Teachers who wish to incorporate critical thinking into the regular content of their curricula face a dual challenge: they themselves must not only be comfortable with CT and its relation to the content that they teach, but also have an idea of how to pass this knowledge and understanding to their students. Van Gelder (2004) offers a series of "lessons" for teachers to keep in mind when teaching critical thinking. One of these is "practice makes perfect" – critical thinking is a set of skills, and skills must be repeated over and over if they are to be mastered. In this context, the hierarchical format of the program presented here is one of its strengths, because the knowledge and skills learned in earlier lessons are continually drawn upon, reused and rehearsed in the later ones. This program also reflects Van Gelder's recommendation that the practice of CT be done *explicitly*, with teachers addressing it openly as a topic in its own right rather than expecting students to come by it independently based solely on the insertion of an implicit "critical emphasis" into their regular school content. Willingham (2008) also notes the importance of practice to attaining the "the ability to think critically," but he stresses too that this ability also depends on a familiarity with the domain knowledge about which one is thinking. It is therefore important that, though this program teaches CT explicitly and discretely from the mathematical content, it nevertheless teaches the two *together*, providing the students with a background of contextual domain knowledge within which to practice their critical skills.

*Critical thinking via probability and statistics – the question of transfer*

Transfer – i.e. the ability to take information acquired in one context and apply it to another – is one of the main goals of teaching higher-order thinking skills like critical thinking, though transfer within and between disciplines is difficult to put into practice (Bransford et al., 2000, El-Sheikh, 2001). Transfer remains one of the greatest challenges to teaching and learning critical thinking, and though it certainly occurs to some extent, it does not do so nearly as often as educators would like (Van Gelder, Bissett & Cumming, 2004; Zohar, 1996) Swartz divides transfer into several different levels, including "transfer *within* a restricted field of study to new examples within the field, transfer *across* disciplinary boundaries, and transfer *into* the thinking practices in which we engage in our everyday reasoning" (Swartz, 1992) To these categories he also adds an injunction to help students "develop a 'spirit of critical thinking'," by which he means "good thinking attitudes and dispositions that prompt us to use these skills" (p.37). The question of transfer is certainly pertinent to the assessment of an infusion approach to teaching critical thinking. When faced with a program that teaches students to use critical thinking skills when addressing problems of statistics and probability, one might rightly ask: "but will they be able to use these skills anywhere else?" Because this study was focused on the efficacy of combining CT with statistics and probability (i.e. first and foremost with whether the skills I had added were being learned at all), it did not check thoroughly and specifically for signs that what the students had learned was being transferred to contexts external to the mathematics classroom. However, the study *did* yield data upon which I may base some tentative suppositions about the manner and extent of the transfer that occurred, and from which I may derive some suggestions for future changes and additions to the program.

A language of critical thinking

One unexpected element that arose from the data and can be seen as a step in the direction of transfer is the students' acquisition of a *CT vocabulary*. Language is a central component of mathematics and of mathematical education; it both supports the thinking processes and is a medium for teaching mathematics and shaping knowledge. Since the learner's thinking development is determined by language, and since thinking to a large extent consists of language, it is impossible to separate language from learning (Vygotsky,1962; Sfard & Linchevsky,1984; Koenig & Harris, 2005; Zohar, 2000; Zoller,1999). The main use of language in mathematics has to do with the definition of principles and terms, the expression of mathematical ideas in the form of formulas and speech, the solving of mathematical problems in general, and word and geometrical problems in particular (Onosko,1990; Talaska,1992; Tishmann, 2000; Zhang, 2002). In teaching this course, however, I as teacher brought a new set of ideas into the classroom, and introduced these ideas explicitly to the students using a new and specific terminology. The new skills the students learned in the course were closely associated with the recurring words and phrases that went with them, particularly because naming the skills we were using required language of a rather higher level than the students habitually used. This connection is evident in the words of one student, who, when asked if the course had improved his critical thinking skills, answered that "They did improve in a certain sense but not in things that found practical expression," because "It’s awfully hard to speak such a high language." As we have seen in the excerpt presented and analyzed in Example 2, the students' adoption of the 'high language' associated with the practice of critical thinking is evident in such quotes from the lessons as:

*“First we should check the information source’s reliability”, ”The conclusion is not valid because we don’t have all the data”, “Despite all the numerical data, I don’t accept the researcher’s conclusion”, “We may have found a statistical connection, but we didn’t find a causal connection between the factors, so we can’t determine the direction of the connection”*

The language used by the students in class also shows that they had at least gained an understanding of the kinds of questions upon which critical thinking relies. Our in-class discussions were punctuated by student remarks such as:

*“Where is the article taken from? Can we see the article for ourselves? ““Is the article's source reliable? How can we check it? “,“I can’t accept this numerical data”, “Who did this research/ survey?” “We need more information.”*

The development of the students' language of critical thinking is also demonstrated by comparing the student answer sheets from before and after the learning unit. Those obtained from students before the new learning process demonstrate that the basic vocabulary and methods of mathematical thinking already existed. The students were able to use the regular methods of approaching a question in probability, using the two-dimensional table and Bayes' formula for conditional probability. Similar answer sheets completed by students after undergoing the learning unit demonstrate an extensive addition of verbal explanations to the two dimensional table, evidence of a new richness in the students’ language and of the added value of critical thinking in informing their work in probability and statistics. Even without analysing the actual content of what the student has written in the second paper (after the learning unit), the sheer difference in the ratio of words to calculation shows that an additional level of *verbal* thought has been added to the simple calculation of an answer to the question.

Further evidence of transfer (or lack thereof) can be gleaned from the students' interviews. Many of the students certainly claimed that the course had taught them a more critical approach to life that extended beyond the strict confines of statistics and probability. Their adoption of critical thinking was often expressed in terms of having become more "aware," more "cautious" and more "suspicious." One student described critical thinking as being more "aware of information sources that explain things to me; for example, if someone gives me a newspaper, I will be somewhat skeptical." The reference to awareness is echoed by another student, who said that after the course, "I put things into question more. I pay attention to things. I’m aware of them." The student also added "It helps me a lot to know whether the one who conducted the research is someone with power (prepared to pay for a certain experiment, apparently there is something good in the experiment or something bad, one has to check)." A third student defined critical thinking as knowing "Not to trust everything, to check before one decides. Not to believe every survey. To think about everything," and added, "Once I didn’t think about studies and surveys and articles in newspapers, once I used to believe everything, now I check everything." While these quotes give little indication of how *successfully* these students will apply their newfound critical tools, they do at least suggest that they now "recognize the importance of good thinking and have the initiative to seek better judgment" – a dispositional aspect of critical thinking that *combines* with cognitive aspects to determine what Ku calls "a person's actual thinking performance (Case, 2005; Ku, 2009, P.71). While the students' confident use of words like "always" and "everything" in reference to their newfound critical faculties suggests that they believe themselves able to generalize their new knowledge and apply it to other contexts, their interviews indicated several limitations in their ability to do so. Most significantly, nearly all of the students had difficulty citing examples of critical thinking other than those we specifically discussed in class. Our lessons were based largely upon newspaper articles that cite surveys and scientific research and/or endorse products. Accordingly, many of the students said in their interviews that critical thinking is useful "in researches and shopping" and in reading the newspaper. One student said, "There are researches carried out all the time, and every company wants to show that it is the most ideal one and the best, and they do this by influencing the public." Another gave mathematics itself as an example, saying that "in mathematics, when you have all kinds of problems to solve and they taught you one way, it is possible to look for another way and not to say right away that if you were taught one way, that’s all there is." In a third student example that is clearly a repetition, in general terms, of a specific case discussed in class, this student claimed that critical thinking is important:

*Because if, for example, there is a survey that was done on a certain group and is not true at all about the total population, we need criticism in order to think about such things. An example for such a case is a survey that was commissioned on products of a certain company by that company itself and is conducted on a certain target group, while the identity of this specific group is not published in order to present the products in a better light.*

Another potential obstacle to transfer that arose from the interviews is that some of the students – despite the fact that the CT skills were taught explicitly – still had difficulty separating the CT skills they had learned in the course from the mathematical content. One student claimed that critical thinking should be taught together with mathematics "because it’s related because of the percentages and numbers." Another claimed that, "A student who knows mathematics has critical thinking, because he knows how to think, [while] a student who doesn’t know mathematics can’t think critically." A third student acknowledged that mathematics are not strictly necessary "for the critical thinking itself" and that "one can refute a claim even without mathematical arguments," but when asked to name concepts in critical thinking, he answered, "Statistical connections, identifying variables, defining variables, sets, causal connection, conditional probability" indicating a certain level of confusion regarding which terms belong in which topic. Separation of terminology might help to clarify the distinction between skills in the two domains. Such a distinction, as well as overcoming both of the limitations noted above, might be favoured if students were introduced to applications of critical thinking in *other* contexts as well. This could be done on a smaller scale by including examples from other disciplines (history, for example) within the "Probability and Statistics in Daily Life" unit itself. Alternatively, it would also be achieved by 'infusing' critical thinking skills into the curricula of other academic disciplines as well, and thus creating a more "comprehensive critical thinking program" for the students (Resnick, 1987; Swartz, 1992). One noteworthy element reported by several students that influenced their ability to transfer critical thinking outside the classroom was an increase in confidence. One student reported that the unit had "increased [her] confidence" in her own doubts – that before the unit she "didn't say it," but that now "I can say that I am not sure, that I can examine other possibilities." Interestingly, this rise in confidence was expressed several times as a willingness to confront the authority of the teacher. One student thought that critical thinking was applicable to any subject "because in any subject there is a teacher who explains, and with any explanation it is possible to ask questions about the topic that I don’t understand or try to correct the teacher when she is making a mistake." This student also thought that his own critical thinking was not very good, "because I am a shy boy and usually don’t ask questions when I don’t understand something." In contrast, another student, who also thought "the ability for critical thinking is really important" because it can be used "to criticize the teacher," claimed that though this was something he previously "never could do in my life," he now thought that "there is a chance it will happen." The topic of confidence was also raised by a third student, who said that he uses the new concepts he had learned (like "I put something into question") only "with people who are close to me," and that "My criticality depends of how comfortable I feel around people that I am with.

**Concluding Remark**

This paper has reported an investigation into the use of the infusion approach to teach critical thinking skills while also teaching conventional probability and statistics content. The students acquired critical thinking skills that they came to value. These skills were so entangled with the mathematical skills being taught that many students came to see critical thinking as an aspect of mathematical thinking. This “confusion” on the part of some students actually highlights a profound theoretical question into the relationship between critical thinking and mathematical thinking (and statistical thinking, in particular). One strategy for disentangling the two modes of thought might be to focus on the typical expressions of probability, statistics and critical thinking. The development of an identifiable vocabulary of critical thinking has been documented here and is one of the most interesting outcomes of this study because it is a crucial premise to transfer. Despite this, the use of taught skills (whether statistical or critical thinking) in contexts other than those in which the skills were acquired remains uncertain and unevenly realized. Nonetheless, there are elements deriving from the analyses of how the infusion approach in the simultaneous teaching of statistical and CT skills was implemented (like the acquisition of typical expressions), which encourage optimism with respect to transfer as the explicit teaching of critical thinking becomes more pervasive in mathematics and in other subjects.

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