Block Holders' elasticity risk- return efficient frontier

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**Abstract**

Empirical studies indicate that family firms are more efficient and also to take less risk. This analytical study supports theoretically these findings by modeling a risk- return efficient frontier that is based on a new suggested Modified degree of Operating Leverage (MDOL) idiosyncratic risk measure. This efficient frontier is determined by the investment levels and optimal production/ outsourcing decisions.

The traditional DOL considers implicitly the volatility of production as a source risk rather than an endogenous decision variable that is sensitive to exogenous shocks in market demand and supply. MDOL remedies these two pitfalls and thus enables us to generate a practical managerial risk return efficient frontier that is based on optimal production/outsourcing levels. Block holders may prefer an optimal point with lower investment and production that yields lower expected return and risk. Our MDOL measures the impact of the shock not only on the firm’s operating profits but also on the total free cash flow that includes the market value of the assets of the firm. Such findings conform to the COVID-19 pandemic which has exposed the sensitivity of the value of the firm's assets to a shock to the demandfor the firm’s products.

*Keywords*: Operating Leverage, Free Cash Flow, Idiosyncratic Risk, blockholders, Risk-Return Efficient Frontier

*JEL classification*: D24, G39

**1. Introduction**

The strategic risk return behavior of family firm has been investigated intensively in the literature[[1]](#footnote-1). Most empirical studies indicates that family firms takes less risk. Family stock holders as well as other block holders such as entrepreneurs and managers are sensitive to idiosyncratic risk. The degree of operating leverage (DOL) is one of the most recognized and familiar measures of idiosyncratic risk used by blockholders investors such as family equity holders, entrepreneurs, managers, and other major investors in private and public SME firms. The DOL is a simple ratio that measures the elasticity of operating income with respect sales or output. The conventional DOL ignores exogenous shocks as the sources of risk that eventually leads to changes in products' prices and to endogenous changes in the optimal investment and output. Thus, using the conventional DOL as a risk measure ignores exogenous source of risks, such as those revealed during the COVID-19 period, affecting the value of the company’s assets[[2]](#footnote-2).

In this paper, we address the above issues by considering the shock to the demand function as a source of risk and by replacing the operating income in the traditional DOL with the firm's free cash flow (FCF), a cornerstone measure in corporate and project valuations. Capturing the elasticity of FCF to unexpected shocks to the product's demand, rather than merely focusing on the operating income of the firm, enables our MDOL to account for the volatility of the firm's assets.

In our model, *well-diversified* investors prefer the level of investment that maximizes the expected FCF and the value of the firm. In contrast, the *non-well-diversified* stakeholders are not immune to idiosyncratic risk. Therefore, in order to decrease their idiosyncratic risk they may prefer low levels of expected cash flow and risk. Note that the firm's motivation for global diversification is affected by its level of idiosyncratic risk. Previous literature shows that presence of owners with high idiosyncratic risk such as family blockholders, dissuade internationalization (Fernández and Nieto, 2006, Hurwitz et al. 2019).

The rest of the paper is organized as follows. Section 2 presents a literature review. Section 3 discusses the preliminaries. Section 4 defines the model. Section 5 shows anumerical example. Finally, Section 6 presents a summary.

**2. Literature Review**

Risk taking and performance are one of the most important strategic features of firms that can be affected by block holders such as entrepreneurs and family holders. Anderson and Reeb(2003) finds that family firms that are one third of S&P500 better perform than the nonfamily firms. finding for Swedish SMEs and empirical review mainly supports less risk taking and better performance by family firms.

Villalonga and Amit (2005) reports additional value to family firms only when the founder is the chairman. They also reports lower owner manager agency cost in family firms.

Fernández and Nieto (2006) show that there is a connection between the firm's choice to internationalize and the firm's type of ownership. They claim that the presence of corporate blockholders, who have less specific risk compared to family blockholders, increases the level of internationalization of SMEs (small and medium size enterprises) and family SMEs. On the other hand, family ownership reduces the firm's level of internationalization. Such firms tend to be more centralized, and lack the resources and qualified staff to achieve a competitive advantage that can be exploited through more international involvement of the firm. However, Sun and Govind (2017) argue that when turbulence in the market (reflected in the dynamics of sales patterns) is high, diversification leads to greater idiosyncratic risk. On the other hand, when turbulence in the market is low, increasing diversification reduces risk. Jouida and Hellara (2018) also investigate the connection between diversification and capital structure decisions. They report a negative relationship between financial leverage and activity as well as geographic diversification.

Several papers in the literature such as Aharon, Kroll, and Riff (2019), Sarkar (2018), Kroll and Aharon (2014), Booth (1991), and Dotan and Ravid (1985) are related to our work. Kroll and Aharon (2014) construct a model for a modified DOL risk measure. However, they do not cover the idiosyncratic managerial risk of FCF, nor they suggest an optimal investment level. Dotan and Ravid (1985) create a model for a firm seeking to choose the optimal investment and debt level. However, their analysis does not include any risk measures in terms of the DOL’s financial or total leverage. Booth (1991) develops state-contingent risk measures, but the application of such measures in practice is complex. Sarkar (2018) analyzed the DOL of a company with flexibility in investment and production using the real options model for the value maximization of the firm. He derived the optimal DOL out of the optimal production and investment levels that maximize the value of the firm.

Our work differs from Sarkar (2018) as we seek to capture the tendency of family and other blockholders to under-invest and thus lower expected cash flow and idiosyncratic risk, where Sarkar (2018) calculated DOL only at the optimal production and investment levels that maximize the expected cash flow of the firm. Unlike Sarkar (2018), we assume that the variable cost per unit increases with the production level and decreases with the investment level.

According to Kroll and Aharon (2014), the modified DOL is at least two times higher than the traditional one because the lower price due to the shock to demand also reduces the optimal production level.[[3]](#footnote-3) In our model, the modified DOL is much higher than that suggested by Kroll and Aharon (2014), as it considers also the impact of the shock on the value of the firm's assets.[[4]](#footnote-4)

Overall, our model is closely related to that of Kroll and Aharon (2014) and Aharon, Kroll, and Riff (2019). However, Kroll and Aharon (2014) did not consider the FCF that determines the overall value of the firm. Similarly, Aharon, Kroll, and Riff (2019) did not consider the optimal debt policy that offsets the expected tax benefits of debt with expected after-tax bankruptcy costs. Therefore, their models require various inputs that are hard to obtain, empirically. We simplify the model of Aharon, Kroll and Riff (2019) by assuming that the well diversified investors select optimal capital structure and that the capital market is complete and costless, and well diversified investors hedge their risks. In contrast, the blockholders cannot hedge the risk of the specific firm in which they are stakeholders

**3. Preliminaries**

Lev’s (1974) analytical definition of the DOL is as follows:

 , (1)

where *Q* is the production units of the firm, π is the operating income, and FC represents the total fixed costs. Eq. (1) is accurate only under the assumption that the price per unit and the variable cost per unit are constants. However, the endogenous motivation of the firm to change the production level is, in general, a byproduct of an exogenous shock to the demand and supply of inputs and outputs that changes the prices and cost per unit of the firm’s products (Booth, 1991; Kroll and Aharon, 2014; Sarkar, 2018; Aharon, Kroll and Riff, 2019).

The risk measures of operating and financial leverage can be combined to a single risk measure, the Degree of Total Leverage (DTL) risk measure. The traditional DTL measures the elasticity of net income (NI) with respect to output. The traditional DTL is as follows:

 (2)

According to Eq. (2), DTL is a product of DOL and the degree of financial level (DFL). Therefore, an X% increase in DOL can be balanced by the same X% decrease in DFL. This property is the basis of the tradeoff hypothesis between operating and financial leverage.

Income statement generally does not distinct between fixed and variable costs and generally do not report production quantities. Thus, Q in Eq. (2), is not provided or cannot be quantified easily. This situation is particularly true with regard to multiple products. The lack of information about the volume of production has led scholars and researchers to use different proxies for measuring DOL.

**4.** **The Model**

Two types of investors are assumed. The first type, which we call *non-well-diversified* stakeholders, such as managers, entrepreneurs, family investors, and others for whom investment in the firm constitutes a major proportion of their portfolio. They are exposed to the idiosyncratic risk of the firm. Assumingly, these stakeholders have the power to determine the level of the firm's investment and may have an interest in reducing the level of investment to lower their own risk exposure.

The second type, the *well-diversified* investors, hold the market portfolio. If we assume equilibrium and costless complete markets, they can ignore idiosyncratic risk and they can also hedge any risk without any cost. Therefore, *well-diversified* investors determine the value of their investment by the risk-free discounting of the expected FCF. The different goals of the two groups might lead to economic struggles between them that impose additional agency costs on the firm.

The model is a single period. At t0, the investment and capital structure decisions are taken. At t1 the actual production decision is taken. For simplicity, we assume that at t1 all outflows and inflows occur.

Based on the Modigliani and Miller model for the optimal capital structure, we assume that the choice of an optimal level of debt at time t0 is determined by balancing the tax benefits of debt with after-tax bankruptcy costs. Thus, both of these factors are even and can be ignored.

After the investment decision and the implied optimal capital structure are determined, there is an exogenous shock that affects the demand for the firm's products. At this point, the firm must deal with a new actual demand function. This shock is the only random variable in our model. Based on the new demand function, the firm must decide on a new optimal production level[[5]](#footnote-5).

The demand function for the firm's product is . The production level is denoted by Q, while  is the random demand shock that is realized at t1. We assume a perfect, competitive product market in order to simplify the model. Thus, for all Q levels, the price at time t0 is.[[6]](#footnote-6) The expected level of  at t0is zero. Based on this price expectation and production cost function, the expected optimal production, FCF, and its idiosyncratic risk for each level of initial investment (A) at t0 can be estimated.

Fixed costs (FC) are approximated by the following function:

. (4)

 is the minimal given level of fixed cost and *d* is a positive parameter that connects fixed cost to level of investment due to depreciation and maintenance cost. The variable cost per unit function is:

. (5)

The parameters,, , , and  are all positive. The first derivative of the variable cost per unit with respect to the production level is positive. Thus, the non-extreme upper finite optimal level of production is guaranteed. The initial fixed cost  bounds the optimal level of production from below to an internal finite positive level. The first derivative of the variable cost per unit with respect to investment (A) is negative, and the second derivative is positive. The abovementioned impact of investment on the variable cost per unit is the main argument for insourcing.

We assume that the fixed and variable costs also include the expenses that are required to keep the assets without any technical amortization. Thus, if the economy is stable, the expected value of the assets at the end of the period is the same as at the beginning.

Sales minus variable costs defined and denoted by gross operating income (GOI) is:

 . (6)

The gross operating income (GOI) as a function of Q is a parabola. The necessary condition for a positive GOI for non-negative Q is:

 (7)

The first-order condition for Q\* that maximizes GOI is:

. (8)

Plugging Eq. (8) into Eq. (6) and rearranging the terms results in:

. (9)

For the given positive expected optimal production level, the expected FCF is:

, (10)

where *T* is the corporate tax rate.

An optimal positive A\* that maximizes the expected FCF can be obtained by an internal solution of four degree polynomic equation.[[7]](#footnote-7)

Since all of our cash flows are at t1, the firm is abandoned at t0, if and only if the expected maximum FCF is non-positive. This situation is equivalent to:

. (11)

Since fixed expenses reduce FCF but they do not reduce GOI . Thus, in order to eliminate the option to abandon the firm at t0 and to preclude the irrelevance of the problem, we assume and that the firm is not abandon at *t0*.

At t1, the random value of the asset of the firm is where *q* is a factor that reflects the impact of on the expected value of the assets of the firm at t1. It is reasonable to assume that  because negative shocks at t1tend to negatively affect prices in the next periods, and vice versa. The parameter *q* is a major risk factor in our model.

COVID-19 is a classic example that highlights the different impacts of such shocks on various industries. If the shock in the current period is uncorrelated with shocks in the consecutive periods, then . One can view the factor *q* as a reflector of the expected amplitude and length of the economic cycle. In order to eliminate negative expected values, we assume that . The factor q is the "constant elasticity of the asset's value with respect to the product's price." In other words, we assume that regardless the price of the firm's assets, a  change in the price of the product will cause a  change in the final value of the firm’s assets.

Following the ex-post exposure of the ex-ante random shock, the firm uses Eq. (9) to recalculates the new optimal production level that maximizes expected level of FCF as:

. (12)

**The Risk**

Recall that the *non-well-diversified* stakeholders' attitude toward idiosyncratic risk and the level of this risk are relevant determinants in their investment and production decisions.

The optimal production level changes with accordance to the realization of a new level of  that follows the investment decision. The elasticity of the FCF with respect to the optimal production level *Q\**, is the "degree of free cash flow leverage". It is denoted as the modified degree of operating leverage (MDOL(Q\*)).

The expected MDOL(Q\*) is expressed in Eq. (13):

 . (13)

The appendix provides confirmation that the expected  equals:

 . (14)

where is the optimal production level and  is the gross operating income at .

Due to our assumption that bankruptcy costs are balanced by deductible taxes, the expected net operating income (NOI) is equal to the expected earnings before interest and taxes (EBIT). Here, it is also equal to the expected pre-tax-free cash flow (PTFCF).

When either q or A are equal to zero, our MDOL(Q\*) is exactly twice the conventional DOL and equal to the DOL of Kroll and Aharon (2014), under similar perfect market conditions. Further analysis of Eq. (14) shows that MDOL(Q\*) increases with the fixed cost parameters, FC0, and d, and decreases with P0, GOI\*, and NOI\*.

The impact of the investment, *A*, on MDOL(Q*\**) is two sided. Higher investment leads to higher fixed costs but also to lower variable costs. The internal four-degree polynomial solution for optimal A is calculated only numerically and it used to generate the efficient frontier between expected FCF and expected MDOL(Q\*).

The non-well diversified stakeholders cannot eliminate the idiosyncratic risk, but they can hedge the risks due to the portion of their market portfolio holding by utilizing market futures or options instruments. Thus, for these types of investors the optimal investment of the firm is generated by the tangency between their indifference utility curve and the efficient frontier line between expected cash flow and idiosyncratic risk (See **Figure 1**). This optimal level of investment for these *non-well-diversified* stakeholders is generally below the level that maximizes the expected cash flow.[[8]](#footnote-8)

The next subsection presents a numerical example that adhere to reasonable empirical levels which shows the potential gap between the optimal investment of the *well-diversified* and the *non-well-diversified* stakeholders. We also demonstrate the potential impact of various parameters on the optimal investment (A) of each type of investor.

**5. Numerical Example**

**Table 1** presents the key parameters for the efficient frontiers for the FCF versus MDOL(Q\*) and the FCF versus DOL(Q\*).

**[Table 1]**

The manager or financial analyst can estimate all of these parameters easily (excluding q) from the internal financial reports and cost analysis of the firm. The parameter q (which is the impact of the shock to the prices of the products on the market value of the firm) is the only parameter that one should estimate externally.

The efficient frontier is for investment levels in the range of 49.34‒160.97. At the investment level of 160.97, the highest expected FCF of 16.9 is obtained with the optimal production level of 15.13 and MDOL of 9.27. Higher investment levels exceeding the value of 160.97 result in lower expected FCF and more risk (MDOL). The lowest MDOL of 6.60 is obtained at the investment level of 49.34. Lower investment levels reduce the expected FCF and increase the risk. If the traditional DOL is used as the risk measure, then a minimum DOL of 2.17 is obtained at the investment level of 104.2, and the efficient frontier is reduced to investment levels of 104.2‒160.97. The efficient frontiers of DOL and MDOL are plotted in **Figure 1**.

**Table 2** lists the implied results for the efficient frontiers between FCF and MDOL(Q\*). **Table 2** and **Figure 1** show that the active part of the FCF vs MDOL efficient frontier in our example is significantly to the right of FCF vs DOL efficient frontier. At the investment level of 160.97, which maximizes expected cash flow, the MDOL’s frontier is 313% above the DOL, and when the investment level is only 49.3, it is 179% above the DOL. These results are logical, because a small change in the price of products can make a large change in the value of the firm and can lead quickly to bankruptcy. Evidence supporting these results appeared in the case of the effect of COVID-19 on the transportation and energy industries. We plot the hypothetical utility indifference curve (UU) of the block-holders to show that the tangency between UU and the efficient frontier can be at a much lower investment level than the one that maximizes the expected FCF.

**[Table 2 & Figure 1]**

**6. Summary**

Previous studies indicates that family firms take less risk. This analytical paper provides a theoretical micro model that provides stack holder a risk –return efficient frontier on which they can practically make their optimal combination of invest-production/outsourcing strategy.

Block holders such as family investors, entrepreneurs and other stakeholders in SME and other public or private firms are sensitive to idiosyncratic risk and thus my prefer to lower the expected return of the firm in order to gain lower idiosyncratic risk.

The traditional degree of operating leverage (DOL) measures idiosyncratic risk by calculating the elasticity of operating earnings with respect to production levels, but erroneously ignores exogenous risk factors by assuming implicitly that the price of the firm’s products remains fixed.

This paper extends the traditional DOL by measuring the elasticity of the free cash flow (FCF) with respect to endogenous changes in the optimal production level (Q\*) that occur due to exogenous shocks to the demand for the product. Our new suggested measure, the degree of free cash flow leverage MDOL(Q\*), also measures the risk due to capital gains or losses, not only the risk of operating income. The higher the investment level of the firm, the greater the difference between our MDOL(Q\*) and the traditional DOL. Previous literature shows that the presence of less diversified owners such as family stockholders is negatively related to the firm's level of internationalization (Fernández and Nieto, 2006).

The impact of COVID-19 demonstrates that a shock to the demand for a product leads to a magnified shock to the firm's assets value. According to the efficient frontier between FCF and MDOL(Q\*), equity block-holders and managers have an incentive to lower investment and production level.

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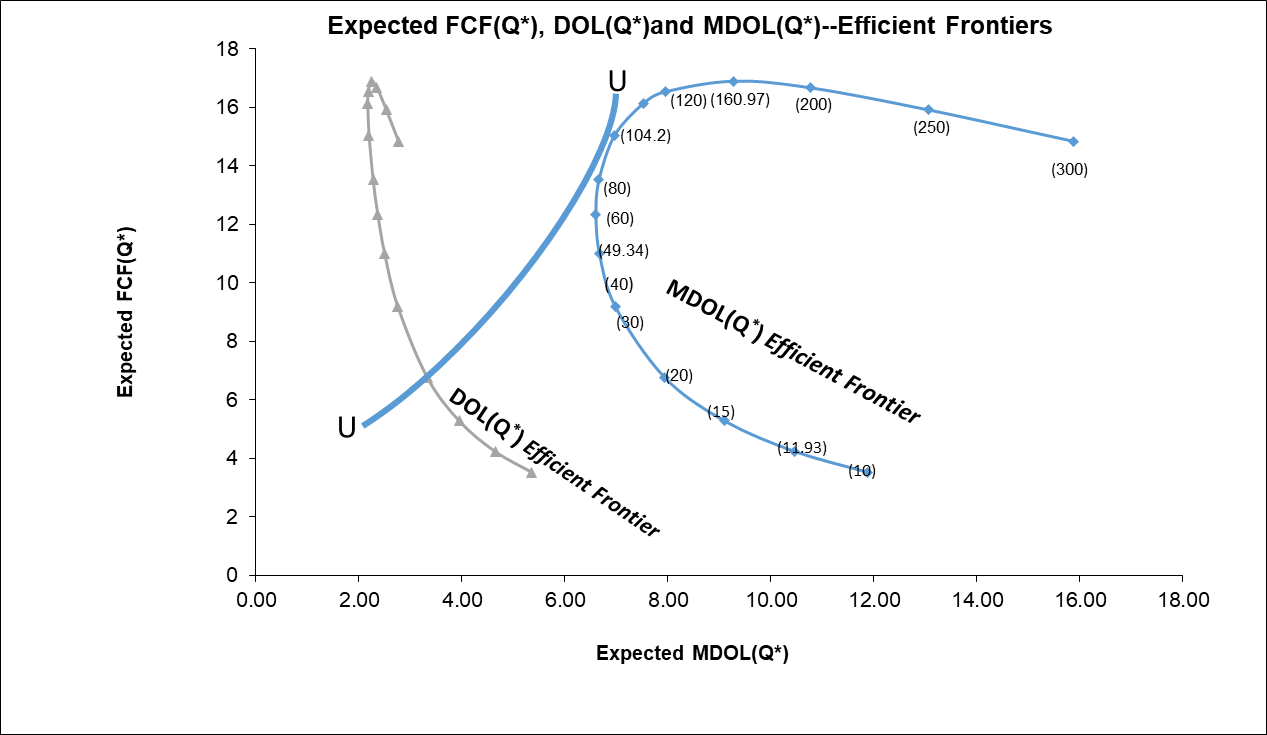
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|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 1: The Key Parameters for the Numerical Example** | | | | | | | | | |
| **Notation** | **P0** | **FC0** | **d** | **C0** | **C1** | **b0** | **b1** | **q** | **T** |
| **Value** | 10 | 20 | 0.05 | 3 | 0.2 | 1.5 | 0.05 | 0.5 | 0.25 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Table 2: Expected FCF, optimal production, MDOL, DOL and profit margin as a function of investment level** | | | | | | | | | |
|  | **Investment A** | **Optimal Q\*** | **FCF=NI** | | **MDOL** | | **DOL** | **Profit Margin** | **DFCF/DOL** |
|  |  |  |  | |  | |  |  |  |
|  | 10 | 9.17 | | 3.53 | | 11.88 | 5.35 | 3.9% | 2.22 |
|  | 11.93 | 9.43 | 4.24 | | 10.46 | | 4.64 | 4.5% | 2.25 |
|  | 15 | 9.81 | 5.28 | | 9.10 | | 3.95 | 5.4% | 2.31 |
|  | 20 | 10.36 | 6.78 | | 7.93 | | 3.32 | 6.5% | 2.39 |
|  | 30 | 11.25 | 9.19 | | 6.98 | | 2.76 | 8.2% | 2.53 |
|  | 40 | 11.94 | 11.01 | | 6.67 | | 2.50 | 9.2% | 2.67 |
| **Optimal** **MDOL(Q\*)** | 49.34 | 12.47 | 12.34 | | 6.60 | | 2.37 | 9.9% | 2.79 |
|  | 60 | 12.95 | 13.52 | | 6.66 | | 2.28 | 10.4% | 2.93 |
|  | 80 | 13.65 | 15.05 | | 6.97 | | 2.20 | 11.0% | 3.17 |
| **Min DOL(Q\*)** | 104.20 | 14.26 | 16.13 | | 7.52 | | 2.17 | 11.3% | 3.46 |
|  | 120 | 14.56 | 16.53 | | 7.95 | | 2.18 | 11.4% | 3.65 |
| **Max Expected FCF** | 160.97 | 15.13 | 16.90 | | 9.27 | | 2.25 | 11.2% | 4.13 |
|  | 200 | 15.50 | 16.67 | | 10.76 | | 2.35 | 10.8% | 4.58 |
|  | 250 | 15.83 | 15.92 | | 13.06 | | 2.53 | 10.1% | 5.16 |
|  | 300 | 16.07 | 14.84 | | 15.87 | | 2.77 | 9.2% | 5.73 |

**Figure 1: Efficient FCF vs MDOL and DOL Efficient Frontier generated by different levels of investments and implied optimal production levels**

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**Appendix**

 from Eq. (13) is presented again below:

(a-1) 

When the realized, the function  should be rewritten as follows:

(a-2) 

The derivative of  with respect tois:

(a-3) 

(a-4) 

Using Eq. a-1- a-4 to obtain for any A.

(a-5) 

1. For a review and finding in Swden see Naldi, Nordqvist, Sjöberg and Wiklund (2007) [↑](#footnote-ref-1)
2. Note that COVID-19 pandemic has severely harmed sectors such as oil, transportation as well as many other companies. See, for example, the recent works of Goodell and Huynh (2020) and Mazur, Dang and Vega (2020). [↑](#footnote-ref-2)
3. In the case of a perfectly competitive product market, the modified DOL is twice the traditional one, but it increases when the elasticity of the demand function is negative. See Kroll and Aharon (2014). [↑](#footnote-ref-3)
4. Abdoh and Varela (2017) show that higher competition increases idiosyncratic risk. When one firm suffers from losses caused by specific cost shocks, other firms might benefit by diverting market shares to them. [↑](#footnote-ref-4)
5. We assume that production function includes partial outsourcing elements according to specific cost benefit considerations as given by Sen and Zhu (1996) [↑](#footnote-ref-5)
6. Sarkar (2018) as well as Kroll and Aharon (2014) assume a decreasing demand curve. Both find that it increases the sensitivity of the expected cash flow to the optimal production level. [↑](#footnote-ref-6)
7. We will demonstrate such solutions only numerically. [↑](#footnote-ref-7)
8. Anderson, Duru and Reeb (2012) demonstrate empirically the underinvestment tendency of family firms. [↑](#footnote-ref-8)