**Students’ Mistakes in Logical Connectives in Solving Equations** **and Inequalities and Teachers’ Assessment of These Mistakes**

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**Abstract**

The aim of the current study is to focus students’ mistakes in using and writing of logical connectives in simplification of algebraic equations and inequalities, and teachers assessment and awareness to these mistakes. The study was conducted among 50 ninth grade students and among 63 mathematics practicing teachers. The data collected from two questionnaires, students’ questionnaire consists items in inequalities and equations and teachers’ questionnaire that consists students’ solution of different items. The data analyzed according to interpretative theory. The findings indicate about common mistakes in students’ utilizing of the mathematical logical connectives or, and, if … then in the manipulation of algebraic expression. In addition, the findings indicate that teachers did not aware for students’ mistakes in writing the mathematical logical connectives, the most mistake identified among students was ignoring the logical connective completely. Moreover, the findings indicated that teachers assess solution with wrong using of mathematical logical connectives as correct solution.

**Keywords**: Logical connectives, algebra, inequalities, practicing mathematics teachers.

**Introduction**

Understanding simplification and solving equations and inequalities have been emphasized in the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). It is considered as one of the basic mathematical procedures to be studied at the secondary school level and it involves a variety of basic algebraic and arithmetic skills, such as perform rote operations with algebraic symbols and applying the quadratic formula (Li, 2007). However, this importantly solving equations and inequalities considered as difficult topics for students (Cai & Moyer, 2008). Students’ difficulties in simplification of algebraic equations and inequalities, manipulating algebraic expressions was focused in different research (e.g. El-khateeb, 2016; Samuel, Mulenga & Angel, 2016; Poon & Leung, 2010). The research discussed different aspects such as the students’ understanding of the equal sign and their ability to solve equations (Knuth, Stephens, McNeil, & Alibali, 2006) and students’ strategies in solving inequalities (Tsamir & Almog, 2001). However, teachers’ understanding of equations and equation solving is not among researchers’ major focuses Doerr (2004), more specifically there is a need for focusing students using of logical connective and teachers’ assessment of their students’ utilizing of logical connective. Therefore, the current study conducted to examine students’ mistakes in logical connective and teachers’ responses to these mistakes.

**Background**

**Solving equations and inequalities**

Algebraic equations simply mean those typical types of equations (linear, quadratic, exponential, rational, etc.) introduced in the secondary school algebra curricula. The inequality is a mathematical sentence built from expressions using one or more of the Symbols (<, >, ≤ or ≥) to compare two quantities (El-khateeb, 2016. P. 124). Solving an equation means to find the numerical values of the unknown variable that make the equality a true statement. Solving equations and inequalities are considered to be an important topic in studying algebra specifically in studying properties and applications on functions, which require students to be aware and to understand method of finding the solution set different types for each inequality and equation (El-khateeb, 2016). Student have different difficulties in solving equations and inequality, such as: an adequate understanding of the meaning of the equal sign in solving equations (Knuth et al. 2006); inadequate understanding of manipulating algebraic expressions and statements (Samuel et al., 2016); Lack of symbolic understanding of variables and coefficients within an equation (Kilpatrick & Izsak, 2008) and changing the direction of inequality when multiplying by a negative number and for conceptual errors (El-khateeb, 2016).

**Logical connectives**

Propositional is “a sentence that is either TRUE or FALSE (but not both)” (Remsing, 2005. P. 2). A sentence that contains a ﬁnite number of variables and becomes a proposition when speciﬁc values are substituted for the variables is called a predicate (or open sentence) (Remsing, 2005. P. 15). The symbols ¬, ˄, ˅, ⇒and ⇔ will be called propositional connectives, any sentence built up by application of these connectives has a truth value that depends on the truth values of the constituent sentences (Mendelson, 2009 p. 3). The translating from natural language statement to formal logic consider as difficult for students and using one connective in place of another is one of the major errors between students (Barker-Plummer, Cox, Dale, & Etchemendy, 2008). Strannegård, Ulfsbäcker, Hedqvist and Gärling (2010) refers these difficulties that in English “or” does not always correspond the connective ∨, since “or” sometimes translates into an exclusive or and sometimes into an inclusive or. Similarly, the English construction “if... then...” does not always correspond to the connective→. An ability to understand logical connectives and set operations is vital, then students need systematic treatment of the logical and set-theoretic connectives (Dreyfus & Eisenberg, 1985).

**Teachers’ mathematical content and pedagogical knowledge**

Content knowledge (CK) includes the structure of knowledge, facts, theories, and principles in the field (Shulman, 1986). The mathematical content knowledge includes common content knowledge and specialized content knowledge (Ball, Thame & Phelps, 2008). The former relates to the content of the curriculum, concepts, procedures and the ability to read and write concept and notions correctly. Specialized content knowledge refers to the knowledge and skills unique to teaching (Delaney, Ball, Hill, Schilling, & Zopf, 2008). It includes an understanding of mathematical structures, which enables the treatment of tasks that require significant mathematical resources (Ball et al., 2008). Pedagogical content knowledge (PCK) is the knowledge needed to make subject matter reachable to students Shulman (1986), it is combines an understanding of both content and pedagogy (Ball, Lubienski, & Mewborn, 2001). It is comprising an awareness of students’ difficulties and misconceptions about the concepts being taught, understanding different ways in taught and represented specific content, and an understanding of the teaching methods that make learning easy or difficult Shulman (1986). Ball et al., (2008) separated mathematical pedagogicalcontent knowledge into two subcategories: knowledge of content and teaching and knowledge of content and students*.* The earlier combines knowledge about teaching with knowledge about mathematics. Teachers need to be aware to the instruction design, the various representations of the explanatory concept, and how to evaluate these representations (Ball et al., 2008). The latter, is a type of pedagogical content knowledge that combines an understanding of students which include awareness of how students think, know, and learn this specific mathematical content (Hill, Ball & Schilling, 2008).

Teachers MCK and MPCK about equation and inequalities solving should include mathematical procedures, algorithms, routines, skills, conceptual understanding, and procedure knowledge. Studying mathematics teachers’ conceptions of these issues could provide a better understanding of their teaching practices and influences on their students’ learning.

**The study’s questions**

1. To what extent ninth grade students have difficulties in logical connective? and what are their major difficulties?
2. How do teachers respond to students’ mistakes in logical connective? How they assess student’s mistakes?

**Method**

**Participants**: The study was conducted among 50 ninth grade students and among 63 mathematics practicing teachers. The students are from two nine grade classes from north Israel, the student in each class consider with different mathematical achievements, but the achievements of each class in mathematics consider as in average according to national mathematical exams in Israel. The practicing teachers were participated in voluntary bases; all of them teaching mathematics for ninth grade; A description of the participants' background is displayed in Table 1.

Table 1. *Distribution of the Participants According to Background Variables*

|  |  |  |
| --- | --- | --- |
| Variable | Categories | Practicing teachers |
| Gender  | Female | 84% |
| Male | 16% |
| Level of mathematics at high school | Basic  | 1.6% |
| Intermediate  | 42.8% |
| Advanced  | 46% |
|  |  |
| Teacher training institution  | College | 74.6% |
| University | 20% |
|  |  |  |
| Teaching experience  | 1-5 years | 60.3% |
| 6-10 years | 19% |
| 11-15 years | 6.3% |
| More than 17 years | 12.7% |
| Current school | Elementary | 44.9% |
| Secondary | 55.1% |

**Data source**: The data was collected from two sources: students’ questionnaire and teachers’ questionnaire.

Students’ questionnaire: The questionnaire consists ten items, the items are about existence statements and algebraic simplification which need using of propositional connectives. Following some examples from the questionnaire:

1. If  then
2. If  then 
3. Solve the equation (x2-3x+2)2 + (x2-6x+5)2=0
4. Solve the equation (x2-3x+2)2 · (x2-6x+5)2=0
5. Solve the inequality x2 > -1

Teachers' questionnaire:  The questionnaire contains ten items, each item includes existence statements and algebraic simplification which need using of propositional connectives. Each question followed with different solutions and the teachers need to evaluate by points from 0 to 10 each and explain their decision. Following some examples from the questionnaire:

1. When the question: solve the inequality x2 > 1 for ninth grade students we get the following eight answers:
	1. x2>1 ⇒ x> ±1
	2. x2>1 ⇒ -1>x>1
	3. x2 >1 ⇒ x>1 also x>-1
	4. x2>1 ⇔ x>1 or x<-1
	5. x2>1 ⇔ x>1 and x<-1
	6. x2>1 ⇔ x>1, x<-1
	7. x2>1 ⇔ x≠ ±1 and x≠0
	8. x2>1 ⇔ x≠1 and x≠-1 and x≠0
2. Below students’ solution for the following equation:

 (x2-7x+12)2 + (x2-4x+3)2=0

(x2-7x+12)2+(x2-4x+3)2=0 ⇔

 x2-7x+12=0, x2-4x+3=0

 (x-3) (x-4), (x-3)(x-1)

 x=3, x=4 , x=3, x=1

**Data analyses**

The data obtained from students’ questionnaire The analyses conducted in two phases, in the first phase we analyzed students’ solutions, through this phase we categorized students’ solutions to four categories: correct answer, correct algebraic manipulation but incomplete explanations of the answers (or, and, if), wrong answer – mistakes in algebraic manipulation and non-solved. In the second phase, we focused the second category which focused the mistakes in logical connective, the data was analyzed according to the constant comparison method (Glaser & Strauss, 1967). We identiﬁed, grouped and categorized mistakes’ types in the logical connective, categories were derived from the data set and were compared with the rest of the data set. For each category we calculated the percentage of its appearance in students’ solutions.

The data obtained from teachers’ questionnaire, used to collocated the means of scores for each items, and the frequencies of assessment by full scores among the teachers across the different years of experience in teaching.

**Findings**

First we will present the findings obtained from students’ difficulties in solving equations and inequalities with focusing the difficulties in logical connective; then we present the findings obtained from teachers’ questionnaire and their assessment to the students’ solutions.

**Students difficulties in solving equations and inequalities**

Students solution of equations and inequalities can be categorizing to four categories: correct answer; correct algebraic manipulation but incomplete explanations of the answers (or, and, if), wrong answer and non-solved. Table 2 presented the distribution of solution categories over all the 10 items over all the 50 students.

Table 2. Distribution of solution categories over all the 10 items over all students (n= 50)

|  |  |  |
| --- | --- | --- |
| Category | Distribution | Examples from students’ solutions |
| Correct answer | 12% | x2 > 1x>1 or x< -1 |
| Correct algebraic manipulation but incomplete explanations of the answers (or, and, if) | 38% | (x2-3x+2)2 + (x2-6x+5)2=0(x2-3x+2)2+(x2-6x+5)2=0 ⇔x2-3x+2=0, x2-6x+5=0(x-1) (x-2), (x-5)(x-1)x=1, x=2 , x=5, x=1 |
| Wrong answer – mistakes in algebraic manipulation  | 32% | $$\frac{1}{2}\leq \frac{x+1}{2}+x+3\leq 11$$$$1\leq x+1+2x+6\leq 22$$1$\leq 3x+7\leq 22$1$\leq 3x \leq 15$1$\leq x \leq 5$ |
| Non solved | 18% | - |

The findings obtained from Table 2 emphasized that difficulties in logical connective is the main difficulty in solving equations and inequalities among students. The findings indicated that more than third of the students’ solutions were in the logical connective. Focusing students’ mistakes in the logical connective we identified five types of mistackes: (1) Ignoring the logical connective, the students did not write any logical connective while doing algebraic manipulation; (2) ignoring the logical connective and replaced them by commas, the students positioned commas when there is a need for logical connective; (3) replacement the logical connective “or” by “and” and verse versa, the students did not distinguish the correct logical connective words; (4) replacement the logical connective “and” by “also”, the students use the word also as a logical connective; (5) wrong interpretation of the logical connective, the students use a logical connective words but they made wrong interpretation of these word, they did not understand the meaning of “and” , “or”, so the get wrong final answers. Table 3 present the distribution of mistakes in mathematical logical connectives.

Table 3. Distribution of the types of mistakes in mathematical logical connectives

|  |  |  |
| --- | --- | --- |
| Category | Student belong to identified category | Examples from students’ solutions |
| Ignoring the logical connective | 64% | $$\frac{1}{2}\leq \frac{x+1}{2}+x+3\leq 11$$$$1\leq x+1+2x+6\leq 22$$1$\leq 3x+7$ $ 3x+7\leq 22$-6$\leq 3x$ $3x \leq 15$-2$\leq x x \leq 5$ |
| Ignoring the logical connective and replaced them by commas | 12 % | x2-3x+2)2 + (x2-6x+5)2=0(x2-3x+2)2+(x2-6x+5)2=0 ⇔x2-3x+2=0, x2-6x+5=0(x-1) (x-2), (x-5)(x-1)x=1, x=2, x=5, x=1 |
| Replacement the logical connective “or” by “and” and verse versa | 2% | X2>1X2=1X=±1x>1 and x<-1 |
| Replacement the logical connective “and” by “also”  | 10% | (x2-3x+2)2 + (x2-6x+5)2=0x2-3x+2=0 also x2-6x+5=0(x-1)(x-2)=0 also (x-1)(x-5)=0x1=1, x2=2 also x1=1, x2=5x1=1, x2=2 also x2=5 |
| wrong interpretation of the logical connective | 12% | (x2-3x+2)2 + (x2-6x+5)2=0(x2-3x+2)2+(x2-6x+5)2=0 ⇔x2-3x+2=0 and x2-6x+5=0(x-1) (x-2)=0 and (x-5)(x-1)=0x1=1 x2=2 and x3=5 x4=1The answer is x= 1,2,5 |

The findings presented in Table 3 present different types of mistakes in logical connective. Almost two third of students ignored the logical connective, they get the final simplest equation or inequalities and did not connect between them, for that they did not succeed to get the final answers.

**Teachers’ assessment of students’ mistakes in mathematical logical connectives**

The findings revealed that teachers assess different answers with mistakes in the logical connectives as a complete answers and with full scores. The mean of scores that teachers give for all the items in the questionnaire with wrong in logical connective ranged between 5.26 points to 9 points. Table 4 presented different items in equations and inequalities, student answers with mistakes in the logical connectives and distributions of teachers who ranked the solution as completed with 10 points (full scores).

Table 4. Distribution of teachers’ assessment to solution with mistakes in logical connective as correct solution (full scores).

|  |  |  |  |
| --- | --- | --- | --- |
| Level of algebraic manipulation | Examples of Items | students’ solutions | Correct solution(Full scores) |
| Simple algebraic manipulations | x2>1 | x2>1 ⇔ x>1 also x<-1 | 8% |
| x2>1 ⇔ x>1, x<-1 | 28.6% |
| x2>1 ⇔ x>1 and x<-1 | 22.2% |
| More Complicated algebraic manipulation  |  | ⇔x2-3x+2=0 , x2-6x+5=0(x-2)(x-1)=0, (x-1)(x-5)=0⇔ x=2, x=1, x=1, x=5 | 44.4% |
|  | $(x^{2}-4x-22)=\frac{x^{2}-4}{x-2}$=$$\frac{(x-2)(x+2)}{x-2}=x+2$$$$x^{2}-5x-24= \frac{5\pm \sqrt{25+96}}{2}$$=$\frac{5\pm 11}{2}=8,-3$ | 17.5% |
|  | So So the answer is 5 also 17 | 63.5% |

The findings indicate that some teachers assess students’ solutions with mistakes in logical connectives as completed answers. The findings indicated that the level algebraic manipulation effect teachers’ assessment. The findings in Table 4 indicate that the teachers ignore the logical connective when the item need more algebraic manipulation, such the fourth item ( ) the 63.5% considered the answers as complete while the first item (x2>1) with the same mistake in the logical connectives only fifth of the teachers give full scores.

The findings indicate that teachers’ teaching experience affect their assessment of students mistakes in logical connective through solving equations and inequalities; Figure 1 present the distribution of teachers with different years teaching experience (1-5 years; 6-10 years; 11-15 years and above 16 years) and their assessment of students mistakes in logical connective.

S1, S2 and S3: Needed a simple algebraic manipulation

C1, C2 and C3: Needed a complicated algebraic manipulation

Figure 1: Distribution of the teachers' assessment according to their teaching experience

Figure 1 present that teachers with more teaching experience mostly ignoring students’ mistakes in logical connective and teacher with lower years’ experience (1-5) more considered the mistakes in the logical connective and did not give full scores.

**Discussion**

The aim of the current study is to focus students’ mistakes in using and writing of logical connectives in solving of algebraic equations and inequalities, and examine teachers’ assessment and awareness to these mistakes. The main findings indicated that student succeeded in manipulation of algebraic expressions but they have difficulties in writing the logical connective correctly. In addition, the findings revealed that teachers assess different answers with mistakes in the logical connectives as a complete answers and with full scores.

The current study’s findings according to students’ mistakes in solving of algebraic equations and inequalities indicated that the most problem among students was in the logical connective. The different types of mistakes in logical connective addressed that student did not aware to the final answers, they did not asses their solution if it the correct answers (Vaiyavutjamai & Clements, 2006). Focusing the types of mistakes in logical connective reveals that the most common mistake was ignoring the logical connective, almost two third of students ignored the logical connective. In which students ignore the use of logical connective, students either did not put a logical connective or are supported by Almog and Ilany’s (2012) study which conducted among student in grade 12. Similar difficulties reported by Tsamir and Almog’s (2001), they found that the ignoring of use the connective words was revealed in applying the square-root property to inequalities; For example, students applying the square-root property to x2> 81 would provide the following solution x> ±9, instead of x < – 9 or x > 9. The lowest distribution of errors among students was the use of wrong word between the inequalities expressions in their solutions they used “or” when they should have used “and”, and vice versa, Neimark (1970) described this mistake as interpreting set union (A or B) as set intersect (A and B).

The findings from student questionnaires emphasized El-khateebs’ (2016) recommendation, that teachers must explain and discuss the meaning of the logical connective, such the meaning of the word (or) when writing the solution set. However, the findings of the current study according to the teachers’ assessment of students’ solutions algebraic equations and inequalities, indicated that teachers did not aware to students’ mistakes in logical connective. Teachers assessed solutions with mistakes in logical connective ae complete solutions with full scores; particularly, when the questions are more complicated. This phenomenon was common more among teachers with more experience in teaching. While Li’s (2007) study reveals three topic areas in solving equations in which teachers’ mathematical subject matter understanding should be strengthened: the balancing method, the concept of equivalent equations, and the properties of linear equations in their general forms. We suggest to added a fourth topic that emphasizing the logical connective.

We think that teachers’ responses to the mistakes in the logical connective and students’ difficulties in this topic are related. So the current study’s recommendation is to strengthening the correct presenting of logical connective among students; in addition to enhancing teachers’ awareness for the important of correct writing of the logical connective words. We recommend to work with teachers and with students, because teachers’ knowledge for teaching and students’ understanding are tied together.

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