**The effect of ethnomathematics through ornamentation on students’ learning congruent triangles**

**Abstract**: Ethnomathematics makes school mathematics more relevant and meaningful for students. The current research aims to study the effect of using Islamic ornamentation on learning the topic of congruent triangles. For this aim, 30 tenth grade students engaged in ethno-mathematics learning about congruent triangles based on Islamic ornamentation. The data was gathered from :(a) videotaping and transcribing students’ learning, (b) students’ answers on two parallel questionnaires that included proof questions reflecting the three congruence theorems. the students were required to answer one of the questionnaires before the learning process and the other after the learning process. The main results indicated that the students succeeded in constructing the concept congruence and congruent triangles through the learning process. In addition, they succeeded to arrive at and formulate the three congruence theorems. Moreover, the findings obtained from the questionnaires indicated that the students improved their proving processes as a result of ethno-mathematics-based learning. Furthermore, paired sample t-test indicated significant differences between the students’ mean scores before and after the learning process.

Keywords: congruence, congruent triangles, ethnomathematics, Islamic ornamentation.

**Introduction**

D’Ambrosio (1985) identified Ethnomathematics as “the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" (p. 45). The adapting of ethnomathematical perspective for developing informal mathematical curricula takes care of students’ cultural links or contextualized problems (D’Ambrosio, 2002). This function of ethnomathematics is satisfied since the implementation of a culturally relevant pedagogy in the school curriculum helps to develop students’ cognitive and emotional levels (Rosa & Orey, 2010). Irwin (2001) reported that using contextualized problems among primary students enhanced their understanding of decimal numbers. Massarwe, Verner and Bshouty (2010) reported that constructing geometrical ornaments and the discovery of their mathematical properties was recognized as a meaningful and enjoyable among high school students. In addition, the constructing of geometrical ornaments can foster creativity and lead to deep inquiry in both mathematical and cultural aspects (Massarwe, Verner & Bshouty, 2011). Furthermore, Widada, Herawaty and Lubis (2018) found that ethnomathematics-oriented materials improved the mathematical understanding of the participating students.

Geometry is considered the core of culture and ethnomathematics (Gerdes, 2005), where art connects between the two of them. Art is considered an important code of cultural groups, where it serves as a powerful tool for teachers in the classroom. This is specifically true for the Islamic art whose most dominants aspect is the ornamentation (Fukushima, 2004). In the current study, we used the Islamic geometrical ornamentation to design learning ethno-mathematics for triangle congruence. The triangle congruence theorems are one of the basic topics in classical geometry (Laudano & Vincenzi, 2017), however students have difficulties in understanding the congruence (Wu, 2005). Little research has been done on the teaching of congruence; at least, English (Jones, Fujita, & Miyazaki, 2013) or on its learning, especially regarding reasoning and proof (Wang, Wang & An, 2018). The importance, the difficulties and the little research done on triangle congruence led us to choose this topic for the current study. The main aim of the current study is to examine the effect of a ethno-mathematics learning in triangle congruence based on Islamic geometrical ornamentation on understanding the triangle congruence among Muslim students who live in Israel.

**Theoretical background**

**Ethnic mathematics**

D’Ambrosio (1990) identified ethnomathematics according to three sections that comprised the term: ethno, mathema and tics. The prefix ethno refers to the socio-cultural context which includes codes and symbols. Mathema means to explain, to know, to activate, to infer and to model, and tics is derived from techné and has the same root as technique. Ethnomatematics bridges between culture and education, so it is perceived as a lens for viewing and understanding mathematics as a cultural product (Sugianto, Abdullah, & Widodo, 2019). Such as the geometry of transformation, as reflected in the Batak society, the craftsman used principles of rotation, translation, dilation and reflection are in making gorga motifs (Ditasona, 2018). For that, ethnic mathematics makes school mathematics more relevant and meaningful to students, motivates them and develop their self-confidence (Amit & Abu Qouder, 2015). Besides, it develops students' intellectual, social, emotional, and political learning by using their unique cultural referrals to convey their knowledge, skills and attitudes (Rosa & Orey, 2011). Ethnomathematic Several researchers adapted the ethnomathematics for learning- teaching several mathematical concepts, such as linear equation, using Ethnomathematics in selling buying tradition in Peterongan traditional market in Central Java as a context enhancing students’ understanding in linear equation system in two variables (Nursyahidah, Saputro, & Rubowo, 2018).

**Islamic Geometric ornamentation**

The term ornament comes from the Latin Ornare, which has the meaning of adorning (Ditasona, 2018). Ornaments is defined as the “geometric patterns of cultural value composed of basic units repeated under different transformations” (Massarwe, Verner & Bshouty, 2011, p. 1). The ornament is a product of art made for decoration (Gustami, 1980). It is considered as the core of Islamic architectural heritage and can be categorized into three types: geometric, arabesque, and calligraphic patterns (Ahmad, Rashid & Naz, 2018). The geometrical ornamental designs are based on the principles of symmetry and repetition (Shafiq, 2014). The repetition is the most effective and important theme for geometrical patterns, where ornamental designs are generally generated by the repetition of square and triangle shapes (Cenani & Cagdas, 2006). Most of the geometric motifs are based on the hypothesis that each pattern can be repeated and expanded infinitely (Burkhart, 2009). The symmetry is the key factor in the derivation of both geometrical and floral patterns, where in bilateral symmetry, the figure is mirrored through the vertical axis (Cenani & Cagdas, 2006). Another property of Islamic ornaments is the two-dimensionality, where most Islamic design is 2D although it is not only applied to flat surfaces. In addition, the patterns themselves rarely have shading in the background (Ajam, 2013). The use of ornaments have appositive effect in students learning geometry, Massarwe, Verner and Bshouty, (2011) indicated that students became aware of ornaments not only as decorations, but also as interesting geometrical objects.

**Congruent triangles**

Clapham and Nicholson (2009) identified congruent figures in geometric by “Two geometrical figures are congruent if they are identical in shape and size, or if one shape mirrors the other in shape and size” (p. 87). Triangles are congruent if six conditions are met: the three sides are equal and the three angles are equal (Shatnawi, 2008). Sufficient conditions for the congruence is one of four conditions: S.S.S., A.S.A., S.A.S., and R.H.S; which are taught in secondary grades and called congruence theorems (Jones, Fujita, & Miyazaki, 2013). The minimal number of conditions sufficient for congruence is a key concept to understand congruence, where many students find it difficult to justify the congruence (Leung, Ding, Leung & Wong, 2014). The triangles congruence is considered a core topic in school geometry. This consideration is due to the usefulness of the sufficient conditions for the congruence triangles to proving geometrical theorems. In addition, the triangles congruence is connected with similar triangles (Jones et al., 2013).

**Research aim and questions**

Little research has been done on students’ learning of the congruence concept (Jones et al., 2013), which is especially true for low-achieving students. The congruent triangles’ topic is considered difficult for students, (Wu, 2005) which makes Ethnomathematics a candidate to overcome its difficulty since Ethnomathematics makes school mathematics more relevant and meaningful for students (Sugianto et al., 2019). The current research aims to study the effect of the use of Islamic ornamentation on learning the congruent triangles among Moslems students with mathematical difficulties. By answering the following questions:

1. What characterizes students’ definitions of congruence and congruent triangles?
2. What characterizes students' sequence of learning processes of congruent triangles based on Islamic ornamentation?
3. To what extent students’ ethno-mathematics learning of congruent triangles based on Islamic ornamentation affects their knowledge in congruent triangles?

**Method**

To meet the aim and objectives of the study, we used a mixed-method design with qualitative quantitative and components. In the qualitative analysis, we follow the students’ ethno-mathematics learning process of congruent triangles based on Islamic ornamentation, in addition to following students’ identification of the congruent triangles and their strategies in proving claims about congruent triangles. The quantitative analysis is based on a quasi-experimental (pre-post) design, where it was used to examine the effects of participating in the learning process about students’ solving congruent triangles questions.

**Participants**

The research was conducted among 30 students in 10th grade (15-16 years). The students are considered as students with difficulties in mathematics. They formally learned the congruent triangles topic in eighth and ninth grades, including the three congruence theorems and proving claims about congruent triangles according to these theorems. However, according to their mathematics teacher’s diagnosis and to the pre-test results, they did not succeed in solving congruent triangles questions. Through the study, the participants worked in groups of 4-5 students, we videoed three groups. Group A: Marah, Taqwa, Reem, Saja and Nagam; Group B: Abed, Aya, Doa’a, Noor and Zahran and Group C: Nader, Rula, Majed and Rawan. In the finding section focused the first group and did not mentioned the third group’s work to avoid repletion.

**Data sources and procedure**

The data was collected from two sources, video recording and questionnaires.

**Video recordings***:* Video recording was taken of the three groups along the ethno-mathematics learning of congruent triangles based on Islamic ornamentation. All of the video recordings were transcribed.

**Questionnaires**: Two parallel questionnaires A and B were used in the present research, where each questionnaire consisted of five questions. The first question consisted of two parts about the definition of congruence and congruent triangles; the second, third and fourth questions were proof questions based on first, second and third theorems of congruent triangles; the fifth question was also a proof question but combined between the three theorems. Two of the tasks are presented in the results section.

The students were requested to solve questionnaire A, then they were engaged in ethno-mathematics-based learning of the congruent triangles in six meetings. Each meeting lasted for 60 minutes. Afterwards, the students were requested to solve the second questionnaire B.

**Data analyses**

**Video recordings**:

We analyzed students’ learning depending on conceptual analysis, as it enables us to study students’ learning according to the instructional goals (Thompson, 2008). In more detail, we organized the transcribed data from the video recording according to instructional goals: The first was related to congruence and congruent triangles, while the second was related to the insufficient conditions for congruent triangles. The third to five phases were related to the three congruent triangles theorems. For the learning of students related to each goal, we describe ways of understanding ideas that are targeted by the instruction. In this description we follow students’ learning processes as imagining, connecting, inferring, and understanding situations in particular ways (Thompson & Saldanha, 2003).

**Questionnaires**: We analyzed the data obtained from the questionnaires in two methods. The first, which was based on the constant comparison method (Glaser & Strauss, 1998), to examine the students' solution strategies. The second method included using statistical tests to examine the data before and after the ethno-mathematics learning.

**Ethno-mathematics activities**

Tenth grade students learned congruence and congruent triangles based on ethno-mathematics that utilized Islamic ornamentation, where the activities were prepared by the researchers. The learning included concrete and abstract processes, where the students were engaged in these process with copying, cutting, transforming shapes and solving abstractly. Table 1 presents a summary of each meeting.

Table 1: The objects and the contents of each meet in the ethno-mathematics learning

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| --- | --- |
| Object | Examples of activities through the meeting |
| Meeting 1:  -To be familiar with the Islamic ornamentation and its patterns.  -To expose the concept of congruence in general and connect it with Islamic ornamentation.  -To use geometric transformations (rotation, reflection and translation. | C:\Users\Dell PC\Desktop\صور زخرفة\FB_IMG_1515137169908.jpg -Exhibition and discussion of the different types of Islamic ornamentation, such as animal, plants and geometric ornamentations.  -Finding the geometric shapes in the ornamentation.  -Cutting different shapes.  and discussing their congruence.  - Discussing the meaning of congruence.  - Using different transformations as a mathematics process that enables the movement from one shape to another in the ornamentation.  - Analyzing geometrical properties of ornaments, as well as constructing them. |
| Meeting 2:  -To recognize the congruent triangles.  -To recognize the symmetry between sides and angles in congruent triangles.  -To write precisely the properties in congruent triangles. | 7-Cutting the triangles and discuss which of them are congruent.  9-Examining the congruent between the concrete triangle and triangles in the ornamentation.  - Discussing the reasons why triangles are congruent.  -Drawing congruent triangles.  -Presenting the congruent notion and practising writing the properties in congruent triangles. |
| Meeting 3:  -To find out the sufficient conditions for two triangles to be congruent.  -To find out the sufficient conditions related to the first theorem (S, A, S).  -To formulate the first theorem (S, A, S).  -To write a claim and its explanation in the frame of the mathematical proof.  -To solve different exercises based on the first congruence theorem. | 1ef4061a404df9161fb62a5bf45e90c3 -Inquiry about incongruent triangles in the ornamentation, which triangles are equal in one side; equal in one side and one angle or equal in three angles.  212- Inquiry by drawing ornamentation, measuring the angles, cutting the triangles and comparing between them.  -Distinguishing congruent triangles in ornamentation based on (S, A, S).  -Writing the proof processes of congruent triangles based on (S, A, S). |
| Meeting 4:  -To find out the sufficient conditions for two triangles to be congruent.  -To find out the sufficient conditions related to the first theorem (A, S, A).  -To formulate the first theorem (A,S, A).  -To write a claim and its explanation in the frame of the mathematical proof.  -To solve different exercises based on the first congruence theorem. | 19-Inquiry about congruent triangles according to (A,S,A) through drawing triangles in the ornamentation.  - Inquiry about congruent triangles according to (A,S,A) through cutting and pasting triangles.  890-Distinguishing congruent triangles based on (A, S, A) in ornamentation.  -Writing the proof phases of congruent triangles based on (A, S, A). |
| Meeting 5:  -To find out the sufficient conditions for two triangles to be congruent.  -To find out the sufficient conditions related to the first theorem (S, S, S).  -To formulate the first theorem (S, S, S).  -To write a claim and its explanation in the frame of the mathematical proof.  -To solve different exercises based on the first congruence theorem. | 091-Drawing ornaments that include equal-side triangles.  - Drawing ornaments that include triangles with different sides.  63-Inquiry about congruent triangles according to (S, S, S) through cutting and pasting triangles.  - Distinguishing congruent triangles based on (S, S, S) in the ornamentation.  - Writing the proof process of congruent triangles based on (S, S, S). |
| Meeting 6:  -To solve various question based on congruent triangles.  -To choose an appropriate theorem for proving that triangles are congruent.  -To write sufficient conditions for proving that triangles are congruent. | Capture2-Completing different processes in a given proof.  0000-Distinguishing data about triangles from the figure and writing it as conditions for congruence/incongruence.  -Solving question based on the three congruent theorems. |

**Findings**

In this section, we will present the findings according to the instructional goals (congruence and congruent triangles, the insufficient conditions for congruent triangles, and each of the three congruent triangles theorems). The findings are based on conceptual analyses (Thompson, 2008) of students' discourses and activities through the ethno-mathematics learning. Besides, the findings target students’ solutions in questionnaires A and B.

**Processes of Students’ Learning of congruent triangles through the Islamic ornamentation**

The students’ discourse throughout the Islamic-ornamentation-based learning, indicated that they succeeded, through imagining, connecting, inferring, and understanding, to define the concept of congruence and congruent triangles. Doing that, they found and formulated the sufficient conditions for two triangles to be congruent; i.e. the three theorems of congruent triangles (S, A, S), (A, S, A) and (S, S, S). Besides, the students succeeded in carrying out justification activities that depended on the theorems. Following, we detail the feature of each phase in students’ learning. To avoid repetition, we focus on one group, describing what happened in the other two groups when different processes occurred.

**Defining the congruence in general and congruence of triangles in particular**

**Defining the congruence in general**

The Analyses of students’ discourse in the three groups indicated that the students in each group defined, at the beginning of the learning process, the congruence concept in terms of equality of areas or shapes’ covering, where the description of covering was without details, as presented in Episode 1 and 2.

Episode 1: Group A’s Definition of congruence as equality and covering of shapes

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| [2] | Marah: | Congruence is that one shape covers another one. |
| [3] | Taqwa: | That means that the two shapes are equal |

Episode 1 shows that Marah [2] and Taqwa [3] provided an incomplete definition of congruence for they did not detail the covering must be precise.

Episode 2: Group B’s definition of congruence as equality of shapes’ areas

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| --- | --- | --- |
| [8] | Abed | These rhombuses [pointed to the rhombuses in the ornamentation] are congruent. |
| [9] | Doa’a | Yeh, because they have the same area. |

In Episode 2, Doa’a incorrectly connected the congruence to the equality of shapes’ area.

The students then identified congruence concept, through their concrete work in ornamentation, the repetition and the symmetry of different shapes in the ornamentation, which helped the students to arrive at a precise definition. To do that, the students cut shapes from the ornamentation and tried to move these shapes to cover other shapes. Through this process, the students needed to use different transformation (rotation, reflection and translating), as presented in Episode 3.

Episode 3: Group A’ inquiry of the congruent shapes using ornamentation

|  |  |  |
| --- | --- | --- |
| [16] | Reem: | Look at the squares [She cut some squares in the ornamentation] |
| [19] | Taqwa: | Put these squares here. |
| [21] | Saja: | They are the same, they are congruent [Pointing at two squares with minor difference]. |
| [22] | Nagam: | Why? These squares aren’t congruent |
| [23] | Saja: | Because they do not exactly cover each other. |
| [24] | Taqwa: | The congruence is when the two shapes exactly cover each other. |
| [26] | Saja: | The two shapes have the same length of sides. |
| [29] | Reem: | When we compare between the two rhombuses, do we have to do direct coverage? [she pointed at two opposite rhombuses] |
| [36] | Nagam: | You can do translating… |
| [37] | Saja: | Ahh, we can rotate this rhombus to get this rhombus. |

Episode 3 shows that students arrived, through working with ornamentation, at the exact definition of congruence, which is presented in Taqwa’s inference [24]. They also arrived at the features of two congruent shapes, which is presented in Saja’s conclusion [26]. Also, through imagination, Nagam [36] and Saja [37] indicate the utilizing transformation supports the examining of congruent shapes.

**Defining the congruence of triangles**

Through concrete and visual working ornamentation, the students in all groups arrived at the definition of congruent triangles. The students cut and covered triangles in the ornamentation and drew ornamentations using triangles and their reflections. They concluded, as presented in Episode 4, that there are many congruent triangles.

Episode 4: The definition of congruent triangles in Group A

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| --- | --- | --- |
| [54] | Nagam: | 7The triangles 1 and 3 are congruent because they cover each other completely [after cutting the numbered triangles in the ornamentation] |
| [68] | Saja: | There are other different congruent triangles in the ornamentation. |
| [74] | Marah: | The vertex of the triangle must be D [She drew a triangle that is congruent to the triangle ABC] |
| [82] | Nagam: | I drew a triangle that in the opposite position of the ABC triangle. |
| [83] | Teacher: | You can check if they are congruent. |
| [84] | Nagam: | I can do reflection. |
| [91] | Saja: | All these triangles are congruent. |
| [94] | Saja: | All these triangles are equal in their sides and angles. |

Episode 4 shows that Nagam [54] understood through her concrete experience the meaning of congruent triangles which enabled her to defined correctly congruent triangles. Saja [68] too showed her understanding by indicating that there are different congruent triangles to the ABC triangle. Marah [74] inferred the position of the vertex in order for the two triangles to be congruent. Nagam [82] showed her understanding by indicating that she succeeded in drawing congruent triangles that were in different positions. This indicates that she knew the conditions for congruent triangles. Nagam [84], through imagination, argued that reflection could help her in checking the congruence of triangles. Saja [91 & 94] formulated the properties of congruent triangles.

Analysis of all students’ answers in questioners 1 and 2 indicated changes in their definition of congruent triangles. Before the learning process, the definitions of students were categorized into six categories: Equal sides, equal angles, similarities, complete covering and covering in the sides and angles. However, after the learning process, we identified four categories. Table 1 presents the distribution of students' categories of definitions before and after the learning process.

Table1: students' definitions of the congruent triangles before and after the learning process

|  |  |  |
| --- | --- | --- |
| Category | Before the learning | After the learning |
| Equal sides: When the sides of the triangles are equal, they are congruent. | 6.6% | 0 |
| Equal angles: When the angles of the triangles are equal, they are congruent. | 13.3% | 6.6% |
| Similarities: when there is a similarity between triangles, they are congruent. | 20% | 10% |
| Coverage of sides and angles: When the sides and angles of two triangles cover each other, the two triangles are congruent. | 10% | 33.3% |
| Complete Cover: Congruent triangles that each one covers the other precisely. | 6.6% | 50% |
| Without answer | 43.3% | 0 |

Table 1 shows that students before the learning process did not provide definitions of the congruent triangles or the definitions were related to similarity. After the learning process, students’ definitions were related to coverage of triangles.

**Investigating the sufficient conditions of congruent triangles using Islamic ornamentation**

The investigation of ornamentations led students in all groups to discover sufficient conditions for congruent triangles. Through cutting and covering, students found which conditions do not lead to the congruence of triangle: equality in one side; equality in one side and one angle or equality in three angles. Episode 5 presents students’ investigation regarding the insufficient conditions for the congruence of triangles.

Episode 5: Insufficient conditions for congruent triangles

|  |  |  |
| --- | --- | --- |
| [102] | Reem: | 1ef4061a404df9161fb62a5bf45e90c3If there are corresponding sides that are equal in the triangles BDE and BDA |
| [105] | Nagam: | ED |
| [107] | Saja: | No, it is BD |
| [111] | Reem: | In this ornamentation, the triangles are not congruent; one is a big ornament and one is a small ornament. |
| [116] | Reem: | 68Two equal corresponding sides in two triangles is not a sufficient condition for congruence. |
| [117] | Marah: | Look at this ornamentation, we need to find if all corresponding angles and sides are equal in triangles ABC and BDC. |
| [119] | Saja: | The side BC. |
| [123] | Nagam: | The angles B and D. |
| [126] | Saja: | One side and one angle is sufficient for congruence. |
| [127] | Marah: | No, not enough, one is big and one is small. |
| [130] | Saja: | Now the third ornamentation. We need to check if the angles in the triangles ABC and AED are equal. |
| [132] | Saja: | We have here two parallel lines, so the corresponding angles are equal. |
| [133] | Marah: | All the corresponding angles are equal. |
| [135] | Nagam: | But the triangles are not congruent; one is big and the other is small. |
| [137] | Saja: | That means three equal angles is not sufficient for congruent triangles |

Episode 5 presented the insufficient condition for congruent in triangles. Looking at one big and one small ornaments, Saja [107] found that BD is a mutual side in the two triangles BDE and BDA; Reem [111] showed understanding the concept of congruence, claiming that the triangles are not congruent, emphasizing in [116] that one side is insufficient condition for congruent triangles. Similarly, Saja [126] and Marah [127] inferred that one side and one angle were insufficient conditions for congruent triangles. Nagam [135] explained why three equal angles are insufficient for congruence, while Saja [137] inferred an appropriate conclusion.

**The first congruent triangles theorem (S, A, S)**

Students in all groups performed a construction task to draw the ornamentation, then they performed cutting and covering of the triangles to verify their congruence. The students concluded that two sides and their included angle are sufficient for the congruence of triangles. After that, they succeeded in formulating the first theorem (S, A, S), as presented in Episode 6.

Episode 6: investigation of the first congruence theorem (S, A, S)

|  |  |  |
| --- | --- | --- |
| [139] | Reem: | 212If there are equal corresponding angles and sides in the triangles AEB and AEC in the ornamentation. |
| [140] | Saja: | EC and BE |
| [142] | Reem | According to the ornamentation, the length of each one of their lengths is half of the big square’s side. |
| [143] | Nagam: | AE and AE, AE is mutual. |
| [145] | Saja: | The angle E is equal to 90. |
| [147] | Taqwa: | So that means, we have two sides and their included angle, which are equal. |
| [148] | Teacher | What about the congruence? |
| [149] | Saja: | Yes |
| [150] | Taqwa: | They are congruent. What can we say about the conditions? |
| [151] | Saja: | Two equal sides |
| [153] | Taqwa: | And angle |
| [154] | Saja: | The angle must be in the middle. |
| [157] | Nagam: | Three elements, two sides and one angle between the sides. |

Episode 6 shows that Saja [140,145] and Nagam [143] were able to infer, through connecting between the ornamentation characteristics and the geometric shapes, the equal sides and equal angles between the triangles in the ornamentation. Saja [151 and 154] Taqwa [153] inferred the conditions for the congruence of the triangles while Nagam [157] formulated the conclusion of first congruence theorem.

**The second congruent triangles theory (A, S, A)**

Drawing triangles’ ornamentation by using protector and ruler, the students had difficulties in in drawing triangles according to two angles and one side. They overcame this difficulty by the teacher’s guidance. After the drawing, the students cut and covered the ornamentation with triangles. Doing that, they arrived at the second theorem of congruent triangles (A, S, A). At the end, the students formalize their knowledge by writing the proof of the second theorem (A, S, A). Episode 7 presents students’ discussion in performing the previous task.

Episode 7: Inquiry the second congruent triangle (A, S, A)

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| --- | --- | --- |
| [160] | Saja: | We need to draw triangles with angles 50 and 60, and side with 5 cm length between them. |
| [161] | Taqwa: | How can we do it? |
| [162] | Teacher: | You must use protractor for drawing the angles. |
| [163] | Nagam: | Does it measure the sides? |
| [164] | Taqwa: | No it for angles. |
| [165] | Teacher: | This point is the zero point and the rays assign any angle we want… |
| [167] | Nagam: | First, we draw a segment of length 5 cm. |
| [177] | Saja: | So, we need to put the protractor on the end point of the segment [she drew while talking] and here 50 [using the protractor]. We assign a mark, and we assign an angle of 60 on the other side of the segment [again using the protractor]. Now we draw the lines that connect these two points. |
| [178] | Marah: | We want to cut one triangle. |
| Students drew several triangles with the same method | | |
| [181] | Reem: | They are congruent. |
| [186] | Teacher: | What is the condition for congruence? |
| [187] | Nagam: | There have the same angles |
| [189] | Saja: | Two angles. |
| [191] | Marah: | And one side between the angles. |
| [195] | Taqwa: | We can conclude that with three conditions; two angles and one side between them, we get congruence of triangles. |

Episode 7 presents students’ difficulties in drawing angles, as Taqwa’s difficulty [161], but they overcame these difficulties with the teacher’s guidance, as seen in Saja [177]. Nagam [163] tried to connect between the geometric tool and the task. Marah [178] and Reem [181] used cutting and covering to prove the congruence. Nagam [187], Saga [189] and Marah [191] inferred the conditions of the second theorem, while Taqwa [195] formulated this theorem.

**The third theorem of triangles’ congruence (S, S, S)**

Students in all groups arrived at the third congruence theorem through their engagement in the drawing of ornamentations. Here too, they used ornaments’ cutting and covering, and afterward wrote the corresponding sides of the congruent triangles. The students also discussed the different options of sides’ lengths of the congruent triangles; i.e. the cases of isosceles triangle, equilateral triangle and different-side triangle. Doing that they concluded the third congruence theorem. Episodes 8 and 9 represent the group’s discussion regarding the isosceles and equilateral cases (Episode 8) and the different-sides case (Episode 9).

Episode 8: Discussing the third congruence theory regarding the isosceles and equilateral cases

|  |  |  |
| --- | --- | --- |
| [306] | Doaa | Draw a triangle, each side is 3 |
| [Students drew and cut different equilateral triangles] | | |
| [309] | Abed | All these triangles [He put the triangles one above the other] are congruent. |
| [314] | Aya | We need to write the corresponding sides. |
| [318] | Noor | DE=AB; DF=AC; EF=BC |
| [321] | Aya | Now we need to draw triangles with sides 4,3,4 |
|  |  | Students drew and cut different isosceles triangles. |
| [329] | Doaa | The triangles are congruent. |
| [339] | Doaa | It is sufficient that three sides in two triangles are equal to have congruence. |

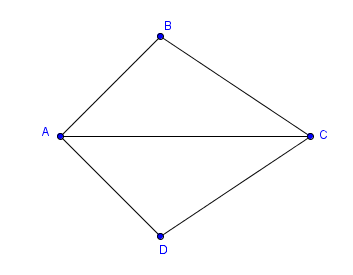
Episode 8 shows that students arrived at the sufficient condition (S, S, S). At the beginning, starting from an equilateral triangle, Doaa [306] showed her understanding of the task, while Abed [309] showed understanding of the congruence concept. The students; Aya [321] and Doa’a [329], discussed the congruence of isosceles triangles. As a result of this discussion, they inferred and formulated the sufficient conditions as presented in Doa’a’s [339] declaration.

Episode 9: Discussing the third congruence theory regarding the isosceles and equilateral cases

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| --- | --- | --- |
| [200] | Marah: | This triangle (She drew a triangle in the ornamentation). |
| [201] | Nagam: | 678See these triangles (She cut and covered one triangle with the other). |
| [207] | Teacher: | Try to name the triangles in the ornamentation and write the corresponding sides. |
| [208] | Nagam: | [Pointing at two triangles in the ornamentation] This is DEF and this is ABC. |
| [211] | Marah: | AB=DE |
| [212] | Taqwa: | DF=AC |
| [213] | Saja: | EF=BC |
| [215] | Marah: | These triangles DEF and ABC are with different lengths. |
| [218] | Reem: | They are congruent. |
| [223] | Marah: | When three corresponding sides are equal in two triangles, they are congruent. |

Episode 9 presents student inferring the third congruence theorem related to triangle. They did that through drawing, as Marah [200], through cutting and covering, as Nagam's [201]. These processes were followed by writing the corresponding sides in the congruent triangles [211-213]. At the end of the activity, Marah [223] formalized the third congruence theory (S, S, S).

**The impact of working with ornamentations on students’ proving processes**

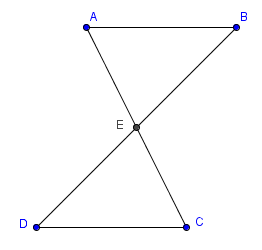
 The results obtained from analyzing students’ proving processes indicated positive influence. Before working with ornamentations, the students wrote claims without justification, they added data that was not given in the task could be concluded from it. Moreover, they wrote the congruence theorem without all the sufficient conditions or writing the sufficient conditions without indicating the specific congruence theorem. In addition, some student before working with ornamentations made use of wrong justification. We exemplify these processes according to Task 1 and 2 presented below.

Task 1: AC is an angle bisector of ∢BAD and ∢BCD

Prove that:

ABC ≅ ΔADC

BC = DC

Task 2: Given:

∢C = ∢ A

E is the middle of AC

Prove that E is the midpoint of BD

Table 2: Proving processes before working with ornamentations

|  |  |  |
| --- | --- | --- |
| Category | Examples from Figure 1.a and Figure 1.b | Explaining of the example |
| Wiring claim without explanation | Figure 1.a | C1=C2; A1=A2  The triangles are congruent according to (A, S, A).  The proof did not include explanations about the properties of the triangle |
| Writing the congruent theory only |  | (A, S, A)  Writing the congruent theory only without writing the phases of the proof |
| Additional data | Figure 1.a | BC=DC; AB=AD  Students added the qualities between the sides that did not give in the question. |
| Absent one of the condition | Figure 1.b | AE=EC Given  ∢E1= ∢E2 opposite angles  ∆ABE=∆EDC  It is congruent according to (A, S, A)  Three is a need for more one condition |
| Unmentioned the congruent theory | Figure 1.b | ∢A=∢C Given  ∢E1= ∢E2 opposite angles  AE=CE Given  AED and DCE are congruent |
| Writing the wrong explanation | Figure 1.b | ∢A=∢C Given  ∢E1= ∢E2 opposite angles  AE=CE mutual  side  The explanation for the final claim is wrong |

The processes mentioned in Table 2 are examples of the justification processes related to the three congruence theorems. Analysis of students’ answers on questionnaires A and B indicated a positive effect for the Islamic-ornamentation-based learning process. This was reflected in improving the geometric proving process related to the congruent triangles. Paired Simple t-test indicated significant difference (*t* (9) = - 17.14, p <.001) between students’ mean scores in questionnaire B (*M* = 88.6, *SD* = 11.57) and A (*M* = 15.9, *SD* = 10.137) in favor of questionnaire B.

**Discussion**

The current research aimed to study the effect of the use of Islamic ornamentations on learning the congruent triangles topic by tenth grade Moslem low achieving students. The students, as observed in the pre-test results, had different difficulties in defining the concept of congruence and congruent triangles, and in the proving process related to this topic. Most of the difficulties observed in the pre-test were students’ writing of claims without justification, adding data not present in the task and concluding the congruence theorem without writing the sufficient conditions or writing the sufficient conditions but not indicating the specific congruence theorem. Students’ difficulties were expected as previous researchers point at them (e.g., Wu, 2005). Specifically, most of these difficulties were also reported in Wang et al (2018) as difficulties of 8th grade students. The previous researches’ results constituted the rationale of the current study, which points at one of the contributions of this research.

The Islamic ornamentation that is the core of the ethno-mathematics learning in the current study includes different geometry transformation: Reflection, translation and rotation. The students engaged in these transformations through their drawing, connecting between the ornaments and geometric shapes, imagining transformations on these shapes, and inferring the conditions for the theorems. Thus the ornamentations enabled the students to advance from concrete to abstract thinking. This is in line with Gravemeijer (2008) who argues that realistic mathematics enables to make mathematics consider different aspects of society, including the cultural aspect, through the concrete things that can be observed or understood by learners through the process of mathematization as mathematics is a reflection of the real world through an empirical abstraction process. In the present research, the ornamentations were realistic ethno-mathematics tools that enabled ‘empirical abstraction process’.

The main findings indicated that sequences of activities based on Islamic ornamentation had a positive effect on learning the congruent triangles topic among Moslems students. The students succeeded to define the congruent triangles, infer the conditions for congruent triangles, formulate the corresponding theorems, and engage in proving based on the congruence theorems. Students’ success in proving claims can be attributed to their use of the imagining, connecting, inferring processes, which enabled them to understand of the minimal number of conditions sufficient for congruence, that is considered a key concept to understand congruence (Leung et al., 2014). The study results emphasize the positive effect of integration of ethnomathematics activities on students’ learning of geometric concepts and relations; here the congruence theorems. As such, it supports the different studies that emphasized the effect of ethnomathematics on learning various mathematical concepts and subjects. Specifically, ethnomathematics had a positive effect on the understanding of the and linear equation system in two variables (Nursyahidah et al., 2018); in learning fractions via pattern that are familiar to students after the second grade (Sankaran, Sampath, & Sivaswamy, 2009). In learning problems solving among senior high school students (Widada et al., 2019). The previous literature and the present study confirm that ethomathemarics could be utilized for learning the different mathematical topics.

The findings related to students’ processes in learning the congruence theorems through working with ornamentations indicated that the students succeeded in formulating the congruent triangles theorems and the proofing claims based on the congruent triangles. These findings can be explained by two main factors. First, the ornamentations were related to the culture of the students, and thus it influenced positively motivation for learning and inquiring, which is supported in different studies (e.g. Amit & Abu Qouder, 2015; Massarwe et al., 2010). This increase of students’ motivation affected positively their learning strategies, cognitive resources and learning achievement (Pekrun & Perry, 2002). The second factor is related to concrete and abstract activities in the ornamentation and different features of the ornamentation, such as repetitive and symmetry of shapes. Thus the concrete and abstract activities that design in the current study considered as a supportive learning environment for students, it enables students to physically interact with objects through feeling, seeing, and touching (Carbonneau & Marley, 2012). The students’ success in proof questions in the questionnaire B can be connected to their use of geometric transformations. Our findings support those of Fan, Qi, Liu, Wang and Lin’s (2017) who reported that geometric transformations can help improve students’ ability to solve geometric proof problems.

For conclusion, we found that ethnomathematics activities are fruitful for students’ concrete and abstract learning processes of geometric concepts and relations, which emphasizes the importance of designing ethnomathematics activities for the various mathematical topics. We join the call for designing mathematics curriculum related to students' ethno-background, to which will help build a bridge between the students’ background knowledge and formal mathematics.

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