Culture, Utility and Economic Growth.
The Case of Mexico's Indigenous People

Abstract

This article presents a new way of approaching the relationship between culture and economic theory. First, a review of the literature on the relationship between culture and economics is presented (more than 200 articles were reviewed). Subsequently, a subset of what is understood as culture is linked to the utility function and/or with constraints. Then, a rigorous model of economic growth is developed by means of this linkage, which is used to analyze the poverty of Mexico's indigenous communities, complemented by empirical and historical evidence of this relationship.

I. Introduction

In Mexico, approximately 6% of the population aged 5 years or older are indigenous, according to the 2010 Census[1]. This represents 6.3 million people categorized as such by the language criterion, which implies that they speak and use one of the recognized indigenous languages. The history of indigenous Mexicans has made them one of the poorest groups of Mexicans for hundreds of years. The question to be answered is: if, in the long term, technology is a public good and institutions serve all of Mexico's people equally, why have these communities been the poorest, with per capita economic growth lower than all other Mexicans? Everything indicates that the cause of their poverty does not seem to lie in the economic realm, but rather in the cultural realm.

However, to analyze culture and its relationship with economic behavior, it is necessary to clearly define what is understood as culture and how to introduce it in an economic growth model. Therefore, an evaluation of the literature on the relationship between economics and culture was first conducted. Presented next and based on the evaluation results, is a method of incorporating culture in the utility function and/or its constraints. Subsequently, an

[1] Expanded Population and Housing Census 2010, 10% sample of the Population and Housing Census, 2010 Census.

economic growth model is presented in which culture is incorporated throughout the utility function, such that it rigorously explains the systemic poverty of Mexican indigenous communities.

II. Culture and economic theory

A total of 234 academic articles were reviewed, selected according to the following criteria: the title contains the words ‘culture’ or ‘cultural’; the article was published between 2000 and August 2017; the article was published in one of the 50 most-cited economics journals, according to Thompson (2016), or in six other peer-reviewed publications that repeatedly presented the search term[2]. The following results were obtained:

Commented [Trans1]: Please note that this citation is not included in the references that were provided in a separate document.

Of all the articles reviewed, 57 have an explicit definition of culture - 18 different definitions were used, some of them similar. Nineteen articles are devoted to *Economics of the Arts and Literature*; 44 have a compound definition (such as *Ecological Culture*, *Culture of Crime*, etc.), and the remaining 112 articles have no explicit definition. Of these, 66 are empirical in nature and the vast majority of them use the term ‘culture’ to mean nationality, language, religion or ethnicity. The remaining 48 articles (conceptual and theoretical) use the word ‘culture’ more loosely than the empirical models because they had no need to measure it. They use culture as a synonym for *beliefs*, *values*, *norms*, *inheritance of ancestors*, etc., and in some cases the term also includes places and environments.

Lastly, but very importantly: 140 articles use some cultural components as exogenous factors and examine their influence on an economic issue, with nationality being the most frequently used meaning of culture; 29 articles analyze components of culture from different perspectives, focusing mainly on institutional or corporate culture, or on the measurement of culture; 25 articles explore changes in a specific cultural component for very diverse topics, such as altruism, family characteristics, etc.; 12 articles use components of culture as endogenous variables, of which seven are about game theory and the other five through some

[2] The additional publications are: *Journal of Economic Issues*, *Journal of Socio-Economics*, *European Economic Review*, *Journal of Economic Behavior & Organization* and *Journal of Development*. In all, there are 236 articles that meet the criteria used, but two were excluded for using the word ‘culture’ in a proper name (Cultural Revolution) and as a type of technology (tissue culture).

kind of optimization; the remaining nine articles analyze the relationship between culture and ecosystems, mainly to determine how to value the ecosystem in social terms. The above summary demonstrates that 65% of the articles address ways in which exogenous cultural components condition various economic topics.

The conclusions of the analysis are that a) the term ‘culture’ is used to explain circumstances that economic theory cannot address, primarily the differences that can be seen in the economic behaviors of individuals and companies that are assumed to be equal (at least theoretically) under different environments and situations such as different regions, ethnic groups, nationalities, religions, beliefs, values and more. For this reason, there is great interest in the subject. Most analyses use some component of culture as an exogenous variable to explain its relationship with some economic issue; and b) the analysis also reveals that this term does not have a widely recognized definition, so the term is used with different definitions or without any definition at all, sometimes using culture as a ‘black box’ to explain divergent economic behaviors. Therefore, to examine the relationship between culture and economics in depth, it is necessary to clearly define what is meant by culture. But above all, it is necessary to incorporate the term in formal models.

III. Culture

One way of removing ambiguity from the term ‘culture’ will be to retain the only property shared by all definitions. In fact, the only distinction on which all the articles analyzed agree, is that culture is composed of factors shared by groups of individuals; that is, they are social factors. In this article, we will restrict social factors to only those that influence individuals through their preferences (utility function) and/or constraints. This will eliminate generality from the ‘culture’ term, since according to the results of the bibliographic analysis, not all culture influences the utility function and/or the constraints. The advantage is that it provides a way of incorporating culture into formal models.

III. A. Culture and the utility function.

Generally speaking, the decisions of individuals are influenced by one or more factors. In that sense, an influencing factor (IF) is defined as any circumstance identifiable by the individual that affects his order of preferences, but that does not affect the basic assumptions about this order. Going forward, by basic assumptions we mean the following: transitivity, completeness, continuity, convexity and strict monotonicity of preferences. Having IF retain these basic assumptions is a requirement of rationality, regardless of the underlying IF philosophy.

For example, if an IF affects an individual (say, by some divine revelation) who then goes from being a consumer to being a hermit, the individual's preferences will change substantially. But this would not require changing any of the assumptions, since comparisons between baskets of goods will no longer be made using the baskets from the individual's consumerist period. An example of a weakly accepted IF would be a society that shares the belief that "humans should not eat red meat because they do not have claws". This would be an IF, but there would be no manner in which decisions based on this belief would fail to meet the basic assumptions, even though the belief does not seem to have any basis. People would simply stop consuming red meat and would replace it with something else.

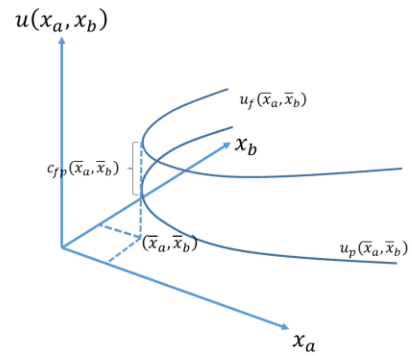
The Factorial Utility Function (FUF) relies on the comparison of intra-individual utilities (that is, pertaining to the same individual), which is mainly used in intertemporal utility functions, as well as between asset categories. We will add one more in this article - the comparison of utilities according to influencing factors.

The idea of intra-comparability for one individual according to influencing factors can easily be explained by first studying a single IF on x consumption baskets. Let $u_p(x)$ be the primitive utility function (without the IF), and let $u_f(x)$ be the final utility function (with the IF); both functions being continuous and quasi-concave, since the assumptions continue to be fulfilled. Given that the ranges of both functions lie on the real number line, then the influence of the factor can be defined as $c_{pf}(x) = u_f(x) - u_p(x)$, where $c_{pf}(x)$ is a continuous function (since both utility functions are continuous) that identifies the influence of the factor in terms of utility. Since there are two utility functions, the primitive and the final, then two indifference curves traverse each $\overline{x} \in x$ basket. The comparison between the two, $c_{pf}(\overline{x})$, is the vertical distance between both functions at point $\overline{x}$ (see Figure 1 for the two-goods case). The utility function that considers the IF will be:

(1) $$u_f(x) = u_p(x) + c_{pf}(x)$$

where $c_{pf}(x) = u_f(x) - u_p(x)$, which will be expressed as $\Delta u_1(x)$. We will call this function the Factorial Utility Function (FUF).

Figure 1



Let us now assume two IFs. When we have more than one factor, there can be many types of relationships between them. For example, in some cases IF(1) may be incompatible with IF(2). Or they may be interdependent, in which case one factor would affect the influence of the other factor on the utility function, and vice versa. When analyzing a factor, in order to determine whether there are other factors, the variable $Z(i)$ is defined, with $i = 1{,}2$, where $Z(1)$ indicates that only the first factor is active and $Z(2)$ that both factors are active. Since the other factor is active, we define the influence of each factor using equation (1) as: $u_i(Z(2), x) = u_p(x) + c_{ip}(Z(2), x)$ with $i = 1{,}2$. Note that $u_1(Z(1), x)$ is not necessarily equal to $u_1(Z(2), x)$, since factor 2 may have some relationship with the influence of the first factor. To add the influence of the factors, it is important to bear in mind that the utility of each is the primitive plus the net increase by the factor, as defined in equation (1). In this sense, to obtain the total utility, we must add the primitive utility to the net increase in utility for each influencing factor, given the influence of the other factor, that is: $u_f(Z(2), x) = u_p(x) + c_{1p}(Z(2), x) + c_{2p}(Z(2), x)$. Using the utility definitions for each factor we have:

$$(2) \qquad u_F(Z(2), x) = u_p(x) + \left[u_1(Z(2), x) - u_p(x)\right] + \left[u_2(Z(2), x) - u_p(x)\right]$$

which would be the factorial utility for two influencing factors. The formal derivation for the n factors case is included in the appendix.

It is important to note that the influencing factors are exogenous for the individual, although their effects on utility may be different for each person. In this sense, the individual is not allowed to decide on the effect of these factors endogenously; that is, cognitive dissonance is not allowed, which for some authors is a sign of irrationality (Lester and Yangb, 2009).

The advantage of the FUF is that it is possible a) to separate or break down the utility function to show the influence of the factors, b) to rigorously analyze this influence, and c) that when carrying out these two tasks, the axioms and assumptions of the utility function are fulfilled. In the Appendix, we demonstrate that the only way to do this is through the FUF.

Obviously, the form of the FUF will depend on the characteristics of the IFs. For example, if IFs only affect a group of goods that are not in the primitive utility function, or if the affected goods are strongly independent of the unaffected ones, then those IFs can be represented by an additive function. If the IFs can be represented by a utility function similar to the primitive function, such as the Cobb-Douglas function, the analysis would be simpler than if they were different utility functions. But a very important point is that, when constructing the FUF, it is necessary to clearly specify what is meant by culture. This prevents the definition of culture from becoming a black box, as established in the bibliographic analysis in Section II.

III. B. Utility function versus constraint

So far, we have assumed that the IF or IFs have been internalized by the individual, so they appear in their utility function. The other extreme would be the influencing factors that are followed for some expected net gain or because of some kind of constraint. The value *do not* *steal* can cause many people not to do so out of conviction, because respecting the property of others is a moral imperative; as such, stealing would imply a decrease in utility. However, other people will evaluate stealing according to the probability of being caught and punished, thus following a function of expected utility. And, of course, there is the intermediate case, where people decide according to their own moral values and the probability of being caught. Knowing when an IF acts as a part of the utility function (is internalized) or when it is a constraint can be very important, because this distinction affects the indirect utility function differently. Given the statements of this section, the poverty situation of Mexican indigenous people will be analyzed using a FUF.

IV. Indigenous communities in Mexico

The poverty of indigenous societies in Mexico is evident: more than 50% of the municipalities with majority indigenous populations face the highest levels of marginalization, according to Mexico's 2010 Marginalization Index; of these municipalities, 93% have at least a high level of marginalization. Of the Mexican municipalities without majority indigenous populations, the levels of marginalization are 14% (highest) and 47% (high). Even though indigenous communities have faced the same economic and institutional situations as the non-indigenous population for hundreds of years, their growth has been consistently lower. What has caused that poverty?

An initial factor would be discrimination. According to the National Discrimination Survey (*Encuesta Nacional de Discriminación* - END, 2017), dark skin color engenders discrimination in Mexico, as does speaking an indigenous language (Martínez Casas *et al*, 2014, and Acharya and Barragán, 2012). However, this situation will only be briefly analyzed in the model, as we will focus on internal factors. Below are some characteristics of indigenous communities, in order to identify the IFs that affect their economic behavior. The analysis will focus on rural areas (populations of less than 25,000 inhabitants), which is where 62.1% of Mexico's indigenous population lives. This is their traditional environment, since the indigenous people who have permanently migrated elsewhere abandon their languages over time (Ordorica, *et al*, 2009, Yoshioka, 2010). In this case, they would no longer be considered indigenous according to the definition we are using in this study.

Income and consumption - The average income for indigenous persons reported in the 2010 Census was 22% lower than that of non-indigenous people in the same municipality. Likewise, no evidence was found that income among indigenous communities is more homogeneous than that of non-indigenous people, nor among the main ethnic groups. Regarding consumption, the average level of ownership of personal (movable) and real (immovable) property reported in the 2010 Census (refrigerator, stove, quality of home building materials, etc.) is lower for all assets, on average per municipality, for indigenous people than for non-indigenous people. Likewise, there is no evidence of any IF that affects their consumption. They consume less because they have lower incomes than non-indigenous people, not because of cultural issues[3].

TABLE 1. TYPE OF JOB AND WORK ACTIVITY IN RURAL AREAS OF MEXICO (PROPORTIONS)

TYPE OF JOB	INDIGENOUS PEOPLE	NON INDIGENOUS	WORK ACTIVITY	INDIGENOUS PEOPLE	NON INDIGENOUS
Employee	15.6	38.8	Maize and beans	41.4	11.5
Laborer or Assistant	20.9	24.3	Other agriculture	23.4	23.5
Employer	0.8	2.0	Construction	7.8	10.1
Self-employed	47.2	27.5	Craftsmen	3.4	1.9
Unpaid relative	15.4	6.4	Shopkeepers	3.4	6.4

Source: Census 2010

Work — Agriculture is the main occupation of the indigenous communities, which they usually combine with other secondary tasks. According to ethnographic sources[4], agriculture

[3] A case in which IFs exist for consumption is that of the Mennonites, and especially the Amish communities, where their IFs do not allow them to use certain goods, such as cell phones, automobiles, televisions, etc.
[4] By ethnographic sources, we refer to *Comisión Nacional para el Desarrollo de los Pueblos Indígenas* - CDI (2013), and Millan and Valle (2003), which are several ethnographic studies of the main ethnic groups in Mexico.

and artisan production are generally carried out without economic specialization using and traditional technology[5].

Regarding economic specialization, 47.2% of the economically active rural indigenous population is self-employed, and 15.4% work as unpaid family members. These proportions are much higher than those of the non-indigenous population (see Table 1), and represent work that produces goods not part of a production process. That is, more than half of the indigenous economically active population (EAP) does not specialize. In contrast, there is practically no evidence of businesses comprised of indigenous people that incorporate factory discipline (Clark, 1994), in the sense that some member of the community, as an entrepreneur seeking to maximize their own economic benefit, decides how their employees should do things, as well as when and where.

Traditional technology is difficult to determine empirically, but its use by indigenous communities is recognized in ethnographic texts, such as corn and beans produced with traditional technology. In addition, the proportion of people engaged in this activity is much higher in the indigenous sector than in the non-indigenous sector (see Table 1), which is due to a combination of cultural factors and personal consumption.

However, the example of artisanal goods is where the absence of specialization and the use of traditional technology is best appreciated. Although only this is the principal activity for only 3.4% of indigenous people, it is a secondary activity for most of them. Ethnographic sources reveal that the artisanal production is carried out in family workshops, using traditional technology and without economic specialization. Likewise, communities tend to specialize in a certain craft, which is more due to cultural issues than to economic efficiency, since each family produces them independently.

[5] Traditional technology could incorporate economic specialization, so to avoid confusion, we will define it as the use of individual processes and physical capital similar or equal to that used by their parents, regardless of whether there is economic specialization or not.

Influencing factors — Considering the above, two IFs can be identified: the first is that rural indigenous communities manifest a tendency to work with no economic specialization, or similarly, they demonstrate an adversity to factory discipline. The second IF is that they reveal a preference for using traditional technology. However, to understand the logic of these IFs requires an explanation of their traditional government structure.

Government — The system of government related to their culture, called the system of rotating positions (Chance and Taylor, 1985), is a hierarchical structure of positions for the civil and religious administration of the community in which individuals are chosen annually to perform various functions. The most important positions are full-time, but are usually not remunerated, so there are clear incentives not to keep occupying the position. However, those individuals who have fulfilled the duties entrusted to them as adults, become people with much influence on community decisions, and are the candidates for council membership. In some cases, this is the highest authority for social, legal and economic issues, along with the general assembly constituted by most of the community[6].

There are three important issues relative to the type of government and family relationships: a) if the individuals holding the positions are chosen for having more income than others, this system would foster economic equality; b) these communities are gerontocratic, as demonstrated by the age of the council members who have the greatest influence on communal decisions; and c) there is a high level of reciprocity within indigenous communities (Millán and Valle, 2003). Presented below is the hypothesis for the reasons that led indigenous communities to demonstrate preferences for the two IFs, and the logic of the system of rotating responsibilities.

IV. A. IFs as a strategy for producing public goods

The hypothesis is as follows: a society producing the same goods with the two IFs and reciprocity, will trend towards economic equality, because in essence, each indigenous family would be equal to other families. Similarly, important decisions are shared by means

[6] The use of the system of intra- and inter-ethnic rotating positions varies widely throughout the country, and some communities have stopped using it.

of the general assembly and carried out through the hierarchical system of rotating responsibilities. A society functioning under this system would be very efficient in producing public goods. In the case of the indigenous Mexicans, the movement towards equality was initially driven by exogenous factors, but later it was used as a strategy for defending their lands (that is, for the production of a public good) that lasted more than 250 years. Over time, however, the IFs and rotating responsibilities became an essential part of their culture. Before presenting the historical evidence, it is necessary to clarify the relationship between the efficient provision of a public good, and the IFs and system of government.

As is known, market solutions are not efficient for the optimal production of public goods. However, families in indigenous communities, producing the same goods under a system with IFs and reciprocity, theoretically would reach economic equality. In that case, voluntary contributions would not lead to a Pareto optimum (Cornes and Sandler, 1996), but it would be the same for everyone. In this manner, if production of a certain amount of the public good is desired, its cost is simply calculated and divided equally among the members. Again, the result would not necessarily be a Pareto optimum, because the preferences are not the same. But when public goods are produced, individual preferences are subordinated to the collective interest since decision-making is done in the general assembly, usually under a face-to-face voting system where individuals monitor each other. Instead of or in addition to the general assembly, there may be a system of government directed by a council of elders appointed for their elevated contributions to the production of public goods. All this would minimize the problem of the *free rider*, and the administration and execution of the public good would be greatly facilitated.

The hypothesis does not purport strict economic equality among individuals, but requires that at least they have tended towards economic equality when the production of public goods was of great importance. Currently, there is no observable economic equality or homogeneity among ethnic groups, but IFs, the system of rotating duties, and lower income and consumption can indeed be observed. Given the above, the historical evidence for the development of this strategy to produce public goods is presented below. The context of the analysis is the central and southern regions of what was formerly called New Spain, and that currently is part of Mexico and Guatemala - a region known as Mesoamerica - in the period after the Spanish conquest of that territory.

IV. B. The Indian villages

In the mid-sixteenth century, during New Spain's so-called Colonial period (1535-1810), the Indian villages [7] were created. These were communities composed of indigenous families, called *maceguales*, that were self-administered and had a governor, also indigenous, called a *cacique* who acted as an intermediary between the Spanish and the indigenous population, especially for tax collection. Each *macegual* family had its own plot of land with usufruct rights that was not private property, but was inalienable and hereditary as long as the land was cultivated; the villages also had communal lands.

The Indian villages were communities whose main economic activity, in addition to the sustenance of its inhabitants, was the production of public goods and for tax payment. Taxes were paid in-kind, and with money and labor, to the Spanish crown, to businesses and to the Catholic Church within and beyond the communities.

From the time they were first implemented until the mid-17th century, there were several circumstances that led the indigenous peoples towards economic homogenization: a) these communities were treated as a group. For example, the payment of taxes was calculated according to the number of village inhabitants, but it was a collective payment made on everyone's behalf by the *cacique*, and was not done by each individual *macegual*; b) the Catholic Church was heavily involved in the village's economic life, and many clergy had a vision of communal organization closely related to first-century Christian communities. Evidence of this is Vasco de Quiroga, bishop of Michoacán, who established several villages whose rules were based on Thomas More's *Utopia*, where everyone was economically equal, although with Catholic variations (Lynch 2012), and; there were no economic incentives for private enterprise within the Indian villages, so many indigenous people migrated away from the villages in search of better opportunities, even though there was significant discrimination against them.

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[7] The discussion on this topic is very brief and does not address many details. A complete explanation of the Indian villages in central Mexico can be found in Gibson (1964) and Lockhart (1992).

Thus, the payment of community taxes, the intervention of the church and the lack of economic incentives forced these communities towards economies of equality. But in addition, when facing strong forces to deprive them of their lands - their main sustenance - the indigenous people adopted this economic equality as a strategy to mount an effective defense. We delve deeper into this point in the following paragraphs.

Public goods — At that time there were two situations that encouraged the production of public goods: religion and legal defense. For various reasons, indigenous people adopted a high level of religiosity, with many physical manifestations (Israel, 1975, Lynch 2012), and the funding of all religious activities was the responsibility of the *maceguales*. But their principal public good was their legal defense mechanism, since a military defense was not a viable option[8]. The Indian villages faced many problems, primarily the always present possibility of losing their lands due to legal or illegal actions by settlers and agricultural enterprises, as well as exploitation by their *caciques*, excessive tax collection and territorial conflicts with other communities. The Indian villages made intensive used of the judicial system, and all its members participated directly or indirectly in it. It is important to note that individually, the *macegual* was seen as a "miserable" citizen (Ruiz Medrano, 2010), but collectively, they gained much strength. Obviously, all this resulted in high costs that were borne by the community, and many trials were lost precisely because the village could not afford the cost.

However, the incentives changed over time. The need for communal labor for large buildings (mainly churches and convents) decreased, and migration became a good option, mainly because the business climate improved. In addition, many of indigenous people learned various trades, and people from the larger Indian villages scattered and integrated with the rest of the economy. But there were also indigenous groups that decided to form new, smaller and more remote Indian villages (Lockhart, 1992), and many of the smaller villages that were far from economic centers continued with few changes. According to Gibson (1964), during

[8] The full history of the legal defense of indigenous peoples can be found in Ruiz Medrano (2010), and for a detailed explanation of these trials and their participants, see Kellogg (1995).

the 18th and early 19th centuries, Indian villages increased in number but decreased in size and economic significance.

The *macegualles* still living in the Indian villages maintained their lifestyle, because the threats against their native lands never disappeared. Their economic life continued without economic specialization and with the same technology. Their alternative was to migrate and integrate into the broader economy, but they did not, revealing their affinity for that way of life. As time passed, those that did leave their Indian villages and the villages that were absorbed by cities stopped speaking their native languages (Chance, 1976). As such, it is very likely that the indigenous communities existing today descended from the Indian villages that survived.

From independence to the present — After Mexico's independence, the Indian villages disappeared in a legal sense. The liberal government promoted private property ownership and the use of modern technology. However, many of the villages successfully refused to change because of their ability to defend themselves (Buve 1992). At that time, there is already evidence of IFs in their culture. Foremost was their very refusal to change and their opposition to private property ownership, as well as their resistance to technological change, which had already become a type of technology with factory discipline imported from the English industrial revolution. But there were also declarations by politicians and liberal intellectuals accusing the indigenous peoples of being a burden to modernity, of *despising free enterprise and new technology*, and other similar characteristics (González, 1996).

Due to their resistance to change, the attacks on their lands resumed and as a result, the villages reinforced their communal defense, taking advantage of the circumstances to strengthen their own internal structures such as the system of rotating responsibility, which was consolidated at that time, according to Chance and Taylor (1985).

In the early 1930s, various governments began to distribute land to the peasants, including to the Indian villages that had lost their lands to large landowners towards the end of the 19th century and earlier in the 20th century. The distribution imparted usufruct land rights, not private property, but inalienable and inheritable (de Grammont *et al*, 2009). For those who could demonstrate their possession of a property, which was true for most Indian villages, these were designated as communal lands. Currently, indigenous communities maintain this type of land tenure, much as they did in the colonial period.

There is no direct evidence that the present-day indigenous communities are the direct descendants of the Indian villages, but there is evidence that the indigenous communities that have survived to date are ones that adopted strategies for the production of public goods. This does not mean that all the communities that adopted these strategies have survived, but rather that those that did survive definitely followed these strategies. So, in this sense, the public goods production strategy was successful.

IV. C. From strategy to culture

The persistence of this way of life over many generations in the Indian villages, became entrenched as part of their culture, and generated the IFs. This idea is supported by two points: a) the defense strategy has been absorbed into their traditions, customs, taboos and myths, losing much of its main purpose[9]; and b) the indigenous communities have a very strong connection with the past, because as already mentioned, they are gerontocratic societies with only one method of cultural transmission – vertically, from father to son[10] – that ensures the intergenerational cultural transmission of the IFs[11]. Now that we understand the IFs of the indigenous communities and their origins, we can present the economic growth model.

V. Economic growth model

In this section, we present an economic growth model for a large society with economic specialization and incentives for the development of new technologies. Subsequently, a price-taking community within that society is introduced, along with a function of factorial utility with the two IFs.

[9] However, Ruiz Medrano (2010) presents cases of indigenous communities today that continue to pursue legal processes to defend their lands.
[10] The models of cultural transmission assume that the transfer can be vertical (from parents to children), horizontal (between peers) and oblique (television, social networks, etc.). See Bisin and Verdier (2001).
[11] For example, in the Bisin and Verdier (2001) cultural transmission model, if only vertical transmission is considered, the model would indicate that the probability of transmission from parents to children is 100%.

V. A. Basic model

The basic assumptions of the model are: overlapping generations, absence of physical capital, perfect competition and information, zero transaction costs as well as perfect labor mobility, and where inquiry is possible since accumulated knowledge generates economic opportunity. Society S consists of a large number of families, each with a father and a son, both of whom live for two periods. In the first period, they are young dependents of their parents, committed to educating themselves to enter the workforce and also to developing new technology. This is the period in which knowledge expansion happens. No further detail on the incentives to produce new technology is offered here, since they are not our main interest and this will facilitate the analysis of the factors of social influence. In the second period, the youths have become parents and apply their knowledge at work. They eventually die without leaving an inheritance, so they do not need to save. As usual, the static equilibrium state is first determined and then the generations are added.

Static equilibrium — At the beginning of the second period, each father $i \in S$ is granted a unit of work ($L_i = 1$) measured in time and perfectly divisible (that is, he can devote time to different activities), thereby receiving one or more salaries depending on the number of jobs held. The father's income is used to support himself and his child. For the static economy, we omit the time subscript.

Demand: Following the Grossman and Helpman model (1991), the utility function for the representative family is a CES-type model (Dixit-Stiglitz) that contains a large number of comparable substitute goods:

$$u = \left[\sum_{i=1}^{N+Q} x_i^{\alpha}\right]^{\frac{1}{\alpha}} \tag{3}$$

where x_i is the consumption of good i, with $i = 1,2,\cdots,N+Q$, and $0 < \alpha < 1$. The number of goods to consume is $N+Q$, where N represents existing goods and Q represents new goods developed at the end of the previous period. The family maximization program will be:

$$(4) \qquad max\Omega = \left[\sum_{i=1}^{N+Q} x_i^{\alpha}\right]^{\frac{1}{\alpha}} - \mu\left[\sum_{i=1}^{N+Q} p_i x_i - E\right]$$

where p_i is the price of good x_i, and E is the total spending. For simplicity, we will assume equal prices from this point on, so that demand for each good has the following form:

$$(5) \qquad x_i = \frac{E}{p}\frac{1}{(N+Q\,)}$$

Production: Businesses do not own property nor are there transaction costs, so workers only join together to form companies and produce a certain good, seeking to maximize gross profit[12]. It is assumed that good 1 was the first to be invented, then 2 and so on. The first goods are the simplest to produce. In order to produce good 1, only one type of work is needed. For the second, two types of specialized workers are needed, and so on up to $N+Q$ consumer goods, indicating that the latest goods require greater specialization. The production function of any company (we omit the company subscript) in the industry for good h, with $h = 1,2,\cdots,N+Q$ will be:

$$(6) \qquad x_h = \emptyset_h\, min[L_1, L_2, \cdots, L_h]$$

where h will indicate, in addition to the industry, the number of specialties needed to produce the good. The $\emptyset_h$ parameter refers to the industry's technology updates, which are rented to companies that develop them, where the total cost of the update is $\gamma_h x_h$, a function of production where γ_h is the unit price of the update. This means that although technology is free, its application has a cost proportional to the level of production. The fact that businesses use the latest update means that they are on the Production-Possibility Frontier (PPF). In this

[12] Maximizing benefits per worker or maximizing total benefits are equal, since the production function presents constant returns to scale.

sense, the cost function for any company in industry h will be: $CT_h = \sum_{j=1}^{h} w_{jh} L_{jh} + \gamma_h x_h$, where w_{jh} is the wages of specialty j in industry h, with $j = 1,2,\cdots,h$.

Given production function (6), in the optimum $L_{ih} = L_{jh}$, for all $i,j = 1,2,\cdots,h$, so the production function of industry h will be:

(7) $$x_h = \emptyset_h \, L_h^d$$

where L_h^d is the total work contracted for the period. In this sense, for the same production quantity, the work demand for good $h = 1$ will be $L_1^d = L_{1h}$; for good $h = 2$, the work demand will be $L_2^d = (1/2)(L_{1h} + L_{2h})$; and so on until good $h = N + Q$ whose demand will be $L_j^d = (1/(N+Q))\left(\sum_{k=1}^{N+Q} L_{kh}\right)$. With this, L_h^d will represent the total work demand for industry h. Note that, for the same quantity of product, $L_i^d = L_j^d$ for all of industry $i,j = 1,2,\cdots,N+Q$, but the difference is that industry i requires i specializations and industry j requires j specializations. With perfect competition and information, and due to the form of the production function, wages will be the same for all specialties. With the additional assumption that the cost of industry h updates is the same for all businesses, and substituting the cost function, we have:

$$CT_h = \sum_{j=1}^{h} w_h L_{ih} + \gamma x_h = w L_h^d + \gamma_h x_h = w_h \left(\frac{x_h}{\emptyset_h}\right) + \gamma_h x_h = \left(\frac{w_h}{\emptyset_h} + \gamma_h\right) x_h$$

where the updates will be purchased if, for the same level of production and wage, the total cost is lower, that is:

(8) $$\gamma_{ht} - \gamma_{ht-1} \leq w_h \left(\frac{\emptyset_{ht} - \emptyset_{ht-1}}{\emptyset_{ht}\emptyset_{ht-1}}\right)$$

Furthermore, the production function has constant returns to scale, so for each industry h there will be an indeterminate number of businesses. The profit for any of those companies, because of perfect competition, will be zero, which implies that price is equal to marginal cost, that is $p_h = [(w_h/\emptyset_h) + \gamma_h]$ or rather $w_h = \emptyset_h(p_h - \gamma_h)$.

Due to the production function, all industries are equal, regardless of their degree of specialization, so if demand is the same for each good, according to function (5) prices and production will therefore be the same in any industry h.

New updates: The production function of technology updates for each business will be: $\emptyset = min(L_{kID}/A)$, with $k = 1,2,\cdots,K$, where K are the specializations. As the production function has constant returns to scale, we can then refer to the research and development (R&D) industry, so that $L_{ID} = \sum_{j=1}^{K} L_{kID}$ will be the total R&D work time for the entire period, whose L_{ID} workers are devoted to keeping the companies updated. Updated businesses will maximize their production, subject to demand, or rather subject to equation (8), in which case their profits will also be zero, and price will equal marginal cost, so that $\gamma = Aw$. Substituting the previous equality in the wage and price equations, we have $(w/p) = [\emptyset/(1 + \emptyset A)]$.

Youths: The youths are focused on learning in order to specialize in the $N + Q$ specialties and/or in research and development (R&D). The youths focused on R&D learn the knowledge for the period (K_{t-1}), and one sector is dedicated to researching all the specializations required to update the technology of the $N + Q$ goods, and another sector develops the new products indicated by Q. The number of people dedicated to specializations or to R&D will depend on K_{t-1} and on short-term opportunities, but the profits of all companies will be zero in the long run. The implementation of the updates is the same for existing companies as well as for new products. We assume that the costs of learning, specialization and research are equal, measured in terms of their consumption, and that their parents pay for it, so it does not appear explicitly.

Static equilibrium: The father's income will come from the work he does in any of the businesses in the $N + Q + 1$ industries, so that for all $i \in S$, then $E_i = w$ since $L_i = 1$. In this scenario, the total payment for work will be the sum of the wages for the number of workers in the $N + Q + 1$ industries, that is:

$$\sum_{i=1}^{N+Q} wL_i^d + wL_{ID}^d = w\left[\sum_{i=1}^{N+Q} L_i^d + L_{ID}^d\right] = w\left[L^d + L_{ID}^d\right] = wL$$

where L is the number of parents, which is also the total labor supply, which is fixed. The equation above implies that the labor market is in equilibrium. Since all individuals earn the same wages and the goods have the same value, the total demand of the $N + Q$ goods can be expressed as $L\sum_{i=1}^{N+Q} p_i x_i = LE = wL$. To determine equilibrium in each industry, we equate total demand for each good (5) with its supply (11), so that $[EL/(N + Q)p] = \emptyset\, L_i^d$. By substituting the price and clearing L_i^d, while taking into account that $E = w$ and that $(N + Q)L_i^d = L^d$, we have $L = (1 + \emptyset A)L^d$. As such, the production of consumer goods, with prices equal to p and the previous equation, is:

$$\sum_{i=1}^{N+Q} p_i x_i = pL^d\emptyset = \frac{(1 + \emptyset A)}{\emptyset} wL^d\emptyset = w(1 + \emptyset A)L^d = wL$$

which implies that total production is equal to total expenditure and is equal to the income of all of the families, indicating that the market for goods is in equilibrium. The indirect utility function for the representative family will be:

(9) $$V = (N + Q)^{\frac{1-\alpha}{\alpha}}\left(\frac{w}{p}\right) = (N + Q)^{\frac{1-\alpha}{\alpha}}\left(\frac{\emptyset}{1-A\emptyset}\right)$$

which is increasing both in the number of goods and in the updates, and which will be the way to measure economic growth.

Dynamic equilibrium — The dynamic equilibrium, as the model is proposed, will simply be the transfer of knowledge capital K_t, which will depend on capital in the previous period, K_{t-1} and the level of research. This will be a function of the possibility of both updates (equation 8) and new products.

V. B. Influencing factors on the factorial utility function for the indigenous communities
We will now assume that within society (S) another small, price-taking community exists that will be represented as (I).

Building the FUF for the indigenous people case - The FUF that will be used to represent the preferences of a typical individual in society I, is based on equation (2). To this we will add the assumptions derived from the specific case we are analyzing. As already mentioned, the IFs will be about the aversion to specialization and the preference for traditional technology. The first thing to consider for building the FUF is that empirical evidence shows that IFs do not affect the $N + Q$ consumer goods, so it is possible to represent the increase in utility as an additive function. That is, for any factor j, $\Delta u_j = u_j - u_p = u_p + f(\cdot) - u_p = f(\cdot)$, where $f(\cdot)$ would be a function that represents the influence of factor j. The first IF is the *aversion to economic specialization*, which indicates that all parents in community I will prefer less specialization; that is, $L_{h-1} \succ L_h$, where h is the industry, but also the number of specialties in that industry. The aversion to specialization can be represented by a function with decreasing specialization, such as $\Delta u_1 = Z\left[A_h - \frac{h^2}{2}\right]$, where Z is the factor's degree of importance, or the degree of traditionalism of society I , and A_h is a parameter such that, if h is small, the utility will be positive, but if h is large, the utility would be negative.

The second factor is the *aversion to updated technology*. The net increase in utility is defined as: $\Delta u_\emptyset = Z\left[A_\emptyset - \frac{\emptyset_I^2}{2}\right]$, where $\emptyset_I$ is the technology of society I, Z is the degree of traditionalism, and $A_\emptyset$ is a parameter such that, for small values of $\emptyset_I$ the utility will be positive, and for high values of the update, the utility would be negative. Likewise, all updates $\emptyset_{t-j}$ that produce net benefits less than the latest update $\emptyset_t$ will have a price of zero, since their demand will be zero. This implies that if a technology with zero price is used, then $\emptyset_I \leq$

(w/p). Lastly, we assume that if a company in industry $h > 1$ (with specialization) is formed by individuals from community I, they may choose the technology used. But, if in that company there is one or more individuals from society S, the technology will have the latest updates. Adding in the IFs described, according to function (2), the FUF would be as follows:

$$u_{If} = \left[\sum_{i=1}^{n} x_i^{\alpha}\right]^{\frac{1}{\alpha}} + Z\left[A - \frac{h^2}{2} - \frac{\emptyset_I^2}{2}\right] \qquad (10)$$

where $A = A_L + A_\emptyset$. Given that IFs do not change the marginal rate of substitution of consumer goods, we can substitute indirect utility function (9) in function (10), thus yielding the maximization program:

$$\max_{L,\emptyset_I} \mathrm{V} = B\emptyset_I + Z\left[A - \frac{h^2}{2} - \frac{\emptyset_I^2}{2}\right] \qquad (11)$$

where $B = (N+Q)^{\frac{1-\alpha}{\alpha}}$. The result is that society I will be entirely dedicated to producing $h = 1$, which is possible given our assumption that community I is small compared to S, so its total supply of good $h = 1$ will not exceed demand. This result is due to the fact that the individuals of society I prefer to produce without specialization and the individuals of society S are indifferent regarding the type of productive activity. Because of competition, the price of $h = 1$ can not be greater than p, so any conditioning of income by the IFs would go directly to wages. By maximizing (11) with regard to $\emptyset_I$, the result will be a technology $\emptyset_I^* = (B/Z)$ with an upper limit given by $\emptyset_I^* \leq (w/p)$. Substituting the previous result in (11), we have the indirect function of utility of the members of society I:

$$V_I = \frac{1}{2}\frac{B^2}{Z} + Z\left(A - \frac{1}{2}\right) \qquad (12)$$

that will increase due to the existence of more products, but without the updates of society S, because price decreases due to technology updates are compensated by decreases in the price of $h = 1$, which is the good that they produce and they sell.

Both static and dynamic equilibrium are guaranteed since the only difference with the model with no indigenous community is that income will be lower for the members of I, since $\emptyset_I^* < (w/p)$ and there will be more people dedicated to producing that good, because the parents of the community that produce it will not be in the PPF. However, as the number of goods tends to increase each period, the demand for $h = 1$ will decrease until the supply of that industry's goods produced by society I exceeds demand, in which case the members of society I will have to form companies. Likewise, given that the technology of society I is inferior to that used by society S, it will not be interested in belonging to the R&D sector since the technology updates they are going to use already exist, and that this sector needs specialized workers. Lastly, two very important points to conclude the model are the welfare analysis, in order to assess whether or not they are in a Pareto optimum, and the discrimination directed towards these societies, where it is demonstrated that this could also be examined as an IF.

Welfare - Since the IIs are internalized, that is, they are within their utility function, every father in the indigenous society will be maximizing his utility subject to prices and the quantity of his labor. That is, fathers in I are producing $h = 1$ and when maximizing with respect to $\emptyset_I$, no other individual is affected, so they are in a Pareto optimum. They are not in the PPF, but given their preferences, they are willing to reduce their consumption for a lifestyle that incorporates their IFs.

Discrimination - For the model, we will consider discrimination to be an IF, such that any $i \in S$ does not like to work with some $i \in I$, although he is willing to do it for higher wages. The source of the IF comes from the New Spain era, because the society of that time was structured into castes, with indigenous people on the lowest rung (Israel 1975). The discrimination IF will not be derived from an FUF. It will simply be assumed that the wages of any S people who work with I people will be $w_S^I = w(1 + \delta_S)$, and that the wages of any

I people who work with S people will be $w_I^S = w(1 - \delta_I)$, so that what is taken away from the $i \in I$ compensates for what is given to the $i \in S$.

VI. Analysis of the Model

The model can now be used to perform the analysis of the relationship between culture and economic growth that was not possible without a formal model. First, the empirical relationship between the degree of traditionalism and each ethnic group's income is shown, in order to reveal the relationship between culture and poverty. Subsequently, the degree of traditionalism Z is analyzed as an endogenous variable of the model, as is the indigenous communities' reactions to these changes.

Tradition versus income — According to the model, a decrease in the degree of traditionalism Z would imply a higher income, since $(\partial \emptyset_I / \partial Z) < 0$. The most important indigenous ethnic groups were selected to analyze this relationship, representing more than 95% of all indigenous people in Mexico. The indicator to be used to approximate the degree of traditionalism is explained next.

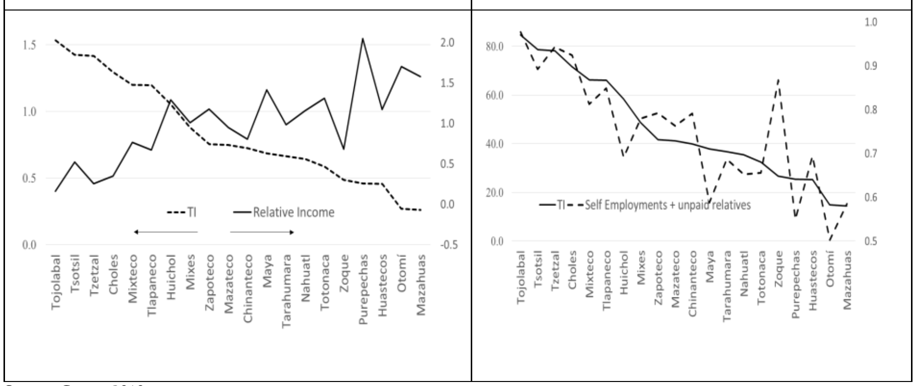
According to the classification used in this study, a person belonging to ethnic group i will also speak dialect i. In most rural, indigenous communities, the dialect is the main form of communication, so the native language is the first language that children learn. Some ethnic groups further instill the use of the dialect in children, so they don't learn Spanish until they attend a bilingual school. Other ethnic groups, however, encourage learning Spanish before entering school. The intention to use the dialect more extensively depends fundamentally on an objective - the transmission of culture.

This intention to more extensively learn the dialect will be used as an approximation of the degree of traditionalism Z. The great advantage of this indicator is that it does not depend on current income, school attendance, the fertility rate, nor on migration. It depends rather on the importance of the transmission of culture. We will use the Traditionalism Index (TI) to measure the intention of the parents and the general community to transmit the native language to children. To measure TI, we calculated the proportion of children three to six years old who speak an indigenous language and do not speak Spanish, in rural indigenous communities, by ethnicity.

The 2010 Census figures for average income by ethnic group were used to determine the average income of all the indigenous communities analyzed, which we will call I2010. As illustrated in Figure 2.A, the relationship between TI and I2010 is negative, as expected, and their correlation is 75% in absolute terms. This test was performed with other related income measures, such as the Marginality Index (MI), and the results were similar.

FIGURE 2

(A) CONSUMPTION (MI 2010) VERSUS TRADITION (TI) | (B) NON-SPECIALIZED WORK VERSUS TRADITION (TI)



Source: Census 2010.

In the same sense, there is also a relationship between TI and the absence of specialization, which is measured by the proportion of self-employed workers and unpaid relatives, by indigenous municipality and ethnic group; the correlation of 75% can be observed in Figure 2.B.

Figure 2.A indicates that the degree of traditionalism can largely explain the discrepancies in relative income between ethnic groups, and the difference in the incomes of indigenous versus non-indigenous people. As such, it can be affirmed that through the IFs, culture is a cause of poverty, which is what was proposed at the beginning of section IV.

Economic growth in indigenous societies — If the indigenous economy is in a Pareto optimum, it means that it is efficiently using its resources, according to their preferences, which include their IFs. However, this economy does not generate economic growth, because it does not develop new goods nor does it update technology, so its economic growth will depend on the growth of society S, although it will always be lower. According to the model, this difference between the economic growth of both societies depends on the degree of traditionalism (Z), which up to now we have assumed to be constant. However, the degree of traditionalism is expected to change. If we compare the results of a study on Mexican indigenous societies conducted in the 1930s (Basauri, 1940) with data from the 2010 Census, we can see that native language use has dropped among all ethnic groups, although in different ways, which is an indication that (Z) has also decreased. Therefore, we will analyze the behavior of this variable as endogenous to the model.

The degree of traditionalism as an endogenous variable — Following the IFs has an opportunity cost, which can be measured by the difference in income between society I and society S. If Z were a function of the income gap, then Z would be an endogenous variable in the model. Suppose that $Z = \theta(\emptyset_I / E_S)$, so that as the income gap widens $(E_I/\emptyset_S)$, the Z variable will decrease by a magnitude of θ. As such, this last parameter would indicate the sensitivity of society I's reaction to changes in the income gap. Substituting Z in equation (11), assuming $h = 1$, and maximizing with respect to $\emptyset_I$, we have:

OPTIMUM TECHNOLOGY	OPTIMUM DEGREE OF TRADITIONALISM
$\emptyset_I^* = \left[\frac{BE_S + \theta\left(A - \frac{1}{2}\right)}{\frac{2}{3}\theta}\right]^{\frac{1}{2}}$	$Z^* = \left[\frac{3}{2}\frac{\theta}{E_S}\left[B + \frac{\theta}{E_S}\left(A - \frac{1}{2}\right)\right]\right]^{\frac{1}{2}}$

If the income of individuals in society S (E_S) rises, then the technology of the indigenous sector will also rise and Z will fall. Note that, as θ decreases, so will the intensity of changes in the indigenous community enacted to cope with changes in E_S. In this manner, θ will reflect how reluctant ethnic groups are to modify their degree of traditionalism.

When comparing the same 1930s study with 2010 census data, it can be seen that the income gap has grown significantly, but with different characteristics among ethnic groups. In this sense, as the income gap widens, the degree of traditionalism falls, and community I will seek to increase their income. According to equation (13), the way to do this is to update technology, but there are also other ways, such as creating *ad hoc* companies, or by temporary or permanent migration. The ways in which indigenous communities react to the decrease in Z are discussed below.

Technology updates — Significant differences can be observed in the production methods of indigenous communities in the 1930s compared to their current methods. However, those differences also depend on the product type: for example, there is not much difference in textile production, since today they continue to use manual looms (Giovannini, 2015). But major differences are observed in the production of items made from wood, since they currently use relatively modern tools for this. In agriculture, the technology used now is very similar to the technology of the 1930s, but inter- and intra-ethnic differences can currently be observed regarding technology and even distribution channels.

Indigenous enterprises — Empirical evidence reveals an absence of neoclassical businesses in indigenous communities, but many of these communities have organized associations or production cooperatives with multiple objectives, be they economic, ecological, cultural, and/or to reduce distribution costs (Giovannini, 2015), and some are successful companies (Klooster and Mercado-Celis, 2016). The most common business type among indigenous communities are for agricultural production, and were formed as communities with communal land and individual plots, as mentioned at the end of section IV.B. In some cases, one can see some economic specialization in this type of business, but most workers continue to work with no economic specialization. Consequently, the production function of this type of company is different from the one we have used (equation 6), since they combine a small amount of specialized work with work without any specialization. The problem is that, since their production function is different, they cannot necessarily use the existing technological updates efficiently, and the research companies focused on updating the technology of the indigenous companies would not be profitable due to the small size of company I and to their already low income. As such, these associations and cooperatives are limited to producing a small number of product types without specialization. For example, the designs of their artisanal crafts do not tend to change, as their products are still mostly ornamental. This is largely due to the fact that updating the design of products that the indigenous people can sell in society S could be very difficult, due to the cultural differences.

Migration — We can divide migration into two types: temporary and permanent. According to the 2010 Census, most indigenous peoples who live relatively near an economic center migrate temporarily, so they combine work (without specialization) in their villages, with specialized work outside the community. This situation can be analyzed by extending the proposed model in order to recognize that these communities have economic growth due to technology updates. Assuming that members of community I have the opportunity to decide if they commit part of their work capacity to companies with specialization, then their income will be $\left({}^{w}/_{p}\right)l$, where l is the time spent working in that type of company. For their work in the community, in companies of industry $h = 1$, their income would be $\emptyset_I(1 - l)$, since their total work capacity is the combination of the two. In this case, the maximization program will be:

$$(12) \qquad \max_{l,\emptyset_I} \mathrm{V} = \left[B\left[\emptyset_I(1-l) + \left(\frac{w}{p}\right)(1-\delta_I)l\right]\right] + \left[A - \frac{Z}{2}l^2 - \frac{Z}{2}\emptyset_I^2\right]$$

with regards to l and $\emptyset_I$, and where $Z \in [0, \infty)$. Upon maximizing (12) for l and $\emptyset_I$, the result is:

LABOR SUPPLY OUTSIDE OF THE COMMUNITY	TECHNOLOGY USED IN THE COMMUNITY
$1 - l^* = Z\left[\dfrac{B\left(\frac{w}{p}\right)(1-\delta_I) - Z}{B^2 - Z^2}\right]$	$\emptyset_I^* = B\left[\dfrac{B\left(\frac{w}{p}\right)(1-\delta_I) - Z}{B^2 - Z^2}\right]$

Let us also assume that there are many goods, such that B is a large number with respect to Z and $\left(\frac{w}{p}\right)(1-\delta_I)$. Note that if $Z = 0$, which would imply a lack of IFs, then $\emptyset_I^* =$

$\frac{w}{p}$ and $1 - l^* = 0$. Furthermore, if $Z \to \infty$, then $1 - l^* = 1$ and $\emptyset_I^* = 0$. Likewise, $(\partial \emptyset_I^* / \partial Z) < 0$ and $(\partial (1 - l^*) / \partial Z) < 0$, which means that the lower the degree of traditionalism, the more time they will work in a company of an industry with specialization.

This result explains Figure 1B, where the degree of traditionalism is related to the proportion of self-employed people. These results are interesting because they combine the economic sphere, represented by the consumption scale B and income $(w/p)(1 - \delta_I)$, with culture, measured by the degree of traditionalism Z. In this pattern, the indigenous economy will be growing not only by producing new goods, but also by $1 - l^*$ of the growth of economy S due to economic modernization.

Regarding permanent or long-term migration, evidence shows that it has increased, mainly to Mexico's economic centers (Acharya and Barragán, 2012) and to the USA. The increase has been both in the number of migrants, as well as in their geographic and ethnic distribution (Fox and Rivera-Salgado, 2004). However, according to the model, if people from the indigenous communities have internalized their culture and are in a Pareto optimum, then why do they migrate permanently? Migration significantly affects their IFs, since they must work in industries with specialization and with updated technology. Obviously, one would think that a person's income could be so low that he is forced to migrate in order to increase it. But in that case, an income increase could be achieved by adopting updated technology or temporarily migrating, thus decreasing the participation of a person's culture in his utility function, but not eliminating it entirely.

The answer could be that people who migrate permanently have not really internalized the IFs, but rather they follow them within the community out of obligation, so migrating would be the optimal solution. The analysis of culture with IFs as a constraint was presented in section III.B. To examine this situation, let us suppose that the culture within an indigenous community indicates that they must work without economic specialization and without technology updates, so that an individual will maximize equation (10) subject to $h_R = 1$ and $\emptyset_I \leq \emptyset_{IR}$, where $\emptyset_{IR}$ is the most up-to-date technology that can be used, and that one may be punished for not complying with both restrictions, so it behooves the person to follow them.

If the individual has not internalized the IFs such that $Z = 0$, then his indirect utility would be $V_R = B\emptyset_{IR}$. If V_M were the indirect utility of migration, also with $Z = 0$, then $V_M = B\left({}^{W}/_{p}\right)(1 - \delta_I)$, so that $V_R < V_M$ if discrimination is not too elevated, in which case he would be willing to migrate. To this must be added that the cost of migrating has currently decreased, as migrants use social networks and receive help not only from individuals who have already migrated, but also from organizations created to make the change less costly in terms of utility (this is analyzed in depth in the book edited by Fox and Rivera-Salgado, 2004).

VII. Conclusions

In this article, we have analyzed the relationship between culture and economics in order to explain the poverty of Mexico's indigenous communities using an economic growth model in which the main reasons for weak growth are cultural factors. The difference with other studies on the relationship between culture and economics has been the manner in which culture is specified. In this study, culture is interpreted here as social influencing factors (IFs) that affect the utility function and/or the constraints on an individual. This provides the great advantage of being able to incorporate culture into formal models of economic theory. As such, the methods for incorporating these social influences are explained and formalized in the appendix. This enabled the formal derivation of the Factorial Utility Function (FUF), a utility function that explicitly incorporates social factors. In the specific case of Mexico's indigenous communities, the internal IFs have been the absence of economic specialization and the use of traditional technology, while the external IF has been discrimination.

The main conclusions regarding the relationship between culture and economics are first, that the problem of defining culture is avoided by using the FUF. The second conclusion is that the FUF is a tool that enables the introduction of culture into mathematical models, as well as a formal interpretation of the cultural components and their relationship with other variables. In the case of Mexico's indigenous communities, their IFs were deduced from Mexico's 2010 Economic Census data and from ethnographic sources. This article presented the origin and evolution of these IFs throughout history, and integrated them into the economic growth model. This has enabled an analysis of the influence of IFs on their economic growth, achieving results that would be difficult without a formal model.

Lastly, the model of economic growth that was presented shows, like other similar models, that increasing knowledge is the only way to grow. As Joel Mokyr says in his book *Culture of Growth*, a culture oriented to developing knowledge in order to transform the economy is the engine of economic growth. Indigenous cultures in Mexico have shown an impressive organizational capacity to defend themselves from a society that has discriminated against them, and a significant ability to adapt that has ultimately enabled them to survive. However, this is not observed in the economic arena. In fact, if we assume that IFs are internalized in their utility function, indigenous communities are at a Pareto optimum, but they are not growing.

Appendix – Existence of the Factorial Utility Function

Let $\boldsymbol{X} \subseteq \mathbb{R}_{+}^{L}$ represent a closed and convex set of goods and services, hours and type of work, etc. Let $\boldsymbol{x} \in \boldsymbol{X}$ represent baskets or groups of goods of size L, and we define an influencing factor as any circumstance identifiable by an individual that affects his ordering of preferences, but not the basic assumptions about his ordering. We assume that for the group of $\boldsymbol{X}$ there are $n+2$ preference relationships: the original or primitive $\succcurlyeq_p$ that does not consider any influence factor, n preference relationships of $\succcurlyeq_i$, with $i = 1,2, \cdots n,$, where each is primitive preference relationship influenced by the factor i. In this regard, we assume that all preference relationships fulfill the basic assumptions[13]. The fact that a preference ordering that considers each factor incorporates the primitive preference relationship is specifically to preserve the basic assumptions and enable comparisons. Otherwise, it would be unlikely that each influencing factor alone could develop a preference relationship that fulfills the basic assumptions. Lastly, let $\succcurlyeq_f$ represent the final preference relationship, which is the preference relationship that reflects all IFs.

In some cases, factors i and j with $i, j = 1,2, \cdots n$ and $i \neq j$, can be incompatible with each other. In other words, factors i and j can be interdependent, in which case factor i could affect both $\succcurlyeq_i$ and $\succcurlyeq_j$; that is, they are dependent – j is needed for i to exist, etc. To simplify the explanation, we will define a vector $\boldsymbol{Z}$, of n size, where if $Z_i = 1$, it's because factor i is active while $Z_i = 0$ is not. When this factor is active, it will affect the $\succcurlyeq_i$ preference relationship and possibly other IFs. Obviously, for two baskets $\boldsymbol{x}, \boldsymbol{y} \in X$, if $\boldsymbol{Z} = \boldsymbol{0}$ (i.e., $Z_i = 0$ for all $i = 1,2, \cdots, n$) then $\boldsymbol{x} \succ_p \boldsymbol{y} \Longrightarrow \boldsymbol{x} \succ_i \boldsymbol{y}$, for the primitive relationship and with $i = F, 1,2, \cdots, n$. Lastly, $\boldsymbol{Z}(h)$ with $h \in [0, n]$ indicates that the first h factors are active.

Based on the Existence of the Utility Function Theorem (Jehle and Reny, 2000), each preference relationship can be assigned a utility function $u_f(\boldsymbol{x}, \boldsymbol{Z})$, $u_i(\boldsymbol{x}, \boldsymbol{Z})$ and $u_p(\boldsymbol{x})$, with $\boldsymbol{x} \in \boldsymbol{X}$, and where vector $\boldsymbol{Z}$ indicates active IFs. For example, $u_i[\boldsymbol{Z}(1), x]$ could be different

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[13] **Completeness**: for all $x, y \in X$, $x \succcurlyeq y$ or $y \succcurlyeq x$
Transitivity: for all $x, y, z \in X$, if $x \succcurlyeq y$ and $y \succcurlyeq z$, thus $x \succcurlyeq z$
Continuity: for all $x \in \mathbb{R}_{+}^{n}$, *at least as good as* set $\succcurlyeq (x)$ and *no better than* set $\preccurlyeq (x^0)$ are closed in $\mathbb{R}_{+}^{n}$
Strict monotonicity: for all $x, y \in \mathbb{R}_{+}^{n}$, if $x \geq y$ thus $x \succcurlyeq y$; and if $x > y$ thus $x \succ y$.

from $u_i[\boldsymbol{Z}(h), x]$, with $h > 1$, which would reflect the relationship with the other factors, or that factor 1 does not influence preferences unless factor 2 is active, in which case $u_1[\boldsymbol{Z}(1), \boldsymbol{x}] = u_p$, but $u_1[\boldsymbol{Z}(2), \boldsymbol{x}] \neq u_1[\boldsymbol{Z}(1), \boldsymbol{x}]$.

Definition 1. Total inter-comparisons. To enable a total inter-comparison, only transformations with equal or increasing monotonicity are allowed for all useful functions, that is, $u_i(\boldsymbol{x}, \boldsymbol{Z}) > u_i(\boldsymbol{y}, \boldsymbol{Z})$ if and only if $\emptyset[u_i(\boldsymbol{x}, \boldsymbol{Z})] > \emptyset[u_i(\boldsymbol{y}, \boldsymbol{Z})]$ for $i = p, f, 1, \cdots n$, where $\emptyset$ is any transformation with positive monotonicity (*intra-individual full cardinal comparisons*; see Bossert and Weymark, 2004). This is the same as assuming that the vector used to construct the utility function is the same for all utility functions, which is generally the unit vector $\boldsymbol{e} \equiv (1, .., 1) \in \mathbb{R}_+^L$. With this, we can define the intra-comparisons.

Definition 2. Comparison within the same basket under different factors. The expression $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$ means that basket $\boldsymbol{x}$ influenced by factor i is preferred over the same basket $\boldsymbol{x}$ influenced by factor j, given other active h factors.

Proposition 1. Equivalent Utility. Let $u_i[\boldsymbol{x}, \boldsymbol{Z}(i, h)]$ and $u_j[\boldsymbol{x}, \boldsymbol{Z}(j, h)]$, with $i, j = 1, 2, \cdots, n$ and $h \in (0, n-2)$ and $i \neq j \neq h$ represent two utility functions valued in the same basket $\boldsymbol{x} \in X$, where $u_i[\boldsymbol{x}, \boldsymbol{Z}(i, h)]$ incorporates the factor i in addition to the active h factors; and the function $u_j[\boldsymbol{x}, \boldsymbol{Z}(j, h)]$ incorporates the factor j in addition to the active h factors. There is a continuous function $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i, j, h)]$ that we will call the *equivalent utility* such that $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i, j, h)] = u_i[\boldsymbol{x}, \boldsymbol{Z}(i, h)] - u_j[\boldsymbol{x}, \boldsymbol{Z}(j, h)]$, where $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i, j, h)]$ represents the difference between factor i and factor j, in terms of utility, given other h factors acting on vector $\boldsymbol{x}$. This is $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i, j, h)] > 0$ if and only if $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$.

Proof. First, we will prove that if $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h \Rightarrow c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i, j, h)] > 0$. According to the Utility Function Existence Theorem, it is possible to assign a utility function $u_i[\boldsymbol{x}, \boldsymbol{Z}(i, h)]$, $\mathbb{R}_+^L \to \mathbb{R}$ with $i = f, p, 1, 2, \cdots n$ for each relationship, such that $u_i[\boldsymbol{x}, \boldsymbol{Z}(i, h)]\boldsymbol{e} \sim \boldsymbol{x}$, where $\boldsymbol{e}$ (already defined) is the unit vector, and likewise for $u_j[\boldsymbol{x}, \boldsymbol{Z}(j, h)]$. If influencing factors i and j are active and given other h influencing factors, then without losing generality, if

$(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$ then $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]\boldsymbol{e} \backsim (\boldsymbol{x})_i \succ (\boldsymbol{x})_j \backsim u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]\boldsymbol{e}$, and $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]\boldsymbol{e} > u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]$, with $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] > 0$.

Now we will prove the inverse, that is, $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] > 0 \Longrightarrow (\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$. We know that if $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] > 0$, then $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] > u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]$, so by the inverse of the Utility Function Theorem, $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]\boldsymbol{e} \succ u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]\boldsymbol{e}$, which means that $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$. Lastly, since the addition of continuous functions is another continuous function (Apostol 1974, Theorem 4.18), then $\mathrm{c_{ij}}[\mathbf{x}, \mathbf{Z}(\mathrm{i,j,h})]$ is a continuous function.

Note that $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$ is not a preference relationship, nor does $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)]$ reflect the order of that relationship, since they do not necessarily fulfill the basic assumptions. They are simply comparisons between two preference structures. The reason that we can work with them is because, if $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] > u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)] \Rightarrow c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] > 0$ because both utility functions lie on the real line and $c_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] > 0 \Leftrightarrow (\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$ due to total comparability; that is, $n+2$ level curves pass through every point $\boldsymbol{x} \epsilon \boldsymbol{X}$. If level curve i displays a higher utility than level curve j, that is, $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] > u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]$, then $(\boldsymbol{x})_i^h \succ (\boldsymbol{x})_j^h$ and it can be said that basket $\boldsymbol{x}$ is preferred under factor i than under factor j.

By means of the equivalent utility proposition, each utility function $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]$ is defined as $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] = u_p(\boldsymbol{x}) + c_{ip}[\boldsymbol{x}, \boldsymbol{Z}(i,h)]$, where $c_{ip}[\boldsymbol{x}, \boldsymbol{Z}(i,h)]$ is the equivalent utility between primitive utility $u_p(\boldsymbol{x})$ and the primitive utility plus factor i, $u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]$.

Proposition 2: Aggregation of factors. The utility function incorporating factors i and j, given other active factors h for $i,j = 1,2,\cdots,n$ is defined as:

$$u_{ij}[\boldsymbol{x}, \boldsymbol{Z}(i,j,h)] = u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] + u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)] - u_p(\boldsymbol{x})$$

For the case that aggregates the n factors:

$$u_f[\boldsymbol{x}, \boldsymbol{Z}(n)] = u_p(\boldsymbol{x}) + \sum_{i=1}^{n} u_i[\boldsymbol{x}, \boldsymbol{Z}(n)] - (n)u_p(\boldsymbol{x})$$

Proof. Let the following sets with $y \in \mathbb{R}_+$: $A_i = \{y/y \leq u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)]\}$ and $A_j = \{y/y \leq u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)]\}$. The union of both sets, by definition, is $(A_i \cup A_j) = A_i + A_j - (A_i \cap A_j)$ in which $(A_i \cap A_j) = u_p(\boldsymbol{x})$. By substituting we have $(A_i \cup A_j) = u_i[\boldsymbol{x}, \boldsymbol{Z}(i,h)] + u_j[\boldsymbol{x}, \boldsymbol{Z}(j,h)] - u_p(\boldsymbol{x})$. For the n factors, we begin with the first factor:

$A_1[\boldsymbol{x}, \boldsymbol{Z}(1)] = (A_1) = u_1[\boldsymbol{x}, \boldsymbol{Z}(1)]$
$A_{12}[\boldsymbol{x}, \boldsymbol{Z}(2)] = (A_1 \cup A_2) = u_1[\boldsymbol{x}, \boldsymbol{Z}(2)] + u_2[\boldsymbol{x}, \boldsymbol{Z}(2)] - u_p(\boldsymbol{x})$
$A_{123}[\boldsymbol{x}, \boldsymbol{Z}(3)] = (A_{p12} \cup A_3) = u_{12}[\boldsymbol{x}, \boldsymbol{Z}(3)] + u_3[\boldsymbol{x}, \boldsymbol{Z}(3)] - 2u_p(\boldsymbol{x})$

And so on until:

$$u_f[\boldsymbol{x}, \boldsymbol{Z}(n)] = (A_1 \cup A_{\mathrm{I}-n} \cup A_n) = \sum_{i=1}^{n} u_i[\boldsymbol{x}, \boldsymbol{Z}(n)] - (n-1)u_p(\boldsymbol{x}) \blacksquare$$

From the previous propositions, we can build the factorial utility function.

Teorema (Función de Utilidad Factorial). If there are n on the primitive preference relationship, according to propositions 1 and 2, the utility function $u_f(\boldsymbol{x}, \boldsymbol{Z})$ will be equal to $u_p(\boldsymbol{x}) + \sum_{i=1}^{n} \Delta u_i \ (\boldsymbol{x}, \boldsymbol{Z})$, where $\Delta u_i(\boldsymbol{x}, \boldsymbol{Z}) = [u_i(\boldsymbol{x}, \boldsymbol{Z}) - u_p(\boldsymbol{x})]$ for $i = 1,2, \cdots, n$, or some monotonic transformation of this function.

Proof. By proposition 2, we know that

$$u_f[\boldsymbol{Z}(n), \boldsymbol{x}] = \sum_{i=1}^{n} u_i[\boldsymbol{Z}(n), \boldsymbol{x}] - (n-1)u_p(\boldsymbol{x})$$

Adding and subtracting u_p, we have:

$$u_I[Z(n), \boldsymbol{x}] = u_p(\boldsymbol{x}) + \sum_{i=1}^{n} \{ u_i[\boldsymbol{Z}(n), \boldsymbol{x}] - u_p(\boldsymbol{x}) \}$$

Inverse. let $u_f[\boldsymbol{Z}(n), \boldsymbol{x}]$ be any utility function that includes influencing factors such as those defined, and consider $u_{f(n-1)}[\boldsymbol{x}, \boldsymbol{Z}(n-1)]$ as the utility function without the final factor. Due to *equivalent utility*, there is a function such that $c_{n(n-1)}(\boldsymbol{x}) = u_f[\boldsymbol{Z}(n), \boldsymbol{x}] - u_{f(n-1)1}[\boldsymbol{Z}(n-1), \boldsymbol{x}]$, so that $u_{fn}[\boldsymbol{Z}(n), \boldsymbol{x}] = u_{F(n-1)}[\boldsymbol{Z}(n-1), \boldsymbol{x}] + \Delta u_{n(n-1)}[\boldsymbol{x}, \boldsymbol{Z}(n)]$. Repeating the procedure we arrive at $u_f(\boldsymbol{x}, \boldsymbol{Z}) = u_p(\boldsymbol{x}) + \sum_{i=1}^{n} \Delta u_i \ [\boldsymbol{x}, \boldsymbol{Z}(n)]$ ■