Is the Beijing Olympic swimming centre unstable?

The swimming centre built for the Beijing Olympics is an extraordinarily beautiful sight, especially lit up at night, when it looks like a transparent box full of bubbles. Its designers, Arup, were keen to capture the spirit of the aquatic sports played inside but wanted also to give the building a natural, organic look.

They began by looking at shapes that can tile a wall, like squares or equilateral triangles or hexagons, but decided that these were all too regular and didn't capture the organic quality they were after. They explored other ways in which nature packs things together, like crystals or cell structures in plant tissue. In all these structures there are examples of the sort of shapes that Archimedes discovered made such good footballs, but Arup were particularly drawn to the shapes made by lots of bubbles packed together to make foam.

Considering that it took until 1884 to prove that the sphere is the most efficient shape for a single bubble, it may not come as a surprise that sticking more than one bubble together to make foam leads to some tough questions that are still vexing mathematicians today. If you have two bubbles that contain the same volume of air, what shape do they make when they join together? The rule is always that bubbles are lazy and look for shapes with the least energy. Energy is proportional to surface area, so they try to make a shape that has the smallest surface area of soap film. Since two joined bubbles share a boundary, they can make a shape with smaller surface area than just two bubbles touching.

If you blow bubbles, and two bubbles of the same volume fuse together, then the combination looks like this:

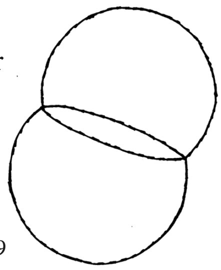

FIGURE 2.09

The two partial spheres will meet at an angle of 120° and be separated by a flat wall. This is certainly a stable state – if it wasn't, nature wouldn't let the bubbles stay as they are. But the question is whether there might be another shape that has even less surface area, and therefore less energy, which would make it even more efficient. It might require putting some energy into the bubbles to take them out of their current stable state, but perhaps there is an even lower energy state that two bubbles could assume. For example, perhaps the two fused bubbles could be bettered by some weird configuration with less energy where one bubble takes the shape of a bagel and wraps itself round the other bubble, squeezing it into the shape of a monkey nut (Figure 2.10).

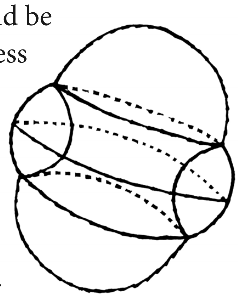

FIGURE 2.10

The first proof that the fused bubbles couldn't be bettered was announced in 1995. Although mathematicians don't really like asking for help from a computer, because that doesn't appeal to their sense of elegance and beauty, they needed one to check through the extensive numerical calculations that were involved in their proof.

Five years later, a pencil-and-paper proof of the double bubble conjecture was announced. It actually proved a more general conjecture: if the bubbles do not enclose the same volume, but one is smaller than the other, then the bubbles fuse together so that the wall between the bubbles is no longer flat but bends into the big bubble. The wall is part of a third sphere and meets the two spherical bubbles in such a way that the three soap films have angles of 120° between them (Figure 2.11 and 2.12).

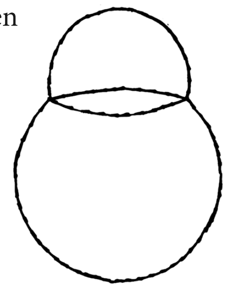

FIGURE 2.11

In fact, this 120° property turns out to be a general rule for the way soap bubbles fuse together. It was first discovered by Belgian scientist Joseph Plateau, who was born in 1801. While he was doing research into the effect of light on the eye, he stared at the Sun for half a minute, and by the age of 40 he was blind. Then, with the help of relatives and colleagues, he switched to investigating the shape of bubbles.

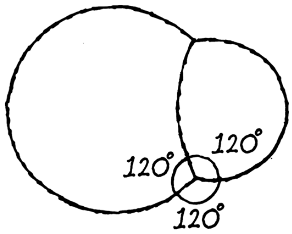

FIGURE 2.12

Plateau began by dipping wire frames into bubble mixture and examining the different shapes that appeared. For example, when you dip a wire frame in the shape of a cube into the mixture, you get 13 walls which meet at a square in the middle (Figure 2.13).

Except that it isn't quite a square – the edges bulge out. As Plateau explored the various shapes that appeared in different wire frames, he began to formulate a set of rules for how bubbles join together.

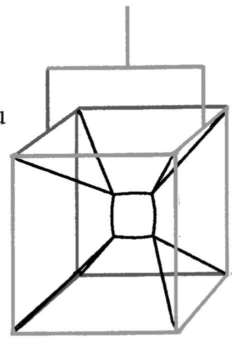
FIGURE 2.13

The first rule was that soap films always meet in threes at an angle of 120°. The edge formed by these three walls is called a Plateau border in his honour. The second rule was about the way these borders can meet. Plateau borders meet in fours at an angle of about 109.47° ($\cos^{-1} -\frac{1}{3}$, to be precise). If you take a tetrahedron and draw lines from the four vertices to the centre, you get the configuration of the four Plateau borders in foam (Figure 2.14). So the edges in the bulging square at the centre of the cube wire frame actually meet at 109.47°.

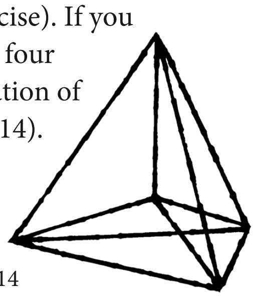
FIGURE 2.14

Any bubble that didn't satisfy Plateau's rules was believed to be unstable and would therefore collapse to a stable configuration that did satisfy these rules. It was not until 1976 that Jean Taylor finally proved that the shape of bubbles in foam had to satisfy the rules laid down by Plateau. Her work tells us how the bubbles connect together, but what about the actual shapes of the bubbles in foam? Because bubbles are lazy, the way to the answer is to find the shapes that enclose a given amount of air in each bubble in the foam while minimizing the surface area of soap film.

Honey bees have already worked out the answer to the problem in two dimensions. The reason they construct their hives using hexagons is that this uses the least amount of wax to enclose a fixed amount of honey in each cell. Yet again, it was only a very recent breakthrough that confirmed the honeycomb theorem: there is no other two-dimensional structure that can beat the hexagonal honeycomb for efficiency.

Once we step up to three-dimensional structures, though, things become less clear. In 1887 the famous British physicist Lord Kelvin suggested that one of Archimedes' footballs was the key to minimizing the surface area of the bubbles. He believed that while the hexagon was the building block of the efficient beehive, the truncated octahedron – a shape made by cutting the six corners off a standard octahedron – was the key to constructing foam:

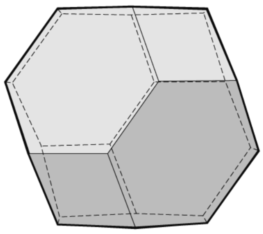

FIGURE 2.15

The rules that Plateau developed for how bubbles in foam must meet show that the edges and faces are not actually flat, but curve. For example, the edges of a square meet at 90°, but by the second of Plateau's rules that isn't permitted. Instead, the edges of the square bulge out as they do in the cube wire frame, so that the two soap films meet at the requisite 109.47°.