Gregorio Baldin

POINTS, ATOMS AND RAYS OF LIGHT: HISTORY OF A CONTROVERSY FROM MERSENNE TO HOBBES

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INDEX

OBITUARIES

ESSAY REVIEWS

Galilæana, XV, 2018, pp. 31-60

Gregorio Baldin

POINTS, ATOMS AND RAYS OF LIGHT
History of a Controversy from Mersenne to Hobbes *

SUMMARY

Paris, 1635: an interesting mathematical and scientific debate develops within Father Mersenne's circle. The problem concerns the ontological nature of the point and is known as the '*problème de Poysson*', taking its name from Jean-Baptiste Poysson de la Benerie. However, from the epistolary exchanges taking place among the philosophers and scientists involved, we deduce that the true author of the *quaestio* is Father Mersenne, and this hypothesis seems to be confirmed by the presence of a similar problem in Mersenne's works dated 1625. The topic was widely debated by important philosophers of the time, such as Campanella and Gassendi, and following the traces of this discussion we are able to uncover the history of a controversy which developed from Mersenne to Hobbes. Moreover, on detailed examination of the problem, different aspects and perspectives emerge, which involve not only mathematics and physics, but also optics and the nature of light.

Keywords: Hobbes, Mersenne, Poysson, Gassendi, Campanella, atoms, light and matter, optics.

Quaestio singularis: an introduction

Among the questions debated within the 'Mersenne circle',[1] there was one in particular that stimulated the interest of the colleagues of the Min-

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* I would like to thank Johanna Louw, who translated my text into English.

[1] On Mersenne's circle see Robert Lenoble, *Mersenne ou la naissance du mécanisme*, Paris, Vrin, 1943, pp. 590 ff.; Jean-Robert Armogathe, *Le groupe de Mersenne et la vie académique*

im friar, namely, that of the famous '*problème de Poysson*',[2] which addresses the ontological nature of the mathematical point. The problem derives its name from Jean-Baptiste Poysson de la Benerie, who was, at least in a formal sense, the originator of the theme under discussion. And yet, in a letter dated 15 November 1635, Ismaël Boulliau suggested to Nicolas-Claude Fabri de Peiresc that the true author of the question was Father Mersenne himself.[3] Obviously, Poysson had assumed the role of promoting the debate, which stimulated great interest in the Parisian intellectual milieu.

The subject was not new at all. It had, on the contrary, been thoroughly examined in antiquity[4] and even during the Middle Ages,[5] but

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parisienne, «Dix-septième siècle», 175, 1992, pp. 131-139; René Taton, *Le P. Marin Mersenne et la communauté scientifique au XVII^e siècle*, in Jean-Marie Constant – Anne Fillon, *Quatrième centenaire de la naissance de Marin Mersenne (Actes du colloque)*, Le Mans, 1994, pp. 13-25; Armand Beaulieu, *Mersenne. Le grand minime*, Bruxelles, Fondation Nicolas-Claude de Peiresc, 1995, pp. 63, 176-185; Jean-Pierre Maury, *À l'origine de la recherche scientifique: Mersenne*, Paris, Vuibert, 2003, pp. 154-156. See also, John Lewis, *Galileo in France. French Reactions to the Theories and Trial of Galileo*, New York, Peter Lang, 2006, pp. 113-140; Marc Fumaroli, *La République des Lettres*, Paris, Gallimard, 2015, pp. 74 ff. On Hobbes' relationship with Mersenne's circle, see Gregorio Baldin, *La 'réflexion de l'arc' et le* conatus*: aux origines de la physique de Hobbes*, «Philosophical Enquiries», 7, 2016, pp. 15-42.

2 Bernard Rochot was the first scholar to examine the problem in greater detail, see Bernard Rochot, *Une discussion théorique au temps de Mersenne: le problème de Poysson 635-1636)*, «Revue d'Histoire des Sciences», 2, 1948, pp. 80-89, which nevertheless reminds us (*ibid.*, p. 80) that the subject was also alluded to in the imposing essay on atomism authored by Lasswitz. See Kurd Lasswitz, *Geschichte der Atomistik vom Mittelalter bis Newton*, Georg Olms, Hildesheim 1963[2], 2 vols., vol. II, p. 129. More recent treatments of the question include: Paolo Ponzio, *Tommaso Campanella e la* Quaestio Singularis *di Jean-Baptiste Poysson*, «Physis», 34, 1997, pp. 71-97, Ruth Hagengruber, *La fondazione del punto matematico nella filosofia di Tommaso Campanella*, «Bruniana & Campanelliana», III, 1997, pp. 77-92: 86-87; Ead., *Tommaso Campanella. Eine Philosophie der Ähnlichkeit*, Sankt Augustin, Academia Verlag, 1994, pp. 63 ff.; Germana Ernst, *Tommaso Campanella. The Book and the Body of Nature*, Dordrecht-Heidelberg-London-New York, Springer, 2010 (original edition 2002), pp. 244-245. The only reference to Hobbes' possible interest in the problem is found in an article by Médina: see José Médina, *Mathématique et philosophie chez Thomas Hobbes: une pensée du mouvement en mouvement*, in Jauffrey Berthier – Nicolas Dubos – Arnaud Milanese – Jean Terrel (eds.), *Lectures de Hobbes*, Paris, Ellipses, 2013, pp. 85-132: 103 and note.

3 «Je vous diray confidemment que ce n'est point autre que le P. Mersenne, à qui telle question est nee dans l'esprit. Il m'a autresfois proposé quelque chose de semblable». Ismaël Boulliaud à Nicolas-Claude Fabri de Peiresc, 15 November 1635, *La Correspondance du P. Marin Mersenne*, publiée par Paul Tannery, Cornelis de Waard *et alii*, Paris, Édition Universitaires de France et Centre National de la Recherche Scientifique, 1932-1986, 16 vols. (henceforth *CM)*, vol. V, p. 474.

4 Aristotle wrote: «nothing that is continuous can be composed of indivisibles: e.g. a line cannot be composed of points, the line being continuous and the point indivisible» Aristotle, *Physics*, VI, 231 a; Engl. Transl. in *The Complete Works of Aristotle*, ed. by J. Barnes, Princeton, Princeton University Press, 1991, 2 vols., vol. I, p. 94.

5 See John E. Murdoch, *Infinity and continuity*, in Norman Kretzmann – Anthony Kenny – Jan Pinborg (eds.), *The Cambridge History of Later Medieval Philosophy*, Cambridge,

acquired primary significance during the 17th century, when atomist philosophy regained popularity, thereby providing an opportunity for a reconsideration of the structure of matter and the *continuum*.[6] The problem was circulated on a note or *'placard'*,[7] (of which all trace has been lost), written in Latin, and presented in the following form:

QUESTION: *Whether there is some perfectly logical, perfectly mathematical and perfectly perceptible demonstration proving that it is possible to have a magnitude with a certain extension, which coincides at a certain time and place with a truly mathematical point, a point whose parts are nil and, yet, in which distinct parts are found.*[8]

Cambridge University Press, 1982, pp. 564-591. William of Ockham restated the Aristotelian thesis, strongly denying that points have any actual existence (see William of Ockham, *Expositionis in libros artis logicae Prooemium et Expositio in librum Porphyrii de praedicabilibus*, in *Opera Philosophica et Theologica*, New York, St. Bonaventurae, 1974-1988, 7+10 vols., vol. II, pp. 205 ff.; Marilyn McCord Adams, *William Ockham*, Notre Dame, Notre Dame University Press, 1987, 2 vols., vol. I, pp. 201 ff.). The Ockhamist position was also held by Jean Buridan, Albert of Saxony, and by the Mertonians, such as Thomas Bradwardine and William of Heytesbury (see Murdoch, *Infinity and continuity* (cit. note 5), pp. 573-575). Conversely, the existence of the indivisibles was defended by Henry of Harclay, Walter Chatton, Geraldus Odonis and Nicolas Bonet (*ibid.*, pp. 575-576). See also, Id., *Beyond Aristotle: indivisibles and infinite divisibility in the later Middle Ages*, in Cristophe Grellard – Aurélien Robert (eds.), *Atomism in Late Medieval Philosophy and Theology*, Leiden/Boston, Brill, 2009, pp. 15-38. On the problem of the continuum and *minima naturalia* see also the classic essay by Maier: Anneliese Maier, *Scienza e filosofia nel Medioevo. Saggi sui secoli XIII e XIV*, Milano, Jaca Book, 1984, pp. 271 ff.

6 Giordano Bruno, for example, addressed the subject of the point and composition of the *continuum* on several occasions. In *De la causa*, Bruno, while examining this theme, suggested a genetic conception of geometry, which can also subsequently be seen in Campanella and Hobbes: «Se dalla potenza non è differente l'atto, è necessario che in quello il punto, la linea, la superficie e il corpo non differiscano: perché cossì quella linea è superficie, come la linea, movendosi, può essere superficie; cossì quella superficie è mossa ed è fatta corpo, come la superficie può moversi e, con il suo flusso, può farsi corpo. È necessario dunque che il punto ne l'infinito non differisca dal corpo, perché il punto, scorrendo dall'esser punto, si fa linea; scorrendo da l'esser linea, si fa superficie; scorrendo da l'esser superficie, si fa corpo; il punto, dunque, perché è in potenza ad esser corpo, non differisce da l'esser corpo dove la potenza e l'atto è una medesima cosa». Giordano Bruno, *De la causa, principio e uno* in *Dialoghi filosofici italiani*, ed. by G. Gentile (amended and revised by Giovanni Aquilecchia), Firenze, Sansoni, 1985, pp. 320-321. Bruno returned to the composition of the continuum in *De l'infinito universo e mondi* (*ibid.*, p. 513). See Michele Ciliberto, *Il lessico di Giordano Bruno*, Roma, Ed. dell'Ateneo & Bizzarri, 1979, 2 vols., vol. I, pp. 991-992. See also the entry *Punto*, ed. by M. Matteoli, in Michele Ciliberto (ed.), *Giordano Bruno. Parole, concetti, immagini*, Pisa, Ed. della Normale, 2014, 3 vols., vol. II, pp. 1614-1616.

7 See Beaulieu, *Mersenne. Le grand minime* (cit. note 1), p. 63.

8 «QUAESTIO: *Utrum sit aliqua demonstratio perfectè logica, perfectè mathematica, perfectè sensibilis, qua probetur dari magnitudinem latitudinis non expertem, quae aliquando et alicubi sit in puncto vere mathematico et cujus puncti nullae sint partes et tamen in eodem ipsa habeat partes extra partes*». Tommaso Campanella to Jean-Baptiste Poisson de la Benerie, Paris, 7 July 1635, in Tommaso Campanella, *Lettere*, ed. by G. Ernst, Firenze, Olschki, 2010, p. 416.

The text is not immediately intelligible, but the terminology employed allows us to deduce the core of the problem. If we adhere to the Euclidean definition of the point, «A point is that which has no part»,[9] the solution suggested by Aristotle, according to which geometrical figures are merely the product of a process of abstraction, would appear to be the most natural. However, if we pass from the realm of geometry into that of physics – reflecting on the composition of matter and its infinite divisibility – we are confronted with a dilemma: either we assume that matter consists of physical points, namely atoms, or we are obliged to conceive of it as being composed of *indivisibles*,[10] without extension. In the latter case, however, the problem becomes even more difficult, as we must try to understand and explain how unextended points are able to form that is, by contrast, extended.[11]

Although the question was only explicitly stated in 1635, there are already traces of it in a work that Mersenne had published a decade before this date: *La Vérité des Sciences*.

1. *1625*: 'Mersenne problem'

It is not difficult to believe that Mersenne was the real author of the *quaestio* since in his 1625 work *La Vérité des Sciences* this theme provides

[9] See Euclid, *Elements*, I, Def. I. The Euclidean definition can be interpreted as treating the point as unextended, but also as a unit, which is retained without its parts, by Plato (Plato, *Sophist*, 245a; *Republic*, 526a).

[10] On the indivisibles in Cavalieri, see: Léon Brunschvicg, *Les étapes de la philosophie mathématique*, Paris, Blanchard, 1993[3], pp. 162-167, Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles*, Bologna, Cremonese Ed., 1980, Kirsti Andersen, *Cavalieri's Method of Indivisibles*, «Archive for History of the Exact Science», 28, 1985, pp. 292-367, Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in Seventeenth Century*, Oxford, Oxford University Press, 1996, pp. 34 ff., Michel Blay, *Penser avec l'infini*, Paris, Vuibert, 2010, pp. 48-53. On the transition from the concept of the indivisible to that of the Leibnizian *infinitesimal*, see Antoni Malet, *From Indivisibles to Infinitesimals*, Bellaterra, Universitat Autònoma de Barcelona, 1996, especially pp. 11-50. On the 'prehistory' of the concept of the indivisible, see Jean Celeyrette, *From Aristotle to the Classical Age, the Debate Around Indivisibles*, in Vincent Jullien (ed.), *Seventeenth-Century Indivisibles Revisited*, Heidelberg-New York-Dordrecht-London, Springer, 2015, pp. 19-30. On the controversy regarding the indivisibles, see Egidio Festa, *Quelques aspects de la controverse sur les indivisibles*, in Massimo Bucciantini – Maurizio Torrini (eds.), *Geometria e atomismo nella scuola galileiana (Atti del convegno)*, Firenze, Olschki, 1992, pp. 193-206. On the indivisibles in Torricelli, see François De Gandt, *Les indivisibles de Torricelli*, in Id., *L'œuvre de Torricelli: science galiléenne et nouvelle géométrie*, Paris, Les Belles Lettres, 1989, pp. 147-206.

[11] This involves a recasting of sorts of Zeno's classic paradox on the impossibility of the existence of extension. See Rochot, *Une discussion théorique au temps de Mersenne* (cit. note 2), pp. 80-89.

the focus of the discussion between the *Sceptic* and the *Christian philosopher*, two of the three protagonists in the dialogue. The *Pyrrhonist* asserts the impossibility of treating mathematico-geometrical reasoning as beyond dispute, citing as an example the difficulty of defining the mathematical point ontologically:

Iamais ie ne me treuve plus embarassé que quand ie pense à ce point mathematique, & me semble qu'il vaut mieus dire qu'il n'est point que de se peiner davantage pour l'entendre, car soit qui vous le mettiez, ou que vous le niez, i'y voy de si grandes difficultez, qu'elles sont insurmontables, puisque si on le met, il faut en admettre une infinité en chaque ligne, non pas qui la composent (veu qu'il n'est pas possible qu'un indivisible produise un divisible), mais qui unissent les parties de chaque ligne, lesquelles sont infinies.[12]

He goes on to say:

Ceste infinité de parties me travaille aussi grandement, car il faudroit qu'il y eût dans chaque corps, & dans chaque ligne un nombre infini d'infinitez, puisque quand une ligne seroit divisée en parties infinies, chaque parcelle prise à part seroit encore composée d'une infinité de parties. De plus il y auroit dans chaque corps plus petit un milion de fois qu'un ciron, une infinité de corps infinis en parties, dans la surface des quels il y auroit plusieurs infinitez des plans infinis & autant d'infinitez d'indivisibles, ce qui semble fort estrange, & contre toute sorte de vérité.[13]

The question raised by the Sceptic is interesting and significant, not merely because Mersenne expresses the *vexata quaestio* clearly, but also because in this case he anticipates the subjects that are subsequently addressed by other authors, and in particular Gassendi. Moreover, the reply given by the Mersennian *Philosophe* is remarkable; he asserts that:

[...] car comme c'est une proprieté essentielle à Dieu que d'estre indivisible, & infini, c'est aussi une proprieté inseparable de la quantité qu'elle ait des parties, & des points infinis en multitude: car s'il arrivoit autrement, elle ne seroit plus quantité non plus que Dieu ne seroit plus Dieu, s'il venoit à estre divisible, ou fini.[14]

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12 Marin Mersenne, *La Vérité des Sciences. Contre les Sceptiques ou Pyrrhoniens*, Paris, Toussainct du Bray, 1625 (Faksimile-Neudruck: Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag, 1969), p. 725.

13 *Ibid.*, pp. 725-726.

14 *Ibid.*, p. 727.

Mersenne begins by developing the theme of the entity and the nature of the mathematical point, alluding peripherally to the concept of the *indivisible*. He subsequently operates a conceptual shift, bringing the physical into consideration; he thus claims that any quantity can be divided into infinite points, with the result that any given quantity, and therefore also a body a million times smaller than a mite, is always divisible into parts in which there are always an infinite number of planes and, accordingly, an infinite number of indivisibles.

2. *1635*: The responses provided by Campanella and Boulliau

Mersenne's correspondence shows that the first scholar to reply to Poysson's problem was Tommaso Campanella, who arrived in Paris in 1634 and immediately met with Mersenne.[15] On 7 July 1635, Campanella sent a letter to Jean-Baptiste Poysson de la Benerie who, as we know, was officially the proponent of the theme under discussion. In the first place, the Italian Dominican expresses his bewilderment regarding the phrasing of the question,[16] and not without some justification since it involves the overlapping of two quite separate spheres: mathematical knowledge and perceptible knowledge, which relate to different objects. Nevertheless, Campanella attempted to develop a solution to the *quaestio* by reflecting on the concepts of place and body:

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[15] Mersenne wished to meet Campanella in person, in order to effect a sort of reconciliation, after vigorously attacking him ten years earlier, in his *Quaestiones in Genesim* (see Marin Mersenne, *Quaestiones in Genesim*, Paris, Sebastien Cramoisy, 1623, col. 130-131, 707 and 939-940). In November 1634, on Campanella's arrival in Paris, Mersenne had written to Peiresc that he intended to meet him shortly, but from a letter written the following May, it would appear that his expectation remained unfulfilled: «je vis le R. P. Campanella 4 heures durant ou environ pour la deuxième fois; où j'ay appris qu'il ne nous apprendra rien dans les sciences. L'on m'avoit dit qu'il sçavoit merveille dans la musique dont il m'a mesme dit qu'il avoit escrit, mais l'interrogeant je n'ay pas trouvé qu'il sceust seulement ce que c'est de l'octave; au reste il a une heureuse memoire et une feconde imagination». Mersenne à Nicolas-Claude Fabri de Peiresc, 23 May 1635, *CM*, vol. V, p. 209. A more positive judgement is revealed in the next communication, in which Mersenne acknowledges that: «cet excellent homme a un grand entendement». Mersenne à Nicolas-Claude Fabri de Peiresc, 25 May 1635, *CM*, vol. V, pp. 213-214. See also: Lenoble, *Mersenne ou la naissance du mécanisme* (cit. note 1), pp. 40-42; Ponzio, *Tommaso Campanella e la* Quaestio Singularis *di Jean-Baptiste Poysson* (cit. note 2), pp. 72-76.

[16] «Vel non exacte clareque quaeritur, vel mihi non intelligitur». Tommaso Campanella à Jean-Baptiste Poysson de la Benerie, 7 July 1635 (*CM*, vol. V, p. 285), in Campanella, *Lettere* (cit. note 8), p. 416. See also: Ponzio, *Tommaso Campanella e la* Quaestio Singularis *di Jean-Baptiste Poysson* (cit. note 2), p. 83.

I maintain that everything which, in a specific space and time, possesses a magnitude, is a body in a place. The place is therefore a surface surrounding it, or an incorporeal space, or immobile base, a space which interpenetrates bodies; and I say the object positioned there, is physical. Thus, the magnitude having width also has depth, which fills the space, or it is surrounded by a surface. For this reason, it cannot exist in a point if not perhaps tangent to this point.[17]

Although the magnitude (*magnitudo)* of a body is inextricably linked to the Aristotelian definition of *place*,[18] the outcome is that the point, by its very nature, does not possess this characteristic. Campanella continues his discussion, arriving at the demonstration he defines as 'perceptible'. He determines that 'two things' are imperceptible to the senses, namely «the maximum and the minimum», which cannot be 'gathered' by the intellect «or by analogy or syllogism alone, and not through true intuition».[19] As a consequence, it is impossible to conceive of «a point that is perceptible to the senses, either existing by itself, or in another. In fact, that which exists is a part rendered tangible by the surrounding parts or the parts that it itself surrounds. Thus, it cannot be perceived by the senses if it is not divisible».[20] Conversely, according to Campanella, «it is impossible to conceive of an indivisible point» because «either it is placed in the mental world, or in the corporeal» and, in the latter case, the point must have its own position «in relation to the other coexisting bodies, to the right, to the left, above, below, to the east and to the west, and to the poles of the earth, to the infinite parts of the world, to which we may draw infinite lines from this point».[21] The Italian philosopher nevertheless admits that it is possible to imagine «a magnitude existing

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[17] «Quidquid est alicubi vel aliquando magnitudinis ritu puto esse corpus in loco. Locus autem vel est superficies ambientis, vel spatium incorporeum, immobile basis, intranea corporum, tunc assero locatum esse corporum. Igitur magnitudo habens latitudinem, habet eam profunditatem qua replet spatium vel circumdatur a superficie. Igitur non potest in puncto localiter, nisi forsan et tangens punctualiter». Tommaso Campanella to Jean-Baptiste Poysson de la Benerie, Paris, 7 July 1635, in Campanella, *Lettere* (cit. note 8), p. 416.

[18] See Aristotle, *Physics*, 212a, 5-15.

[19] «Duo enim sunt nobis imperceptibilia, maximum et minimum [...]. Propterea et intellectus neque minima entia neque maxima infinitaque recte percipit, nisi per similitudinem syllogizando tantùm, non autem intuendo». Tommaso Campanella to Jean-Baptiste Poysson de la Benerie, Paris, 7 July 1635, in Campanella, *Lettere* (cit. note 8) p. 417.

[20] «Igitur non datur punctum sensibile neque existens neque inexistens. Quod enim inexistit». *Ibid.*

[21] «Neque potest intelligi punctum indivisibile. Vel enim ponitur in mundo mentali vel in corporali. Si in corporali, ubicumque ponatur, habet respectum ad corpora coexistentia à dextris, à sinistris, superis, inferis, ad ortum et ad occasum et ad polos mundi, imo ad infinitas mundi partes, ad quas ex illo puncto duci possunt lineae infinitae». *Ibid.*

in a point», as if it were in some way tangent to the point, «but not because the parts of this magnitude are contained within the point, but because they are outside it, since they are not touched by the same point except exactly at one point, and not by the parts conjunct to it, which are not involved in contact with the point».[22]

The explanation advanced by Campanella presents some interesting ideas, although they are far from clearly formulated due to their complexity and the obscurity of the logic employed by the author. He finds that the mathematical point is not conceivable of itself, but only by analogy,[23] since according to Campanella (and, as will be seen below, even according to Gassendi and Hobbes) the point, in common with all other geometrical objects, does not exist in nature since it is no more than the outcome of a process of abstraction.[24]

After receiving Campanella's reply, Mersenne refers the question to Peiresc and Gassendi, and also commits himself to finding a response. Unfortunately, no trace of this response remains, but it is nevertheless interesting to consider the solution advanced by Ismaël Boulliau, which is found in one of his letters to Peiresc, since Gassendi states that the reply given by Boulliau was very similar to that of Mersenne.[25] More-

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[22] «Potest tame intelligi magnitudo super punctum quod esset corporis vel lineae extremum punctualiter tantùm existens. Non autem quod partes illius magnitudinis sint in illo puncto, sed extra, cùm non nisi in puncto tangatur solum a puncto; non autem a cognatis partibus quae sunt extra tactum puncti». *Ibid*.

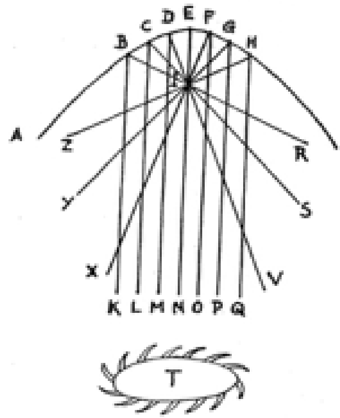
[23] On the process of analogy used by Campanella to justify geometrical concepts, see Hagengruber, *La fondazione del punto matematico nella filosofia di Tommaso Campanella* (cit. note 2), pp. 84 ff.

[24] On the strictly mathematical interpretation of the point in Campanella, see Ponzio, *Tommaso Campanella e la* Quaestio Singularis *di Jean-Baptiste Poysson* (cit. note 2), pp. 94 ff. Cees Leijenhorst has emphasised the convergence of the positions adopted by Hobbes and Campanella regarding the status of formal certainty in geometry (see Cees Leijenhorst, *Motion, monks and golden mountains: Campanella and Hobbes on perception and cognition*, «Bruniana & Campanelliana», III, 1997, pp. 93-121: 118). Emilio Sergio has emphasised that the genetic understanding of geometrical figures, and also the idea that they are conceived via a process of analogy, displays remarkable similarities to the standpoint assumed, later on, by Hobbes. Cf. Emilio Sergio, *Verità matematiche e forme della natura da Galileo a Newton*, Roma, Aracne, 2006, pp. 115-116. It is interesting to draw attention to the analogies between what Campanella writes in his *Metaphysica* and the works by Gassendi and Hobbes analysed below: «Dicunt (*the mathematicians*) lineas fieri ex fluxu puncti; superficiem ex fluxu lineae: corpus ex superficiei: sphaeram ex transitu dimidii circumvoluti circuli, quae quidem in natura non sunt: sed proponuntur nobis solum ad notitiam praeconcipiendam item linea de polo ad polum ducunt, impossibilem quidem re, sed imaginatione: quoniam trascendunt et penetrant animo corpora, ut possint illa metiri». Tommaso Campanella, *Metafisica*, Lib. V, Cap. II, Art. II, ed. by G. Di Napoli, Bologna, Zanichelli, 1967, 3 vols., vol. I, p. 372. More generally, on the mathematical understanding of Campanella see Hagengruber, *Tommaso Campanella* (cit. note 2), pp. 149 ff.

[25] Gassendi noted that the solutions offered by Boulliau and Mersenne were very close.

over, a significant innovation emerges in a letter written by Boulliau as the debate moves out of the strictly mathematical and physical domain into the optical. This represents a remarkable shift, which also occurs in Mersenne's *Harmonie Universelle* and, subsequently, in the works of Hobbes.

As we have stated, it was Boulliau who suggested that 'Poysson's problem' was, in fact, an idea formed by Mersenne, and the same Boulliau wrote to Peiresc that he was aware of the result arrived at by Gassendi, whom he had met by chance.[26] The author subsequently relies on his consideration of hyperbolic mirrors, in order to derive his solution to the question: «je luy *(i.e. to Mersenne)* donnay l'exemple des rayons du Soleil et de toute espece tombant sur le verre taillé en parfaicte et mathematique hyperbole, car il est certain qu'ils s'assembleront dans l'umbilic de la section en un poinct mathematique après la refraction».[27] The same reasoning may be applied to the parabola and it leads us to an assessment of the magnitude of the rays coinciding in a 'truly mathematical' point:



Car vous sçavez que toute ligne menee parallelement à l'axe tombant dedans la section, faict angles egaux à la touchante avec celle qui est menee de l'umbilic à la touchante, et qui rencontre la première parallele à l'axe, et ainsy toute l'espece[28] et tous les rayons se rassembleront en un poinct mathematique. Or et cette lumiere et cette espece de longueur et largeur, qui rassemblent dans le poinct mathematique sans confusion de parties, car après le poinct, lorsque [le] cone lumineux s'eslargira, alors les parties se verrons distinctement.

Il y a donc quelque magnitude qui a largeur qui se peut rencontrer en un poinct vrayment mathematique, qui n'ayt point de parties, et toutesfois dans

Thus, in December 1635 he writes to Mersenne that: «Hoc solum dico circa illam, cujus ipse authoris videris (et in quam etiam Bullialdus noster, ut accipio, inciderat) [...]». Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, p. 532. Cf. also de Waard's note, *ibid.*, pp. 288-289.

[26] Ismaël Boulliau to Nicolas-Claude Fabri de Peiresc, 15 November 1635, *CM*, vol. V, p. 474.

[27] *Ibid.*, p. 475.

[28] Here, Boulliau shows his support for the theory of species (*species*), which would, however, be contested by Mersenne and Hobbes.

icelluy la magnitude aura ses parties quantitatives les unes hors des autres et non confuses.[29]

Poysson's reflections on this problem encouraged Boulliau to continue his investigations in optics and the nature of light,[30] but our attention should remain focused on the passage cited. If we analyse the phenomenon of the refraction (or reflection, in the second case) of the sun's rays on a hyperbolic, or parabolic, surface, we notice that they are reflected into the point that constitutes the fire of the parabolic mirror *(see figure*[31]*)*. Hence, Boulliau's conclusion is that we must acknowledge that «il y a donc quelque magnitude qui a largeur qui se peut rencontrer en un poinct vrayment mathematique, qui n'ayt point des parties, et toutefois dans icelluy la magnitude aura ses parties quantitatives les uns hors des autres et non confuses».[32] Boulliau's explanation reveals a significant aspect of the problem, which concerns optics and, in particular, the relationship between geometrical optics and physical optics, a theme that would provide the focus of Hobbes' thinking.[33] As we will see below, this subject is highly conspicuous in Mersenne's *Harmonie Universelle*, which displays remarkable similarities with the thought of Hobbes. However, we must first analyse the answer given by an author whose understanding of geometrical entities is very close to that of Hobbes, namely Pierre Gassendi.

3. *1635*: Gassendi's solution

The solution suggested by Boulliau differs markedly from that of Gassendi, who had offered his opinions a number of times during the debate, in his letters to Mersenne and Peiresc. The first of these letters by Gassendi expresses itself in a quite enigmatic fashion,[34] to the extent

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29 *Ibid.*

30 Boulliau wrote a work on the subject: Ismaël Boulliau, *De Natura Lucis, authore Ismaele Bullialdo*, Parisiis 1638. Cf. *CM*, vol. V, p. 476 note. On Boulliau, see: Henk J.M. Nellen, *Ismaël Boulliau. Astronome, épistolier, nouvelliste, et intermédiaire scientifique*, Amsterdam, Holland University Press, 1994 (on *De Natura Lucis*, pp. 71 ff.).

31 The figure is taken from the work: *Astrologia gallica* (1661) by Jean Baptiste Morin, who had just addressed Poysson's problem in book IV, pp. 108-109). See also, *CM*, vol. VI, pp. 36 ff.

32 Ismaël Boulliaud to Nicolas-Claude Fabri de Peiresc, 15 November 1635, *CM*, vol. V, p. 475.

33 See Alan Shapiro, *Kinematics Optics: A Study of the Wave Theory of Light in the Seventeenth Century*, «Archive for History of Exact Sciences», 11, 1973, pp. 134-266: 160-161.

34 See *Pierre Gassend to Mersenne*, 2 November 1635, *CM*, vol. V, pp. 444-453. Gassendi had argued that the answer to the problem should coincide with Plato's 'chiasm' solution

that Mersenne asks him to go back to work on the question and express himself more clearly. However, before responding to him, Gassendi sends Peiresc his critical observations on the explanation advanced by Boulliau. He holds that it is logically untenable. Hence, if we consider the mathematical point to be unextended and indivisible, it is entirely reasonable for us to assert that there are an infinite number of geometrical lines passing through it, but this does not allow us to maintain that there are physical rays intersecting at this point, namely three-dimensional solids, as in Boulliau's contrasting claim.

[...] je ne voy point que si bien plusieurs lignes mathématiques, qui ne sont qu'en l'imagination, peuvent se rencontrer en un point mathématique, qui n'est aussi qu'une supposition des mathématiciens, toutesfois plusieurs lignes physiques, sensibles et corporelles, puissent se loger en un point mathématique et autre que physique, sensible et corporel, et par conséquent ayant tousjours quelque grandeur, quoyqu'imperceptible à noz *(sic)* sens. Et certes, je m'estonne un peu que ce brave homme (*scil. Boulliau*) ayant advoué auparavant que la lumière est une substance corporelle, il vueille après que plusieurs rayons, c'est à dire plusieurs corps, se rencontrent en un mesme poinct mathématique, c'est à dire penètrent et soient en mesme lieu; ce qui n'est pas possible par nature. [...] Je veux dire pour cela que là où le miroir brulant reunist beaucoup de rayons, il ne les confond point pour cela, et ne les réduit pas en un mesme point, mais en un plus petit espace, lequel certainment pourra estre pris par nostre sens pour un point, mais qui néantmoins sera tousjours en soy divisible en autant de parties qu'il y aura des rayons comprimez et reduits dans petite capacité [...].[35]

According to Gassendi, all geometrical figures are creations «of the imagination», in other words, abstractions produced by the mental functions.[36] The same idea is restated in the letter to Mersenne dated 13 December 1635, in which Gassendi elaborates an interesting argument concerning physical rays and parabolic mirrors.[37] He begins by reiterating

(*ibid.*, p. 445), referring to a rather hermetic passage in the *Timaeus* (Plato, *Timaeus*, 36 a ff.) in which the Greek philosopher describes the creation of the world as the work of god, according to the criteria of mathematical harmony.

35 Pierre Gassendi to Nicolas-Claude Fabri de Peiresc, 30 November 1635, *CM*, vol. V, pp. 508-509.

36 On Gassendi's analysis of Poysson's problem, see Stephen Gaukroger, *The Emergence of a Scientific Culture*, Oxford, Oxford University Press, 2006, pp. 268-271. More generally, on the Gassendian understanding of the point, see also Saul Fisher, *Pierre Gassendi's Philosophy and Science*, Leiden/Boston, Brill, 2005, pp. 224-231.

37 Gassendi also addresses parabolic and hyperbolic mirrors peripherally in his *Syntagma*.

that the mathematical point is merely a fiction or hypothesis and that nothing of this sort exists in nature.[38] Moreover, he uses the opportunity to present his own understanding of geometry,[39] conceived as a mere mental construct evolving from certain initial hypotheses, or first principles, a standpoint analogous to that adopted by Hobbes:

[...] the mathematicians also describe indivisible points and magnitudes, not because they come into contact with them, but in order that, when we are about to use points, lines or surfaces, we understand that we assign them names that are all the more appropriate since they are appropriate for the definitions that we ascribe to them. So, therefore, whatever proof we construct in relation to indivisible points, lines and surfaces, it is very clear that the physical points, lines and surfaces are an elaboration of things that never cast off the model of the divisible. [...] I am addressing this question simply so that you understand that I am not saying something absurd when I consider the points, lines and surfaces described by mathematicians to be pure hypotheses and that it may happen of things that do not exist.[40]

Gassendi, on the other hand, maintains that the sun's refracted rays are focused on a *physical*, and not mathematical, point which is accordingly endowed with extension:

I now maintain that the point in which the parabolically reflected or hyperbolically refracted rays converge will never be a mathematical point but

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See Pierre Gassendi, *Syntagma, Opera Omnia*, Lyon, Anisson & Devenet, 1658, 6 vols., vol. I, p. 431 b.

38 Cf. Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, p. 533.

39 On mathematics in Gassendi, see Bernard Rochot, *Gassendi et les mathématiques*, «Revue d'histoire des sciences et de leurs applications», 10, 1957, pp. 69-78; Tullio Gregory, *Scetticismo ed empirismo. Studio su Gassendi*, Roma-Bari, Laterza, 1961, pp. 46-47, pp. 72 ff., 158-161; Pierre Magnard, *La mathématique mystique de Pierre Gassendi*, in Sylvia Murr (ed.), *Gassendi et l'Europe*, Paris, Vrin, 1997, pp. 21-29.

40 «Mathematici puncta ac magnitudines describunt individuas, non sane quod taleis indigitent usquam, sed ut puncta aut lineas aut superficies usurpaturi, intelligamus tantò congruentius attribui illis haec nomina, quantò iis, quae de illis traduntur, definitionibus congruerint magis. Hinc quidquid de punctis, lineis et superficiebus individuis demonstratur, praeclare succedit quando explicantur physicis punctis, lineis ac superficiebus, quae non exuunt unquam rationem dividui; demonstrationesque in istis tanto veriores efficiuntur quanto sua tenuitate propius illis accesserint. Solent pari ratione illi hypotheseis statuere in rerum caelestium doctrina, quales tamen sic se habere, ut statuunt, non asserant, concentricos, epicyclos, deferenteis, aequanteis, et alia id genus; et faciunt tamen quod ex ipsis calculus mathematicus intelligatur procedatque. Sane cùm ad eandem aliqui assumant Telluris motum, caeteri quietem, quarum opinionum oportet falsam esse alterutram, vides tamen ut calculus ex hypothesi utraque texatur. Quod attingo solum ut intelligas nihil me dicere absurdi cum puncta, lineas et superficies a Mathematicis definitas pro meris habeo hypothesibus, quaeque fieri possint de rebus, quarum nulla sit existentia». Cf. Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, pp. 533-534.

only physical, because in a point with no parts there will therefore never be a magnitude with parts outside the parts.[41]

Thus, when it is stated that the reflected or refracted rays of the sun intersect at a single mathematical point, this is only what our senses are capable of perceiving. In reality, the physical rays converge in a space which, although minimal, is not a mathematical point at all, each physical ray occupying a space that, although microscopic, is still extended. In order to explain his theory, Gassendi uses an example; like loose hair spread over the shoulders and which, when gathered together, can be condensed to the space of a single digit «using a band», in the same way the physical rays are concentrated in a tiny space, but nevertheless occupy a specific point in space.[42] Therefore, if we take the smallest animal that we know of, such as a mite, it will still be composed of parts, and its legs will themselves be divisible into several microscopic parts.[43]

If we then consider a small space no larger than the leg of a mite where the rays are concentrated, even though it may equally be divided into millions of even smaller spaces, each ray will possess its own area not confused to the extent that each of the converging rays will occupy its own distinct little area.[44]

Gassendi, in conformity with his atomist orientation,[45] is far from assuming that matter is infinitely divisible into mathematical points, but

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[41] «Dico jam punctum in quod radij seu parabolicῶs reflexi, seu hyperbolicῶs refracti concurrant, non fore unquam mathematicum punctum, sed duntaxat physicum, quare et in puncto partium experte numquam fore magnitudinem parteis extra parteis habentem». Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, p. 534.

[42] *Ibid.*, p. 536.

[43] «Verum si magni, opinor, ducet, si focum vel umbilicum deducere potuerit ad exilitatem animalculi illius, quod *Acari* dicitur, vel decimae illius partis. Sanc hoc animalculum pro puncto pene est sensui, ac illius saltem portio decima tam minuta est, et simil puncto, ut nemo sit illam amplius divisurus. Porro cum vel resectus Acari pediculus habeatur sensui ac humanae industriae individuus, cogita tamen quam sit amplior ipsa natura subtilitas, quae resolvere illam potest in milliones aliquot particularum illarum, ex quibus ipsum contexuit». *Ibid.*, p. 536.

[44] «Itaque esto spatiolum non amplius pede Acari, in quod radij confluant; cum id pari jure possit dividi in milliones aliquot minorum adhuc spatiolorum, habebunt in eo radij singuli regiones suas inconfusas, adeo ut quotquot radij coibunt, habituri sint regiunculas inter se distinctas». *Ibid.*, p. 536.

[45] On the understanding of matter in Gassendi, see Olivier Bloch, *La philosophie de Gassendi: nominalisme, matérialisme et métaphysique*, The Hague, Martinus Nijhoff, 1971, pp. 210-229, Marco Messeri, *Causa e spiegazione: la fisica di Pierre Gassendi*, Milano, FrancoAngeli, 1985, pp. 74-93; Margaret J. Osler, *Divine Will and Mechanical Philosophy. Gassendi and Descartes on Contingency and Necessity in the Created World*, Cambridge,

takes the opposing view that there are final physical elements, the atoms, «for nature ultimately ceases somewhere and does not prolong the process of division and disaggregation to infinity».[46]

And do you see (it is not generally remarked) how narrow are the paths into which they are condensed, how they are compressed, how it can be that the parts of the sun, and every other thing that passes through a small hole or through the lens of an optical tube converge in such a way that their position, their colour and the variety of their parts is preserved? In general, however, we do not pay attention to the subtlety of nature and measure everything according to what the senses are able to perceive or the hand produce.[47]

Gassendi asserts that nature is not capable of being divided infinitely and, at the same time, that each physical ray occupies a specific space, almost wishing to treat every point occupied by each of the sun's rays as a sort of atom of light.

The theme addressing the nature of the point is also reopened, in passing, in the *Objections* raised by Gassendi to Descartes' *Méditations*. While criticising the possibility that the soul is joined to the body at a point, Gassendi asserts that the imaginary nature of the mathematical point differs substantially from that of the physical point.[48] In the latter, it is in fact possible for the nerves and the spirits of the animal flowing through the nerves to converge; «their convergence cannot, however, end in a mathematical point; for these are bodies and not mathematical

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Cambridge University Press, 1994, pp. 180 ff.; Gaukroger, *The Emergence of a Scientific Culture* (cit. note 36), pp. 262-276. Antonio Clericuzio has focused particularly on Gassendi's *materia actuosa*, a concept which – according to the author – distances the latter from a rigid mechanist perspective. See Antonio Clericuzio, *Elements, Principles and Corpuscles. A Study of Atomism and Chemistry in Seventeenth Century*, Dordrecht, Kluwer, 2000, pp. 63 ff.

46 «Nam natura aliquo usque tandem procedit neque divisionem resolutionemve in infinitur molitur». Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, p. 536.

47 «Et vides, quod vulgo id non advertatur, in quas cogantur angustias qui urgentur, quamobrem fiat ut trajectae solis caeterarumque rerum species foraminulo aut per lenteis optici tubi, ita decussentur, ut situs et color ac varietas partium ita perfecte conservetur? Sed nempe vulgo non attendunt ad subtilitatem naturae, et metiuntur omnia ex ijs quae vel sensus dispicere, vel manus potest elaborare». *Ibid.*, p. 537.

48 «An dices te cerebri partem pro puncto accipere? Incredibile sane; fed esto punctum. Si illud quidem Physicum sit, eadem remanet difficultas, quia tale punctum extensum est, neque partibus prorsus caret. Si Mathematicum, nosti primùm id nisi imaginatione non dari. Sed detur, vel fingatur potius dari in cerebro Mathematicum punctum, cui tu adjungaris, & in quo existas: vide quam futura sit inutilis fictitio». *Obiectiones Quintae*, in René Descartes, *Œuvres de René Descartes*, publiées par Charles Adam et Paul Tannery, Paris, Vrin, 1982-92 (henceforth *AT*), 11 vols., vol. VII, p. 340.

lines»[49] and Gassendi, when developing his argument, employs a vocabulary that is very close to that already found in his letters as well as in the letters of Campanella.[50]

The philosopher therefore distinguishes two radically different notions of the point – 'mathematical' and 'physical' –[51] each with a completely different ontological status. The mathematical point is accordingly either a mere creation of the mind or, inversely, the physical point (or atom) is a real entity, endowed with magnitude, occupying a space.

4. *1636-37*: Points and rays of light. The *Harmonie Universelle*

Father Mersenne's response to the 'Poysson's problem' has been lost and we know only that he had offered a solution similar to that of Bouliau.[52] Nevertheless, if we turn to the second volume of the *Harmonie Universelle*, we find traces of this explanation, where Mersenne addresses the issue: *De l'utilité de l'harmonie et des autres parties des mathématiques*. Here, he emphasises the importance for preachers of studying mathematics and addresses a burning question, namely, the possibility of explaining transubstantiation using an analogy with parabolic mirrors. The subject of the parabolic mirror had already been dealt with in the

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49 *Ibid.*, pp. 340-341.

50 «Et ut demus coire, spiritus per illos traducti exire e nervis aut subire nervos non poterunt, utpote cum corpora sint, & corpus esse in non loco, feu transire per non locum, cujusmodi est punctum mathematicum, non possit. Et quamvis demus esse, & transire posse, attamen tu, in puncto existens, in quo non sunt plagae dextra, sinistra, superior, inferior, aut alia, dijudicare non potes unde adveniant, aut quid renuncient». *Ibid.*, p. 341.

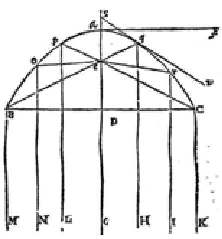
51 Even in the *Syntagma*, when he comes to address the subject of the point in relation to the composition of the *continuum*, Gassendi rejects the possibility of transferring the arguments, of a purely abstract nature, elaborated in the field of geometry into the realm of physics: «Quod dico autem eo sensu, ac fine, ideò est, vt intelligamus non licere perpetuò transferre in Physicam quicquid Geometrae abstractè demonstrant». Gassendi, *Syntagma* (cit. note 37), vol. I, p. 265. Again, when defining the concept of the atom he writes: «Adnotare autem lubet dici Ἄτομον, non ut vulgo putant, (& quidam eruditi interpretantur) quòd partibus careat, & magnitudine omni destituatur, sitque proinde aliud nihil, quàm punctum Mathematicum; sed quod ita solida, &, utita dicam, dura, compactáque sit; ut divisioni, sectionive, & plagae nullum locum faciat; seu quod nulla vis in natura sit, quae dividere illam possit». *Ibid.*, p. 256 b. See Gregory, *Scetticismo ed empirismo* (cit. note 39), pp. 158-160, Gaukroger, *The Emergence of a Scientific Culture* (cit. note 36), p. 269.

52 See Pierre Gassendi to Mersenne, 13 December 1635, *CM*, vol. V, p. 532. Cf. the note by de Waard, *ibid.*, pp. 288-289.

Impiété des Deistes,[53] but here it is approached from a completely different perspective:

> Les Predicateurs peuvent aussi user de ces figures pour exprimer les mysteres de la Foy, par exemple, pour monstrer qu'il est aisé de croire que le corps du Sauveur peut estre contenu sous chaque parcelle de l'hostie consacrée, puisque la plus grande estenduë de lumiere que l'on puisse s'imaginer peut estre reduite à un poinct par la glace du miroir parabolique qui reflechit tous les rayons paralleles dans son foyer, de sorte que nulle partie de lumiere ne peut frapper la glace, quoy qu'elle fust aussi grande que le firmament, qui ne soit contenuë dans le poinct du dit foyer. Et si l'on ajoûte que ce poinct lumineux envoye ses rayons sur toute la glace, & qu'il semble quasi se reproduire soy mesme autant de fois qu'il y a de parties & de poincts dans ladite glace, c'est à dire une infinité de fois, l'on aura un moyen d'expliquer comme un mesme corps peut estre en plusieurs lieux.[54]

The theme is analysed in greater detail in the first volume of the *Harmonie*, in which Mersenne recommends that light should be treated as an *accident*. This choice arose out of the need to counter a difficulty raised previously by Boulliau and Gassendi, namely, how to evaluate the concentration of the sun's rays, which are, accordingly, physical rays, at a point, coinciding with the fire of the parabolic mirror.



> Or encore qu'il soit tres-difficile de s'imaginer comment toute la lumiere qui passe par le plan BC, *(voir figure)* (quoy qu'on la suppose aussi large que le Ciel) peut estre rassemblée dans un point, attendu qu'il n'y a nul point dans ladite surface qui n'en soit couvert & rempli, & consequemment que ladite lumiere est continuë sans aucune pore & sans aucune vuide, & que ce rassemblement au point *e* ne se peut faire sans penetration d'une infinité de rayons qui se condensent iusques à l'infini, neantmoins il est ce me semble encore plus difficile de comprendre comment tout le solide de l'air qui va frapper la glace *aCB*, se reflechit au point *e*; car l'on peut dire que

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[53] Cf. Marin Mersenne, *Impiété des Deistes, Athées et Libertins*, Paris, Pierre Bilaine, 1624, t. II, pp. 169-174.

[54] Marin Mersenne, *Harmonie Universelle*, vol. II, Paris, Pierre Ballard, 1637, *(Livre De l'vtilité de l'Harmonie et des autres parties des Mathematiques)*, pp. 5-6. The consecutive pages up to *proposition IV* (*ibid.*, p. 37) describe the reflection and refraction of the sun's rays on hyperbolic and parabolic surfaces. Mersenne also refers here to the *miroir brûlant* «du R. P. Bonaventure Iesuate, Professeur des Mathematiques dans l'Université de Boulogne». (*ibid.*, p. 32). Mersenne refers to the work by Bonaventura Cavalieri: Bonaventura Cavalieri, *Lo specchio ustorio, overo Trattato delle Settioni Coniche, et alcuni loro mirabili effetti*, Bologna, Clemente Ferroni, 1632.

la lumiere est un accident qui n'est pas tellement determiné aux lieux, qu'il ne puisse occuper & couvrir tantost un plus grand lieu, & tantost un moindre: mais l'air est un corps, dont les differentes parties ne peuvent naturellement se penetrer: & bien qu'il eust une infinité de petits espaces vuides, neantmoins il ne peut estre reduit à un point comme la lumiere.[55]

Mersenne proposes to treat light as an 'accident' because he is faced with an apparently insuperable difficulty. If we consider light to be a physical being – in other words, like a body – or, conversely, if we suppose that it is coincident with the air, which is itself physical, it is difficult to comprehend how the rays of that light, which are physical in nature and therefore three-dimensional, can be reduced and concentrated in a mathematical point, the point that constitutes the fire of the parabolic mirror. Mersenne was not innovating in defining light as an accident,[56] but it is natural to ask what he meant by this term. In fact, if light is incapable of being contained by a physical body, and if it is not coextensive with the air either (as had already been emphasised by Aristotle[57]), then it must necessarily be identified with the movement propagated through the medium, as affirmed, moreover, by Descartes in his *Dioptrique*.[58]

5. *1640-43*: Optics and geometry in Hobbes

We have noted, in the first section, that during the years 1634-1636, Hobbes maintained close contact with Mersenne and his circle.[59] In this connection, it is interesting to observe the recurrence, in certain texts by Hobbes, of themes that had already been addressed by Mersenne, in *Harmonie Universelle*.

Hobbes addressed the issues affecting the relationship between geometrical optics and physical optics on a number of occasions, focusing particularly on the idea of the point, in order to elaborate an argument concerning the light ray. He took the opportunity to reflect on the na-

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55 Mersenne, *Harmonie Universelle* (cit. note 54), vol. I, Book I, p. 49.

56 Cf. Thomas Aquinas, *Scriptum super Libros Sententiarum*, Bologna, Edizioni Studio Domenicano, 2000, 10 vols., vol. III, p. 642.

57 Cf. Aristotle, *De Anima*, II (B), 7, 418b-419b.

58 Cf. Descartes, *La Dioptrique*, *AT*, vol. VI, p. 197.

59 On Hobbes and Mersenne, see: Gregorio Baldin, *Hobbes e Galileo. Metodo, materia e scienza del moto*, Firenze, Olschki, 2017, pp. 1-55.

ture of the ray,[60] stating in the *Tractatus Opticus II* that the term 'ray', if used inappropriately, could result in two types of errors:

> Given that the nature of the luminous body is such that its movement or action is diffused in every direction in straight lines, philosophers are accustomed to describing all of these lines using a term derived from the spokes of a wheel, ray, as if, since it is the source in A *(see figure)*, one ray is AB, another AC, and another AE etc. And, therefore, owing to the use of this term ray, they were liable to fall into two errors: the first, that the ray was a body; the second, that it was a mathematical line, in other words, as they maintain, a length without width.[61]

The first error consists in treating the ray as if it were a body; and indeed, Hobbes believes that light is not coincident with the environment through which the movement is propagated, but in fact coincides with the movement itself.[62] However, he asserts that this movement cannot

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[60] See Thomas Hobbes, *Tractatus Opticus I*, in *Thomae Hobbes Malmesburiensis Opera Philosophica quae Latine Scripsit Omnia*, ed. by W. Molesworth, London, Johannem Bohn, 1839-45, 5 vols. (henceforth *OL*), vol. V, pp. 221-222.

[61] «Cum natura Lucidi talis sit ut motus sive actio, ab ipso undique secundum lineas rectas diffundatur, solent Philosophi unamquamque dictarum rectarum, vocabulo sumpto a radiis Rotarum, appellare Radium, ut si Lucidum sit in A, unus radius sit AB, alter AC, alter AE, etc. Pronum autem erat ab illa raddi appellatione, in duos incidere errores: unum, quod Radius esset Corpus; alterum, quod esset linea Mathematica, hoc est, ut putant, Longitudinem sine Latitudinem». Thomas Hobbes, *Tractatus Opticus II*, chap. II, § 1 (British Library, Ms. Harley 6796, ff. 193-266*r* et *v*), f. 204*r* (first complete edition by Franco Alessio, Franco Alessio, *Thomas Hobbes: Tractatus Opticus*, «Rivista critica di storia della filosofia», XVIII, 1963, pp. 147-228: 159-160).

[62] «Cum vero directa haec motûs a lucido propagatio, non sit ipsum Corpus per quod motus propagatur (nam differentia magna est jnter ipsum aerem et motum in aerem) neque aliud corpus praeter ipsum, non potest radius lucis dici corpus, ut radius rotae ligneae lignum, sed tantum via motûs propagati». Hobbes, *Tractatus Opticus II* (cit. note 61), chap. II, § 3, f. 20/p. 161. On the Hobbesian understanding of light, see Jean Bernhardt, *Hobbes et le mouvement de la lumière*, «Revue d'histoire des sciences», 30, 1977, pp. 3-24; Id., *L'œuvre de Hobbes en optique et en théorie de la vision*, in Bernard Willms *et alii*, *Hobbes Oggi*, Milano, FrancoAngeli, 1990, pp. 245-268; Elaine C. Stroud, *Thomas Hobbes' A Minute or First Draught of the Optiques: a critical edition*, PhD Dissertation, University of Winsconsin-Madison, 1983, Id., *Light and Vision: Two Complementary Aspects of Optics in Hobbes' Unpublished Manuscript* A Minute or First Draught of the Optiques, in *Hobbes Oggi* (cit. note 62), pp. 269-277, José Médina, *Nature de la lumière et science de l'optique chez Hobbes*, in Christian Biet – Vincent Julien (eds.), *Le siècle de la lumière 1600-1715*, Paris, ENS Éditions, 1997, pp. 33-48; Jan Prins, *Hobbes on light and vision*, in Tom Sorell (ed.) *The Cambridge Companion to Hobbes*, Cambridge, Cambridge University Press, 1996, pp. 129-156 and, in particular, Franco Giudice, *Luce e visione. Thomas Hobbes e la scienza dell'ottica*, Firenze, Olschki, 1999, Antoni Malet, *The Power of Images: Mathematics and Metaphysics in Hobbes's Optics*, «Studies in the History

be propagated without a medium that is necessarily coincident with a physical entity endowed with dimensions.

The second error, on the other hand, consists in identifying the ray with a mathematical line, having length only, and no width, an impossible and contradictory phenomenon according to Hobbes, since the movement of the light, like any other movement, presupposes a body and a physical space.

In the same paragraph, the philosopher seizes the opportunity to compose a digression on geometrical figures, which, like rays, have dimensions, although these dimensions are not taken into consideration in the mathematical proof.[63]

Nevertheless, before analysing the Hobbesian understanding of geometrical entities in greater detail, it should be emphasised that Hobbes had addressed the same problem raised by Mersenne in *Harmonie Universelle* concerning the nature of light. Light cannot be a body, nor is it coincident with the air, but rather coincides with the movement through which it is propagated through the air. The ray should not, however, be conceived as completely lacking in dimensions, since no movement may be propagated without the existence of a medium, which is of necessity physical and corporeal. This explains the necessity of identifying the ray with a mathematical line, and the requirement to take into account its physical, and not merely its geometrical, nature.

Based on these reflections on optics, Hobbes elaborates his interesting observations on the nature of geometrical entities, from which we are able to deduce not only the close link between geometry and optics, but also the essentially 'physical' nature of Hobbesian geometry. As we have already seen, in the *Tractatus Opticus II*, while addressing the light ray, Hobbes maintains that the *point*, the *line* and the *surface* are not dimensionless, despite the fact that these dimensions are not dealt with in the geometrical proof. This view is shared by Gassendi, who, in his *Objections* to the *Méditations*, had asserted his belief that it would be «false and imaginary» to conceive the nature of the triangle as an entity

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and Philosophy of Science», 32, 2001, pp. 303-333: 319-320. Malet argues that the Hobbesian optics must be considered in relation to Hobbes' metaphysics. At the same time, he maintains that the possibility of integrating geometry into physics – in other words, the creation of a mathematical physics, in the manner of Galileo – was never part of Hobbes' philosophical project. On this point, my position differs sharply from that of Malet: cf. BALDIN, *Hobbes e Galileo* (cit. note 59), pp. 100 ff.

[63] «[...] neque dicitur aliquid punctum vel linea, vel superficie mathematica propterea quod dimensionibus careat, sed quia in argumentum non assumuntur». Hobbes, *Tractatus Opticus II* (cit. note 61), chap. II, § 3, f. 205*r*/p. 161.

«in which it is assumed that it is composed of lines of no width, that it contains a space without depth, and that it terminates in three points without any parts».[64]

This theme recurs in *De motu, loco et tempore*, where Hobbes re-examines the question in the context of his dispute with Thomas White regarding the problem of infinity. White had established an analogy between mathematical infinity and theological infinity,[65] and Hobbes takes the opportunity within his critique to develop an interesting analysis of the nature of geometrical entities. Extending the argument of Sextus Empiricus,[66] he states that the *line*, the *surface* and *body* (or *solid*), are not contained in each other, as a smaller magnitude is contained in a larger or infinite magnitude. But a certain approach would be required when measuring geometrical objects:

> However, three methods were to be used in the measurement of each body, the first called length, the second width, the third volume; the first method was that traced by the movement of a body whose quantity is not taken into consideration, and the geometers used it to identify this body, with a small mark, which the Greeks call "σ[τ]ιγμήν"and "κέντρον", and the Latins, "point", and not because the point is so small that it has no quantity, but because they wanted the quantity to appear so small that it could be deemed to be absolutely non-existent.[67]

The same line of reasoning was applied to the «fine line that the Greeks call γραμμήν, which is consequently not devoid of length, but

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[64] «Dicendum hîc praeterea foret de falsa illâ trianguli naturâ, quae supponitur constare ex lineis, quae latitudine careant, continere aream, quae profunditate, terminari ad tria puncta, quae omnibus partibus. Attamen nimium evagaremur». *Objectiones Quintae, AT*, vol. VII, p. 322.

[65] «Quo exemplo hoc unum clarum factum video, nimirum ipsum aeque intelligere quid sint linea, superficies, et corpus, et quid sint substantiae abstractae, nec magis vidisse quomodo lineam superficie, superficies incorpore esse dicatur, quam quomodo substantia incorporea contineat corporeas, ut eas valeat». Thomas Hobbes, *Critique du* De Mundo *de Thomas White*, ed. by J. Jacquot and H. Whitmore Jones, Paris, Vrin, 1973 (henceforth *MLT*), II, 8, p. 114. Cf. Thomas White, *De Mundo Dialogi tres*, Paris, Moreau, 1642, p. 21.

[66] Cf. Sextus Empiricus, *Adversus mathematicos*, Paris, apud Martinum Iuvenem, 1569, p. 76.

[67] «Sed cum in omni corpore mensurando triplici via eundum erat, 1a longitudo, 2a latitudo, 3a crassities appellabatur, primavia erat quam designat motu suo corpus aliquod cuius quantitas non consideratur, ideoque u[s]i sunt Geometrae ad tale corpus designandum, notâ parvulâ, quam Graeci σ[τ]ιγμήν et κέντρον, latini punctum vocant, non quod punctum ità exiguum esse possit ut nullam habeat quantitatem, sed quod quantitatem apparere ibi minimam voluerunt ubi nulla omnino erat consideranda». Hobbes, *MLT*, II, 8, pp. 114-115.

considered as such».[68] It is produced by the movement of a point, as well as the surface, which is produced by the movement of a line and the body delineated in turn by the movement of the surface. It follows that:

> The line and the surface are not things contained in the body being measured, like the parts, but like the different considerations adopted by those who measure, and for this reason they are called mathematical, or theoretical, owing to the fact that these theoretical terms are useful to the theory used by the geometers, who wished to heed the definitions of lines and surfaces, and who wished to ensure that the fine surfaces of the figures (which had to be drawn into the material because they were required for their proofs), existing in the physical lines, were not entered into the calculation of the surfaces, and that the outer layer of the bodies whose surfaces they were measuring were not counted among the parts of those solids. Once this is understood, it is easy to see that the line is not included in the surface, nor the surface in the body, and that the length of the lines is considered without width, as everyone is accustomed to doing, and that both are conceived as having no thickness, as in the case of someone who sells a field without measuring its depth.[69]

The view of geometry presented by Hobbes is, in certain respects, similar to that of Gassendi, although the latter emphasises, to a far greater degree than the English philosopher, the imaginative nature of the mathematical sciences.[70] Nevertheless, both authors consider them to be «theoretical sciences»;[71] in other words, abstract out of doctrinal necessity, even though they are essentially derived from a physical substrate.

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[68] «exili linea quam Graeci γραμμήν dicunt, quae quidem sine latitudine non est, sed consideratur». *Ibid.*, p. 115.

[69] «Sunt itaque linea et superficies non res aliqua contenta in corpore mensurato, ut partes, sed ut considerationes diversae mensurantium, ideoque dicuntur mathematica, id est doctrinales, ppropterеà quod eae voces doctrinae tantum causâ à Geometris adhibitae sunt, qui definitionibus lineae et superficiei cavere voluerunt ne figurarum (quas demonstrationibus suis necessarias in materia exarare oportebat) exiles superficies in lineis materialibus existentes, ponerentur incalculo superficierum, aut pelliculae corporum quorum superficies mensurabant, inter solidorum ipsorum partes numerarentur. His intellectis, facile est agnoscere lineam in superficiem, et superficiem in corpore non comprehendi, sed viarum longitudinem sine latitudine considerari, ut fieri solet ab omnibus, et utrumque sine crassitie, quaemadmodum faciunt qui vendunt agrum, nullâ factâ profunditas mensionem». *Ibid.*

[70] Cf. for example, GASSENDI, *Exercitationes Paradoxicae adversus Aristoteleos. Liber II*, in *Opera Omnia* (cit. note 37), vol. III, p. 209a.

[71] *Ibid.* Hobbes relationship with mathematics was first considered by Wolfgang Breidert (see WOLFGANG BREIDERT, *Les mathématiques et la méthode mathématique chez Hobbes*, «Revue internationale de philosophie», 129, 1979, pp. 414-431, followed by GIULIO GIORELLO, *Pratica geometrica e immagine della matematica in Thomas Hobbes*, in WILLMS *et alii, Hobbes Oggi* (cit. note 62), pp. 215-255, HARDY GRANT, *Hobbes and mathematics*, in SORELL (ed.), *The*

Hobbes, however, complements this with his own genetic and constructive interpretation of geometry, according to which a solid is an abstract figure produced by the movement of a surface, just as a surface is generated by the movement of a line, and the latter, in its turn, is generated by the movement of a point. Nevertheless, the philosopher makes it clear that due to their very nature, it is not possible to state that the point is contained within the line, nor that the line is contained in the surface nor the surface in the solid, as parts are contained within a physical body, for example.

The notion that a solid is created by the movement of a surface, a surface by the movement of a line and the latter, lastly, by the movement of a point, was shared by several authors,[72] including Mersenne, who, in his *Harmonie Universelle* states: «il n'y a rien au monde corporel qui ne dépende du point, puis que son flux ou mouvement engendre la ligne, que par le mouvement de la ligne la surface est produite, & que le corps est fait par le mouvement de la surface».[73] Another important aspect should, however, be borne in mind: although he assigns geometry the status of an abstract science, Hobbes argues, like Gassendi, that the entities with which this science is concerned have a close relationship with the only objects existing in the world, namely bodies. Geometry as a whole, and all geometrical figures, are in fact formed – by a process of abstraction – out of the physical world. And yet, when the point is treated as a geometrical figure, its

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Cambridge Companion to Hobbes (cit. note 62), pp. 108-128, and the important study of the dispute between Hobbes and Wallis by Jesseph: Douglas M. Jesseph, *Squaring the Circle. The War between Hobbes and Wallis*, Chicago and London, University of Chicago Press, 1999. See also, Emilio Sergio, *Contro il Leviatano. Hobbes e le controversie scientifiche*, Soveria Mannelli, Rubbettino, 2001, pp. 87-226; Sergio, *Verità matematiche e forme della natura* (cit. note 24), pp. 207-254, Médina, *Mathématique et philosophie chez Thomas Hobbes* (cit. note 2); Katherine Dunlop, *Hobbes's Mathematical Thought*, in Aloysius P. Martinich – Kinch Hoekstra (eds.), *The Oxford Handbook of Hobbes*, Oxford, Oxford University Press, 2016, pp. 76-105.

[72] See *supra*, in paragraph 3, Campanella's position in this regard. It must nevertheless be emphasised that Campanella's epistemological standpoint is far removed from that of Hobbes, as far as the epistemological status of mathematics and the sciences is concerned. Regarding the latter, he asserts that «Itaque principia scientiarum sunt nobis historiae. Historiam dico etiam, quod non ab alio audivimus, sed nostris patuit oculis et sensibus: ex eo enim, quod patet historice, ad investigandum quod latet proficiscimur». Campanella, *Metafisica* (cit. note 24), I, p. 367. With regard to mathematics, he openly asserts that this discipline is not a science, thereby distancing himself significantly from Galileo and Hobbes. On the development of a genetic interpretation of geometry in the thought of Hobbes, see: Aldo G. Gargani, *Hobbes e la scienza*, Torino, Einaudi, 1971, pp. 197-208, Jesseph, *Squaring the Circle* (cit. note 71), pp. 73-85, Sergio, *Contro il Leviatano* (cit. note 71), pp. 113-125; Sergio, *Verità matematiche e forme della natura* (cit. note 24), pp. 212-228.

[73] Mersenne, *Harmonie Universelle* (cit. note 54), vol. II, *(De l'utilité de l'Harmonie)*, Book III, pp. 26-27.

extension is not considered and as a result it is viewed as a being of reason. It is nevertheless a body of a specific magnitude, albeit tiny or negligible.[74]

At the same time, in *De motu, loco et tempore*, Hobbes emphasises that it is absolutely inaccurate to view a line as having width and a surface as a collection of lines since «the lines, regardless of their number, arranged by width, do not correspond to any surface; in fact, since each line considered individually has no width, it is also the case that all of the lines taken together, as they are, still have no width because the sum of nothing multiplied any number of times will always be nothing».[75] This, then, is where the crucial question emerges:

> Since the surface has a certain width and, on the other hand, the lines have none, how can we say that the surface is equal to the lines? The same must be said of surfaces in relation to the solid; we cannot draw a line without it having a certain width, and it may therefore be considered equivalent to lines drawn on paper, but not to an infinite number of lines, insofar as they are fine, as these cannot properly be described as lines, but as plane figures. In reality, line refers solely to length, and, even though it does not exist without width, it must however be treated as having no width, as it is in practice conceived by geometers, out of doctrinal necessity.[76]

Hobbes therefore considers it impossible to imagine the actual existence of an object without any dimensions, whether it be a point or a line. According to the philosopher, the concepts of '*res*' and '*corpus*' are coincident and, as a consequence, each object can only exist in a physical form.[77] However, within the realm of geometrical science, which relates to abstract objects, the real or physical dimension of these objects is not taken into account and they are therefore conserved as such out of «doctrinal necessity».

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[74] See Giorello, *Pratica geometrica e immagine della matematica in Thomas Hobbes* (cit. note 71), pp. 225-226; Grant, *Hobbes and mathematics* (cit. note 71), p. 112; Jesseph, *Squaring the Circle* (cit. note 71) pp. 76-77.

[75] «Lineae enim quotcunque, dispositae in latitudinem nullam omnino superficiem adaequant; cùm enim unaquaeque linea sigillatim sine latitudine sit, etiam omnes simul quotcunque fuerint sine latitudine erunt, quia ex nihilo quotiescunque numerato summa effecta erit nihil». Hobbes, *MLT*, XXVII, 21, p. 329.

[76] «Cum ergo superficies latitudinem habeat aliquam, lineae nullam, quomodo dicetur superficies lineis aequalis? Idem dicendum de superficiebus comparatis cum solido; pingi quidem linea sine latitudine aliqua non potest, sed non infinitis utcunque exiles sunt, neque sunt eae propriè loquendo lineae, sed figurae planae linea enim significat meram longitudinem, quae licet nunquam existat sine latitudine, potest tamen sine latitudine considerari, quemadmodum doctrinae causa consideratur a geometris». *Ibid*.

[77] Cf. Hobbes, *Objectiones ad Cartesii Meditationes, Objectio quarta*, in *OL*, vol. V, p. 258.

6. *1655-1672*: Hobbes and the mathematical point

Hobbes re-examines this subject in *De Corpore*, where he presents the same genetic interpretation of geometry. It is described as an abstract science whose objects nevertheless maintain a close connection with the physical world, which is inhabited by bodies in motion:

[...] if, when any body is moved, the magnitude of it be not considered, the way it makes is called a *line*, or one single dimension; and the space, through which it passeth, is called *length;* and the body itself a *point*, and the way of its yearly revolution, the *ecliptic line*.[78]

The same description applies to a '*surface*' and a '*solid*':

But if a body, which is moved, be considered as *long*, and be supposed to be so moved, as that all the several parts of it be understood to make several lines, then the way of every part of that body is called *breadth*, and the space which is made is called *superficies*, consisting of two dimensions, one whereof to every several part of the other is applied whole. Again, if a body be considered as having *superficies*, and be understood to be so moved, that all the several parts of it describe several line, then the way of every part of that body is called *thickness* or *depth*, and the space which is made is called *solid*, consisting of three dimensions, any two whereof are applied whole to every several part of the third.[79]

The Hobbesian interpretation of the mathematical point was not received at all favourably by his contemporaries. In the same year in which the first edition of *De Corpore* appeared, John Wallis, Professor of Mathematics at Oxford, published *Elenchus Geometriae Hobbianae*, a work providing a detailed examination and critique of the principles of Hob-

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78 «Si corporis movetur magnitudo (etsi semper aliqua sit) nulla consideretur, via per quam transit, *linea*, sive *dimensio una* et *simplex* dicitur, spatium autem quod transit *longitudo*, ipsum corpus *punctum* appellatur; eo sensu quo terra *punctum*, et via ejus annua linea eccliptica vocari solet». Hobbes, *De Corpore*, VIII, 12, in *OL*, vol. I, pp. 98-99, Engl. Transl. in *The Collected English Works of Thomas Hobbes*, ed. by W. Molesworth, London, 1839-1845, 12 vols. (henceforth *EW*), vol. I, p. 111.

79 «Quod si corpus, quod movetur, consideretur jam ut *longum*, atque ita moveri supponatur, ut singulae ejus partes singulas lineas conficere intelligantur, via uniuscujusque partis ejus corporis *latitude*, spatium quod conficitur *superficies* vocatur, constans ex duplici dimensione *latitudine* et *longitudine*, quarum altera tota ad alterius partes singular sit applicata. Rursum si corpus consideretur ut habens jam *superficiem*, et ita intelligatur moveri, ut singulae ejus partes singulas conficiant lineas, uniuscujusque partis via corporis illius *crassities* seu *profunditas*, spatium quodconficitur *solidum* vocatur, conflatum ex dimensionibus tribus, quarum quaelibet duae totae applicantur ad singulas partes tertiae». *Ibid.*, p. 99; *EW*, vol. I, p. 111.

besian geometry.[80] The subject of the dispute between Hobbes and Wallis has already been extensively investigated.[81] It is nevertheless worth mentioning the question in order to elaborate on the observations made with regard to the Hobbesian understanding of the point. In fact, one of the most significant objections levelled by Wallis at Hobbes relates precisely to the nature of the point. He emphasises the difficulty of reconciling the Hobbesian interpretation with the Euclidian definition of the point, highlighting what appears to him to be a blatant contradiction. Hobbes did indeed assert the materiality of the real point, but, at the same time demanded that this dimension should not be considered in the case of geometrical figures.[82]

Hobbes reacted to the criticisms offered by Wallis by publishing *Six Lessons to the Professors of Mathematics*, the following year, but his reply failed to resolve the difficulties. He begins his discussion by examining the Euclidian axioms: «the first is point: Σημεῖον, &c. *"Signum est, cujus est pars nulla"*, in other words, a mark is that of which there is no part».[83] Based on this definition, he asserts that his concept of the point is consistent with the Euclidian tradition since for Euclid the point is literally a sign, and must therefore also be visible and potentially divisible into innumerable parts. Rather, that which is indivisible does not have quantity and is, as a result, '*nihil*'.[84] However, this definition of the point as potentially divisible appears to contradict the principles of geometry and Hobbes is consequently forced to explain that the 'quantity' of the point is not taken into consideration as part of the mathematical proof.[85]

80 See John Wallis, *Elenchus Geometriae Hobbianae*, Oxford, Johannis Crook, 1655.

81 See Jesseph, *Squaring the Circle* (cit. note 71); Sergio, *Contro il Leviatano* (cit. note 71), pp. 87-226.

82 «At quaero' num interim consideretur esse corpus? Si non, quorsum corporis mentio in definitione, vel saltem cur ipsa corporeitas non pariter excludatur, atque ipsa magnitudo? Cur non igitur sic definis, *Punctum est corpus quod non consideratur esse corpus, & magnum quod non consideratur esse magnum?*». Wallis, *Elenchus Geometriae Hobbianae* (cit. note 80), p. 7. See also Jesseph, *Squaring the Circle* (cit. note 71), p. 79. On the concept of the point in Hobbesian mathematics, see Grant, *Hobbes and mathematics* (cit. note 71), p. 112 ff. Wallis and Hobbes appear to use two different conceptions of the indivisible since Hobbes preserves the classic meaning of the term, as found in Cavalieri, whereas in Wallis the term is closer to the Leibnizian infinitesimal. Cf. Malet, *From Indivisibles to Infinitesimals* (cit. note 10).

83 Hobbes, *Six Lessons*, *EW*, vol. VII, p. 200.

84 «A mark or as some put instead of it σίγμη, [*scil.* στιγμή] which is a mark with a hot iron, is visible; if visible, then it hath quantity, and consequently may be divided into parts innumerable. That which is indivisible is no quantity; and if a point be not quantity seeing it is neither substance nor quality, it is nothing». *Ibid.*, pp. 200-201.

85 «[N]o argument in any geometrical demonstration should be taken from the division, quantity, or any part of a point; which is as much as to say, a point is that whose

Finally, the author presents a more precise definition of the point, treating it, not as '*indivisible*', but rather as '*undivided*'.[86] Although division consists of nothing more than an action of the mind, the point can easily be retained when deprived of its parts, it being sufficient to *conceive* it as such.[87]

In *De Principiis et Ratiocinatione Geometrarum* (1666), Hobbes once again analyses the definitions provided by Euclid and begins, logically, with the definition of the point. Having reaffirmed that the correct term is '*undivided*', and not *indivisible*, the author explains why the point cannot be conceived as indivisible.[88] If the point were *indivisible*, «in other words, *non-quantum*», it would literally be '*nihil*'[89] and would, therefore, form lines and surfaces which would themselves be non-quantities. From this, it would be possible to draw the conclusion that all these of figures are '*aequalia*' in relation to each other.[90] Hobbes thus maintains that the point is potentially divisible and seeks to demonstrate this; if a line is divided into two parts, we will have an end at the end of each of

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quantity is not drawn into demonstration of any geometrical conclusion; or, which is all one, whose quantity is not considered». *Ibid.*, p. 201.

86 «An accurate interpreter might make good the definition thus, *a point is that which is undivided*; and this is properly the same with *cujus non est pars*: for there is a great difference between *undivided and indivisible*, that is, between *cujus non est pars*, and *cujus non potest esse pars*. Division is an act of understanding; the understanding is therefore that which maketh parts, and there is no part where there is no consideration but of one». *Ibid.*

87 Hobbes expresses the same idea in *Examinatio et Emendatio Mathematicae Hodiernae* (1660), also written in the context of his dispute with John Wallis. See *OL*, vol. IV, pp. 55-56.

88 On Hobbes' treatment of the indivisibles, see Paolo Mancosu – Ezio Vailati, *Torricelli's Infinitely Long Solid and Its Philosophical Receptions in the Seventeenth Century*, «Isis», 82, 1991, pp. 52-70: 67-68, and Douglas M. Jesseph, *Of analytics and indivisibles: Hobbes on the modern mathematics*, «Revue d'histoire des sciences», 46, 1993, pp. 153-193. See also Jesseph, *Squaring the Circle* (cit. note 71), pp. 185-189. On Cavalieri's influence on Hobbes' mathematics, see also Id., *Hobbes on the Ratios of Motions and Magnitudes. The Central Task of* De Corpore, «Hobbes Studies», 30, 2017, pp. 58-82: 76 ff.

89 «[...] si punctum sit indivisibile, carebit linea omni latitudine: et quia nihil est longum quod non habeat latitudinem, erit linea plane *nihil*. Quanquam enim longitudo lata non sit, longum tamen omne latum est. Videtur etiam Euclidem ipsum in ea opinione fuisse, punctum, quanquam partem actu non habeat, potentia tamen divisibile esse et quantitatem: alioqui non postulasset a puncto ad punctum duci posse lineam rectam: quod impossibile est, nisi linea habeat latitudinem aliquam». Hobbes, *De Principiis et Ratiocinatione Geometrarum*, *OL*, vol. IV, p. 391.

90 «Praeterea si *punctum* indivisibile esset, id est, *non quantum*, id est, nihil: sequetur (supposito, ut nunc supponunt scriptores mathematici, quantitatem dividi in infinitum, ut punctum sit pars lineae infinite exigua) parte infinite exiguam lineae rectae, et quadratum quod sit minima pars quadrati, et cubum qui sit minima pars cubi, esse inter se aequalia». *Ibid.*, pp. 391-392. See Jesseph, *Of analytics and indivisibles* (cit. note 88), p. 187; Sergio, *Contro il Leviatano* (cit. note 71), pp. 154 ff.

the two parts, namely two points. However, the point '*dividens*' can be divided into two quantities (*quantitates)* only when it is considered to be a *quantitas*; since it has been divested of quantity, it will be '*nihil*'.[91] In this manner, Hobbes succeeds in defining a geometry that is entirely dependent on physics and its fundamental elements: the body and motion.[92] The impossibility of imagining a point of absolutely zero magnitude draws attention to the indissoluble link between geometry and the real quantities of physics. The only possible way to conceive geometrical figures is identical to the method we use in measuring bodies since we are unable to envisage a geometrical quantity that is not physically represented by a visible body. This constructive and, at the same time, *corporeal* interpretation provided by Hobbesian geometry is integral to the objective of avoiding «the cavils of the sceptics» and especially, to the goal of rationally proving a number of propositions and the very foundations of geometry, which remained otherwise indemonstrable.[93]

Equally, in *Rosetum Geometricum* (1671), which includes *Censura Brevi Doctrinae Wallisianae De Motu* as an appendix, Hobbes dedicates a great deal of attention to the *continuum*, which was held by Wallis to consist of an infinite number of indivisibles.[94] This is, according to Hobbes, a

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91 «Verum sive ita senserit Euclides, sive aliter, manifestum est punctum divisibile esse, ex eo quod, secta linea in duas partes, habebit utraque pars duos terminos, id est, duo puncta extrema: et per consequens punctum dividens secatur si quantitas sit, in duas quantitates; si nihil sit, in duo nihila. [...]». Hobbes, *De Principiis et Ratiocinatione Geometrarum*, *OL*, vol. IV, p. 391.

92 See Giorello, *Pratica geometrica e immagine della matematica in Thomas Hobbes* (cit. note 71), pp. 225-226. The presence in Hobbes of a geometry characterised by its dependency on physics has recently been emphasised by Médina: Médina, *Mathématique et philosophie chez Thomas Hobbes* (cit. note 2), pp. 96-97. On this question, see also the analysis of the sixth paragraph of chapter XX of *De Corpore* performed by Sergio: Sergio, *Contro il Leviatano* (cit. note 71), pp. 125-147.

93 «I treat of geometry, I thought it necessary in my definitions to express those motions by which lines, superficies, solids, and figures, were drawn and described, little expecting that any professor of geometry should find fault therewith, but on the contrary supposing I might thereby not only avoid the cavils of the sceptics, but also demonstrate divers propositions which on other principles are indemonstrable». Hobbes, *Six Lessons*, *EW*, vol. VII (the epistle dedicatory), pp. 184-185. See Jesseph, *Squaring the Circle* (cit. note 71), pp. 81-82.

94 «"Continuum", inquit, "quodvis, secundum Cavallerii *Geometriam Indivisibilium*, intelligitur ex indivisibilibus numero infinitis constare". Quod deinde explicans, "Hoc est", ait, "ex particulis homogeneis infinite exiguis, numero infinitis, ut linea ax infinitis punctis, hoc est, lineolis infinite exiguis, longitudine aequalibus", etc. Miraberis fortasse tu juxta quam logicam vox *Cavallerius* intrare potest in definitionem quantitas continuae». Hobbes, *Rosetum Geometricum, cum Censura Brevi Doctrinae Wallisianae De Motu*, *OL*, vol. V, p. 85.

misleading definition since each *continuum* should, according to his own definition, always be divisible into parts which are themselves always divisible.[95] As the *continuum* is always divisible, if it were composed of infinite indivisibles, in other words, innumerable parts of no quantity, this would lead to it being composed of 'nihil' of infinite number, with the result that it would also be a 'nihil'.[96] Moreover, how could there be commensurable quantities if the continuum were composed of these indivisibles? In the work *Lux Mathematica* (1672), it is the same definition of indivisible as material *ultimum* that is questioned by Hobbes since it represents a limit to the material divisibility inherent in the very notion of a *continuum*.[97]

It should be remembered that, for Hobbes, division is a mental activity and therefore an operation of the intellect carried out over a continuum. The philosopher maintains that there is no idea of the infinite since the human mind is entirely incapable of mastering such a concept.[98] According to Hobbes, it is only possible to have a negative concept of the infinite, which can be used to refer to something incomplete or without limits. Hobbes explains this clearly in the *Principia et Problemata aliquot Geometrica* (1673), in which he analyses the instances where the word is habitually employed:[99] «*Infinite*, however, is used by the mathemati-

[95] «Definitio falsa est, et ex ea sequitur, primo, quantitatem continuam omnino nulla esse. Continuum enim omne divisibile est (ut fatentur geometrae omnes, nec Wallisius negat) in semper divisibilia: et propterea pars infinitissima continui nulla est. Quod autem ex nihil componitur etsi numero infinitis, nihil est: et per consequens, juxta definitione, hanc, continuum nullum est. Secundo, si continuum sit aggregatum, ut ille dicit, ex indivisibilibus, ubi est quantitas discreta? Nam, numerus nihilorum numerus non est: quia numerus numerum additus, vel in numerum multiplicatus, fit major. Sed nihil neque additione nihilorum neque multiplicatione ulla augeri potest, nec divisione minui». *Ibid*.

[96] The same observation may be found in the work *Lux Mathematica* (1672), *OL*, vol. V, p. 115.

[97] «Postremo, quid tam contra lumen naturale immediatum esse potest aut absurdum, quam in serie quantorum infinita datum dicere esse ultimum: aut in publico professore puerilius, quam per *infinitum* intelligere se dicere *indefinitum* aut *quantum est possibile*, cum si sic dixisset id quod demonstrare susceperat demonstare non potuisset? Neque vero istius *quantum est possibile*, sive *indefinitum* datur ultimum. Datum enim non est, quod non expositum est et cognitum. Quantum rerum Conditori possibile est, infinitum est. Quosque autem homo dividere potest ignotum est. Quare ejus quod potest *ultimum* nec datur, nec dari potest». *Ibid*., pp. 149-150.

[98] Hobbes asserts this in almost all of his major works, beginning with his objections to the *Méditations*. See Hobbes, *Obiectiones Tertiae*, *OL*, vol. V, p. 265; see also Id., *MLT*, II, 8, p. 114; Id., *Leviathan*, ed. by N. Malcolm, Oxford, Oxford University Press, 2012, 3 vols., vol. II, pp. 46-47; Id., *De Corpore*: XXVI, 1, *OL*, vol. I, p. 335.

[99] See Hobbes, *Principia et Problemata aliquot geometrica, antehac desperata, nunc breviter explicata et demonstrata*, *OL*, vol. V, pp. 211-214.

cians as *indefinite*»,[100] but, where the infinitely small is concerned, the term may be used incorrectly. In fact, «when it is used for the infinitely small, it is treated as if it were nothing. No quantity is in fact capable of being divided infinitely, in other words into infinite nothings».[101] An infinite division can therefore only be conceived as indefinite; if we are to conceive of an infinite division in action, we must conceive of the division of a given quantity into «infinite nothings»,[102] which is impossible.

7. Conclusion

The trajectory that we have considered developed in several stages and encompassed various aspects orbiting around a central theme, namely the question of the ontological nature of the point, both mathematical and physical. The issue attracted Mersenne's attention from 1625 onwards, as we have seen in our examination of *La Vérité des Sciences*. However, the Minim friar was not only the promoter of the debate concerning the nature of the point, but acted – along with Boulliau – as the chief protagonist of a significant change in perspective regarding this subject. Mersenne in fact brought into the enquiry the parabolic mirror, which directly involves the field of optics and, as a consequence, the relationship between geometrical optics and physical optics. Moreover, Mersenne's solution, which is to treat light as a sort of *accident*, fulfils, as we have seen, the requirement to avoid the difficulty raised by Boulliau and Gassendi, involving the concentration of the sun's rays, and therefore of physical rays, in a single point.

These reflections on optics reveal its crucial importance with regard to the nature of the point, as is already apparent from the first works that Hobbes dedicated to the question. The English philosopher proceeds from the assumption that rays, viewed simultaneously as physical entities and mathematical lines, are no more than an abstrac-

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[100] «*Infinitum* autem a mathematicis saepissime dicitur pro *indefinito*». *Ibid.*, p. 213.

[101] «aliquando pro *indefinite parvo*, modo non sit nihil. Dividi enim in infinitum, id est in nihila, quantitas nulla potest». *Ibid.*

[102] Hobbes, *Principia et Problemata aliquot geometrica*, *OL*, vol. V, p. 213. An interpretation of Cavalieri's indivisibles close to that of Hobbes may be found in Gassendi, who, when addressing the problem of the acute hyperbolic solid and analysing the reflections of «Cavalerius & Torricellius» emphasises the difficulty of treating the continuum as being composed of indivisibles. See Gassendi, *Syntagma* (cit. note 37), vol. I, pp. 264-265.

tion derived from a *continuum* and the point is therefore also defined in the same way, as a pure abstraction. However, he believes that this point has dimensions since it would otherwise not even be possible to imagine it.

The Hobbesian discussion of the nature of the point is understood even more clearly if it is related to his research into the field of optics, and even more so when it is compared to the analyses offered by Mersenne and the scholars who formed part of his circle. The investigation that we have conducted demonstrates that the debate promoted by Mersenne did not merely stimulate the interest of the Parisian intellectual community, but actively contributed to the elaboration and development of the mathematical understanding of Thomas Hobbes.

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