

Analytic Formulation for the Calculation of Leakage Reactance in Wound-Core Transformers

Guillermo A. Diaz, Jhon J. Perez, Enrique E. Mombello, *Senior Member IEEE*, Russby L. Castañeda

Abstract—Wound-core transformers are characterized by their economical and efficient design and are therefore widely used in power distribution systems; for this reason it is of paramount importance to determine their electrical characteristics. One of the most important parameters in these transformers is leakage reactance, since this is what largely determines short-circuit currents and voltage regulation. And yet the authors of this work have found a conspicuous gap in the scientific literature concerning analytical formulations for the calculation of leakage reactance in wound-core transformers. In order to fill this gap, the authors propose a formula in terms of leakage areas and the Rogowski's height of the windings. A geometric model of the active part of the transformer is proposed which allows for the exact calculation of the leakage areas. The results of the proposed formula are compared against simulations with the finite element method in 3D (FEM3D) and with measurements in eight real case studies. It is concluded that although the FEM3D calculates the closest results to measurements, the proposed formula reported sufficiently accurate results for transformer design purposes.

Index Terms—Distribution Transformer, Leakage Reactance, Analytic Formulation, Finite Element Method

I. INTRODUCTION

Leakage impedance is undoubtedly one of the most important parameters in the design of a transformer and it is of vital importance to manufacturers that its final value be as close as possible to the value guaranteed to the customer. Due to production constraints at the manufacturing stage, it is difficult to ensure that the measured impedance value exactly matches the guaranteed value. This is why international standardization organizations such as ANSI have defined the maximum permissible deviations of the measured impedance value from the guaranteed value. According to ANSI IEEE C57.12.00 the tolerance for transformers with impedances greater than 2.5 % is ± 7.5 %, and in the case of transformers with impedances equal to or less than 2.5 %, it is ± 10 % [1]. In the event that a transformer falls outside the tolerance specified by the standard, the purchaser may reject the unit or apply fines to the manufacturer, generating negative economic impacts for both parties.

Short-circuit impedance has two components, the real and the imaginary. The imaginary is known in the literature as *leakage reactance* and makes up the predominant part of the impedance; hence, this work will focus only on the calculation of leakage reactance.

While transformer manufacturers have relied for decades on analytical formulas to calculate the different parameters of transformers, it is indisputable that numerical methods such as the finite element method have the ability to model much more complicated geometries [2], [3]. Despite this, analytical methods are still valuable for their speed, and because they allow for the interrelation of variables to be visualized more clearly [4].

As far as the construction of transformers is concerned, the two most common models for the active part, and specifically the core, are the stacked core and the wound core. Wound-core transformers are typically used for power ratings below 800 kVA and play a fundamental role in linking the distribution network with end users [5]. Figure 1 shows a 3D model of the active part of a single-phase, shell-type wound-core transformer. As can be seen, although the core has a rectangular cross section the geometry of the coils is not completely rectangular for reasons that will be expanded on in section III-B.

Dozens of works can be found in the literature where leakage reactance is calculated both analytically and numerically in transformers with a stacked core and circular windings [2], [3], [6], [7], [8], [9]. However, the authors of this work identified a notable lack of scientific literature dedicated to analytical formulations for the calculation of leakage reactance in wound-core transformers. Several empirical formulas presented in the literature have the drawback that they use unknown constants. Furthermore, these empirical formulations typically do not indicate the extent to which they can reliably be used for the calculation of the leakage reactance.

The formulation developed by Georgilakis in [10] is among the most important works presenting analytical procedures for the calculation of leakage reactance in wound-core transformers. In this work the author propose a highly detailed model of the active part, considering the effect of the cooling ducts and the geometric particularities of the coils. Unfortunately the formulation remains in terms of factors that depend on the number of cooling ducts, which is not disclosed in the work for reasons of confidentiality with the manufacturer.

The fundamental purpose of this paper is to fill the gap identified in the scientific literature by proposing a reliable formulation that does not depend on empirical factors and that clearly indicates its scope of application.

The most important contributions of this work are the following: *a)* To fill a gap in the scientific literature by proposing an analytical formula for the calculation of leakage reactance of wound-core transformers. *b)* To propose a formula whose results are suitable for the design of distribution

Guillermo A. Diaz F. currently works as a professor and researcher of the La Salle University in Bogota Colombia. e-mail: guandiaz@unisalle.edu.co (see <http://www.unisalle.edu.co>).

E. Mombello currently works as professor and researcher of the CONICET - Universidad Nacional de San Juan, San Juan, Argentina.

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transformers, validated by comparison with measurements and FEM3D simulations for eight real case studies. *c)* To propose a geometric model of the coils of the wound-core transformer that is consistent with its construction characteristics; this geometrical model facilitates, on the one hand, the exact calculation of the leakage areas and, on the other, the precise definition of the geometrical scope of the formulation.

II. DESCRIPTION OF THE PROBLEM

Figure 1 shows an isometric view of the active part of a wound-core distribution transformer. As can be seen, the

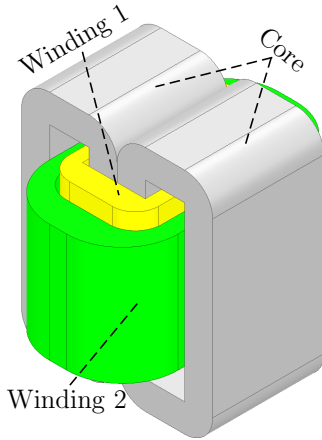


Fig. 1. Isometric view of a single-phase wound-core transformer

windings have two different shapes depending on the region. In the region under the core window the coils are flat, while outside the core window the coils are quasi-circular. An issue that will be explained in detail in section III-B is the fact that the thickness of the quasi-circular region is greater than the thickness of the flat portion. This situation further complicates the direct application of traditional formulas.

These particularities in the geometry of the active part of the transformer mean that the direct application of methods or analytic formulations designed for 2D planar or axisymmetric configurations might cause significant deviations between calculations and measurements. While the scientific literature has devoted great efforts to the analytical calculation of leakage reactance in transformers with circular coils, little attention has been paid to the calculation of leakage reactance in wound-core transformers.

Likewise, although the geometric description of a transformer with circular windings is almost trivial, to the authors' knowledge there is no rigorous and satisfactory geometric definition of the coils for wound-core transformers in the literature. The purpose of this work is to address these deficiencies.

III. LEAKAGE REACTANCE IN TRANSFORMERS WITH QUASI-RECTANGULAR COILS

A. Modified Leakage Reactance Formula

A modification of Del Vecchio's formula in terms of leakage areas is proposed in [11] for the calculation of leakage

reactance in transformers with partially circular windings. The modified formula is presented in (1)

$$X_{md} = \frac{2\pi\mu_0 f S_b}{(h_m + s)(V_b/N)^2} \cdot \left\{ \begin{array}{l} A_g + \frac{A_1 + A_2}{3} \\ + \frac{\pi(w_1^2 - w_2^2)}{6} \end{array} \right\} \quad (1)$$

where s is an empirical factor, f is the rated frequency, S_b is the rated apparent power per leg, V_b is the rated rms phase voltage, h_m is the average height of the windings, N is the number of turns of the reference winding, A_1 is the leakage area of winding 1, A_2 is the leakage area of winding 2 and A_g is the leakage area of the main duct. The main assumption in this work is that it is possible to calculate the leakage reactance of wound-core transformers with reasonable precision by making two additional modifications to (1) (See section III-E).

B. Geometric Description of the Active Part

Figure 2 shows the front and top view of the active part of a wound-core transformer. The internal winding is depicted in yellow and the external winding is depicted in green. The most relevant aspect of the coils in Figure 2 is that

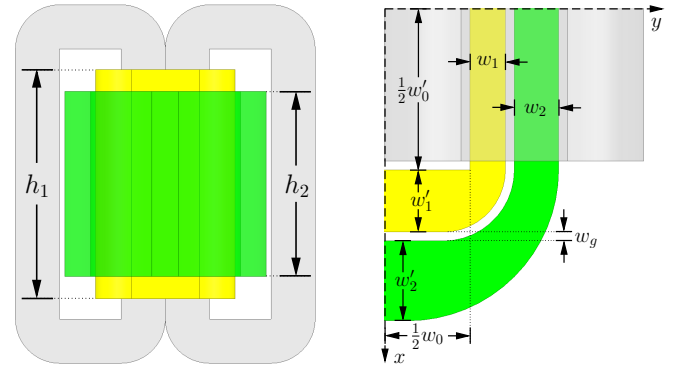


Fig. 2. Front and top views of the active part of a wound-core transformer

they are not circular, but not completely rectangular either. Because of this, these coils will hereafter be referred to as *quasi-rectangular* coils.

Another relevant difference to circular coils is that each of the coils has two different thicknesses. For example, in the case of the internal winding we have the thicknesses w_1 and w'_1 , which will be denoted as the thickness of the flat part and the curved part of winding 1 respectively. Similarly, the thicknesses w_2 and w'_2 are shown for winding 2. There are three fundamental reasons why the thickness of the curved part is greater than that of the flat part:

a) In the curved part, a certain number of cooling ducts are located between one layer and another which increases the thickness of the winding.

b) The output leads of the coils and high voltage taps are located in the curved part of the coils which increases its thickness. Figure 4 shows the HV output leads in detail.

c) In order to improve the mechanical stability of the coils, they are subjected to a pressing process. This pressing is done to the flat part of the coils. After the pressing process, it can be shown that the dimension of the flat part is reduced,

while the dimension of the curved part is increased.

Figure 2 also shows the internal dimensions of winding 1, which are w_0 and w'_0 respectively. It should be noted that the symmetry was used to show only the quarter symmetry model. The thickness of the main leakage duct can also be seen, denoted as w_g .

Up to this point the overall dimensions of the coils have been presented generally; nonetheless, it is essential to provide a geometric description of the curvature of the coils, which will be done using the diagram in Figure 3. The curvature

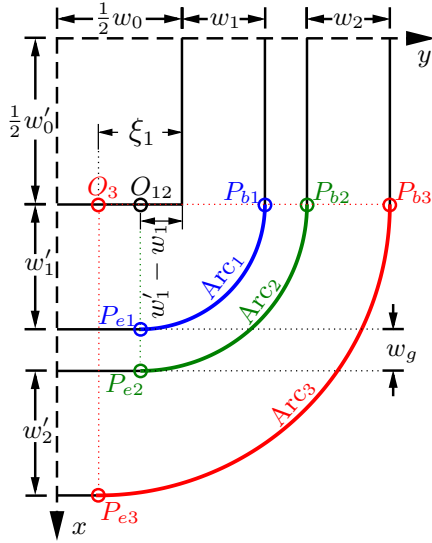


Fig. 3. Geometric model of the coils of a wound-core transformer

of the coils has been modelled using the circumference arcs Arc₁, Arc₂ and Arc₃. Although other types of curves such as ellipses can be used [10], the reasons for using circumference arcs are as follows:

a) The exact calculation of area and perimeter is considerably simpler for circumference arcs.

b) Circumference arcs ensure compliance with the principle of conservation of mass. It is easy to show that at no point will the thickness of the coil exceed its maximum thickness (w'_1 and w'_2 respectively) or be less than its minimum thickness (w_1 and w_2 respectively). In the case of ellipses it is not trivial to verify this condition.

c) A qualitative study was carried out showing that the curvature of the corners of the coils can be adequately represented by means of circumference arcs. See Figure 4.

d) Circumference arcs guarantee that the thickness is exactly w_g at any point of the main leakage duct, which is of great importance since this thickness directly affects the calculation of leakage reactance. In the case of ellipses, it is more difficult to guarantee this condition.

The geometrical description of the arcs that make up the geometry of the coils is presented below:

Arc₁: $\widehat{P_{b1}P_{e1}}$ and center at O_{12} where:

$$P_{b1} \left(\frac{w'_0}{2}, \frac{w_0}{2} + w_1 \right), P_{e1} \left(\frac{w'_0}{2} + w'_1, \frac{w_0}{2} - (w'_1 - w_1) \right)$$

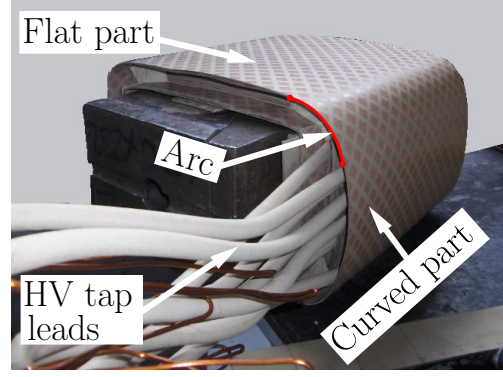


Fig. 4. High voltage coil of a wound-core transformer.

Arc₂: $\widehat{P_{b2}P_{e2}}$ and center at O_{12} where:

$$P_{b2} \left(\frac{w'_0}{2}, \frac{w_0}{2} + w_1 + w_g \right),$$

$$P_{e2} \left(\frac{w'_0}{2} + w'_1 + w_g, \frac{w_0}{2} - (w'_1 - w_1) \right)$$

Arc₃: $\widehat{P_{b3}P_{e3}}$ and center at O_3 where:

$$P_{b3} \left(\frac{w'_0}{2}, \frac{w_0}{2} + w_1 + w_g + w_2 \right),$$

$$P_{e3} \left(\frac{w'_0}{2} + w'_1 + w_g + w'_2, \frac{w_0}{2} - \xi_1 \right)$$

The coordinates of the centers of the arcs are defined as follows:

$$O_{12} \left(\frac{w'_0}{2}, \frac{w_0}{2} - (w'_1 - w_1) \right), O_3 \left(\frac{w'_0}{2}, \frac{w_0}{2} - \xi_1 \right)$$

where ξ_1 is defined by (6). The validity of the expressions proposed in this work must meet the following inequalities to ensure the geometrical consistency of the model and the validity of the mathematical expressions proposed.

$$w'_1 - w_1 \leq \frac{w_0}{2}, w'_1 + w_g + w'_2 - w_2 \leq \frac{w_0}{2} + w_1 + w_g,$$

$$w'_1 \geq w_1, w'_2 \geq w_2 \quad (2)$$

C. Weighted Coil Thicknesses

As can be seen in (1), leakage reactance is expressed as a function of the coils' thicknesses w_1 and w_2 . Since the thickness of the coils in wound-core transformers is not constant, it is necessary to define a weighted thickness so that the thickness of the coil is not overestimated or underestimated. We will denote the weighted thicknesses of windings 1 and 2 as w_{w1} and w_{w2} respectively. Since the thickness of the leakage duct is constant, it is not necessary to define a weighted thickness in this case. The dimensions of Figure 5 will be used for the definition of the weighted thicknesses. We define the weighted thickness of winding 1 as follows

$$w_{w1} = \frac{l_1 w_1 + l'_1 w'_1}{l_{T1}} \quad (3)$$

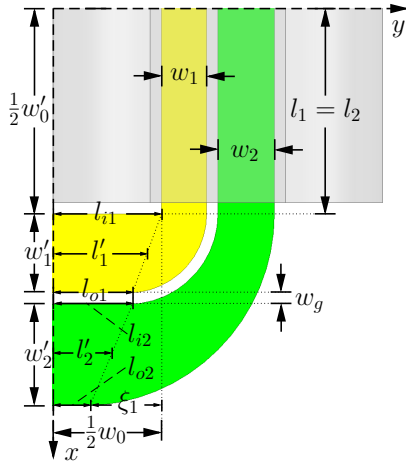


Fig. 5. Schematic diagram showing additional dimensions for the definition of weighted thicknesses.

where l_1 is the winding length with thickness w_1 , l'_1 is the winding length with thickness w'_1 and $l_{T1} = l_1 + l'_1$. It is evident from Figure 5 that $l_1 = \frac{1}{2}w_0$, but l'_1 has not yet been clearly defined. According to Figure 5 it would be incorrect to state that the winding length with thickness w'_1 is l_{i1} or l_{o1} , since in the first case the distance would be underestimated and in the second it would be overestimated. The following criterion is defined to calculate the distance l'_1

$$l'_1 = \frac{l_{i1} + l_{o1}}{2} \quad (4)$$

From the geometry in Figure 5 the internal and external distances of winding 1 can be calculated as follows

$$l_{i1} = \frac{w_0}{2}, l_{o1} = \frac{w_0}{2} - (w'_1 - w_1) \quad (5)$$

Similarly, the following expressions are obtained for the weighted thickness of winding 2

$$w_{w2} = \frac{l_2 w_2 + l'_2 w'_2}{l_{T2}}, l_{T2} = l_2 + l'_2, l_2 = \frac{1}{2}w_0,$$

$$l'_2 = \frac{l_{i2} + l_{o2}}{2}, l_{i2} = l_{o1}, l_{o2} = \frac{w_0}{2} - \xi_1$$

where,

$$\xi_1 = w'_1 + w_g + w'_2 - (w_1 + w_g + w_2) \quad (6)$$

D. Rogowski Height

As can be seen in (1), Del Vecchio includes the empirical factor s which modifies the average height of the windings h_m . The Del Vecchio's formula for reactance is derived from an idealized situation where the field has only an axial component, so the purpose of the empirical factor s is to include the effect of the field's radial component [6].

To the authors' knowledge, the empirical factor s can be reliably applied to stacked-core transformers and circular coils. There is no evidence however that this factor is applicable to wound-core transformers with quasi-rectangular coils. Consequently, the need arises to include the effect of the radial

component of the field adequately. For this sake, the traditional Rogowski factor will be used, which is defined as follows [12]:

$$k_R = 1 - \frac{w_{w1} + w_g + w_{w2}}{\pi h_m} \cdot \left(1 - e^{-\frac{\pi h_m}{w_{w1} + w_g + w_{w2}}}\right) \quad (7)$$

where $h_m = \frac{h_1 + h_2}{2}$ is the average height of the windings, and the Rogowski height is calculated as follows

$$h_R = \frac{h_m}{k_R} \quad (8)$$

In addition to the geometrical constraints set out in (2), the following inequalities related to the formulation of the Rogowski factor [13] must also be met for (7) to give reliable results.

$$w_g < w_{w1} + w_{w2} \text{ and } k_r = \frac{w_{w1} + w_{w2} + w_g}{h_m} < 2 \quad (9)$$

E. Proposed Formulation

The strategy to be followed in proposing the new formulation for the calculation of the leakage reactance in wound-core transformers is as follows:

- The modified Del Vecchio formula (1) is taken as a basis.
- The empirical factor s will be removed and the height h_m will be replaced by the Rogowski height as presented in section III-D.
- The thicknesses w_1 and w_2 will be replaced by the weighted thicknesses w_{w1} and w_{w2} calculated as stated in section III-C.

Applying these modifications to formula (1) gives

$$X_p = \frac{2\pi\mu_0 f S_b}{h_R (V_b/N)^2} \cdot \left\{ \begin{array}{l} A_g + \frac{A_1 + A_2}{3} \\ + \frac{\pi(w_{w1}^2 - w_{w2}^2)}{6} \end{array} \right\} \quad (10)$$

where X_p is the *proposed* leakage reactance.

In order to make the calculations easier, the leakage areas have been divided as shown in Figure 6. The leakage areas of

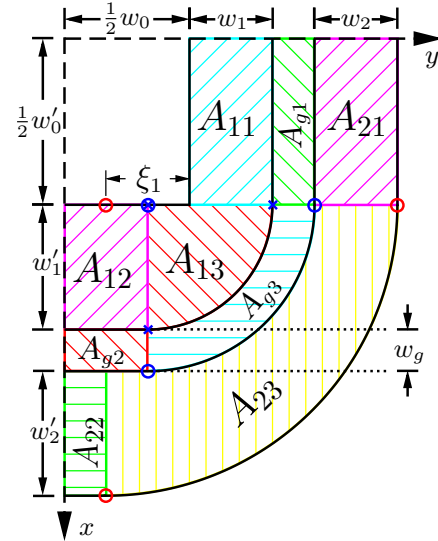


Fig. 6. Schematic diagram showing transformer leakage areas.

windings 1 and 2 and the main duct are defined as follows:

$$A_1 = 4 \cdot (A_{11} + A_{12} + A_{13}), A_g = 4 \cdot (A_{g1} + A_{g2} + A_{g3}),$$

$$A_2 = 4 \cdot (A_{21} + A_{22} + A_{23}) \quad (11)$$

By classical geometry, each subarea is determined exactly. The expressions of the areas that compose the winding are as follows

$$A_{11} = \frac{w_1 w'_0}{2}, A_{12} = w'_1 \left(\frac{w_0}{2} - (w'_1 - w_1) \right), A_{13} = \frac{\pi w'_1{}^2}{4}$$

The corresponding expressions for the subareas of the main duct are

$$A_{g1} = \frac{w_g w'_0}{2}, A_{g2} = w_g \left(\frac{w_0}{2} - (w'_1 - w_1) \right),$$

$$A_{g3} = \frac{\pi (w'_1 + w_g)^2}{4} - \frac{\pi w'_1{}^2}{4}$$

In the case of winding 2, the following expressions are used for the leakage areas

$$A_{21} = \frac{w_2 w'_0}{2}, A_{22} = w'_2 \left(\frac{w_0}{2} - \xi_1 \right),$$

$$A_{23} = \frac{\pi}{4} (w'_1 + w_g + w'_2)^2 - \frac{\pi}{4} (w'_1 + w_g)^2$$

$$- (w'_1 + w_g) (\xi_1 - (w'_1 - w_1))$$

F. Calculation of Leakage Reactance by Means of FEM3D

Due to the particular geometry of the coils of the wound-core transformer, it is not feasible to determine leakage reactance by 2D simulations in either planar or rotational symmetry. In order to make the representation as accurate as possible, the authors constructed a parametric 3D model of the active part of the transformer using the finite element method program ANSYS Maxwell. The most relevant aspects of the model are listed below:

a) The model has nine input parameters that determine the geometry, which are defined as shown in Figure 2. The input parameters are: $w'_0, w'_1, w'_2, w_0, w_1, w_g, w_2, h_1, h_2$.

b) The curvature at the corners of the coils was modeled exactly with the same criteria presented in section III-B

c) Due to the fact that the cooling ducts are located only in the curved part of the coil and that each has a different length according to the layer in which it is located, it was decided not to explicitly model the cooling ducts either in the FEM3D model or in the mathematical formulations. This considerably simplifies the formulations and the FEM3D model. Comparison of the simulated results and the measurements indicates that the effect of these ducts may not be determinant for the calculation of the leakage reactance for this particular type of transformers.

d) A magnetostatic simulation was performed and the per-unit leakage reactance was calculated from the total energy W_T stored in the magnetic field using the following expression

$$X_{3D} = \frac{4\pi f W_T S_b}{V_b^2 I_b^2}$$

where X_{3D} denotes reactance calculated by means of FEM3D and $I_b = S_b/V_b$. $f = 60$ Hz for all case studies.

e) The amperes turns are exactly the same in both windings $NI = N \cdot I_b$ but opposite in direction.

f) The constant core relative permeability used was $\mu_r = 10000$.

g) A maximum error of 0.5 % was established for adaptive

meshing. Figure 7 presents the geometric model and the final mesh that met the error criterion for case study 7.

h) Mesh operations: During the first performance tests of

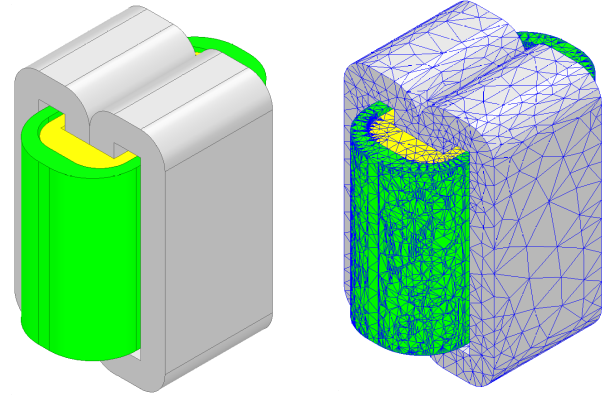


Fig. 7. Geometry and final mesh for case study 7

the model, it was identified that when calculating the areas by means of surface integrals¹ there were discrepancies with respect to the areas calculated by means of the expressions in (11). This discrepancy is due to the fact that when discretizing with tetrahedral elements the curved regions are represented in a polygonal fashion. If the length of the edges of the elements is large compared to the radius of curvature, considerable deviations from the ideal surface may occur. To avoid this inconvenience, a mesh operation was applied to the external curved faces of winding 1 and to the internal curved faces of winding 2. The imposed mesh operation guarantees a maximum mesh deviation of 0.01 mm from the ideal surface. Figure 8 shows the effect of the mesh operation on the curved faces of the model for both windings.

The simulations were performed on a desktop computer

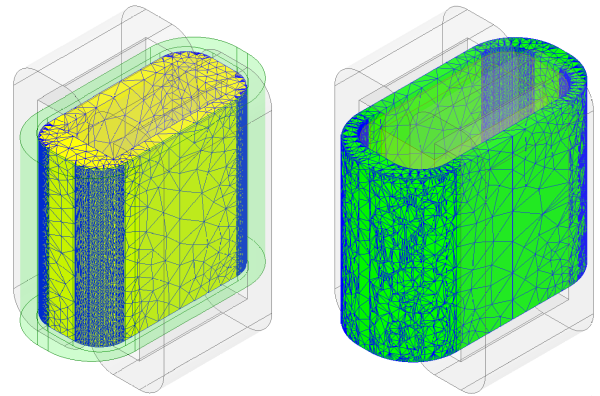


Fig. 8. Meshes of windings 1 and 2 for case study 7

of 2.00 GHz Intel Core i7 and 16.0 GB of RAM memory using ANSYS Maxwell which is part of the suite ANSYS Electromagnetics 2019 R1.

In order to give a general idea of the computational performance of the FEM3D, a table with the most important characteristics of the simulation of case study 7 is presented below.

¹The surface integrals are carried out with the Fields Calculator tool in ANSYS Maxwell

It is important to note that the calculation time presented in

TABLE I
SIMULATION SUMMARY FOR CASE STUDY 7

Parameter	Value
Number of adaptive passes	3
Total calculation time	9 min 15 s
Maximum memory used	1.16 GB
Final number of elements	197049
Final system size of equation system	262317
Final energy stored in field (W_T)	6.8006 J

Table I should not be taken as a reference for comparison with other methods because the simulation was specified to obtain the highest possible accuracy in the calculation of reactance. Taking advantage of the symmetry of the problem, the model could be reduced to its eighth part, theoretically reducing the memory and the calculation time by a factor of eight.

IV. VALIDATION AND RESULTS

This section presents the results obtained when applying the different calculation methods described in the previous sections. The results of each of the methods will be compared with the values measured in real transformers by means of relative error. Tables II and III present constructive information and rated data (S_b and V_b) for the eight case studies analyzed in this work. The measured reactance X_m , the reactance calculated with the proposed formula X_p and the reactance calculated with FEM3D X_{3D} are given at the end of the table. The relative errors of the proposed formula $\varepsilon_{m,p}$ and FEM3D $\varepsilon_{m,3D}$ with respect to the measurements are also reported. The relative error is calculated using the following formula $\varepsilon_{m,i} = (X_i - X_m) / (X_m)$.

Figure 9 graphically shows the leakage reactance values

TABLE II
CONSTRUCTIVE DATA AND RESULTS FOR THE FIRST FOUR CASE STUDIES

Variable (unit)	Case Study 1	Case Study 2	Case Study 3	Case Study 4
S_b (kVA)	15	167.5	37.5	75
V_b (V)	7620	13200	13800	7620
N (-)	1619	770	2070	698
h_m (mm)	224.9	302.8	205.7	223.1
w_0 (mm)	96	198	107	134
w'_0 (mm)	156	259	156	232
w_1 (mm)	13.6	21.2	17.0	19.1
w'_1 (mm)	23.9	38.6	33.3	43.4
w_g (mm)	2.5	2.5	2.5	2.5
w_2 (mm)	16.4	31.8	24.0	25.4
w'_2 (mm)	26.6	58.9	37.2	44.1
X_m (%)	1.23	2.27	2.35	2.36
X_p (%)	1.25	2.36	2.40	2.46
X_{3D} (%)	1.22	2.29	2.34	2.40
$\varepsilon_{m,p}$ (%)	1.20	4.17	1.96	4.29
$\varepsilon_{m,3D}$ (%)	-1.08	0.83	-0.54	1.75

calculated with each method. As can be seen, FEM3D practically overlaps the measurements, while the proposed method tends to overestimate the reactance.

In order to visualize the performance of each methodology with respect to the measurements, relative error graphs of each of the methods are presented in Figure 10. It can be seen that

TABLE III
CONSTRUCTIVE DATA AND RESULTS FOR THE LAST FOUR CASE STUDIES

Variable (unit)	Case Study 5	Case Study 6	Case Study 7	Case Study 8
S_b (kVA)	25	100	167	100
V_b (V)	6000	13800	14400	19920
N (-)	900	1035	1144	1494
h_m (mm)	130.5	193.4	358.5	251.2
w_0 (mm)	98	139	123	185
w'_0 (mm)	186	232	259	259
w_1 (mm)	25.0	20.3	25.0	29.5
w'_1 (mm)	42.3	47.6	57.3	56.5
w_g (mm)	2.5	2.5	3.5	5.0
w_2 (mm)	23.5	30.7	22.0	27.5
w'_2 (mm)	35.2	50.9	39.2	47.5
X_m (%)	2.77	2.79	3.11	3.14
X_p (%)	2.90	2.86	3.08	3.23
X_{3D} (%)	2.77	2.77	3.07	3.16
$\varepsilon_{m,p}$ (%)	4.70	2.46	-0.75	3.09
$\varepsilon_{m,3D}$ (%)	0.00	-0.49	-1.14	0.75

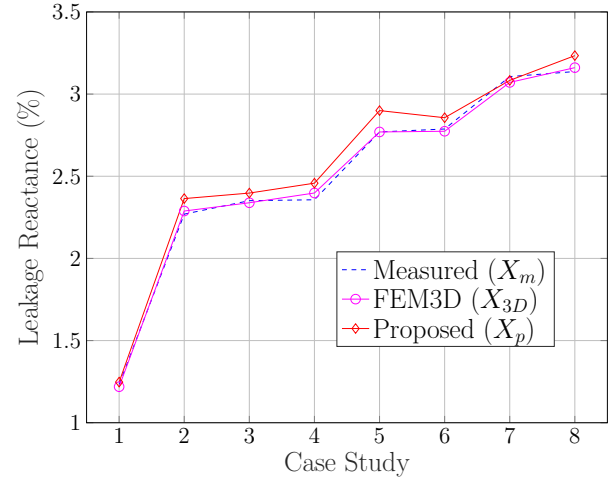


Fig. 9. Leakage reactance calculated using each of the methods.

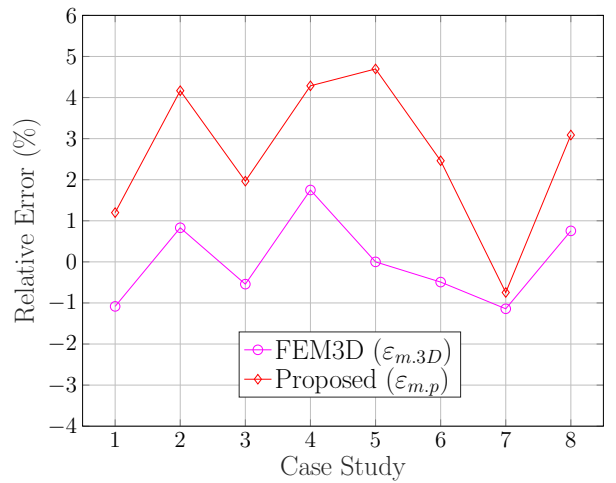


Fig. 10. Relative errors for all case studies.

the relative errors of the proposed formula and FEM3D have similar trends with the exception of case study 5, where the proposed formula reports the highest relative error of all cases

(4.7 %), while for this case FEM3D reports the most accurate result of all. The particularity of this transformer is that it is precisely the shortest, that is, the transformer with the lowest average height h_m . On the other hand, the transformer that reports the smallest relative error with the proposed formula is that of case study 7, which is precisely the tallest of all case studies. Another interesting situation can be observed in case studies 5 and 7. Analyzing the behavior of the variable k_r defined in (9), it is found that case 5 reports the largest value $k_r^{(5)} = 0.46$, while case 7 reports the smallest value among all cases $k_r^{(7)} = 0.17$. The above result is reasonable because the formulation for the calculation of the leakage reactance starts from the idealized case of an infinitely long solenoid where the field has only axial component [6]. This is why it is necessary to introduce the Rogowski factor to include the effect of the radial component of the field. The behavior of the relative error for the proposed formula in case study 5 may be indicative of some limitation of the Rogowski factor to adequately model the radial field in coils of considerably low height. Despite this, the result still remains within the tolerance of ± 7.5 %.

V. CONCLUSIONS

This paper fills a gap in the scientific literature by proposing a specific formulation for the calculation of leakage reactance in wound-core transformers.

The proposed formulation is compared with measurements for eight case studies with different geometries and reactances, finding that the relative error never exceeded 5 %, which is satisfactory for distribution transformer design purposes.

An important contribution of this work is that (2) and (9) clearly specify the scope of the proposed formulations, which is essential for the implementation of the formulas.

The FEM3D method was the best method for calculating leakage reactance, giving relative errors of less than 2 % for all case studies. Even higher precision could be obtained by including the cooling ducts in the model, as shown by Georgilakis in chapter 6.4 of [10].

The authors recommend evaluating the leakage reactance of transformers with considerably low heights (in the order of 10 cm or less) with caution, since case study 5 revealed that the analytical formula, which is based on the Rogowski factor, may present some limitations. For these cases it is also suggested to check the solution with FEM3D.

VI. APPENDIX

Table IV presents the test data for each of the case studies. All case studies are single-phase step-down distribution transformers. The variable S_n represents the rated power of the transformer, V_n is the rated high voltage, V_{scm} is the short-circuit voltage measured on the high-voltage side, P_{scm} is the short-circuit power measured on the high-voltage side, R_m is the resistance in percent, Z_m is the short-circuit impedance and finally X_m is the measured leakage reactance.

It is important to note that the variables V_{scm} and P_{scm} are

TABLE IV
MEASUREMENTS OF EACH CASE STUDY.

Case Study	Tested Units	S_n (kVA)	V_n (kV)	P_{scm} (W)	V_{scm} (V)	R_m (%)	X_m (%)	Z_m (%)
1	2	15	7.62	139.0	117.5	0.93	1.23	1.54
2	1	167.5	13.20	1106.0	312.0	0.66	2.27	2.36
3	11	37.5	13.80	470.7	367.9	1.26	2.35	2.67
4	3	75	7.62	620.3	190.3	0.83	2.36	2.50
5	1	25	6.00	215.0	174.0	0.86	2.77	2.90
6	3	100	13.80	925.0	405.3	0.93	2.79	2.94
7	2	167	14.40	1472.5	465.0	0.88	3.11	3.23
8	2	100	19.92	801.5	645.0	0.80	3.14	3.24

obtained from direct measurements, while R_m , Z_m and X_m are obtained indirectly from the following expressions [7].

$$R_m = \frac{P_{scm}}{S_n}$$

$$Z_m = \frac{V_{scm}}{V_n}$$

$$X_m = \sqrt{Z_m^2 - R_m^2}$$

The general test conditions for all case studies were as follows: *a)* The tests were performed by short circuiting the low-voltage terminals and feeding from the high-voltage side. *b)* All test data refer to a test temperature of 20 degrees Celsius. *c)* The second column of table IV indicates the number of identical units that were tested. The V_{scm} and P_{scm} data correspond to the average value of the units tested. *d)* It should be noted that in this work $V_b = V_n$ and $S_b = S_n$ because they are single-phase units. In the case of three-phase transformers V_b is the phase voltage and S_b the apparent power per leg.

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Russby L. Castañeda Russby L. Castañeda H. Was born in Velez, Colombia, in 1980. She received the B.S. degrees in economy and electrical engineering from the Universidad de La Salle, Bogota DC, Colombia, in 2010 and 2018, respectively and M.Sc. degree in electrical engineering from the Universidad Tecnológica de Pereira, Pereira, Colombia, in 2019. Her areas of interest include power distribution and transformers and electricity markets.

Guillermo A. Diaz F. Was born in Bogota, Colombia, in 1984. He received the B.S. degree in electrical engineering and the Ph.D. degree in electrical engineering in 2006 and 2013, from the Universidad de La Salle and the Universidad Nacional de San Juan respectively. He has worked from 2005 to 2013 at the Siemens Andina Transformers Factory as researcher, electrical designer and programmer. In addition, he has performed a research internship at the High Voltage Institute of RWTH, Aachen, Germany. He is presently at the Faculty of Engineering of the La Salle University, Colombia, as a Titular Professor. His main fields of interest are design, optimization and magnetic/electric modeling of electrical equipment such as transformers, motors and electric generators.

Jhon J. Perez Was born in Bogota, Colombia, in 1980. He received the B.S. degree in electrical engineering and the Ph.D. degree in electrical engineering in 2003 and 2017, from the Universidad de La Salle and the Universidade Estadual de Campinas respectively. He has worked from 2010 at the Faculty of Engineering of the La Salle University, Colombia, as a lecturer and researcher. His main fields of interest are electric machinery, electric markets, renewable energies and socioeconomic impacts of electrification.

Enrique E. Mombello (IEEE M'95-SM'00) was born in Buenos Aires, Argentina, in 1957. He received the B.S. degree in electrical engineering and the Ph.D. degree in electrical engineering in 1982 and 1998, respectively from Universidad Nacional de San Juan. He has more than 20 years of experience in research projects which include topics like electrical transients and resonance processes within transformers as Researcher of the National Council of Technical and Scientific Research (CONICET, Argentina). He has worked from 1989 to 1991 at the High Voltage Institute of RWTH, Aachen, Germany. He is presently at the Instituto de Energia Electrica of the University of San Juan, Argentina, as a Lecturer and Researcher since 1982. His main fields of interest are design, modeling and diagnostics of power transformers, asset management, transformer life management, electromagnetic transients in electric machines and networks, modeling of equipment, corona losses in overhead transmission lines, low frequency electromagnetic fields.