SCAFCA - Symplectic Circle And Finite Cyclic Actions: a new approach

A *symplectic* manifold is a smooth manifold M with a non-degenerate differential 2-form ω. Hamiltonian torus actions on symplectic manifolds may be thought of as a mathematical formulation of Noether's principle "to a symmetry of a physical system there corresponds a conserved quantity". They correlate to tori in the group $\mathrm{Ham}(M,\omega)$ of Hamiltonian symplectomorphisms. Mapping the maximal tori and abelian subgroups of $\mathrm{Ham}(M,\omega)$ requires understanding to which Hamiltonian torus actions the $\mathbb{Z}_n$-actions and the S^1-actions extend. The manifolds that we consider are compact, connected, symplectic, and of dimension four.

Background: In dimension four, the classification theorems of Delzant and Karshon associate to a Hamiltonian action of $(S^1)^2$ or S^1 a combinatorial object, a polygon or a decorated graph, and show that this association is one-to-one (up to isomorphism). It follows from the classifications that such an action is obtained from a minimal model, either the complex projective plane $\mathbb{C}P^2$ or an S^2-bundle over a Riemann surface, by a sequence of equivariant symplectic blowups, in which an embedded invariant ball is replaced with a sphere of self intersection -1. However, Delzant and Karshon's classifications do not give the actions on a fixed symplectic manifold nor the symplectic forms that are invariant with a fixed action.

The classification of $\mathbb{Z}_n$-actions is currently out of reach. The question of whether any homologically trivial $\mathbb{Z}_n$-action on a given symplectic manifold extends to an S^1-action was answered positively by Chen for $\mathbb{C}P^2$ and by the PI and Chiang for S^2-bundles over S^2. On the other hand, the PI and Chiang construct examples of homologically trivial symplectic $\mathbb{Z}_n$-actions that do not extend to Hamiltonian S^1-actions.

Goals: The project's goal is to extend the scope of classification of group actions on symplectic four-manifolds beyond the current boundaries: to classify $\mathbb{Z}_n$-actions according to their extensions to Hamiltonian tori actions, and to add axes to the classification of Hamiltonian S^1-actions: in one, we fix the form and let the G-action vary, and in the other, we fix the action and let the symplectic form deform in the cone of invariant symplectic forms.

Concretely, for $G = S^1$ and $G = \mathbb{Z}_n$, we look at global features, such as the number of maximal torus actions to which the G-action extends and the homotopy type of the space of equivariant symplectomorphisms, and check, in both axes, what are the chambers in which the feature is stable. In another axis, we track the effect of the blowup on the feature. Moreover, each of these questions is prominent on its own.

Methodology: We will combine methods from Gromov's theory of J-holomorphic curves, complex geometry, and equivariant symplectic geometry. The *novelty* is in harnessing the tool of inflation of a symplectic form along the class of a J-holomorphic curve in the equivariant setting. We will use equivariant inflation to get stability of a feature along a path of invariant symplectic forms up to a certain point, and to show that the equivariant symplectic blowup is unique up to equivariant *isotopy*: a deformation in which the class of a form is constant, which is stronger than isomorphism. The inflation tool was used so far only in the non-equivariant setting; it has proven to be ground-breaking in McDuff's proof of the uniqueness up to isotopy of the symplectic blowup in four-manifolds. McDuff used the Seiberg-Witten invariant to obtain the J-holomorphic curves to inflate along for a generic ω-tamed almost complex structure. However, a generic ω-tamed J need not be invariant under the G-action. To overcome this, for $G = S^1$, we obtain invariant J-holomorphic curves from the associated combinatorial models. Still, inflating along these curves does not give enough freedom to deform in all directions of the invariant symplectic cone. Thus, we develop new algorithms for sequences of inflations along the curves. The possibility of getting a path by a sequence of inflations was only opened recently by the tame-to-tame version of the inflation lemma of Chakravarthy, Payette, and Pinsonnault.

Each step of the plan involves many subtleties and special cases and requires creativity and mastering different techniques; completing the plan requires a lot of brain power in the form of post-doctorants and research students, and gathering of experts from different areas of mathematics, through conferences and workshops.

ERC Advanced 2026 call – pre-proposal for the ERC-Support Committee

CV

PERSONAL DETAILS

Family name, First name: Kessler Liat
Researcher unique identifier (ORCID): 0000-0003-2983-5469
URL for web site: https://sites.google.com/view/liatkesslershomepage/home

- **Education and key qualifications**

01/2005 PhD (direct track)
Torus actions on small blow ups of CP^2
Einstein Institute of Mathematics, The Hebrew University of Jerusalem, Israel
Prof. Yael Karshon
1999 Entered the direct track towards Ph.D.
Einstein Institute of Mathematics, The Hebrew University of Jerusalem, Israel

- **Current position(s)**

2019-present Senior Lecturer
Department of Mathematics, Physics and Computer Science, at Oranim
Faculty of Natural Sciences, University of Haifa, Israel
2012-2019 Lecturer
Department of Mathematics, Physics, and Computer Science at Oranim
Faculty of Natural Sciences, University of Haifa, Israel

- **Previous position(s)**

2017 - 2018 Visiting Assistant Professor
Department of Mathematics, Cornell University, USA
2010 - 2012 Postdoctoral fellow
Department of Mathematics, Technion, Israel
2006 - 2010 C.L.E. Moore Instructor
Department of Mathematics, Massachusetts Institute of Technology, USA
2004 - 2006 Courant Instructor/Assistant Professor
Courant Institute of Mathematical Sciences, New York University, USA

RESEARCH ACHIEVEMENTS AND PEER RECOGNITION

Research achievements

Note: For joint publications, the order of the listed authors is according to the alphabetical order of their last names.

1. L. Kessler, Holomorphic shadows in the eyes of model theory, *Transactions of the AMS* **363** (2011), no. 6, 3287-3307.

2. Y. Karshon and L. Kessler, Distinguishing symplectic blowups of the complex projective plane, *Journal of Symplectic Geometry* **15** (2017), no. 4, 1089-1128.

3. R. Chiang and L. Kessler, Cyclic actions on rational ruled symplectic four-manifolds, *Transformation Groups* **24** (2019), no. 4, 987-1000.

4. T. S. Holm and L. Kessler, with an Appendix by Tair Pnini, Circle actions on symplectic four-manifolds, *Communications in Analysis and Geometry* **27** (2019), no. 2, 421-464.

5. R. Chiang and L. Kessler, Homologically trivial symplectic cyclic actions need not extend to Hamiltonian circle actions, *Journal of Topology and Analysis* **12** (2020), no. 4, 1047-1071.

6. T. S. Holm and L. Kessler, The equivariant cohomology of complexity one spaces, *L'Enseignement Mathematique* **65** (2020), no. 3, 457-485.

7. L. Kessler, A symplectic form on the space of embedded symplectic surfaces and its reduction by reparametrizations, *Pacific Journal of Mathematics*, 316 (2022), no. 2, 409-430.

8. I. Charton and L. Kessler, Monotone symplectic six-manifolds that admit a Hamiltonian GKM action are diffeomorphic to smooth Fano threefolds, accepted by *Transformation Groups* (2025).

<u>Submitted:</u>

1. T. S. Holm and L. Kessler, Equivariant cohomology of a complexity-one four-manifold is determined by combinatorial data, *Advances in Mathematics*, (95 pages). Submitted after revisions according to the journal's request. arXiv:1912.05647

2. R. Chiang and L. Kessler, Finite cyclic actions on symplectic blowups of the complex projective plane, *Journal of Symplectic Geometry*, (18 pages). Submitted after revisions according to the journal's request. https://sites.google.com/view/liatkesslershomepage/publications/ckpreprint

3. T. S. Holm, L. Kessler, and S. Tolman, Equivariant cohomology distinguishes four-dimensional Hamiltonian S^1-manifolds up to equivariant diffeomorphism, *International Mathematics Research Notices,* (23 pages). arXiv:2412.14310

Peer recognition

1. Invited lectures in international conferences and Travel grants, recently:
   - AMS Sectional Meeting/ Special Session on Equivariant Cohomology, 2020, USA
   - Workshop on Torus Actions in Topology, 2020, The Fields Institute, Canada.
   - Perspectives in equivariant topology, 2022, University of Cologne, Germany
   - Workshop on Hamiltonian Geometry and Quantization, 2024, The Fields Institute, Canada.

2. Invited presentation and Travel grant for distinguished young scientists to the 3rd Kavli Frontiers of Science Symposium, the U.S. National Academy of Sciences (2017).

3. Recent Research grants:
   - ISF, 2020-2024, Title: Group actions on symplectic manifolds: a study through holomorphic, combinatorial and algebraic methods.
   - NSF-BSF, 2022-2025, Title: Equivariant symplectic geometry,
     Co-PIs: Tara S. Holm (Cornell University), Yael Karshon (Tel Aviv University), Susan Tolman (University of Illinois Urbana-Champaign).

4. Recent Conference grant:
   SwissMAP Research Station in Les Diablerets, 2025, Title: Contemporary trends in Hamiltonian geometry, Co-PIs: Anton Alekseev (Geneva), Ana Cannas da Silva (ETH Zurich), River Chiang (NCKU), Joseph Palmer (Amherst), Daniele Sepe (Medellin).

ADDITIONAL INFORMATION

Career breaks, diverse career paths and major life events

Birth of children: November 2006 (twins), February 2010, July 2014.
Each birth was followed by a maternity leave.
The maternity leave following the birth at 2010 was long due to health complications for the newborn, which required urgent operation and hospitalization in the NICU.

Other contributions to the research community

1. Supervision of Researchers in Early Career Phases:
   • 2016: supervising B.Sc. student Tair Pnini on Research Project;
   • 2022-present: supervising post-doctoral fellow Isabelle Charton;
   • 2024-present: supervising post-doctoral fellow Nikolas Wardenski;
   • 2024-present: supervising M.Sc. student Noa Shimoni.

2. Organizer of the international conference "Contemporary Trends in Hamiltonian Geometry", to be held in 2026.

3. Organizer of the third and fourth Israeli workshop for women in mathematics, and speaker in the second, fifth, and sixth Israeli workshop for women in mathematics, 2016-2019, 2024.

4. BSF scientific advisor for the Regular Research Grant program, 2022-23.

5. Reviewing for refereed Journals, for example: *Israel Journal of Mathematics (2020), Journal of Symplectic Geometry (2021), Compositio Mathematica (2022), Transactions of the American Mathematical Society (2020, 2023, 2024), Mathematische Annalen (2025).*

Why is your project innovative, and what makes you the suitable PI for it?

The project presents a new approach to the classification of group actions on compact symplectic four-manifolds. This approach will yield a so-far-not-approachable complete classification of finite cyclic group actions and a new multi-axial classification of circle actions, fixing the form and letting the action vary, or fixing the action and letting the form vary. For that, I apply tools that were not used before in the equivariant setting, in an original way.

This project is the culmination of my research experience and academic training. I have been studying group actions on symplectic four-manifolds since my PhD thesis. The emphasis was first on Hamiltonian 2-torus actions, later shifted to Hamiltonian circle actions, and eventually to finite cyclic group actions and their extension to torus actions. My papers on extensions of finite cyclic group actions are the leading papers on the subject. In past works, I have used diverse techniques: combinatorial, algebraic, holomorphic, and J-holomorphic; from each, I took another tool. In many of them, I demonstrated creativity; e.g., in constructing counter examples, and in applying model theory (from logic) to study J-holomorphic curves. Recently, in a work-in-progress, I have been working on showing the uniqueness of a symplectic blowup that is equivariant with respect to a Hamiltonian circle action. The breakthrough was realizing how to use the J-holomorphic tool of inflation for that. Finding the right J-holomorphic curves to inflate along requires a thorough understanding of the decorated graphs associated to Hamiltonian circle actions. Devising the inflation algorithms requires creativity. Furthermore, I have realized that these ideas can be applied in a much bigger scope to extend the classification of group actions beyond the current boundaries. This plan involves many sub-steps, each with its own subtleties and special cases, and requires a combined effort of several research assistants. The experience I gained in supervising post-doctoral fellows and research students will help me to lead the combined effort.